

Coupled Dictionary Learning for Image Registration

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Abstract

Dictionary learning is a technique to learn a basis for compact representation of the data which has been widely used in image processing and computer vision applications. Image registration is an important part for many medical image analysis tasks. However, little work has been done on using dictionary learning to solve image registration problem. As a result, I propose to investigate dictionary learning methods and their applications in image registration problems. Multi-modal image registration is a common image registration problem specifically handle multi-modal images which is challenging due to the distinct appearance of biological structures from different modalities. To allow joint analysis of the multi-modal images, I first propose an dictionary learning based image analogies method to transform images of a given modality into the appearance space of another modality. Hence the registration between two images with different modalities can be transformed to a mono-modal image registration. This method generalize standard image analogies by incorporating a coupled dictionary in a sparse representation model to avoid exhaustive search of the whole training set. Furthermore, we consider the problem that register a set of subject image to a common atlas image¹. I propose a coupled dictionary learning (CDL) method for predicting deformation fields based on image appearance different between subject images and atlas image. The deformation is predicted based on the connection between the appearance differences and deformation in a coupled dictionary. Beyond image registration, we extended the coupled dictionary learning method to modeling GTPase activities and cell protrusions of mouse embryonic fibroblasts (MEFs). We tracked a set of boundary points along the orthogonal directions to the cell boundary which can be considered as a special case of curve registration. The coupled dictionary learning is applied to learn a common pattern of the GTPase activation and cell movement. Also, a probabilistic model for robustness coupled dictionary learning is proposed which discriminates between corresponding and non-corresponding patches. Instead of only transforming the appearance of multi-modal images, the proposed method further extend by transforming the appearance at the same time predicting the transformation with coupled dictionary.

1 Introduction

Dictionary learning plays a key role in many computer vision applications. A dictionary, i.e., a set of basis signals, is usually learned in a sparse representation model. Sparse representation is a powerful tool which represents the signal with sparse combinations of atoms in a learned dictionary. As a result, many dictionary learning methods have been introduced in recent literature [1, 22, 21, 13, 15]. In [1], a dictionary is learned for image denoising, while in [13], supervised learning is performed for classification and recognition tasks.

Coupled dictionary learning has been studied in many many literatures [21, 23, 11, 18]. It learns a joint dictionary on two different spaces to establish correspondence of dictionary atoms. In [5, 6], a multi-modal dictionary, also a special case of coupled dictionary, is learned

¹Atlas image is a common reference image here.

from correlative microscopy images and applied to multi-modal registration. In [21, 23], a coupled dictionary is performed for image super-resolution. While in [5, 6, 21, 23], the two parts of the coupled dictionary are assumed equivalent to each other. In [11, 18], the authors establish an explicit mapping between two dictionaries.

Image registration estimates spatial transformations between images (to align them) and is an essential part in analyzing medical images. I first focus on multi-modal image registration problem. Multi-modal registration which register two images from different modalities is very challenging: images should carry distinct information to combine, for example, for correlative microscopy images, knowledge about protein locations (using fluorescence microscopy) and high-resolution structural data (using electron microscopy). Correlative Microscopy is a technology combining different microscopy modalities including conventional light-, confocal- and electron transmission microscopy for improving biological specimens' examination [7]. E.g., fluorescent markers can be used to highlight regions of interest combined with an electron-microscopy image to provide high-resolution structural information of the regions. To allow such joint analysis requires the registration of multi-modal microscopy images.

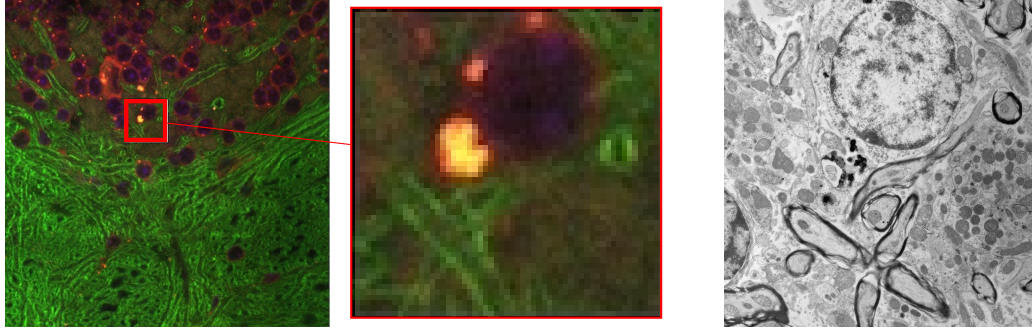
Standard methods for general multi-modal image registration [17] include a) applying advanced similarity measures, such as mutual information [20], b) or transforming a multi-modal to a mono-modal registration [19]. For example, Wachinger introduced entropy images and Laplacian images which are general structural representations [17].

Motivated by the second approach, i.e. transform the modality of image from one to another, I proposed a coupled dictionary based image analogies method to achieve this goal thereby allowing for the reconstruction of a microscopy image in the appearance space of another.

Furthermore, in some medical image analysis applications, such as brain image analysis, we need register subject images to a common atlas (reference) image, i.e, we need obtain the underline deformation fields to transform the subject images to the atlas image. I propose a general framework to estimate the deformation field for a given subject image based on a learned coupled dictionary for the appearance and the corresponding deformation fields. Also the coupled dictionary based method can be applied to different deformation parametrization.

Beyond applying coupled dictionary to image registration, I also propose to learn a coupled dictionary on GTPase activities and cell protrusions of MEFs. By learning the coupled dictionary, I try to find a common spatial and temporal patterns of the GTPase activation corresponds to the cell edge movement.

Moreover, I propose a robust dictionary learning method under a probabilistic model. Instead of directly learning a coupled dictionary from training data, I distinguish between image regions with and without good correspondence in the learning process, and update the learned dictionary iteratively.



(a) Confocal microscopic image and the corresponding resampling of the rectangular region (b) TEM image

Figure 1: Example of Correlative Microscopy. (a) is a stained confocal brain slice, where the red rectangle shows an example of a neuron cell, and the corresponding resampled image of the rectangular region is also shown in (a). (b) is the TEM image corresponds to the rectangular region in (a). The goal is to align the resampled image in (a) to (b).

1.1 Contributions

The main contribution of my work include:

- a sparse representation based image analogies method to simplify the multi-modal registration problem to a mono-modal one;
- a coupled dictionary learning method which joint learning image appearances and deformations;
- a framework for deformation prediction using appearance based on coupled dictionary learning;
- an investigation for coupled dictionary learning under different deformation parametrization methods;
- a coupled dictionary learning method for joint GTPase activation and cell protrusion modeling of MEFs;
- a probabilistic model for robustness coupled dictionary learning is proposed which discriminates between corresponding and non-corresponding patches.

1.2 Thesis

Learning a coupled basis for the compact representation of two spaces can be achieved by coupled dictionary learning. Such dictionaries can be learned to capture appearance differences of different imaging modalities as well as dependencies between image appearance and

deformation. To account for data inconsistencies a robust coupled dictionary can be obtained based on a probabilistic dictionary model. Coupled-dictionaries are useful for example to simplify multi-modal image registration or to study how signaling patterns inside cells relate to cell boundary protrusions and retractions.

The remainder of the document is organized as follows: section 2 describes the image analogies based multi-modal registration. The joint dictionary learning method for both appearances and deformations is explained in section 3. Section 4 presents an application of coupled dictionary learning for modeling GTPase activation and cell protrusion for MEFs. A robust multi-modal dictionary learning method is proposed in section 5. The summary is presented in section 6.

2 Coupled Dictionary Learning for Multi-modal Registration

Before propose the coupled dictionary learning method for multi-modal registration, I first introduce the standard image analogies approach for appearance transformation.

2.1 Image Analogies for Appearance Transformation

The objective for image analogies is to create an image B' from an image B with a similar relation in appearance as a training image set (A, A') [10]. The standard image analogies algorithm achieves the mapping between B and B' by looking up best-matching patches for each image location between A and B which then imply the patch appearance for B' from the corresponding patch A' (A and A' are assumed to be aligned). These best patches are smoothly combined to generate the overall output image B' . Fig. 2 shows an example of image analogies.

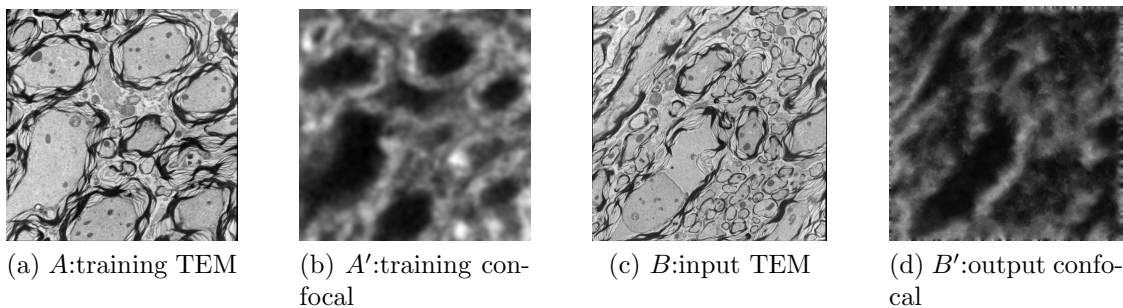


Figure 2: Result of Image Analogies: Based on a training set (A, A') an input image B can be transformed to B' which mimics A' in appearance.

2.2 Coupled Dictionary Learning for Joint Appearance Modeling

To avoid costly lookups and to obtain a more generalizable model with noise-reducing properties we propose an image analogies approach based on sparse representation model. Sparse representation is a technique to reconstruct a signal as linear combination of a few basis signals from a typically over-complete dictionary. A dictionary is a collection of basis signals. The number of dictionary elements in an over-complete dictionary exceeds the dimension of the signal space. Suppose a dictionary D is pre-defined, in order to sparsely represent a signal x , we need to solve the following optimization problem,

$$\hat{\alpha} = \arg \min_{\alpha} \|\alpha\|_0, \quad \text{s.t.} \|x - D\alpha\|_2 \leq \epsilon, \quad (1)$$

where α is a sparse vector that explains x as a linear combination of columns in dictionary D with error ϵ and $\|\cdot\|_0$ indicates the number of non-zero elements in the vector α . Solving (1) is an NP-hard problem. One possible solution of this problem is based on a relaxation that replaces $\|\cdot\|_0$ by $\|\cdot\|_1$, where $\|\cdot\|_1$ is the ℓ_1 norm of a vector. The equivalent Lagrangian form of (1) is

$$\hat{\alpha} = \arg \min_{\alpha} \lambda \|\alpha\|_1 + \|x - D\alpha\|_2^2, \quad (2)$$

which is a convex optimization problem that can be solved efficiently [3, 2].

The crucial part of our proposed method is coupled dictionary for the appearances of images from different modalities. Given sets of corresponding training patches $\{x_i^{(1)}, x_i^{(2)}\}$ we want to estimate the dictionaries themselves as well as the coefficients $\{\alpha_i\}$ for the sparse coding. The problem is non-convex (bilinear in D and α_i). The standard solution approach [8] is alternating minimization, i.e., solving for α_i keeping $\{D^{(1)}, D^{(2)}\}$ fixed and vice versa.

The multi-modal dictionary learning problem decouples from the image reconstruction and requires solving the minimization problem

$$\begin{aligned} \{\hat{D}, \{\hat{\alpha}_i\}\} &= \operatorname{argmin}_{D, \{\alpha_i\}} \sum_{i=1}^N \frac{1}{2} \left\| \begin{pmatrix} x_i^{(1)} \\ x_i^{(2)} \end{pmatrix} - \begin{pmatrix} D^{(1)} \\ D^{(2)} \end{pmatrix} \alpha_i \right\|_2^2 + \lambda \|\alpha_i\|_1 \\ &= \operatorname{argmin}_{D, \{\alpha_i\}} \sum_{i=1}^N \frac{1}{2} \|x_i - D\alpha_i\|_2^2 + \lambda \|\alpha_i\|_1. \end{aligned} \quad (3)$$

Usually to avoid D being arbitrarily large, a common constraint is added to each column of D where the l_2 norm of each column in D is less than or equal to one, i.e. $d_j^T d_j \leq 1, j = 1, \dots, m, D = \{d_1, d_2, \dots, d_m\} \in \mathbb{R}^{n \times m}$. We use a single α in (3) to enforce the correspondence of the dictionaries between two modalities.

2.3 Sparse Representation based Image Analogies

Given two training images A and A' from different modalities, we can transform image B to the other modality by synthesizing B' . This idea is also applied to image colorization and

demosaiing in [14]. We jointly optimize for the coefficients and the reconstructed/denoised image. We formulate images analogies with sparse representation model as,

$$\{\hat{u}^{(1)}, \hat{u}^{(2)}, \{\hat{\alpha}_i\}\} = \underset{\hat{u}^{(1)}, \hat{u}^{(2)}, \{\hat{\alpha}_i\}}{\operatorname{argmin}} \frac{\gamma}{2} \|u^{(1)} - f^{(1)}\|_2^2 + \frac{1}{2} \sum_{i=1}^N \left\| R_i \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} - \begin{pmatrix} D^{(1)} \\ D^{(2)} \end{pmatrix} \alpha_i \right\|_2^2 + \lambda \|\alpha_i\|_1, \quad (4)$$

where we have corresponding multi-modal dictionaries $\{D^{(1)}, D^{(2)}\}$ and only one image $f^{(1)}$ is given and we are seeking a reconstruction of a denoised version of $f^{(1)}$, $u^{(1)}$, as well as the corresponding analogous denoised image $u^{(2)}$ (without the knowledge of $f^{(2)}$). $R_i(u)$ extracts the i th patch from image u . Note that there is only one set of coefficients α_i per patch, which indirectly relates the two reconstructions. The problem is convex (for given $D^{(i)}$) which allows to compute a globally optimal solution.

2.4 Use in Multi-modal Registration

For multi-modal image registration, we (i) reconstruct the “missing” analogous image and (ii) consistently denoise the given image to be registered with [9]. By denoising the target image using the learned dictionary for the target image from the joint dictionary learning step we obtain two consistently denoised images: the denoised target image and the predicted source image. The image registration is applied to the analogous image and the target image. The framework of our proposed method is illustrated in Fig. 3.

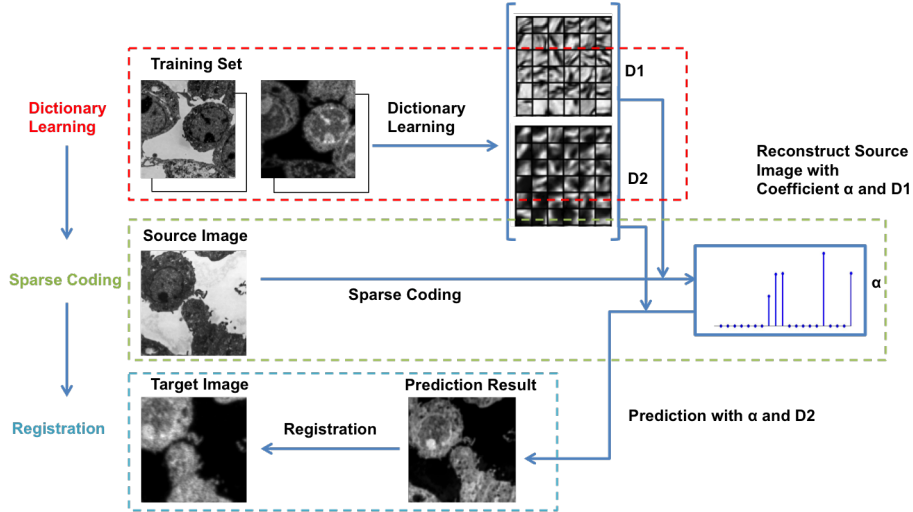


Figure 3: Flowchart of our proposed method. This method has three components: 1. dictionary learning: learning coupled dictionaries for both training images from different modalities; 2. sparse coding: computing sparse coefficients for the learned dictionaries to reconstruct the source image while at the same time using the same coefficients to transfer the source image to another modality; 3. registration: registering both transferred source image and target image.

3 Coupled Dictionary Learning for Deformation Estimation

In this section, we propose a coupled dictionary learning method to estimate deformation field based on image appearance. Rather than estimating deformations by standard image registration methods, we investigate how to obtain a basis of the space of deformations. In particular, we explore how image appearance differences with respect to a common atlas image can be used to predict deformations represented by such a basis. Fig. 4 shows an example of training images for a set of subject images and their common atlas image. Although the dictionary learning method is similar to the method proposed in previous section, the goals of the dictionary learning are different. Here, the coupled dictionary can directly used for deformation prediction based on image appearance, while in previous section, the coupled dictionary is applied to transform the image appearance from one modality to another, and the deformation field is estimated using standard image registration methods for single modality.

3.1 Coupled Dictionary Learning for Joint Appearance and Deformation Modeling

The coupled dictionary is learned on the data from two different spaces, here, image appearance space and deformation parameter space. Using only one set of coefficients imposes the strong assumption that coefficients of the representation of the two spaces are equal. However, this strong assumption sometimes not hold. To relax this assumption, a semi-coupled dictionary learning is proposed [18],

$$\{\hat{D}^1, \hat{D}^2, \{\hat{\alpha}_i^1\}, \{\hat{\alpha}_i^2\}, \hat{W}\} = \underset{D^1, D^2, \{\alpha_i^1\}, \{\alpha_i^2\}, W}{\operatorname{argmin}} \sum_{i=1}^N \frac{1}{2} \|x_i^1 - D^1 \alpha_i^1\|_2^2 + \frac{1}{2} \|x_i^2 - D^2 \alpha_i^2\|_2^2 \quad (5)$$

$$+ \lambda_1 \|\alpha_i^1\|_1 + \lambda_2 \|\alpha_i^2\|_1 + \gamma_1 \|\alpha_i^2 - W \alpha_i^1\|_2^2 + \gamma_2 \|W\|_F^2,$$

where $\lambda_1, \lambda_2, \gamma_1, \gamma_2$ are regularization parameters. Distinct from CDL, W is a matrix to define a mapping between the coefficients in two spaces. Eq. (3) is a special case of Eq. (5) when W equals to the identity and α^1 equals α^2 . Unlike for CDL the columns of the dictionaries D^1 and D^2 are normalized separately. Our experiments show that such a separate normalization is beneficial to jointly compute a basis for appearance differences and deformations. Eq. (5) is not convex with respect to $D^1, D^2, \alpha^1, \alpha^2, W$ jointly, however, it is convex with respect to each of them when others are fixed.

3.2 Deformation Estimation

After we obtaining D^1, D^2 and the linear mapping W from training data x_i^1 and x_i^2 , given a difference image $I = T - S$, where S is an input source image, and T is the common

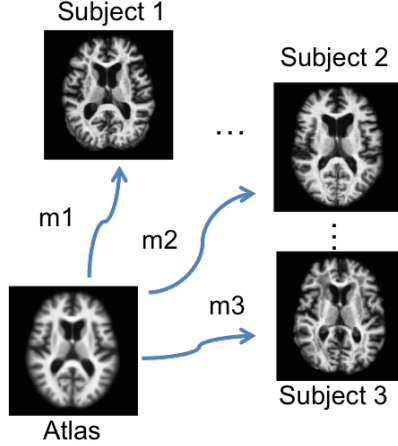


Figure 4: Illustration of training set and atlas. Here, m_i , $i = 1, \dots, n$ are initial momenta generated by atlas construction [16].

target/atlas image, similar to Eq. (5), we solve the following

$$\{\alpha_i^1\} = \underset{\alpha_i^1}{\operatorname{argmin}} \frac{1}{2} \|I_i^1 - D^1 \alpha_i^1\|_2^2 + \lambda_1 \|\alpha_i^1\|_1, \quad (6)$$

where I_i is a patch of I . Eq. (6) is sparse coding problem. The corresponding deformation parameters p_i of I_i can be estimated as,

$$p_i = D^2 W \alpha_i^1.$$

Here p_i are the parameters defining the deformation ϕ_i of the i th patche. After estimating ϕ_i from p_i for all the patches, we can determine the overall ϕ .

Our method is actually learning the relationship between appearance and deformation and obtaining a compact representation of the data. The semi-coupled dictionary learning relaxes the strong coupling between two spaces (here, appearance space and deformation space), and thus obtains a linear relationship resulting in a better fit of the data distribution.

4 Coupled Dictionary Learning for GTPase Activities and Cell Protrusion

We proposed coupled dictionary learning methods to learn a joint basis for multi-modal images or deformations. To further utilize coupled dictionary learning modeling multi-modal data, I propose a method to learn coupled dictionary for GTPase activities and cell protrusion in MEFs. The Previous research have shown the correlations between GTPase activities and cell protrusion [12], however, their method focus on specific regions of MEFs, and the regions were manually selected by experts. Our method learns the bases for GTPase activities and

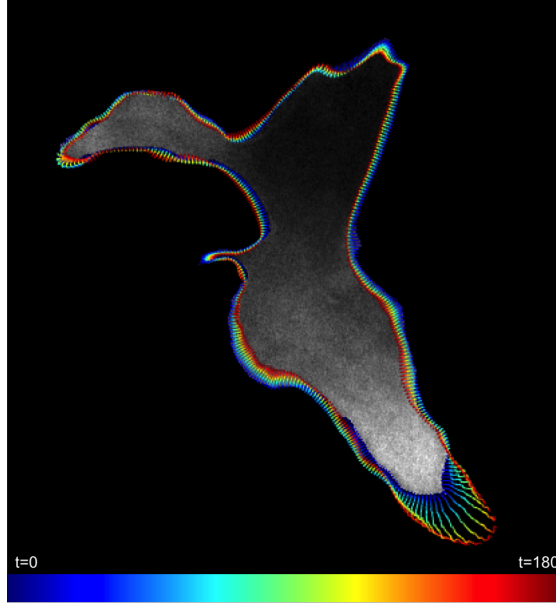


Figure 5: Cell boundary point tracking results. The colormap on the bottom indicates different time points.

cell edge movements automatically, and identifies different regions with different correlations in MEFs based on learned bases automatically.

To investigate the relationship between GTPase activation and cell edge movement, we first tracked a set of boundary points, let $p_{ij}, i \in (1, \dots, n), j \in (1, \dots, m)$ the positions of the i th boundary point at j th time points, and recorded the mean intensity values of a neighborhood of each boundary point as a_{ij} . Assuming that each boundary point move orthogonally to the boundary, the velocity for each boundary point can be obtained from the inner product between the normal vector and position differences in two successive time points $v_{ij} = \langle \vec{n}_{ij}, p_{i,j+1} - p_{ij} \rangle, i \in (1, \dots, n), j \in (1, \dots, m-1)$, where \vec{n}_{ij} is the normal vector at p_{ij} , thus positive v corresponds to cell protrusion and negative v corresponds to cell retraction. Fig. 5 illustrates an example of the boundary points tracking result.

4.1 Coupled Dictionary Learning for Joint GTPase Activities and Cell Protrusion Modeling

Now we know the corresponding cell edge motion vectors $\mathbf{v}_i \in \mathbf{R}^m$ and activation vectors $\mathbf{a}_i \in \mathbf{R}^{m-1}$, the coupled dictionary learning method in previous sections can be applied to \mathbf{v}_i and \mathbf{a}_i to learn a coupled bases to represent the edge velocities and GTPase activities. The bases can be considered as a summary of the relationship between the GTPase activities and edge movement for a single cell (if we learn the dictionary on the boundary points for a single cell) or for cells with the same GTPase activation type (if we learn the dictionary on the boundary points for cells with the same GTPase activation type).

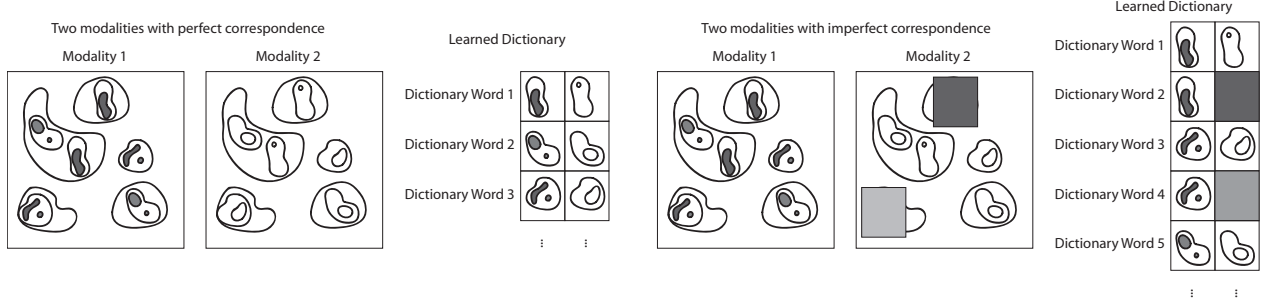


Figure 6: An illustration of perfect (left) and imperfect (right) correspondence between multi-modal images and their learned dictionaries. The imperfect correspondence (gray part in right images) could result in learning an imperfect dictionary (gray dictionary words) which is not desirable. Our goal is to *robustly* recover a compact dictionary of *corresponding* elements.

To further incorporate the spatial information, we can group multiple boundary points in a local neighborhood together to learn the coupled dictionary, the learned dictionary could capture the spatial and temporal pattern of the data.

4.2 Hierarchical Clustering for GTPase Activities and Cell Protrusion Patterns based on Coupled Dictionary Learning

To analysis the interaction between cell boundary points on the edge movement and GTPase activation, we can cluster the boundary points into several groups. Traditional clustering method such as k-means, need determine the number of clusters which is usually difficult to find the optimal k. However, using hierarchical clustering, we avoid choosing a specific k by uncovering the hierarchical structure of the data. The hierarchical clustering arrange the with a tree structure which provide more information about the hierarchical relationship between different data points.

5 Robust Coupled Dictionary Learning

Coupled dictionary learning is challenging: it may fail or provide inferior dictionary quality without sufficient correspondences between modalities in the training data. For example, a low quality image deteriorated by noise in one modality can hardly match a high quality image in another modality. Furthermore, training images are pre-registered. Resulting registration error may harm image correspondence and hence dictionary learning. Fig. 6 shows an example of coupled dictionary learning for both perfect and imperfect corresponding image pairs.

5.1 Confidence Measure

The confidence can be defined as a conditional probability $p(h|x_i)$. Given image patches $\{x_i\}_{i=1}^N$ we want to reconstruct them with our learned coupled dictionary. Here, h is the hypothesis of whether the reconstruction of x_i uses some ‘noise’ dictionary atoms (i.e. non-corresponding dictionary atoms); $h = 1$ indicates that the reconstruction x_i uses ‘noise’ dictionary atoms, while $h = 0$ means reconstructing x_i without ‘noise’ dictionary atoms. Thus a high $p(h = 1|x_i)$ corresponds to high confidence that the reconstruction of x_i is not a good estimation because of the ‘noise’ dictionary atoms are used in the reconstruction and vice versa.

Applying Bayes Rule, $P(h = 1|x_i)$ can be represented as,

$$P(h = 1|x_i) = \frac{P(x_i|h = 1)P(h = 1)}{P(x_i|h = 1)P(h = 1) + P(x_i|h = 0)P(h = 0)}. \quad (7)$$

Assuming the independence of each image patch x_i and that the pixels in each patch follow a Gaussian distribution, for $p(x_i|h)$ we assume

$$\begin{aligned} p(x_i|h = 1, \theta_1) &= \mathcal{N}(x_i; \mu_1, \sigma_1^2), \\ p(x_i|h = 0, \theta_0; D, \alpha_i) &= \mathcal{N}(x_i - D\alpha_i; 0, \sigma_0^2). \end{aligned} \quad (8)$$

The parameters we need to estimate are $\theta_1 = \{\mu_1, \sigma_1\}$ and $\theta_0 = \sigma_0$, as well as the prior probability $p(h)$, where $p(h = 1) = \pi$ and $p(h = 0) = 1 - \pi$.

Based on the assumption of conditional independence of the random variable x_i given h and θ , maximum likelihood (ML) estimation are used for these parameters.

I propose an EM based algorithm which classifies the non-corresponding image patches and updates the coupled dictionary iteratively [5].

6 Summary

This proposal discusses issues related to modeling different coupled spaces with coupled dictionary learning and their applications in image registration and GTPase activation and cell protrusion analysis. I first proposed a coupled dictionary learning method for appearances of images from different modalities to transfer the appearance of one image from one modality to another, thus the multi-modal registration problem simplifies to a mono-modal one. Moreover, I consider the problem to estimate the deformation field from a given subject image to a common atlas image by utilizing the population information. A coupled dictionary is learned from the appearance differences between subject images and atlas image and the corresponding deformations, and the deformation is estimated based on the relationship between the appearances and deformations from the coupled dictionary. Furthermore, coupled dictionary learning is applied to GTPase activation and cell edge movement for MEFs to reveal the common spatial and temporal patterns between GTPase activities and cell protrusion. We also proposed a probabilistic model for robust coupled dictionary learning.

7 Research Status and Tentative Timeline

Table. 1 shows my research plan.

CDL for multi-modal registration	Sec. 2	Completed([6, 4])
CDL for deformation prediction	Sec. 3	Completed(Submitted to ISBI) In preparation to MEDIA
CDL for GTPase activation and cell protrusion modeling	Sec. 4.1	In preparation to MICCAI
Clustering for GTPase activation and cell protrusion modeling	Sec. 4.2	In preparation to MICCAI
Robust CDL	Sec. 5	Completed([5])
Proposal	_____	January 2015
Oral Exam	_____	February 2015
Defense	_____	August 2015

Table 1: Reseach plan

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