Time Value of Money 上

① **Required rate of return is** 要求回报率. 10% v.s.

- affected by the supply and demand of funds in the market;
- the minimum rate of return an investor must receive to accept the investment. D&S ⇒ P.(r)
- usually for particular investment.

② **Discount rate is** 折现率.

- the interest rate we use to discount payments to be made in the future.
- usually used interchangeably with the interest rate.

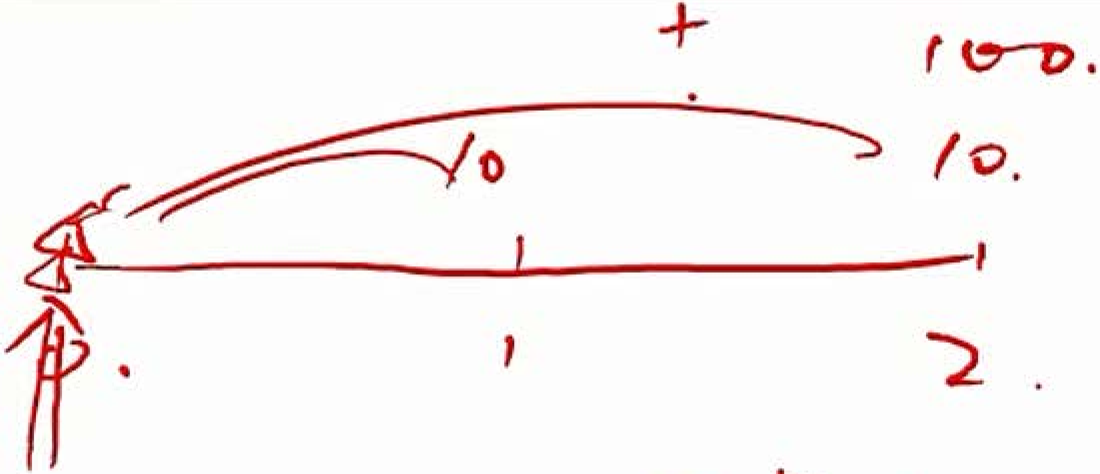
③ **Opportunity cost is** 机会成本.

- also understood as a form of interest rate. It is the value that investors forgo by choosing a particular course of action.

Bond.

$\Updownarrow$

loan.



$$P = \sum \frac{CF_t}{(1+r)^t}$$

$\longrightarrow$ { PRN. / int.

$\Uparrow$

discount rate { yield. / 利r.

Time Value of Money

美国国债

- **Decompose required rate of return:**
  - Nominal risk-free rate = real risk-free rate + expected inflation rate
  - Required interest rate on a security = nominal risk-free rate + default risk premium + liquidity risk premium + maturity risk premium

风险溢价

交易活跃程度. D&S

↑ ↑.

大白菜. D&S ↑↑.

云白菜. D&S. ↓↓.

↓. ↓

Time Value of Money

美国国债

Fisher effect.

- **Decompose required rate of return:**
  - Nominal risk-free rate = real risk-free rate + expected inflation rate
  - Required interest rate on a security = nominal risk-free rate + default risk premium + liquidity risk premium + maturity risk premium

风险溢价

期限

(time)

长

短

7%

5% + 2%

≈ Gwp. Stable.

$R_N = R_R + I^e$

Fisher

汇率

负

2018

2018. 11.

$$\Rightarrow (1+R_N) = (1+R_R)(1+\pi^e) \Leftarrow.$$

$$\cancel{1}+R_N = \cancel{1}+\pi^e+R_R+\underbrace{R_R\times\pi^e}_{\approx 0}$$

$$R_N = R_R+\pi^e. \Leftarrow.$$

$$\begin{cases} ① \quad R_i = \underline{R_f} + \underline{RP} \\ ② \quad R_N = R_R + I^e \end{cases}$$

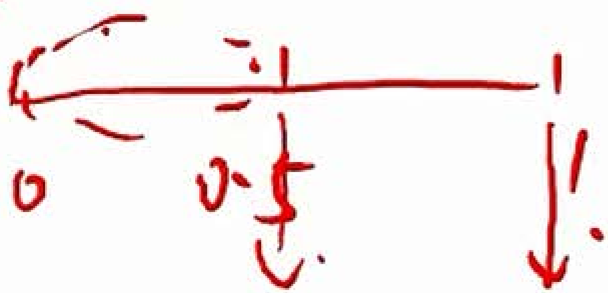
Fisher.

$r = 10\%$

\$1

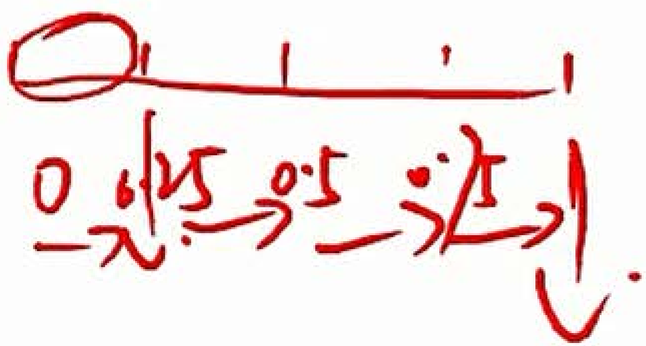
1次. $(1+10\%) - 1 = 10\%$

\$1 5%



2次. $\left(1+\frac{10\%}{2}\right)^2 - 1 = 10.25\%$

\$1 2.5%



4次. $\left(1+\frac{10\%}{4}\right)^4 - 1 = 10.38\%$

1. 计算 r — m.

$$EAR = \boxed{\left(1+\frac{r}{m}\right)^{m}} - 1$$

$m \to \infty$

$$\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{m} = e^{r}.$$

$$\boxed{EAR = e^{r} - 1}$$

Chn 链式运算 9

$1/3 + 2 \times 3.$

AOS. 代数运算 7.

2. 性质：$m\uparrow$，$\varepsilon AR\uparrow$.

Time Value of Money

- A money manager has \$1,000,000 to invest for one year. She has identified two alternative one-year certificates of deposit (CD) shown below:

$EAR = (1+\frac{r}{m})^m - 1$ (×? ✗

	Compounding frequency	Annual interest rate
CD1	Quarterly $m=4$	4.00%
CD2	Continuously max	4.95% 4%

$EAR = e^r - 1$

- Which CD has the highest effective annual rate (EAR) and how much interest will it earn?

	Highest EAR	Interest earned $= 1m \times EAR$
A.	CD1	\$41,902
B.	CD1	\$40,604
C.	CD2	\$50,700

\$100. $r = 10\%$ 2次. 2年.

0　　0.5　　1　　1.5　　2.

$n \times m$

2×2

4

$100 \times \left(1+\frac{10\%}{2}\right)^{2}$

$\underset{PV}{100} \times \left(1+\frac{10\%}{2}\right)^{4} = FV$

Time Value of Money

- Future value (FV): Amount to which investment grows after one or more compounding periods. 终值.
- Present value (PV): Current value of some future cash flow 现值.
- If interests are compounded m times per year, and invest 1 year:

  $FV=PV(1+r/m)^{m}$

- If interests are compounded m times per year, and invest n years:

  $FV=PV(1+ r/m)^{mn}$

  Where: m is the compounding frequency;

  r is the nominal/quoted annual interest rate.

- When we calculate the future value of continuously compounding, the formula is:

$$FV=PV\lim_{m\to\infty}(1+\frac{r}{m})^{nm}=PVe^{nr}$$

Time Value of Money

➢ **What's annuities?** 年金.

- is a finite set of level sequential cash flows.
  - ✓ equal intervals
  - ✓ equal amount of cash flows
  - ✓ same direction

➢ **Elements of annuity：**

- N = number of periods
- I/Y = interest rate per period
- PV = present value
- PMT = amount of each periodic payment
- FV= future value

About ordinary annuities: 普通/后付年金.

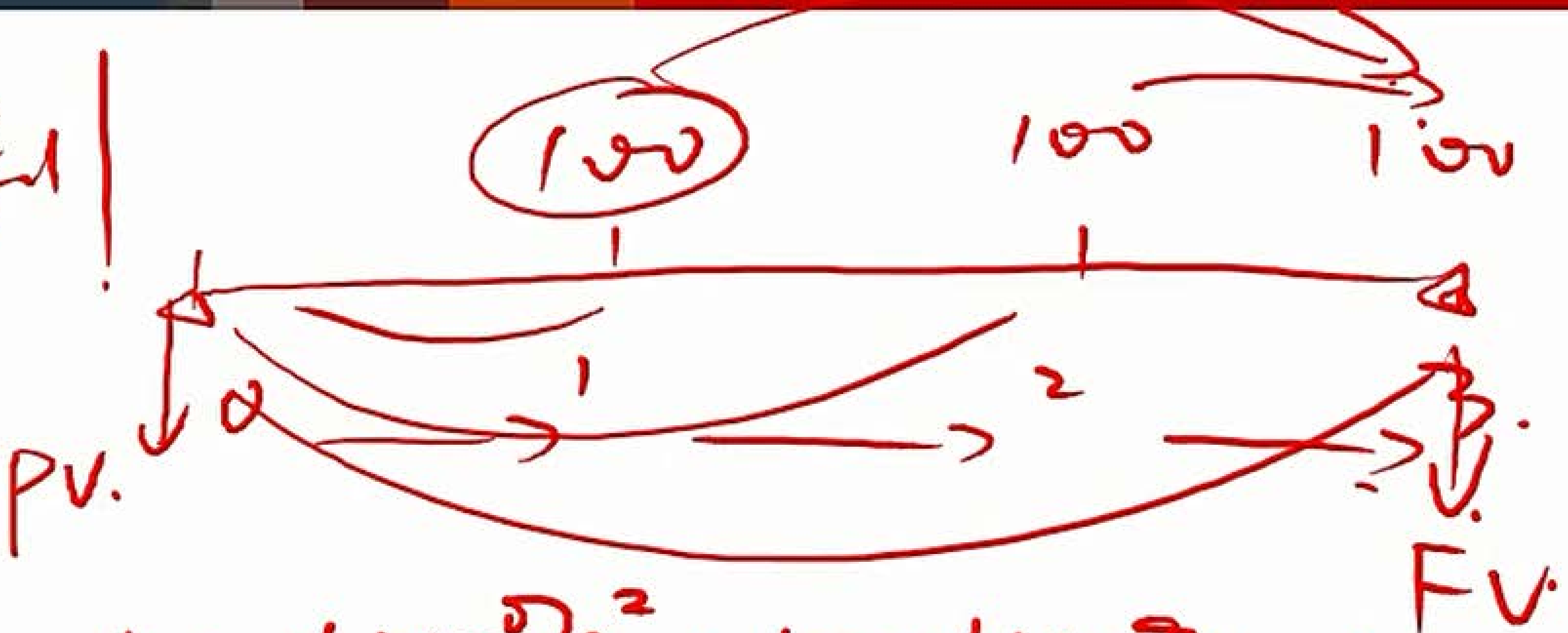
- Tom makes a saving plan with $1000 annual deposit at the end each year. Suppose the deposit rate is 5 percent annually, what's the future value at the end of year five?

- **Correct Answer:**

  - Enter relevant data for calculate.

    ✓ N=5, I/Y=5, PMT=-1,000, PV=0, CPT→FV=5,525.63

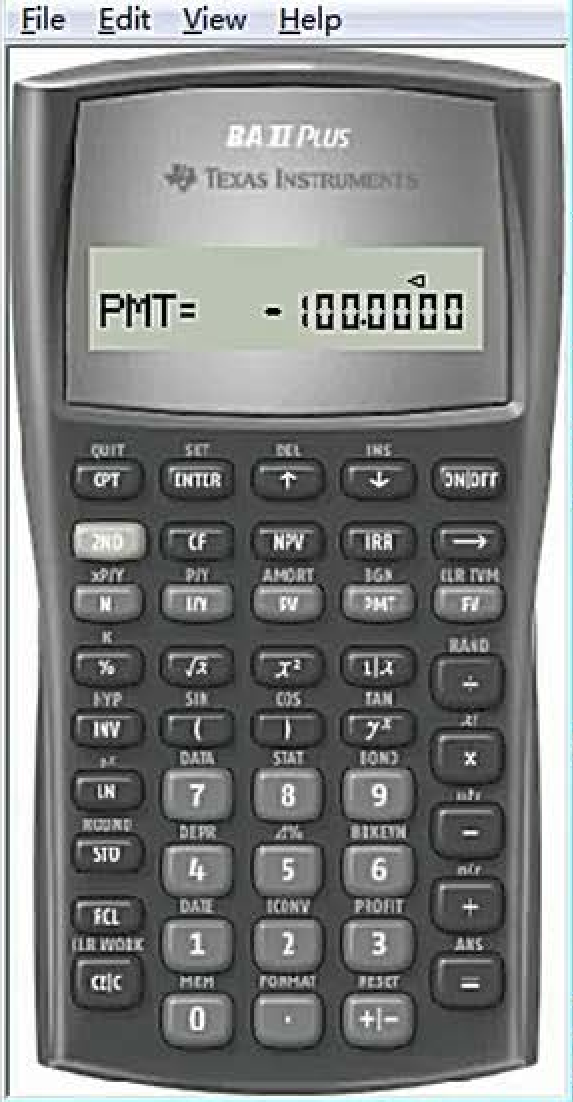
复利

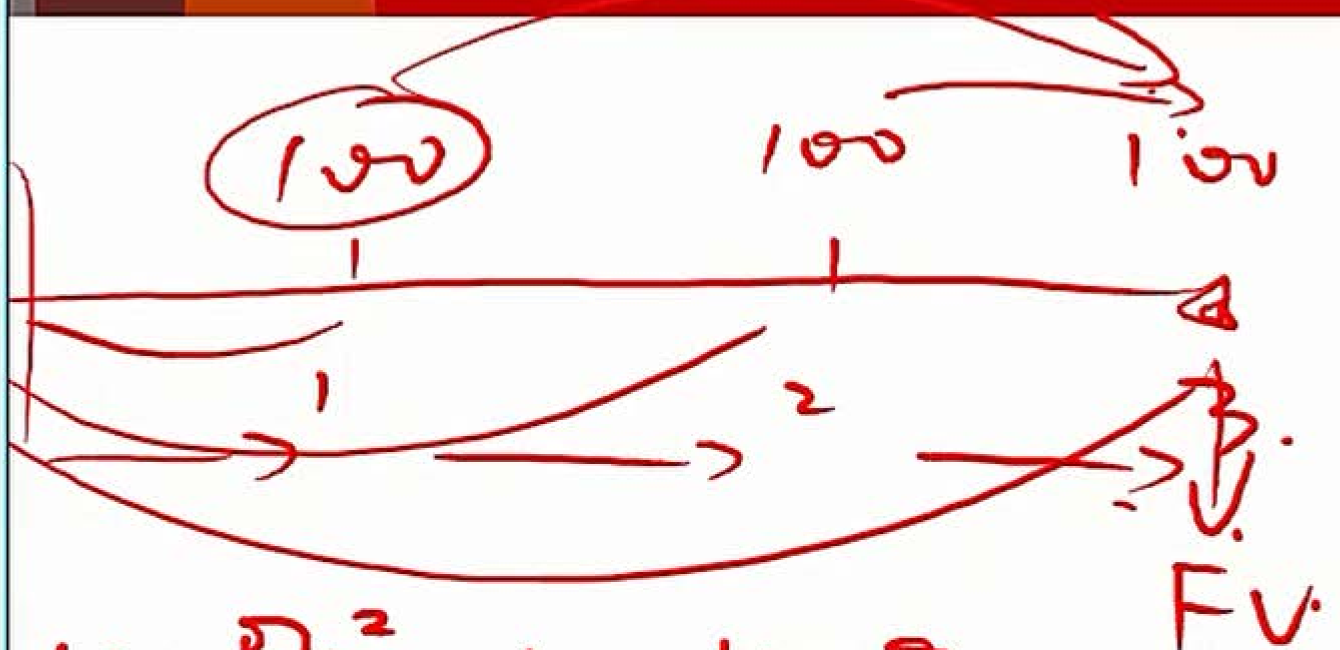


$r=10\%$

$$FV = 100\times(1+10\%)^2 + 100\times(1+10\%) + 100$$

$$PV = \frac{100}{1+10\%} + \frac{100}{(1+10\%)^2} + \frac{100}{(1+10\%)^3}$$





$\times(1+10\%)^2 + 100\times(1+10\%) + 100$

$\ldots 10\% + \frac{100}{(1+10\%)^2} + \frac{100}{(1+10\%)^3}$

"我" outflow　　　"银行" inflow

−100　　　+100

About ordinary annuities: 普通/后付年金.

➢ Tom makes a saving plan with $1000 annual deposit at the end each year. Suppose the deposit rate is 5 percent annually, what's the future value at the end of year five?

1000 1000 1000 1000 1000

0 1 2 3 4 5

FV

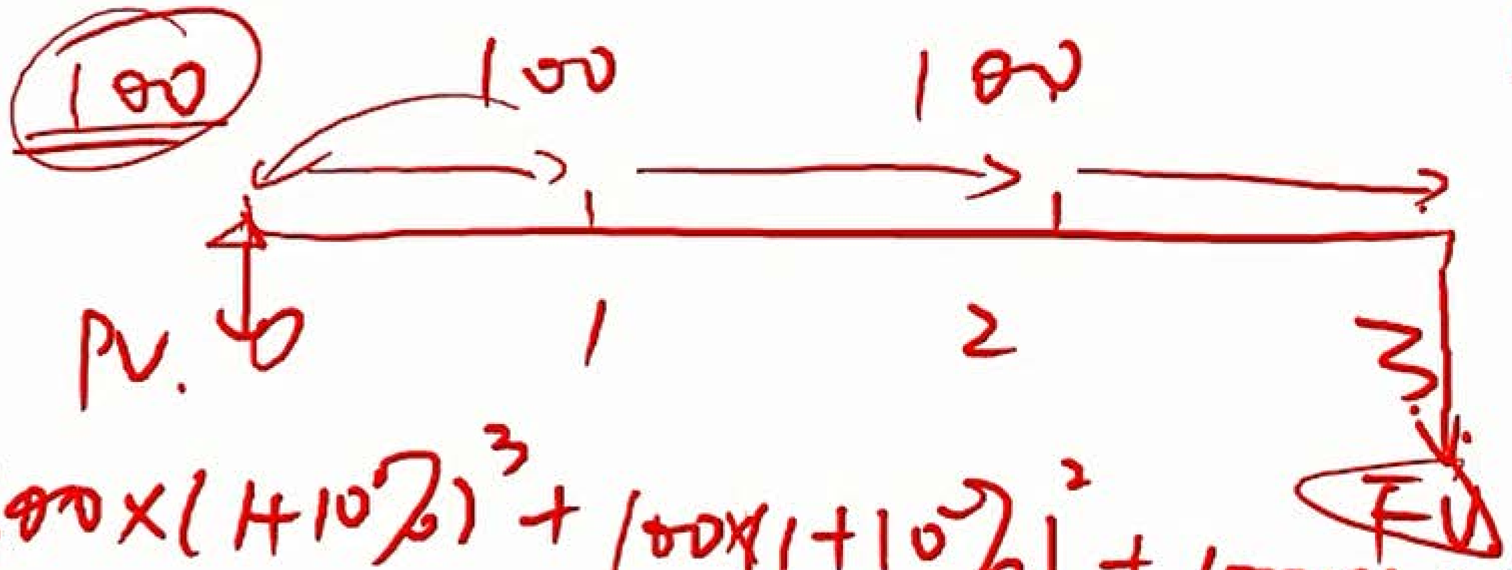
➢ **Correct Answer:**

- Enter relevant data for calculate.

    ✓ N=5, I/Y=5, PMT=-1,000, PV=0, CPT→FV=5,525.63

复利.

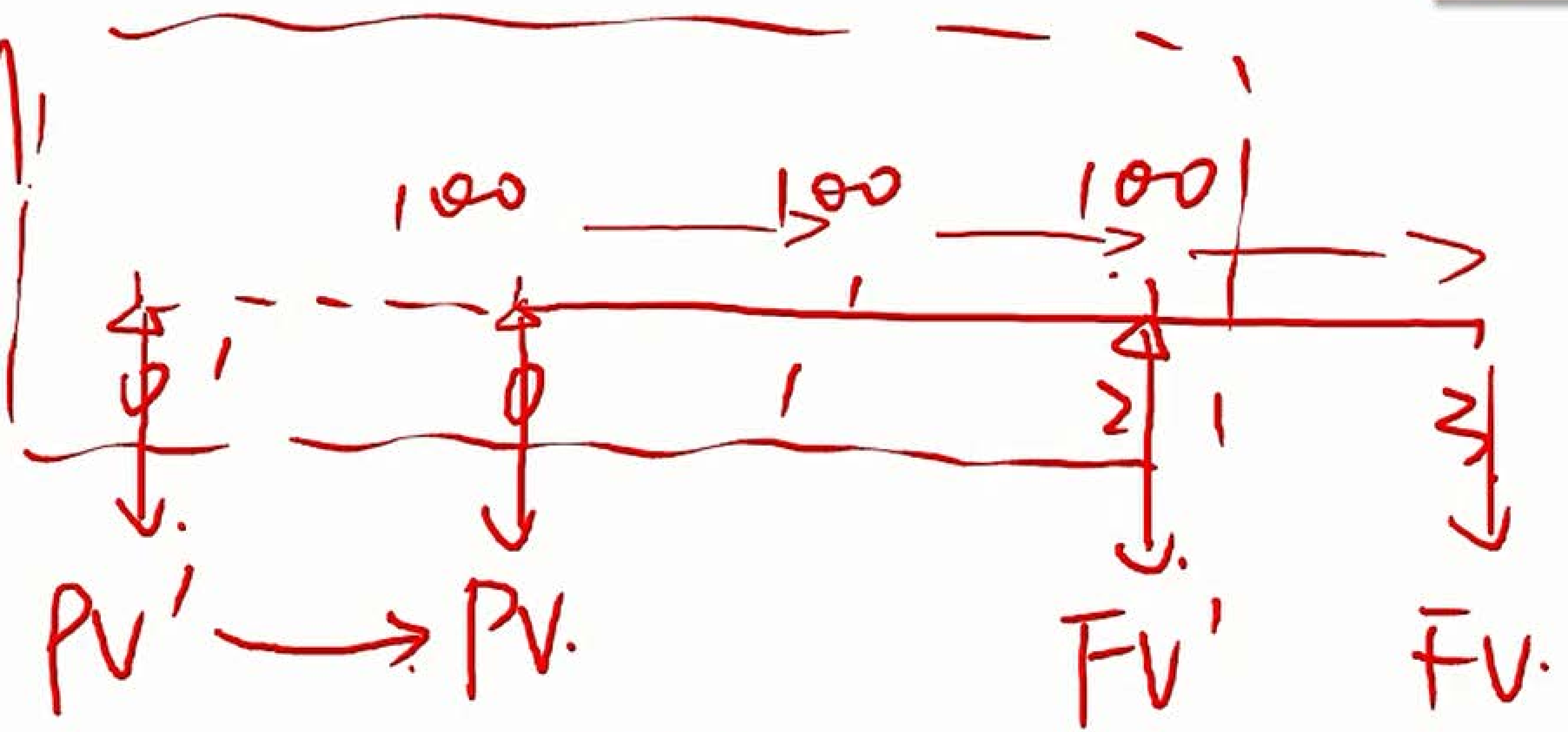
$r=10\%$



$$FV = 100\times(1+10\%)^3 + 100\times(1+10\%)^2 + 100\times(1+10\%)$$

$$PV = 100 + \frac{100}{1+10\%} + \frac{100}{(1+10\%)^2}.$$

复利



$$PV = PV'(1+r)$$

$$FV = FV'(1+r)$$

Time Value of Money

- **About an annuity due** 先付年金.
  - The first cash flow occurs immediately(at t=0)
    - ✓ Example： rental fees, tuition fees, living expenses, etc.
  - **Calculation:**
    - ✓ **Measure 1:** use calculator, put the calculator in the **BGN** mode and input relevant data.
    - ✓ **Measure 2:** treat as an ordinary annuity and simply multiple the resulting PV by (1+I/Y)
      - ◆PV and FV calculation applies, while PMT not.

Time Value of Money 还房贷、摊销

- To see how a lump sum can generate an annuity, assume that we loan $3,170 from the bank today at 10 percent interest. Construct an **amortization** table to show the annuity payments over the next four years.
- **Correct Answer:**

  The amount of the annuity payments : N=4; I/Y=10; PV=-$3,170; FV=0; CPT: PMT=$1,000

Amortization Table					
Time Period	Beginning Balance (1)	Payment (2)	Interest Component (3)=(1)*10%	Principal Component (4)=(2)-(3)	Ending Balance (5)=(1)-(4)
1	3,170	1000	317	683	2,487
2	2,487	1000	248.7	751.3	1,735.7
3	1,735.7	1000	173.57	826.43	909.27
4	909.27	1000	90.93	909.27	0

Time Value of Money

还房贷、摊销表

- To see how a lump sum can generate an annuity, assume that we loan \$3,170 from the bank today at 10 percent interest. Construct an **amortization** table to show the annuity payments over the next four years.

0 1 2 3 4

- **Correct Answer:**

(1) TVM

The amount of the annuity payments : N=4; I/Y=10; PV=-\$3,170; FV=0; CPT: PMT=\$1,000

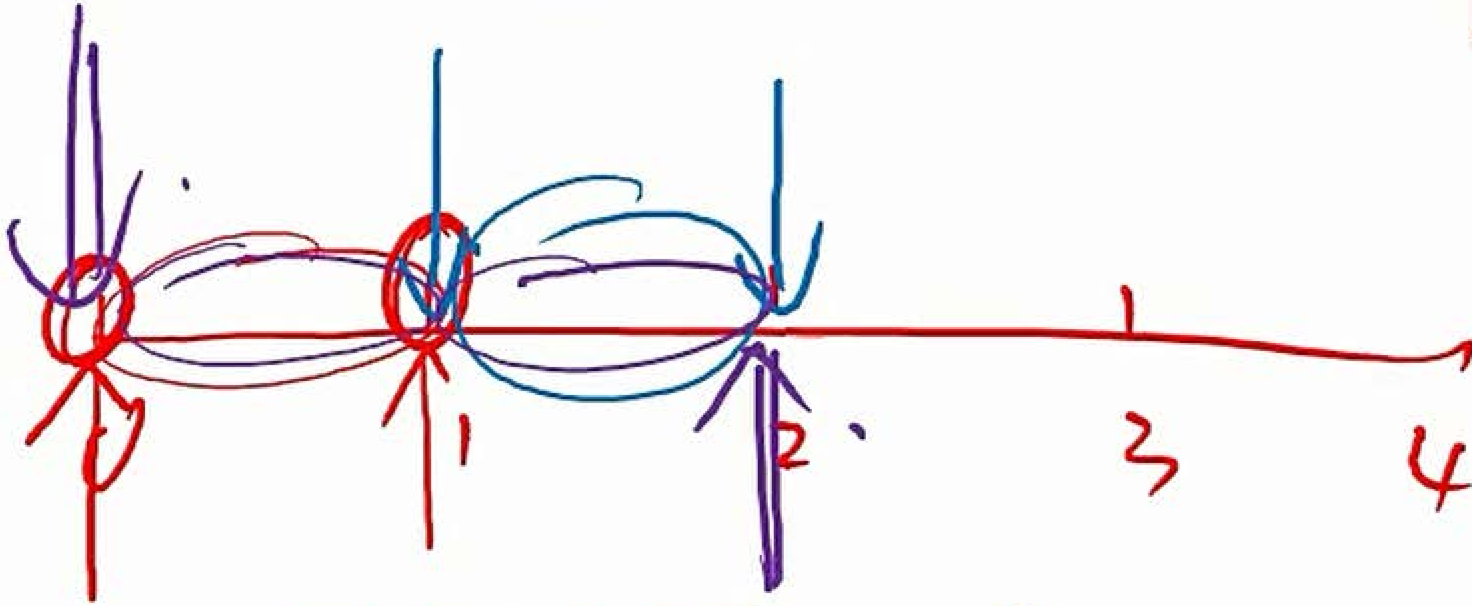
P1=1 P2=1

期末PRN

(2) AMORT

Amortization Table					
Time Period	Beginning Balance (1)	Payment (2)	Interest Component (3)=(1)*10%	Principal Component (4)=(2)-(3)	Ending Balance (5)=(1)-(4)
	期初PRN	PMT	INT	PRN	BAL
1	3,170	1000	317	683	2,487
2	2,487	1000	248.7	751.3	1,735.7
3	1,735.7	1000	173.57	826.43	909.27
4	909.27	1000	90.93	909.27	0

P1=2 P2=2.



P1 = 开始时期. P1=2. P2=2.

P2 = 结束时期. (1-2)

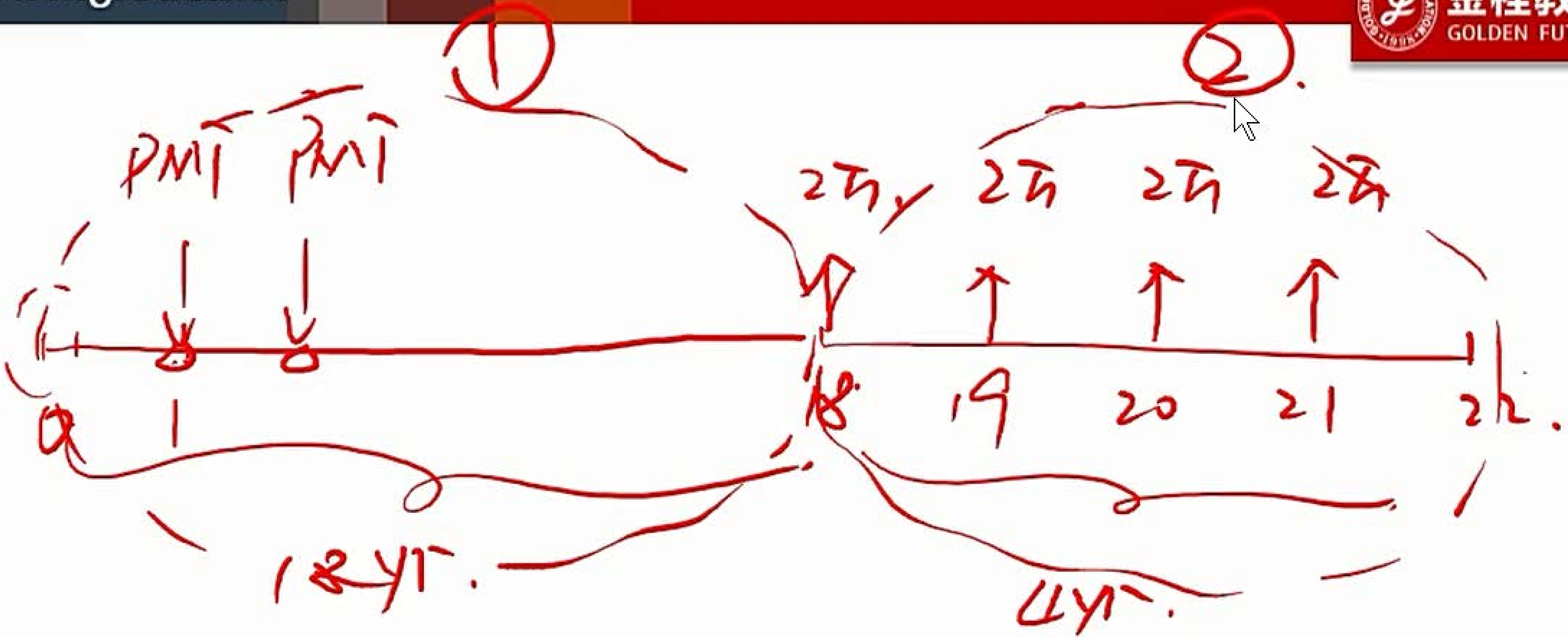
P1=1 P2=1 (0~1).

P1=1 P2=2. (0~2)

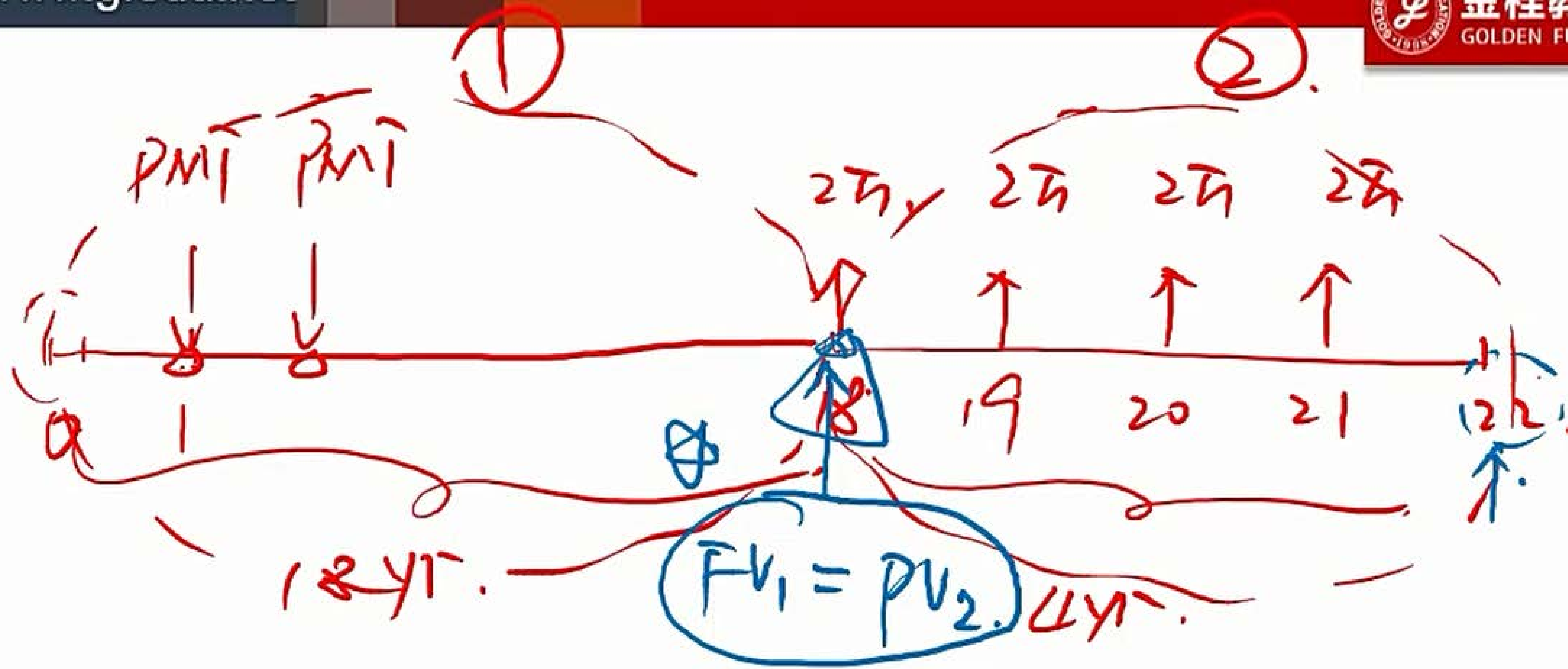
Time Value of Money 养老金/房贷组合.

- A client plans to send a child to college for four years starting 18 years from now. Having set aside money for tuition, she decides to plan for room and board also. She estimates these costs at \$20,000 per year, payable at the beginning of each year, by the time her child goes to college. If she starts next year and makes 18 payments into a saving account paying 5 percent annually, what annual payments must she make?

- **Correct Answers:**
  - Compute PV at t=18 and Set your calculator to the **BGN mode**,
    - ✓ N=4; I/Y=5; FV=0; PMT=-20,000; CPT: PV_{18}=74,465
  - At t=18, PV_{18} =FV(for ordinary annuity), Set your calculator to the **END mode**,
    - ✓ N=18; I/Y=5; PV=0; FV=-74,465; CPT: PMT=2647.



① PMT: N = 18. I/Y = 5. PV = 0 FV.



① PMT: $N = 18$. $I/Y = 5$. $PV = 0$　　$FV_1 = PV_2$.

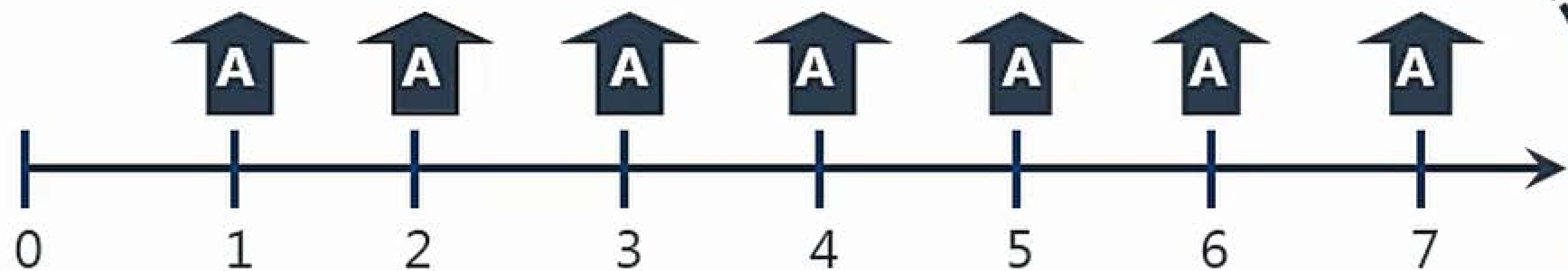
CPT. PMT.

② PV: (BGN). $N = 4$　$I/Y = 5$. $PMT = 2万$　$FV = 0$

CPT. PV_2.

Time Value of Money

- **About perpetuity** 永续年金 —后付 · PV.
  - **Definition:** A perpetuity is a set of level never-ending sequential cash flows, with the first cash flow occurring one period from now.



$PV_1 = A/(1+r)$

$PV_2 = A/(1+r)^2$

$PV_3 = A/(1+r)^3$

$PV_4 = A/(1+r)^4$

etc. etc.

$$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} + \cdots \qquad (1)$$

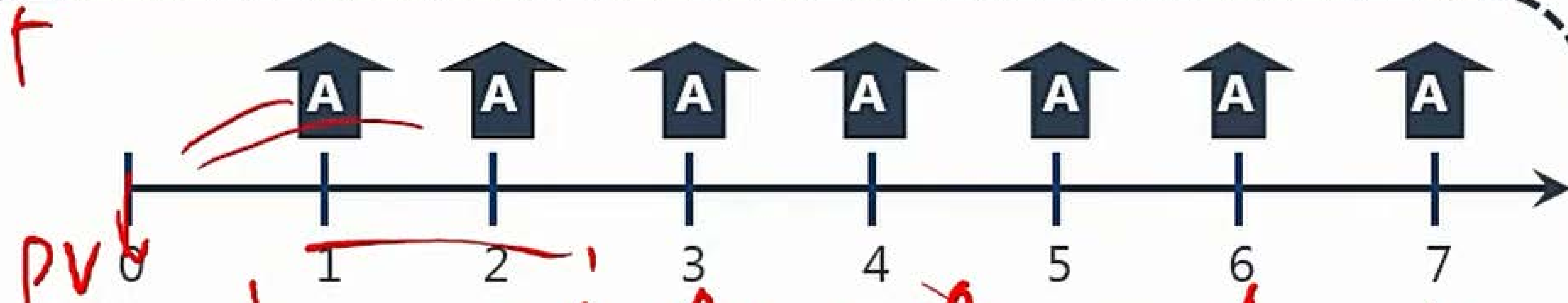
$$(1+r)PV = A + \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots \qquad (2)$$

$$(2)-(1) \qquad r \times PV = A \Rightarrow PV = \frac{A}{r}$$

Time Value of Money

➢ **About perpetuity** 永续年金 —派付. $PV = \frac{CF}{r}$

- **Definition:** A perpetuity is a set of level never-ending sequential cash flows, with the first cash flow occurring one period from now.



$PV_1 = A/(1+r)$
$PV_2 = A/(1+r)^2$
$PV_3 = A/(1+r)^3$
$PV_4 = A/(1+r)^4$
etc.

$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} + \cdots$ ①

$(1+r)PV = A + \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots$ ②

$$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} + \cdots \quad (1) \qquad (1+r)PV = A + \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots \quad (2)$$

② − ① $PV \cdot r = A \Rightarrow PV = \frac{A}{r}$

$$(2)-(1) \qquad r \times PV = A \Rightarrow PV = \frac{A}{r}$$

$PV + PV \cdot r$

1. $\begin{cases} R_i = R_f + RP & \text{3因素} \\ R_N = R_R + \pi^e & \text{(Fisher)} \end{cases}$

2. EAR ☆

(1) 计算

$$EAR = \left(1+\frac{r}{m}\right)^m - 1$$

$$EAR = e^r - 1$$

(2) 性质：m↑，EAR↑.

3、年金.

- 后付.
- 先付.

(TVM).

- (1) 还房贷 / 折旧摊销.
  - ① TVM. ② AMORT.
- (2) 养老金 / 教育金.

Discounted Cash Flow Applications

net present value. 净现值.

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_N}{(1+r)^N} = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

$$NPV = 0 = CF_0 + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \dots + \frac{CF_N}{(1+IRR)^N} = \sum_{t=0}^{N} \frac{CF_t}{(1+IRR)^t}$$

- **IRR (Internal Rate of Return)** 内部收益率.
  - When NPV= 0, the discount rate.
  - Multiple solutions or no solution problem of the IRR calculation (# sign changes)

-100 $\quad 20 \quad 50 \quad 50$ $\quad r = 10\%$

$0 \quad 1 \quad 2 \quad 3$

(收) − (支)

$$\frac{20}{1+10\%} + \frac{50}{(1+10\%)^2} + \frac{50}{(1+10\%)^3} - 100$$

$$= -2.93 < 0$$

NPV < 0 (×)

NPV > 0 (√)

-100	20	50	50
0	1	2	3.

$NPV = 0.$ $\quad r = IRR.$

$$\underbrace{\text{profit.}}_{} = \text{cost.}$$

$$\frac{20.}{1+r} + \frac{50.}{(1+r)^2} + \frac{50}{(1+r)^3} = 100 \ \cancel{101}$$

$$r = \underline{\underline{IRR}}.$$

Discounted Cash Flow Applications

- Project Decision Rule
  - Single project Case
    - ✓ NPV method: Accept it if NPV>0
    - ✓ IRR method: Accept it if IRR>r (required rate of return)
  - Two Projects Case
    - ✓ **Independent Projects** 多个.
      - ◆ Similar to Single projects case
    - ✓ **Mutually Exclusive Projects** 1个.
      - ◆ NPV method: Choose the one with higher NPV
      - ◆ IRR method: Choose the one with higher IRR
      - ◆ When the IRR and NPV rules conflict with each other, NPV method dominates.

$NPV_A > NPV_B$

$IRR_A < IRR_B$

Example

- Calabash Crab House is considering an investment in kitchen-upgrade projects with the following cash flows:

	Project A	Project B
Initial Year	-$10,000	-$9,000
Year 1	2,000	200
Year 2	5,000	-2,000
Year 3	8,000	11,000
Year 4	8,000	15,000

Assuming Calabash has a 12.5 percent cost of capital, which of the following investment decisions has the least justification? Accept:

A. Project B because the net present value (NPV) is higher than that of Project A.
B. Project A because the IRR is higher than the cost of capital.
C. Project A because the internal rate of return (IRR) is higher than that of Project B.

- **Correct answer: C**

Example

single: A(√) B(√).

Two: 多个 A(√)1 B(√)

1个: B. NPV_B = 6687.6086

$NPV_A = 6341.414$

$IRR_A = 24.4164\%$

$IRR_B = 30.8952\%$

- Calabash Crab House is considering an investment in kitchen-upgrade projects with the following cash flows:

	Project A	Project B
Initial Year	-$10,000	-$9,000
Year 1	2,000	200
Year 2	5,000	-2,000
Year 3	8,000	11,000
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A. Project B because the net present value (NPV) is higher than that of Project A.

B. Project A because the IRR is higher than the cost of capital.

C. Project A because the internal rate of return (IRR) is higher than that of Project B.

- **Correct answer: C**

Discounted Cash Flow Applications

$$r_{BD} = \frac{(F - P_0)}{F} \times \frac{360}{t}$$

$$r_{MM} = HPY \times \frac{360}{t}$$

$$HPY = \frac{P_1 - P_0 + CF_1}{P_0}$$

$$EAY = (1 + HPY)^{365/t} - 1$$

$$EAR = (1 + \frac{BEY}{2})^2 - 1$$

Holding Period Return 持有期的益率.

- **Define:** the holding period return is the return that an investor earns over a specified holding period.
- **Formula:** $$HPR=\frac{P_1-P_0+CF_1}{P_0}$$

Example： Jane Peebles purchased a T-bill that matures in 200 days for \$975. The face value of the bill is \$1,000. What's the holding period return of the bond?

$$HPR=\frac{F-P}{P}=\frac{1000-975}{975}=2.564\%$$

\$12. \$15. \$1.

90天.

$$R = \frac{收益}{成本} \rightarrow \begin{cases} \text{Capital gain.} \\ \text{income.} \end{cases}$$

$$\frac{15-12+1}{12} = \text{HPR.}$$

T-bill

Treasury. 国债.

零息债券

① T-bill ≤1年

② T-note. 1~10年.

③ T-bond >10年.

face value.

$P = 920$ $\qquad\qquad$ $F = 1000.$

0 $\qquad\qquad\qquad\qquad$ 90天.

$P < F$ · discount

$$\boxed{HPR = \frac{F - P}{P}}$$

Holding Period Return 持有期收益率

- **Define:** the holding period return is the return that an investor earns over a specified holding period.
- **Formula:** $$HPR=\frac{P_1-P_0+CF_1}{P_0}$$

$t=200$

Example： Jane Peebles purchased a T-bill that matures in 200 days for $975. The face value of the bill is $1,000. What's the holding period return of the bond?

$F=1000$

$P=975$

$$\boxed{HPR=\frac{F-P}{P}}=\frac{1000-975}{975}=2.564\%$$

\$12. \$15. \$1.

annual

90天

400天

$$R = \frac{收益}{成本}$$

Capital gain

income

$$\frac{15-12+1}{12} = HPR$$

~~年化~~

Discounted Cash Flow Applications

Y: yield. E

R: return. E

① $$r_{BD} = \frac{(F - P_0)}{F} \times \frac{360}{t}$$

$$r_{MM} = HPY \times \frac{360}{t}$$

② $$HPY = \frac{P_1 - P_0 + CF_1}{P_0}$$

HPR.

③

$$EAY = (1 + HPY)^{365/t} - 1$$

EAR.

④ $$EAR = (1 + \frac{BEY}{2})^2 - 1$$

$\boxed{\text{T-bill}}$

$P = \underline{\underline{970}}$ $\qquad\qquad\qquad\qquad$ $F = \underline{\underline{1000}}$

0 $\qquad$ ap $\qquad\qquad\qquad\qquad$ $\underline{\underline{90天}}$ / t

$$r_{BD} = \frac{F - \cancel{P}}{\cancel{ap}F} \times \frac{360}{t} \rightarrow 90天$$

$\cancel{单}$t：单

$$HPR = \frac{\overset{1000}{F} - P}{P} = 2\%$$

① $F = ap$

② $F = 1000$

P

HPR 年化 t

HPR

$\Updownarrow$

t

单 $HPR \times \frac{360}{t} = r_{MM}$

money market yield

$$EAR = \left(1+\frac{r}{m}\right)^{m} - 1$$

复 $EAR = (1+HPR)^{\frac{365}{t}} - 1$

BEY:

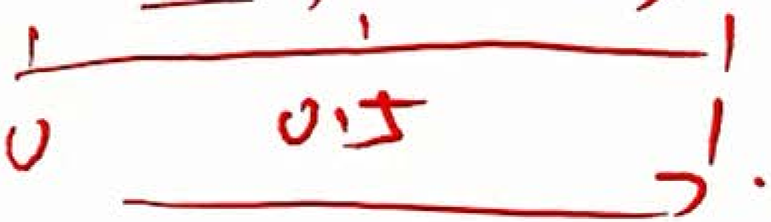
<1> 含义: 1年计息2次的Rx.

$r = 10\% \rightarrow BEY$

$m = 2$.

BEY → 复利

real return.



(2) $EAR = \left(1 + \frac{10\%}{2}\right)^2 - 1$

$$EAR = \left(1 + \frac{BEY}{2}\right)^2 - 1$$

中国：1次. 10.1%

美国：2次. 10%. ⟸ BEY.

① EAR. ② BEY.

Discounted Cash Flow Applications

- The **HPY** is the actual return an investor will receive if the money market instrument is held until maturity.
- The **EAY** is the annualized HPY on the basis of a 365-day year and incorporates the effects of compounding.
- The $\mathbf{r_{MM}}$ is the annualized yield that is based on price and a 360-day year and dose not account for the effects of compounding – it assumes simple interest.

Example

- Jane Peebles purchased a T-bill that matures in 200 days for \$975. The face value of the bill is \$1,000. What's the money market rate of the bond?

  A. 4.615%.

  B. 4.725%.

  C. 4.936%.

$$r_{MM.} = HPR \times \frac{360}{t}$$

$$1000 \leftarrow = \frac{F-P}{P} \times \frac{360}{t}$$

$$P \rightarrow 975 \qquad t \rightarrow 200$$

- **Correct Answer: A.**

$$r_{MM} = HPY \times \frac{360}{t} = 2.56\% \times \frac{360}{200} = 4.615\%$$

- A 175-day T-bill has an effective annual yield of 3.80%. Its bank discount yield is closest to:

  A. 1.80%

  B. 3.65%

  C. 3.71%

- **Correct Answer: B**

① $r_{BD} = \frac{F-P}{F} \times \frac{360}{t}$ $t \to 175$

② $EAR = (1+HPR)^{\frac{365}{t}} - 1 = 3.80\%$ $t \to 175$

③ $HPR = \frac{F-P}{P}$

$$\frac{365}{175}$$

$$(1+HPR)^{\frac{365}{175}} - 1 = 3.8\%$$

$$(1+HPR)^{\frac{365}{175}} = 1.038$$

$$\frac{365}{175}$$

$$(1+HPR)^{\frac{365}{175}} - 1 = 3.8\%$$

$$\frac{\cancel{365}}{\cancel{175}} \times \frac{\cancel{175}}{\cancel{365}}$$

$$(1+HPR) = 1.038^{\frac{175}{365}}$$

$$0.4795$$

$$HPR = 1.8\%$$

T-bill

$P = 970$

$F = 1000$

0

90天 / t

$$r_{BD} = \frac{F - P}{F} \times \frac{360}{t} \rightarrow 90天$$

ap

t：单

$$HPR = \frac{F - P}{P} = 2\%$$

1000

① $F = ap$

② $F = 1000$

p

T-bill

HPR

$r_{MM} \Longleftrightarrow r_{BD}$

EAR

HPR

T & P

对HPR年化

annual 之. 平均.

例

复利

10%　　-2%　　5%　　8%.

4年　0　1　2　3　4

136

$$\sqrt[4]{(1+10\%)(1-2\%)(1+5\%)(1+8\%)}-1$$

4季度

1年

$$(1+10\%)(1-2\%)(1+5\%)(1+8\%)-1$$

平均 R. → (annual) ⇐.

⇒ TWRR → 几何平均R. ⇒

① $(1+R_1)(1+R_2)\cdots$

② 复利

③ >1年 开

=1年 不.

$\sqrt[3]{1 \times 2 \times 3}$.

$\sqrt[4]{(1+10\%)(1-2\%)(1+5\%)(1+8\%)} - 1$

MWRR : IRR

-100	20	50	50
0	1	2	3

NPV = 0 r = IRR.

$$\frac{20}{1+IRR} + \frac{50}{(1+IRR)^2} + \frac{50}{(1+IRR)^3} = 100$$

$\boxed{R}$ - IR — IRR ∈.

Discounted Cash Flow Applications

Stock

同一种 stock

- An investor bought a fund at the beginning of 2015 (t=0) when the fund had a market value of \$100. At the beginning of 2016 (t=1), he bought another share for \$110. And he sold both shares at the beginning of 2017 (t=2), and the market value of the fund was \$150 each. In addition, at the end of each year in the holding period, the stock offered a dividend of \$2 each.
  - What is the money-weighted rate of return?
  - What is the annual time-weighted rate of return?

- **Correct Answer:**
  - Money-weighted rate of return (IRR) =28.53%
  - Time-weighted rate of return (geometric mean return) = 24.40%

MWRR

	0	1	2
	−100	−110 +2	+300 +4

① net CF.

−100	−108.	+304
‖	‖	‖
CF_0	C01	C02

② IRR. :

(CF). 清零.

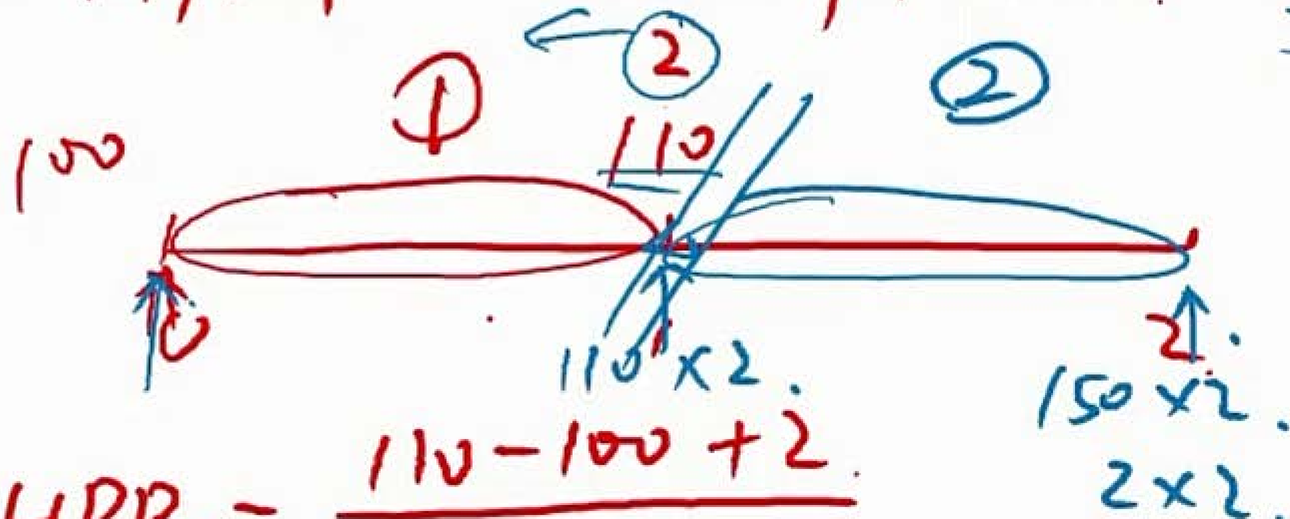
IRR CPT.

TWRR → 几何平均R

① HPR_i　　② 几何平均R

同一种stock

Δ



① $HPR_1 = \dfrac{110-100+2}{100}$

$HPR_2 = \dfrac{150\times 2 - 110\times 2 + 2\times 2}{110\times 2}$

② $TWRR = \sqrt{(1+HPR_1)(1+HPR_2)} - 1$

Discounted Cash Flow Applications

stock

同一种 stock

- An investor bought a fund at the beginning of 2015 (t=0) when the fund had a market value of $100 . At the beginning of 2016 (t=1), he bought another share for $110. And he sold both shares at the beginning of 2017 (t=2), and the market value of the fund was $150 each. In addition, at the end of each year in the holding period, the stock offered a dividend of $2 each.
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  - What is the annual time-weighted rate of return?

- **Correct Answer:**
  - Money-weighted rate of return (IRR) =28.53%
  - Time-weighted rate of return (geometric mean return) = 24.40%

2. 性质.

(1) MWRR. ~~CF~~

TWRR. ~~CF~~

(2) 业绩/选股: TWRR.

Discounted Cash Flow Applications

ΔCF

- Would a client making additions or withdrawals of funds most likely affect their portfolio's:

	Time-weighted return?	Money-weighted return?
A.	No	No
B.	No	Yes
C.	Yes	No

- **Correct Answer: B**
  - The time-weighted return is not affected by cash withdrawals or addition to the portfolio, the money-weighted return measure would be affected by client additions or withdrawals, if a client adds funds at a favorable time the money-weighted return will be elevated.

1、NPV & IRR 计算

2. 各种 yield 计算.

<1> $r_{BD} = \frac{F-P}{F} \times \frac{360}{t}$

<2> $HPR = \frac{P_1 - P_0 + CF_1}{P_0}$; $HPR = \frac{F-P}{P}$

<3> HPR 年化

单 $HPR \times \frac{360}{t} = r_{MM}$

复. $EAR = (1+HPR)^{\frac{365}{t}} - 1$

<4> BEY

<1> 含义.

<2> $EAR = (1+\frac{BEY}{2})^2 - 1$

3. TWRR & MWRR.

↓ 几何平均R　　↓ IRR.

(1) 计算: MWRR: ① net CF ② IRR.

TWRR: ① HPRi ② 几何平均R.

(2) 性质 MWRR: 好　TWRR: 坏

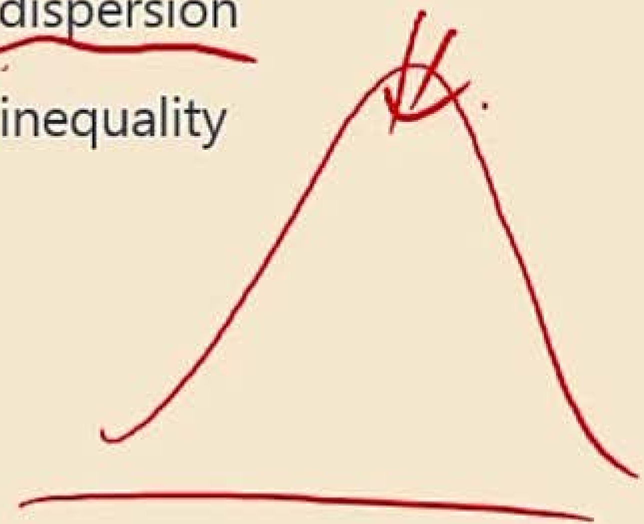
业绩表现: TWRR.

Framework

1. Types of measurement scales

136

2. Measures of central tendency
3. Quantile
4. Measures of dispersion
5. Chebyshev's inequality
6. CV
7. Sharp ratio
8. Skewness
9. Kurtosis



1. Mean.
2. Variance.
3. Skewness. 偏度.
4. kurtosis 峰度.

Statistical Concepts and Market Return

- **Descriptive statistics** 描述性.
  - Quantitatively describe or summarize the important features of large data sets.
- **Inferential statistics** 推断性. 抽样估计.
  - Makes estimations about a large set of data (a population with smaller group of data.

样本 → 总体.

Statistical Concepts and Market Return

- Types of measurement scales: 类别
  1. **Nominal scales** 男生→1 女生→2 2>1
     - ✓ Distinguishing two different things, no order, only has mode
     - ✓ Example: assigning the number 1 to male, the number 2 to female.
  2. **Ordinal scales** (>, <) 排序
     - ✓ Making things in order, but the difference are not meaningful
     - ✓ Example: ranking mutual funds based on their five-year cumulative returns, we might assign the number top1 to 10 for the funds performance.
  3. **Interval scales** (>, <, +, -) 间距.
     - ✓ Subtract is meaningful
     - ✓ Example: temperature
  - **Ratio scales** (>, <, +, -, *, /)
     - ✓ With original point
     - ✓ Example: as is money, if we have twice as much money, then we have twice the purchasing power.

哈

$-10^\circ C$

北.

$0^\circ C$.

上.

$10^\circ C$.

$<$ $<$

Statistical Concepts and Market Return

弱

Types of measurement scales: 类别

男生→1
女生→2
2>1

- 1 **Nominal scales**
  - ✓ Distinguishing two different things, no order, only has mode
  - ✓ Example: assigning the number 1 to male, the number 2 to female.
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- 3 **Interval scales** (>, <, +, -) 间距
  - ✓ Subtract is meaningful
  - ✓ Example: temperature ⇐
- 4 **Ratio scales** (>, <, +, -, *, /)
  - ✓ With original point
  - ✓ Example: as is money, if we have twice as much money, then we have twice the purchasing power.

强

Example

- An analyst gathered the price-earnings ratios (P/E) for the firms in the S&P 500 and then ranked the firms from highest to lowest P/E. She then assigned the number 1 to the group with the lowest P/E ratios, the number 2 to the group with the second lowest P/E ratios, and so on. The measurement scale used by the analyst is *best* described as:

  A. Ratio.

  B. Ordinal.

  C. Interval.

1 2 3 4 - - -

- **Correct Answer: B**

Statistical Concepts and Market Return

- **Population** 总体. 100 120 抽样
  - A population is defined as all members of a specified group.
  - A **parameter** is used to describe the features of a population.

参数. 样本.

- **Sample**
  - A sample is a subset of a population.
  - A **sample statistic** is used to describes the features of a sample.

样本均值 → 总体均值.

样本统计量. Sample size. $n=100$

样本容量.

Statistical Concepts and Market Return

频率分布表.

- **Relative frequency**
  - The relative frequency ~~of observations in an interval is the~~ number of observations(the absolute frequency) in the interval divided by the total number of observations.
- **Frequency Distribution**
  - A frequency distribution is a tabular display of data summarized into a relatively small number of intervals. Frequency distributions permit analyst to evaluate how data are distributed.
- **Cumulative frequency/Cumulative Relative Frequency**
  - The cumulative relative frequency cumulates (adds up) the relative frequencies as we move from the first interval to the last.

Statistical Concepts and Market Return

➢ **Frequency distribution**

Interval Relative	Absolute Frequency	Relative Frequency	Cumulative Absolute Frequency	Cumulative Relative Frequency
-10 - -5	3	0.97%	3	0.97%
-5 – 0	35	11.29%	38	12.26%
0 – 5	176	56.77%	214	69.03%
5 – 10	74	23.87%	288	92.90%
10 - 15	22	7.10%	310	100%
Total	**310**	**100%**		

20人

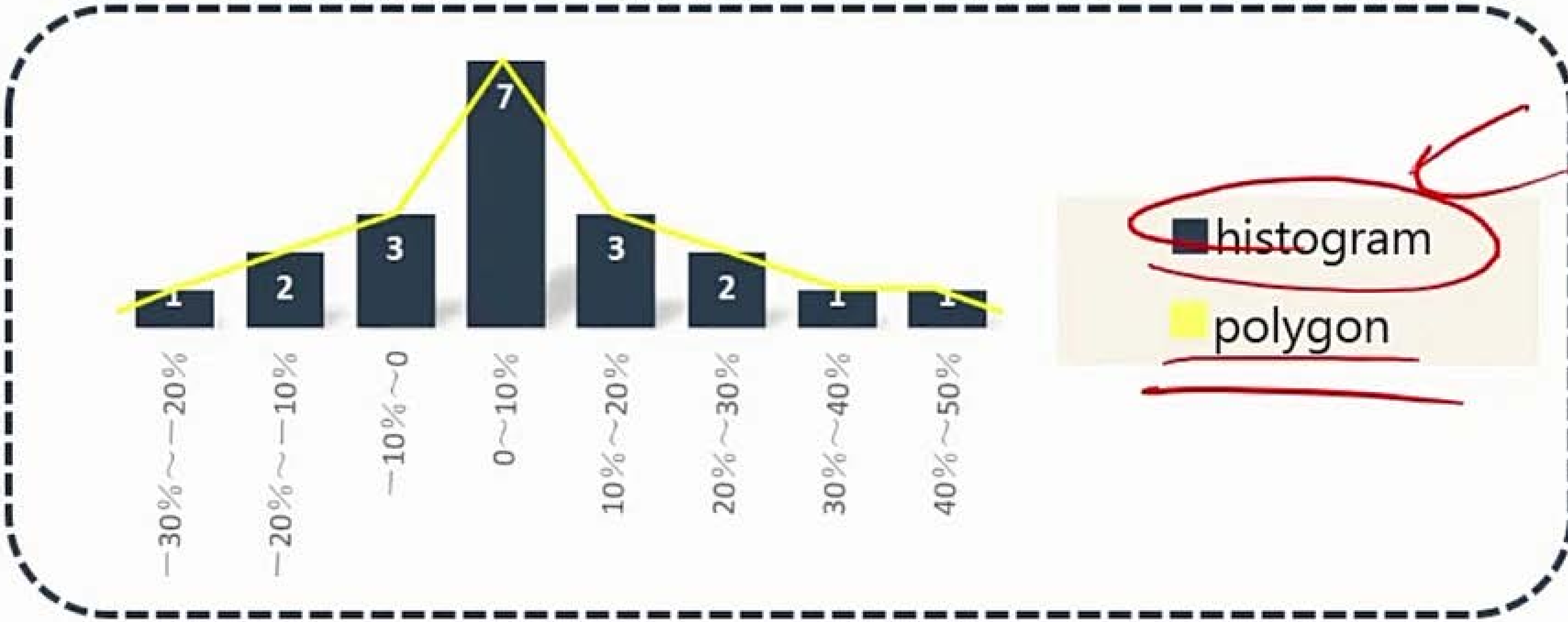
≤

		占比	≤	占比
150-160	10	50%	10	50%
160-170	5	25%	15	75%
170-180	5	25%	20	100%

Statistical Concepts and Market Return

➢ Histogram and Polygon

- A **histogram** is a bar chart of data that have been grouped into a frequency distribution.
- A frequency **polygon** is a graph of frequency distributions obtained by drawing straight lines joining successive points representing the class frequencies.



Example

- An analyst gathered the following information about the annual return on the S&P 500 over the period 1927 to 2016.

Interval	Return interval(%)	Frequency
I	-30.0 to -10.0	10
II	-10.0 to 10.0	34
III	10.0 to 30.0 ≤	30
IV	30.0 to 50.0	16

The relative frequency and the cumulative relative frequency, respectively, for interval III are closest to:

	Relative frequency 占比.	Cumulative relative frequency
A.	20%	82%
B.	33%	36%
C.	33%	82%

$\frac{30}{10+34+30+16}$.

$\frac{10+34+30}{10+34+30+16}$

- **Correct Answer: C.**

Statistical Concepts and Market Return

The Arithmetic Mean:

$$\overline{X} = \frac{\sum_{i=1}^{N} X_i}{n}$$

The Weighted Mean:

$$\overline{X_W} = \sum_{i=1}^{n} w_i X_i = (w_1 X_1 + w_2 X_2 + \cdots + w_n X_n)$$

The Geometric Mean:

$$G = \sqrt[N]{X_1 X_2 X_3 \ldots X_N} = (\prod_{i=1}^{N} X_i)^{1/N}$$

The Harmonic Mean:

$$\overline{X_H} = \frac{n}{\sum_{i=1}^{n} (1/X_i)}$$

Harmonic Mean<= Geometric Mean<=Arithmetic Mean

Mode. 众数.

5个3. 3个2 2个1

Median 中位数. 排序.

1 2 $\frac{3+4}{2}$ 5 6

Statistical Concepts and Market Return

$w_i = \frac{1}{n}$

The Arithmetic Mean: $\overline{X} = \frac{\sum_{i=1}^{N} X_i}{n}$ $= \frac{x_1 + x_2 + \cdots + x_n}{n}$ $= \frac{1}{n}x_1 + \frac{1}{n}x_2 + \cdots + \frac{1}{n}x_n$

算术.

The Weighted Mean: $\overline{X_W} = \sum_{i=1}^{n} w_i X_i = (w_1 X_1 + w_2 X_2 + \cdots + w_n X_n)$

$w_1 x_1 + w_2 x_2 + \cdots + w_n x_n$

加权.

The Geometric Mean: $G = \sqrt[N]{X_1 X_2 X_3 \ldots X_N} = (\prod_{i=1}^{N} X_i)^{1/N}$

① $(1+R_1)(1+R_2)\cdots$

② 数列

几何.

8% -2% 5% 10%

The Harmonic Mean: $X_H = \frac{n}{\sum_{i=1}^{n} (1/X_i)}$

调和

Harmonic Mean<= Geometric Mean<=Arithmetic Mean

$S_1 \quad S_2 \quad S_3 \quad \cdots \quad S_n$

$\$1 \quad \$1 \quad \$1$

$P_1 \quad P_2 \quad P_3$

$$\bar{P} = \frac{\$}{\#} = \frac{\$3}{\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}}$$

$$\boxed{\frac{n}{\frac{1}{P_1} + \frac{1}{P_2} + \cdots + \frac{1}{P_n}}}$$

Example

$(1+14\%)(1-10\%)(1-2\%)-1$

- An analyst obtains the following annual rates of return for a mutual fund:

Year	Return (%)
2008	14
2009	-10
2010	-2

~~annual~~

The fund's holding period return over the three-year period is closest to: 整体3年

A. 0.18%

B. 0.55%

C. 0.67%

几何平均 ⇒ 复利 ⇒ actual R. ⇐ HPR.

- **Correct Answer: B**

- The fund's annual holding period return is closest to: 年化 actual R.

A. 0.18%

B. 0.55%

C. 0.67%

$\sqrt[3]{(1+14\%)(1-10\%)(1-2\%)}-1$

- **Correct Answer: A**

- Which is the most accurate? $A \geq G \geq H$

	Harmonic mean	Arithmetic mean	Geometric mean
A.	13	15	18
B.	15	15	18
C.	13	18	15

- **Correct Answer: C**

Statistical Concepts and Market Return

① HPR; ②

- A hypothetical investment in a single stock initially costs \$100. one year later, the stock is trading at \$200. At the end of the second year, the stock price falls back to the original purchase price of \$100. No dividend are paid during the two-year period. Calculate the arithmetic and geometric mean annual returns.

100 200 100

0 1 2

- **Correct Answer:**

Return in Year1 = 200/100-1 = 100%

Return in Year2 = 100/200-1 = -50%

Arithmetic mean = (100%-50%)/2 = 25%

Geometric mean = $(2.0 \times 0.5)^{1/2} - 1 = 0\%$

$HPR_1 = \frac{200-100}{100} = 100\%$

$HPR_2 = \frac{100-200}{200} = -50\%$

The geometric mean return of 0% accurately reflects that the ending value of the investment in Year2 equals the starting value in Year1. The compound rate of return on the investment is 0%. The arithmetic mean return reflects the average of the one-year returns.

focus on ending value. → 几何平均R

Statistical Concepts and Market Return

➤ **The use of arithmetic mean and geometric mean when determining investment returns**

加权.

- The arithmetic mean is the statistically best estimator of the **next year's returns** given only the three years of return outcomes.
- Since past annual returns are compounded each period, the geometric mean of past annual returns is the appropriate measure of **past performance.**

Statistical Concepts and Market Return

➢ **The use of arithmetic mean and geometric mean when determining investment returns**

加权. 预测明年

- The arithmetic mean is the statistically best estimator of the **next year's returns** given only the three years of return outcomes.
- Since past annual returns are compounded each period, the geometric mean of past annual returns is the appropriate measure of **past performance.**

复利

历史业绩

1、计算.

2、大小. A ≥ G ≥ H

3、判断.

- 算术/加权：预测未来.
- 几何平均：
  - 历史业绩.
  - focus on ending value.

Statistical Concepts and Market Return

- **Quantiles** 分位数

  - Quartile /Quintile/Deciles/Percentile

    ✓ The third quintile: 60%, or there are three-fifths of the observations fall below that value.

  - Calculation $L_y = (n+1)y/100$, Ly is the quantile position expressed in percentage.

Example:

Observers: 5 8 11 12 14 16 16 18 19 21 23

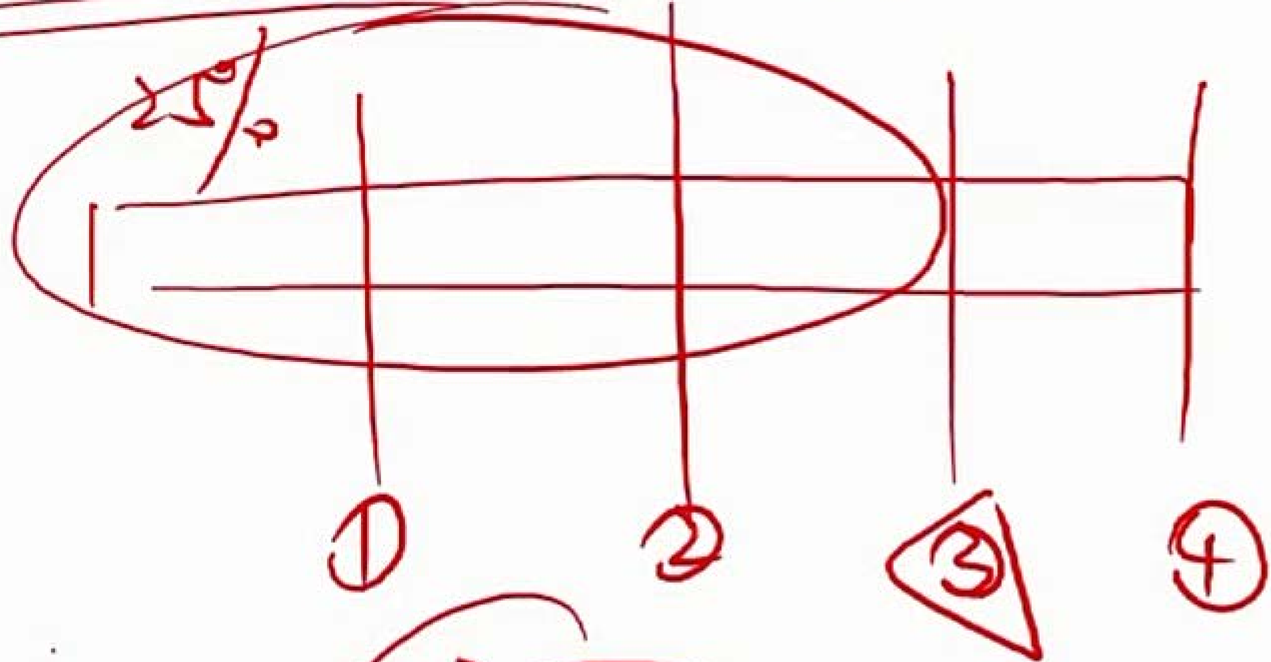
Calculate the third quartile of the data set

$N=11$, $L_y=(11+1)*75\%=9$, i.e. the 9th number is 75%

The third quartiles = 19

第3个四分位数

小→大



上边

75%

第1个五分位数，	20%
2	40%
3	60%
4	80%
5.	100%

Statistical Concepts and Market Return

- **Quantiles** 分位数. 1.含义、2.计算.
  - Quartile /Quintile/Deciles/Percentile ①小→大 ②n=11
    - ✓ The third quintile: 60%, or there are three-fifths of the observations fall below that value. ③公式=(n+1)×%
  - Calculation $L_y = (n+1)y/100$, Ly is the quantile position expressed in percentage. =(11+1)×75%=9.

Example:

Observers: 5 8 11 12 14 16 16 18 19 21 23

Calculate the third quartile of the data set

N=11 , $L_y=(11+1)*75\%=9$, i.e. the 9th number is 75%

The third quartiles = 19

n=10 公式 =(10+1)×75% = 8.25

Example:

8 9

Observers: 5 8 11 12 14 16 16 18 19 21

Calculate the third quartile of the data set

$18+(19-18)\times 0.25$

N=11 , L_y=(11+1)*75%=9, i.e. the 9th number is 75%

The third quartiles = 19

$=18.25$

Statistical Concepts and Market Return

离散

- **Absolute dispersion:** is the amount of variability present without comparison to any reference point or benchmark. 150 − 180

Range = maximum value − minimum value = 180 − 150 = 30

$$MAD = \frac{\sum_{i=1}^{N}\left|X_i - \bar{X}\right|}{n}$$

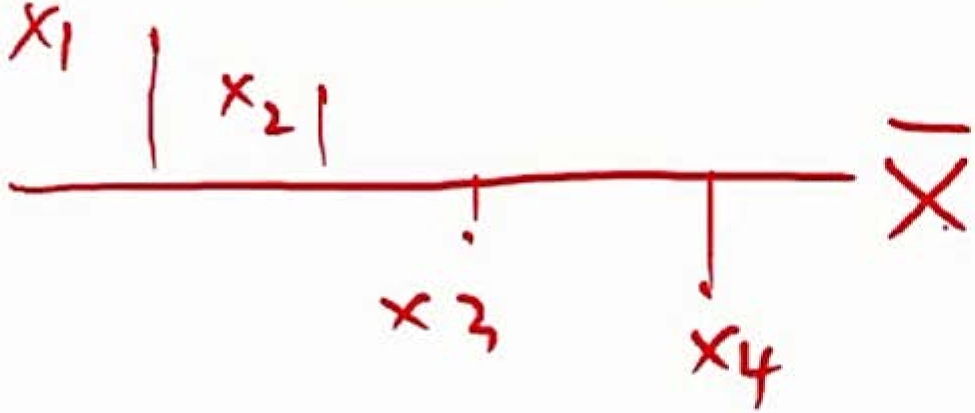
$B \leftarrow \bar{X}$

$$\text{For population: } \sigma^2 = \frac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}$$

$$\text{For sample: } s^2 = \frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}$$

$$\text{Semivariance} = \frac{\sum_{\text{for all } X_i \le \bar{X}}(X_i-\bar{X})^2}{n-1}$$

$$\text{Target Semivariance} = \frac{\sum_{\text{for all } X_i \le B}(X_i-B)^2}{n-1}$$



平均距离

$$MAD = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + |x_3 - \bar{x}| + |x_4 - \bar{x}|}{4}$$

mean absolute deviation.

平均距离

$$MAD = \frac{|x_1-\bar{x}| + |x_2-\bar{x}| + |x_3-\bar{x}| + |x_4-\bar{x}|}{4}$$

mean absolute deviation

方差

Variance

距离平方→平均

	样本	总体
Mean	$\bar{X}$	μ
Var	S^2	σ^2
S.D.	S	σ

$n-1$

样本方差 → 总体方差.

估计量.

?

自由度. free of dfg.

df.

1 2 3

$\bar{x}$

2 ⇐.

$2 \times 3 - (1+2) = 3.$

2

$x_1, x_2 \cdots x_{n-2}\ x_{n-1},\ x_n.$ $\quad \bar{x}.$

$n\bar{x} - (x_1 + x_2 + \cdots + x_{n-1}).$

$df = n-1$

1 2 3 4 5.

清屡.

① S_x σ_x

起子 ←.

② ~~STAT~~

[87]: σ_x & S_x Var ↓.

Statistical Concepts and Market Return

- **Absolute dispersion:** is the amount of variability present without comparison to any reference point or benchmark.

$$\text{Range} = \text{maximum value} - \text{minimum value}$$

$$MAD = \frac{\sum_{i=1}^{N}\left|X_i - \bar{X}\right|}{n}$$

$\bar{X}$ 小→大

$$\text{For population: } \sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

$$\text{For sample: } s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

B

index R

$$\text{Semivariance} = \frac{\sum_{\text{for all } X_i \le \bar{X}}(X_i - \bar{X})^2}{n-1}$$

$$\text{Target Semivariance} = \frac{\sum_{\text{for all } X_i \le B}(X_i - B)^2}{n-1}$$

1、计算 / MAD. 公式.

Var. & S.D.

① 题干

② SHTA: s_x, σ_x

2. 结论: absolute.

Cov

1、含义：变化的方向性 $\begin{cases}\text{同向} & >0 \\ \text{反向} & <0\end{cases}$

X	$\bar{X}$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Y	$\bar{Y}$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$

2. 公式:

$$Var(x) = E\left[(x_i - \bar{x})^2\right]$$

$$= E\left[(x_i - \bar{x})(x_i - \bar{x})\right]$$

$$Cov_{x,y} = E\left[(x_i - \bar{x})(y_i - \bar{y})\right]$$

Probability Concepts

- Calculate the covariance of the returns on Bedolf corporation (R_B) with the returns on Zedock corporation (R_Z), using the following data.

P(AB).

Joint Probability Function of Security B and Security Z Returns (Entries are joint probabilities)		
	R_Z=15%	R_Z=10%
R_B=30%	0.25	0
R_B=15%	0	0.75

- **Solution:**

First, we calculate expected values:

$$E(R_B) = (0.25 \times 30\%) + (0.75 \times 15\%) = 18.75\%$$

$$E(R_Z) = (0.25 \times 15\%) + (0.75 \times 10\%) = 11.25\%$$

Then we find the covariance as follows:

$$Cov(R_B, R_Z) = 0.25 \times (30\% - 18.75\%) \times (15\% - 11.25\%)$$
$$+0.75 \times (15\% - 18.75\%) \times (10\% - 11.25\%)$$
$$= 0.001055 + 0.000352 = 0.001407$$

	Y_1	Y_2
X_1	P_1	0
X_2	0	P_2

$P(X_1 Y_1) = P_1 \quad P(X_2 Y_2) = P_2.$

$$Cov_{X,Y} = E\left[(X_i - \bar{X})(Y_i - \bar{Y})\right]$$

$$= P_1 \times (X_1 - \bar{X})(Y_1 - \bar{Y})$$

$$+ P_2 \times (X_2 - \bar{X})(Y_2 - \bar{Y})$$

	Y_1	Y_2
X_1	P_1	0
X_2	0	P_2

$P(X_1 Y_1) = P_1 \quad P(X_2 Y_2) = P_2.$

$$\mathrm{COV}_{X,Y} = E\left[(X_i - \bar{X})(Y_i - \bar{Y})\right]$$

$$= P_1 \times (X_1 - \bar{X})(Y_1 - \bar{Y})$$

$$+ P_2 \times (X_2 - \bar{X})(Y_2 - \bar{Y})$$

$$\bar{X} = E(X) = X_1 P_1 + X_2 P_2$$

$$\bar{Y} = E(Y) = Y_1 P_1 + Y_2 P_2.$$

x_i

p_1 x_1

p_2 x_2

$Var(x)$

$$\bar{x} = E(x) = x_1 p_1 + x_2 p_2$$

$$Var(x) = E[(x_i - \bar{x})^2]$$

$$= p_1 \times (x_1 - \bar{x})^2$$

$$+ p_2 \times (x_2 - \bar{x})^2$$

$$\begin{array}{c|cc} & X & Y \\ \hline X & \underline{800} & 400 \\ Y & 400 & \underline{600} \end{array}$$

$$\sigma_x^2 = 800$$

$$\sigma_Y^2 = 600$$

$$Cov_{x,Y} = 400$$

2. 公式：

$$\underline{\underline{Var(x)}} = E\left[(x_i - \bar{x})^2\right]$$

$$= E\left[(x_i - \bar{x})\underline{\underline{(x_i - \bar{x})}}\right]$$

$$cov_{x,y} = E\left[(x_i - \bar{x})(y_i - \bar{y})\right]$$

3. $cov_{x,x} = \sigma_x^2$.

4. 范围：$-\infty \sim +\infty$

- **Correlation:** 相关系数 ρ
  - Correlation measures the co-movement (linear association) between two random variables

$$\rho_{XY} = \frac{\mathrm{COV}(X,Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}$$

  - Correlation is a number between −1 and +1
  - Understand the difference between correlation and independence
    - ✓ If $\rho=0$, there is **no linear relationship** between two variables

corr.

1. $\rho_{X,Y} = \dfrac{\mathrm{Cov}_{X,Y}}{\sigma_X \sigma_Y} \Rightarrow \underline{\mathrm{Cov}_{X,Y} = \sigma_X \sigma_Y \rho_{X,Y}}$

2. 含义：线性. $Y = 2X + b$. $Y = -0.5X + c$

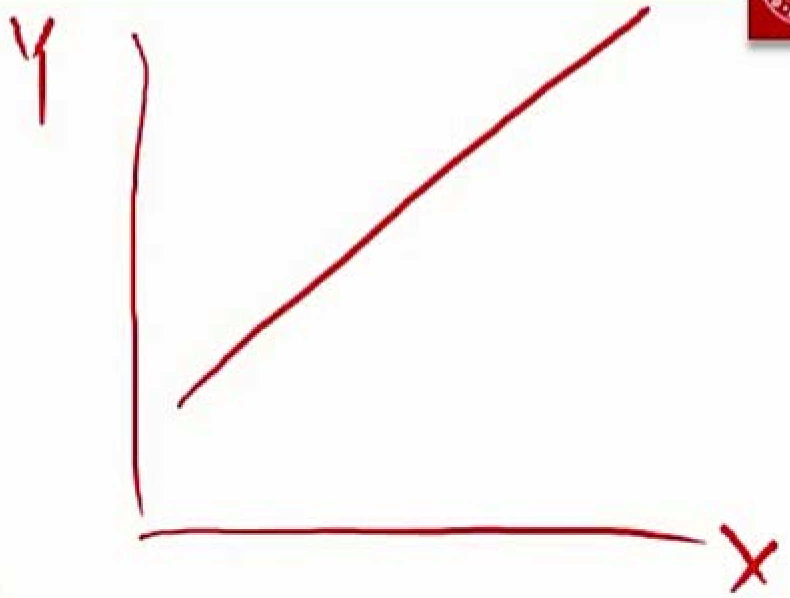
3. 范围：$-1 \sim +1$ no-unit.

4. $+1$, -1, 0.

$Y = 2x + b \Leftarrow$

slope = 2 X

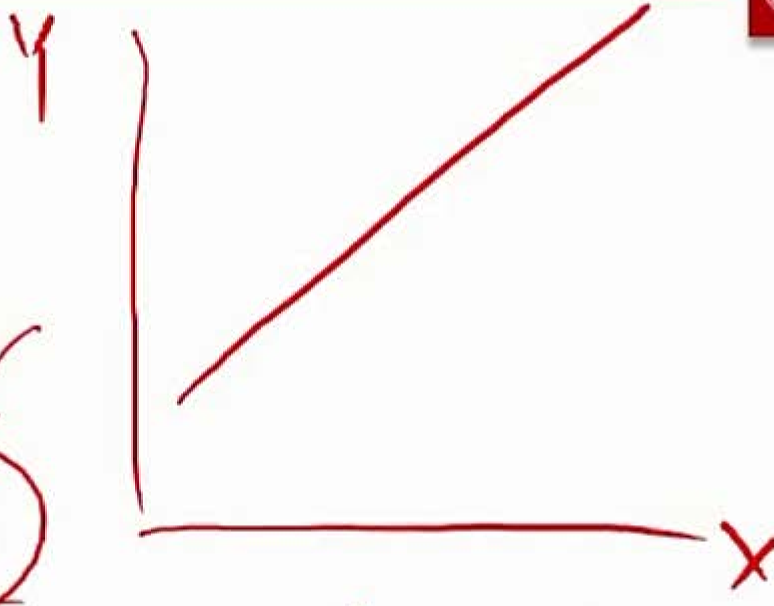
$\uparrow 2 \quad \uparrow 1 \quad \underline{\rho = +1}$



$$\Rightarrow Y = 2x + b \Leftarrow$$

slope = 2

$\uparrow 2$ $\uparrow 1$ $\rho = +1$

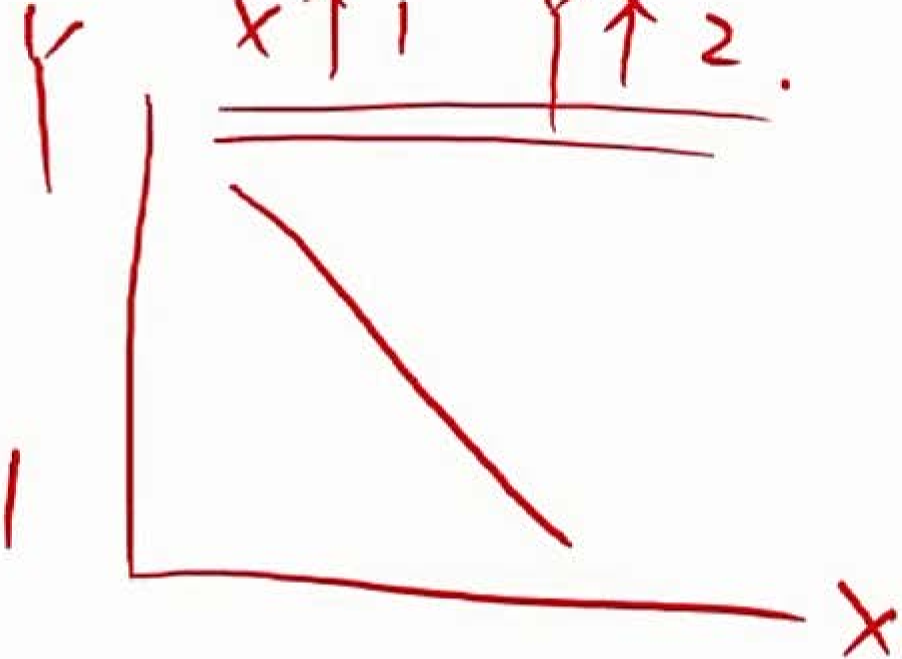


$X \uparrow 1$ $Y \uparrow 2$.

$$Y = \underline{-0.5}x + c$$

slope = -0.5

$\downarrow 0.5$ $\uparrow 1$ $\rho = -1$



$\rho = +1$ 完全正线性关系

$\rho = -1$ 完全负线性关系

$\rho = 0$ 没有线性关系

线~~性~~

$$\rho_{XY} = 0 \;\underset{\longleftarrow}{\not\longrightarrow}\; X、Y \text{ 独立}$$

没有线性关系

$$Y = \cancel{2x} + b$$

$$\underline{XY = 2}.$$

Corr.

1. $\rho_{X,Y} = \dfrac{Cov_{X,Y}}{\sigma_X \sigma_Y} \Rightarrow \underline{Cov_{X,Y} = \sigma_X \sigma_Y \rho_{X,Y}}$

2. 含义：线性. $Y = 2X + b$. $Y = -0.5X + c$

3. 范围：$-1 \sim +1$

   no-unit.

4. $+1$, -1, 0.

   $|\rho| \longrightarrow 1$ 强.

   $ \longrightarrow 0$ 弱.

Probability Concepts

- The covariance of returns for two stocks:

  A. must have a value between -1.0 and +1.0 $\cancel{-1 \sim +1}$ $-\infty \sim +\infty$

  B. must ~~have~~ a value equal to the weighted average of the standard deviations of the returns of the two stocks

  >0 C. will be positive if the actual returns on both stocks are consistently below their expected returns at the same time ✓

  $\cancel{Cov_{1,2} = W_1\sigma_1 + W_2\sigma_2}$

- **Correct Answer: C**

$X \downarrow \quad Y \downarrow$

Example

- The standard deviations of returns of US and Spanish bonds are 24% and 16%, respectively. If the correlation between the two bonds is 0.50, the covariance of returns is *closest* to:
  - A. 0.0192
  - B. 0.1386
  - C. 0.0384

$\sigma_1 = 24\%$, $\sigma_2 = 16\%$, $\rho_{1.2} = 0.5$

$Cov_{1.2} = \sigma_1 \sigma_2 \rho_{1.2}$

- **Correct Answer: A**

- Which of the following correlation coefficients indicates the weakest linear relationship between variables?
  - A. -0.67
  - B. -0.24
  - C. 0.33

$|\rho| \rightarrow 1$ 强

$|\rho| \rightarrow 0$ 弱

- **Correct Answer: B**

Probability Concepts

- Expected return, variance and standard deviation of a portfolio

$$E(r_{\mathrm{p}})=\sum_{i=1}^{n} w_i E(R_i)$$

$E(R_p)$ σ_p^2 / σ_p.

$$\sigma^2{}_p=\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \operatorname{cov}(R_i, R_j)$$ X

1	2.
w_1	w_2.
R_1	R_2.

$E(R_p) = w_1R_1 + w_2R_2$.

2个资产 $\rightarrow \sigma_p^2$.

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\underbrace{\sigma_1\sigma_2\rho_{1,2}}_{cov_{1,2}}.$$

$$\Rightarrow (a+b)^2 = a^2 + b^2 + 2ab$$

$$w_1\sigma_1 \qquad w_2\sigma_2.$$

$$
\begin{array}{c|ccc}
 & w_1 & w_2 & w_3 \\
\hline
w_1 & \sigma_1^2 & cov_{1,2} & cov_{1,3} \\
w_2 & cov_{1,2} & \sigma_2^2 & cov_{2,3} \\
w_3 & cov_{1,3} & cov_{2,3} & \sigma_3^2
\end{array}
$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2$$

$$+ 2w_1w_2 \operatorname{cov}_{1,2}$$

$$+ 2w_1w_3 \operatorname{cov}_{1,3}$$

$$+ 2w_2w_3 \operatorname{cov}_{2,3}.$$

$$\sigma_p^2 = \boxed{w_1^2\sigma_1^2 + w_2^2\sigma_2^2} + 2w_1w_2\sigma_1\sigma_2\,\rho_{1,2}.$$

① $\rho = +1$ $\quad \sigma_p^2 = (w_1\sigma_1 + w_2\sigma_2)^2$

$\sigma_p = w_1\sigma_1 + w_2\sigma_2.$

$+1 \downarrow$ $\quad \sigma_p^2$

$0 \quad \downarrow$

$-1 \quad \downarrow.$

② $\rho = -1$ $\quad \sigma_p^2 = (w_1\sigma_1 - w_2\sigma_2)^2.$

$\sigma_p = |w_1\sigma_1 - w_2\sigma_2| = 0$ no risk

③ $\rho = +1$ σ_p^2 max $\quad \rho = -1$ σ_p^2 min.

$\rho\downarrow$. $\sigma_p^2\downarrow$. risk $\downarrow$. $\Rightarrow$ 分散化 $\uparrow$.

n个资产

① σ_p^2 影响因素：w_i．σ_i^2．ρ_{ij}．cov_{ij}

② n↑．分散化↑

→ σ_p^2． $\begin{cases} \sigma_i^2 \\ cov_{ij} \end{cases}$ ✓

Probability Concepts

- An individual wants to invest $100,000 and is considering the following stocks:

stock	Expected Return	Standard Deviation of Returns
A	12%	15%
B	16%	24%

- The expected correlation of returns for the two stocks is +0.5. If the investor invests $40,000 in Stock A and $60,000 in Stock B, the expected standard deviation of returns on the portfolio will be:
  - A. equal to 20.4%
  - B. less than 20.4%
  - C. greater than 20.4%

σ_p

$W_A = 40\%$ $W_B = 60\%$

$$\sigma_p^2 = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_AW_B\sigma_A\sigma_B\rho_{A,B}$$

$\rightarrow \sigma_p$

- **Correct Answer: B**

Probability Concepts

$\rho = +1$ $\quad \sigma_p = W_A\sigma_A + W_B\sigma_B$

- An individual wants to invest $100,000 and is considering the following stocks:

$= 20.4\%$

stock	Expected Return	Standard Deviation of Returns
A	12%	15%
B	16%	24%

$\rho = 0.5 \downarrow \rightarrow \sigma_p^2 \downarrow \rightarrow \sigma_p \downarrow$

- The expected correlation of returns for the two stocks is +0.5. If the investor invests $40,000 in Stock A and $60,000 in Stock B, the expected standard deviation of returns on the portfolio will be:

  A. equal to 20.4%

  B. less than 20.4%

  C. greater than 20.4%

- **Correct Answer: B**

Example

$W_A = 75\%$ $W_B = 25\%$

- An fund manager has a portfolio of two mutual funds, A and B, 75 percent invested in A, as shown in the following table.

$\sigma_A^2 = 625$

$\sigma_B^2 = 196$

$COV_{A,B} = 120$

Covariance Matrix		
Fund	A	B
A	625	120
B	120	196

The correlation between A and B, and the portfolio standard deviation of return is closest to:

	Correlation between A and B	Portfolio standard deviation of return
A.	0.18	40.80%
B.	0.34	20.22%
C.	0.12	18.00%

$$\rho_{A,B} = \frac{COV_{A,B}}{\sigma_A \sigma_B} = \frac{120}{\sqrt{625} \times \sqrt{196}}$$

$\rightarrow \sigma_p$

- **Correct Answer: B**

$$\sigma_p^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 W_A W_B COV_{A,B}$$

Probability Concepts

贝叶斯公式.

- **Bayes' Formula**

  - $P(AB)=P(A|B)\times P(B) = P(B|A)\times P(A)$

  $$P(A \mid B)=\frac{P(B \mid A)}{P(B)} * P(A)$$

  - Where P(B) can be solved using total probability formula:

    ✓ $P(B)=P(B|W_1)\times P(W_1)+P(B|W_2)\times P(W_2)+\ldots+P(B|W_n)\times P(W_n)$

    ✓ W_i is a set of **mutually exclusive and exhaustive** events

Probability Concepts

贝叶斯公式.

- **Bayes' Formula**
  - $P(AB)=P(A|B)\times P(B)=P(B|A)\times P(A)$

$$\boxed{P(A \mid B)=\frac{P(B \mid A)}{P(B)} * P(A)}$$

  - Where P(B) can be solved using total probability formula:
    - ✓ $P(B)=P(B|W_1)\times P(W_1)+P(B|W_2)\times P(W_2)+\ldots+P(B|W_n)\times P(W_n)$
    - ✓ W_i is a set of **mutually exclusive and exhaustive** events

$$P(AB)=P(A)\times P(B|A)=P(B)\times P(A|B)$$

$$P(A|B)=\frac{P(AB)}{P(B)}=\frac{P(A)\times P(B|A)}{P(B)}$$

Probability Concepts

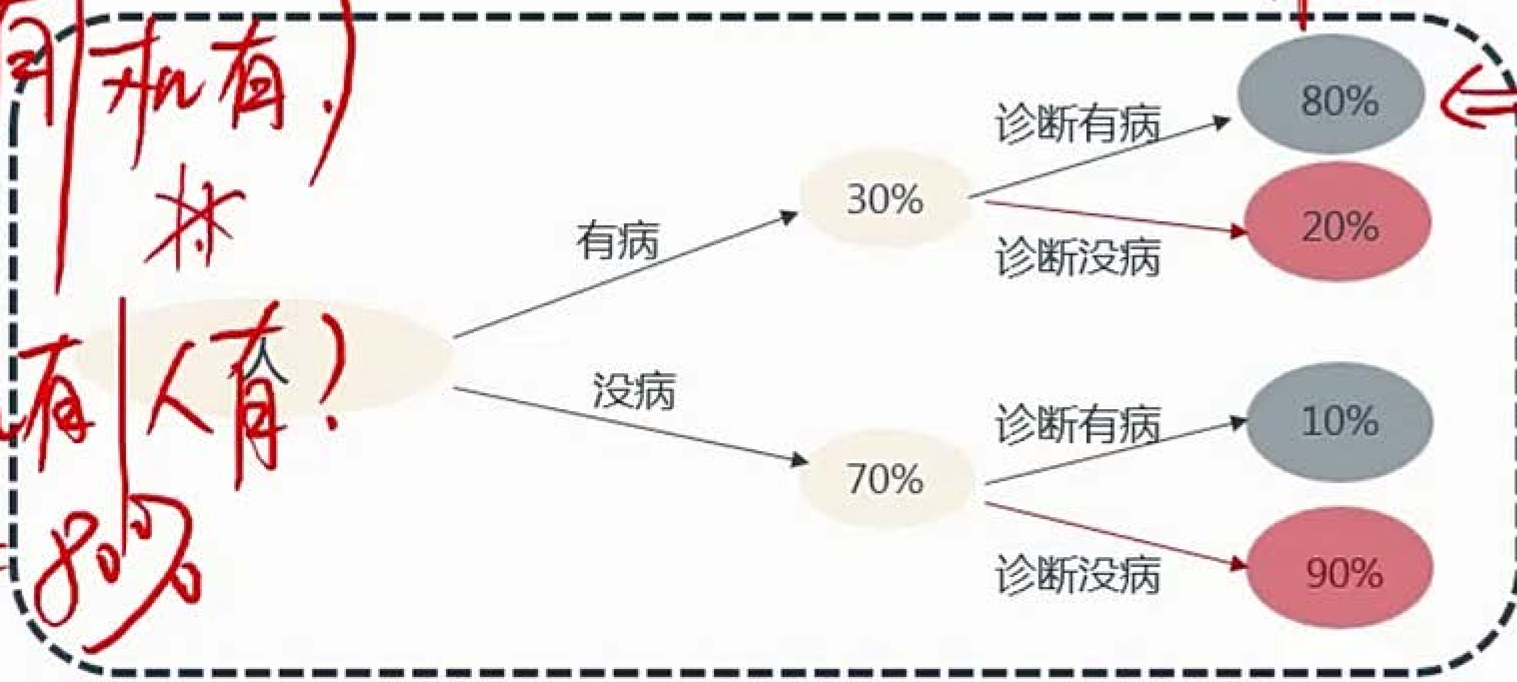
- The probability that a man has got sick is 30%, and if it does, a medical machine will have 80% chance to diagnose the disease. The probability that a man is healthy is 70%, and if it does, a medical machine will have 10% chance to diagnose the disease, and 90% chance not to diagnose disease. What's the probability that the man has actually got sick when the machine diagnoses disease ?

$$P(A/B)=\frac{30\%\times80\%}{30\%\times80\%+70\%\times10\%}=77.42\%$$

$P(人有|机有)$

$P(机有|人有)=80\%$

机.



1. 设事件：人有：A．人没：$\overline{A}$．

机有：B．机没：$\overline{B}$．

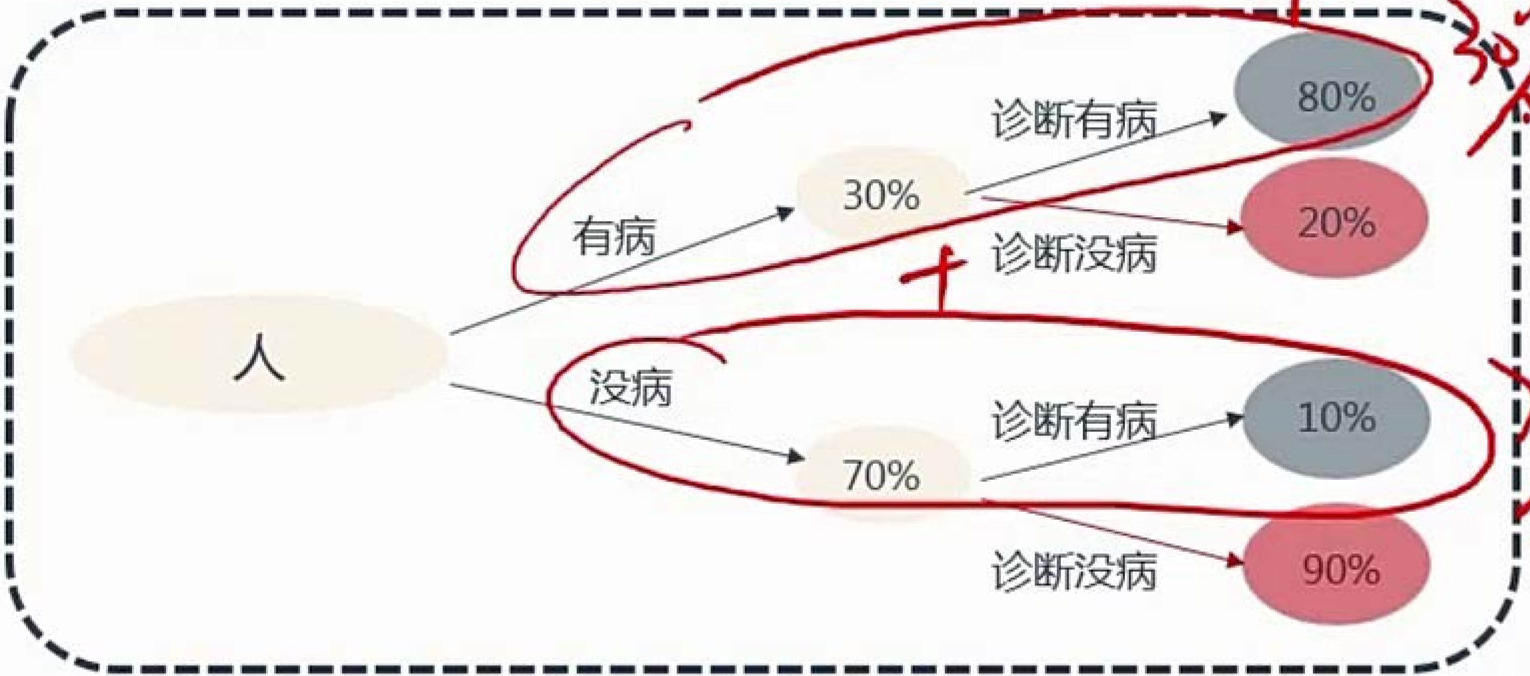
2. $P(A)=30\%$ $P(\overline{A})=70\%$

$P(B|A)=80\%$ $P(\overline{B}|A)=20\%$

$P(B|\overline{A})=10\%$ $P(\overline{B}|\overline{A})=90\%$

$P(A|B)$

$$P(A/B)=\frac{30\%\times 80\%}{30\%\times 80\%+70\%\times 10\%}=77.42\%$$



3. 公式：$P(A|B)$

$$P(AB) = P(A) \times P(B|A) = P(B) \times P(A|B)$$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A) \times P(B|A)}{P(B)}$$

$$= \frac{30\% \times 80\%}{30\% \times 80\% + 70\% \times 10\%}$$

Probability Concepts

- **Multiplication rule:** $n_1 \times n_2 \times \ldots\ldots \times n_k$ 乘法法则
- **Factorial:** $n\,!$
- **Labeling (or Multinomial):** $\dfrac{n\,!}{n_1\,! \times n_2\,! \times K \times n_k\,!}$
- **Combination:** ${}_nC_r = \dfrac{n\,!}{(n-r)\,! \times r\,!}$

  组合.
- **Permutation:** ${}_nP_r = \dfrac{n!}{(n-r)!}$

  排列.

$V_{\cancel{order}}$

10 只

红 5 $5!$

黄 3 $3!$

蓝 2 $2!$

$$\frac{10!}{5! \times 3! \times 2!}$$

~~Order~~

10只

红 7 $7!$

黄 3 $3!$

$$\frac{10!}{7!\,3!}$$

Probability Concepts

- **Multiplication rule:** $n_1 \times n_2 \times \ldots\ldots \times n_k$ 乘法法则
- **Factorial:** $n!$ 阶乘 ⇒ 分层间
- **Labeling (or Multinomial):** $\dfrac{n!}{n_1! \times n_2! \times K \times n_k!}$ 贴标签
- **Combination:** ${}_nC_r = \dfrac{n!}{(n-r)! \times r!}$ 组合 ~~order~~
- **Permutation:** ${}_nP_r = \dfrac{n!}{(n-r)!}$ 排列 order

order

10只

红 7. $7!$

黄 3. $3!$

$$\frac{10!}{7!\,3!}$$

A. B. $\Longrightarrow$ C.

10 个人 $\longrightarrow$ 4 个人

~~order~~

order

$_{10}C_4$

$_{10}P_4$.

一、Prob：2个事件，2个法则.

（一）2个法则.

1、乘法：

$$P(AB)=P(A)\times P(B|A)=P(B)\times P(A|B)$$

2、加法：

$$P(A\text{ or }B)=P(A)+P(B)-P(AB)$$

（二）2个事件.

1、互斥：

$P(AB)=0$

$P(A \text{ or } B)=P(A)+P(B)$

2. 独立.

$P(A|B)=P(A)$ $P(B|A)=P(B)$

$P(AB)=P(A)\times P(B)$.

二. cov. & corr.

(一) cov

1. 含义.
   - 同向 >0.
   - 反向 <0

2. 公式. $Cov_{x,y} = E[(X_i-\bar{X})(Y_i-\bar{Y})]$

3. $Cov_{x,x} = \sigma_x^2$.

4. 范围: $-\infty \sim +\infty$

（二）corr.

1. 公式. $\rho_{x,y} = \dfrac{cov_{x,y}}{\sigma_x \sigma_y}$.

2. 含义：线性.

3. 范围：$-1 \sim +1$

4. $+1$, -1, 0. $|\rho| \to 1$ 强.
$\to 0$ 弱.

三、$E(R_p)$ & σ_p^2（2个资产）

（一）$E(R_p) = w_1R_1 + w_2R_2$.

（二）$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\underbrace{\sigma_1\sigma_2\rho_{1,2}}_{cov_{1,2}}$

1. 计算.

2. 结论

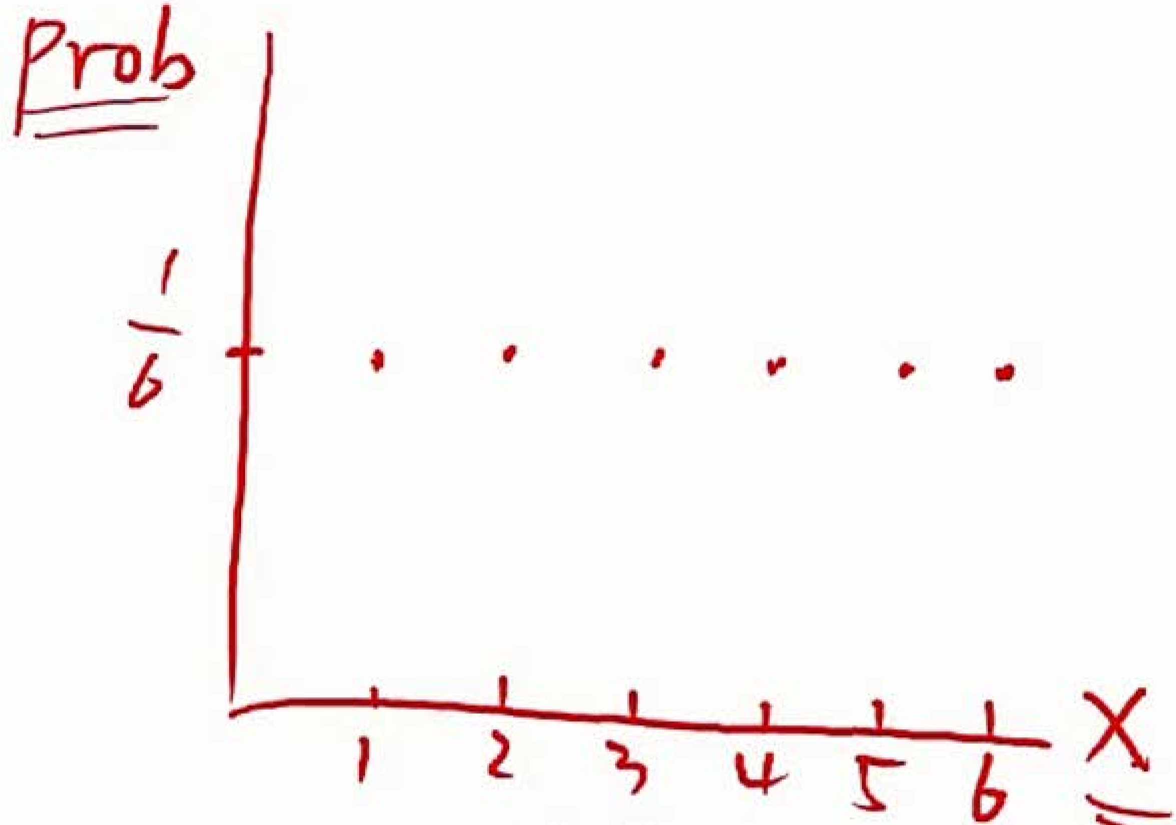
① $\rho = +1$

② $\rho = -1$

③ $\rho\downarrow$. $\sigma_p^2\downarrow$ risk$\downarrow$ 分散化

随机变量 → 概率 → 概率分布.

X : 1 2 3 4 5 6 $\frac{1}{6}$



Common Probability Distributions

- **Probability Distribution**
  - Specifies the probabilities of the possible outcomes of a random variable.

quoted. price. 2.21

离散 可数

- **Discrete and continuous random variables**
  - ① Discrete random variables take on at most a **countable** number of possible outcomes **but do not necessarily to be limited**.
  - ② Continuous random variables: cannot describe the possible outcomes of a continuous random variable Z with a list $z_1, z_2, \ldots$ because the outcome $(z_1 + z_2)/2$, not in the list, would always be possible.
    - ✓ P (x)=0 even though x can happen.
    - ✓ P (x_1<X<x_2)

连续 不可数

ratio / 分数

Common Probability Distributions

密度函数

⇓

图形面积

⇓

Prob.

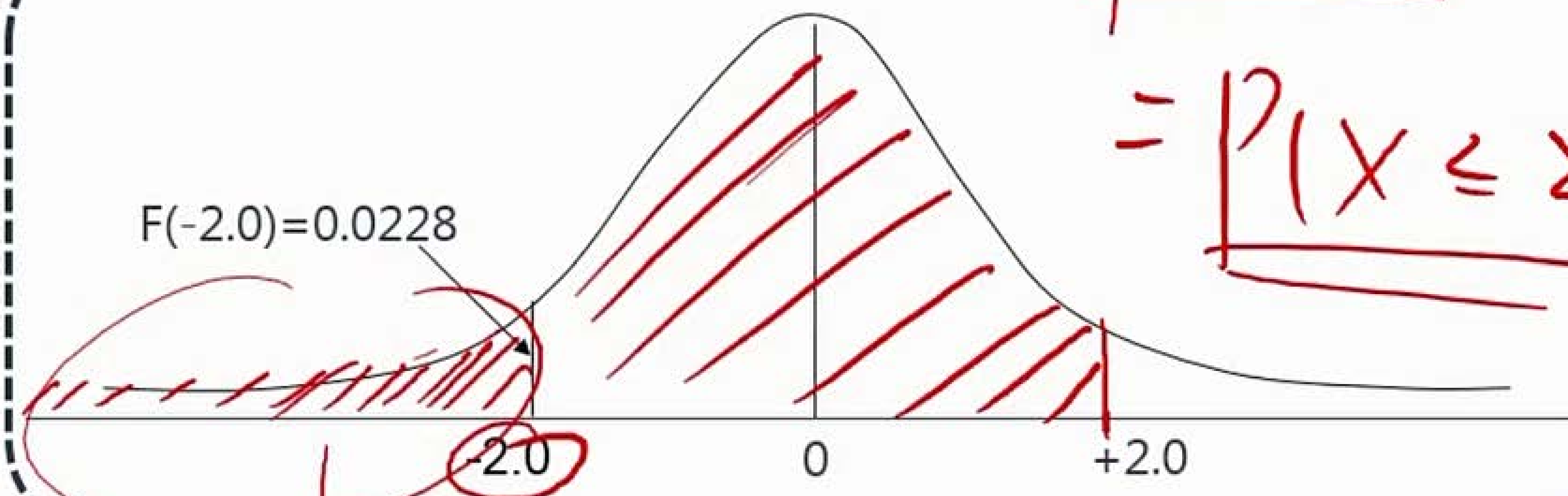
- **Probability function: $p(x)=P(X=x)$**
  - For discrete random variables
  - $0 \le p(x) \le 1$
  - $\Sigma p(x)=1$ $\cancel{\int}$
- **Probability density function (p.d.f) : f(x)**
  - For continuous random variable commonly
- **Cumulative probability function (c.p.f) : F(x)**
  - $F(x)=P(X<=x)$

$\le$ 累积

Common Probability Distributions

- **Probability density function**

$F(2) = P(X \le 2)$



$P(X \le -2) = F(-2)$

Common Probability Distributions

- Which of the following statements about probability distributions is **FALSE**?

  A. For a probability distribution for the number of days the air pollution is above a specified level, $p(x) = 0$ when x cannot occur, or $p(x) > 0$ when it can.

  B. For a probability distribution for the specific level of air pollution on a given day, $p(x) = 0$ even if x can occur.

  C. A cumulative distribution function gives the probability that a random variable takes a value equal to or greater than a given number.

- **Correct Answer: C**

  - A cumulative distribution function gives the probability that a random variable takes a value equal to or *less* than a given number: $P(X \le x)$, or $F(X)$.

Common Probability Distributions

- **Discrete uniform distribution** 离散均匀分布 → 特征.
  - 可数
  - Discrete uniform distribution would be a known, finite number of outcomes equally likely to happen. Every one of n outcomes has equal probability 1/n.
  - prob 一样.
  - For example, rolling a dice will have 6 possible outcomes as X={1,2,3,4,5,6}
    - ✓ In that case, the probability for each outcome is 0.167 [i.e. p(1)=p(2)=p(3)=p(4)=p(5)=p(6)=0.167].

Common Probability Distributions

➢ Binomial distribution

二项式分布

贝努里.

- Bernoulli random variable

$\Rightarrow$ P(Y=1)=p P(Y=0)=1-p

1次

正. 50%

反. 50%

- Binomial random variable，the probability of x successes in n trails

n次

$$p(x) = P(X = x) = {}_nC_x\, p^x (1-p)^{n-x}$$

成功 p.

不成功 1-p.

- Expectations and variances

	Expectation	Variance
Bernoulli random variable (Y)	p	p(1-p)
Binomial random variable (X)	np	np(1-p)

1、贝努里 vs. 二项式.

<1> 相同：2种结果.

<2> 不同：1次. vs. n次.

2、计算Prob：n次，x次成功Prob(p)

$$_nC_x\, p^{x}(1-p)^{n-x}$$

Common Probability Distributions

- **Continuous Uniform Distribution** 连续均匀分布

  概率.

  - All intervals of the same length on the Continuous Uniform Distribution's support are equally probable.

    Prob 一样

    1. 特征.

    - ✓ The support is defined by the two parameters, a and b, which are its minimum and maximum values

    2. 计算 Prob.

- **Properties of Continuous uniform distribution**
  - For all $a \le x_1 < x_2 \le b$:

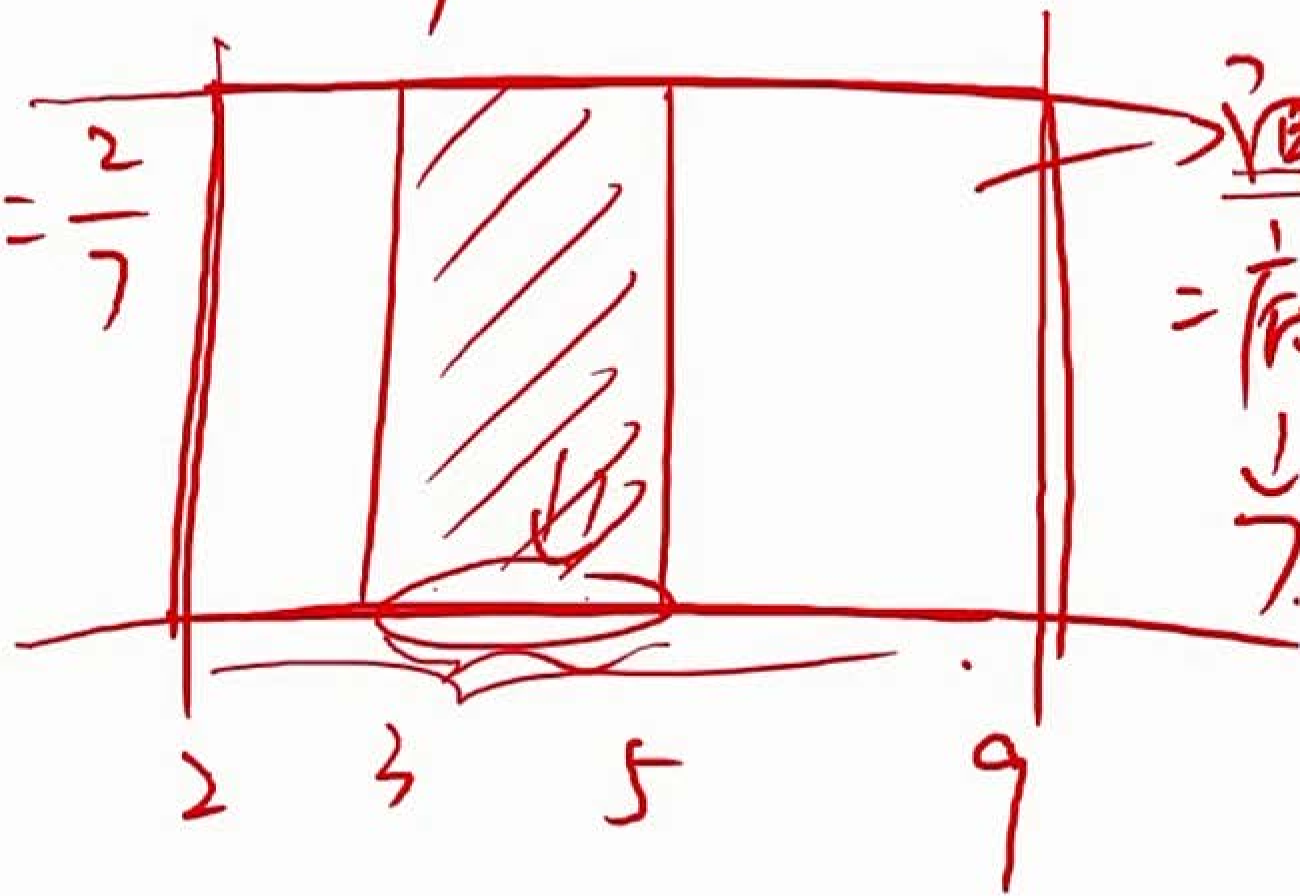
$$P(x_1 \le X \le x_2) = (x_2 - x_1)/(b - a)$$

  - $P(X<a \text{ or } X>b) = 0$

$$P(3\sim5)=\frac{5-3}{9-2}=\frac{2}{7}$$

底 × 高 $=\frac{2}{7}$

底 $\to 2$，高 $\to \frac{1}{7}$



面积 $=1$

$=$ 底 × 高 $=1$

底 $\to 7$，高 $\to \frac{1}{7}$

Common Probability Distributions

- **Continuous Uniform Distribution** 连续均匀分布
  - All intervals of the same length on the Continuous Uniform Distribution's support are equally probable.

    不同参数. Prob 一样 1. 特征.

    - ✓ The support is defined by the two parameters, a and b, which are its minimum and maximum values

2. 计算Prob.

- **Properties of Continuous uniform distribution**
  - For all $a \le x_1 < x_2 \le b$:

$$\Rightarrow P(x_1 \le X \le x_2) = (x_2 - x_1)/(b - a)$$

  - P (X<a or X>b) = 0

Example

- Which of the following statements about probability distributions is TRUE?

  A. A continuous uniform distribution has a lower limit but no upper limit.

  B. A cumulative distribution function defines the probability that a random variable is greater than a given value. $<$ $\le$

  C. A binomial distribution counts the number of successes that occur in a fixed number of independent trials that have mutually exclusive (i.e. yes or no) outcomes.

- **Correct Answer: C**

Example

- A random variable with a finite number of equally likely outcomes is best described by a:

  A. Binomial distribution.

  B. Bernoulli distribution.

  C. Discrete uniform distribution.

有数

均等

离散均匀

- **Correct Answer: C**

Example

$P(\uparrow) = 70\%$ $P(\downarrow) = 30\%$

- Over the last 10 years, a company's annual earnings increased year over year seven times and decreased year over year three times. You decide to model the number of earnings increases for the next decade as a binomial random variable. Assume the estimated probability is the actual probability for the next decade.
  - What is the probability that earning will increase in exactly 5 of the next 10 years?
  - Calculate the expected number of yearly earnings increases during the next 10 years.

$${}_{10}C_5 (70\%)^5 (30\%)^5$$

- **Correct Answer:**

The probability of an earnings increase in a given year is p=0.7 and the number of trials is n=10. The probability of success on a trial

$$P(X=x)={}_{n}C_{x}p^{x}(1-p)^{n-x}={}_{10}C_{5}0.7^{5}0.3^{10-5}=0.1029$$

The expected number of yearly increases is

$$E(X)=np=10\times 0.7=7$$

np = 10 × 70%

Example

→ HF 60%

X → HF 40%

- An analyst has recently determined that only 60 percent of all U.S. pension funds have holdings in hedge funds. In evaluating this probability, a random sample of 50 U.S. pension funds is taken. The number of U.S. pension funds in the sample of 50 that have hedge funds in their portfolio would most accurately be described as:

n.

A. A binomial random variable.

n=50.

B. A Bernoulli random variable.

1>3.

pension.

C. A continuous random variable.

- **Correct Answer: A**

Example

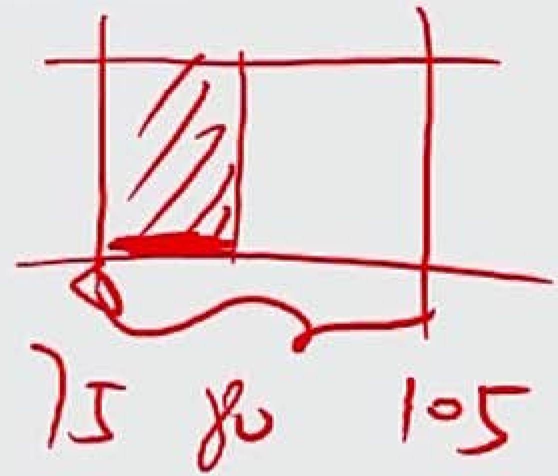
➢ An energy analyst forecasts that the price per barrel of crude oil five years from now will range between USD$75 and USD$105. Assuming a continuous uniform distribution, the probability that the price will be less than USD$80 five years from now is closest to:

A. 5.6%.

B. 16.7%.

C. 44.4%.

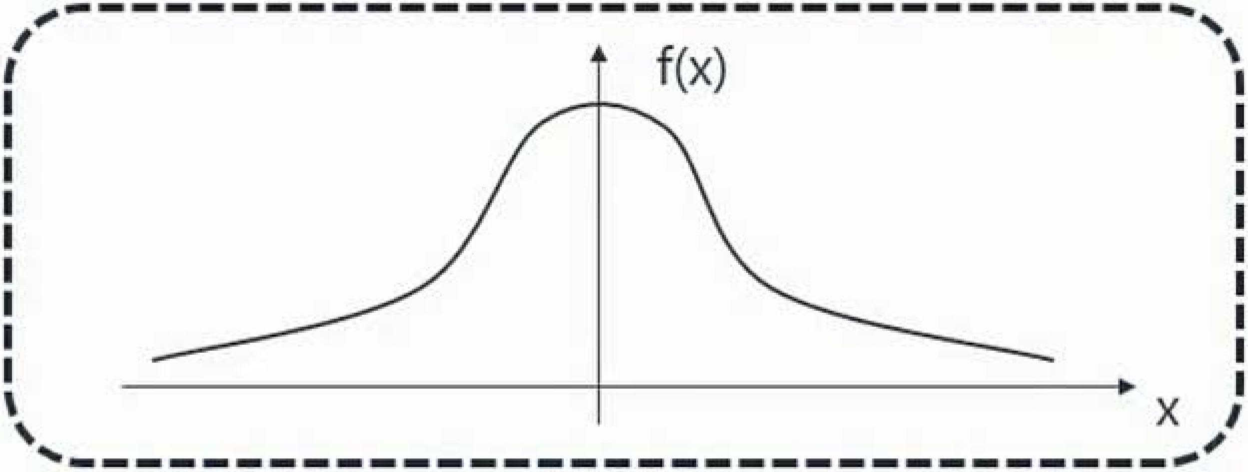
$$P(X < 80) = \frac{80 - 75}{105 - 75}.$$



➢ **Correct Answer: B**

Common Probability Distributions

- **The shape of the density function**

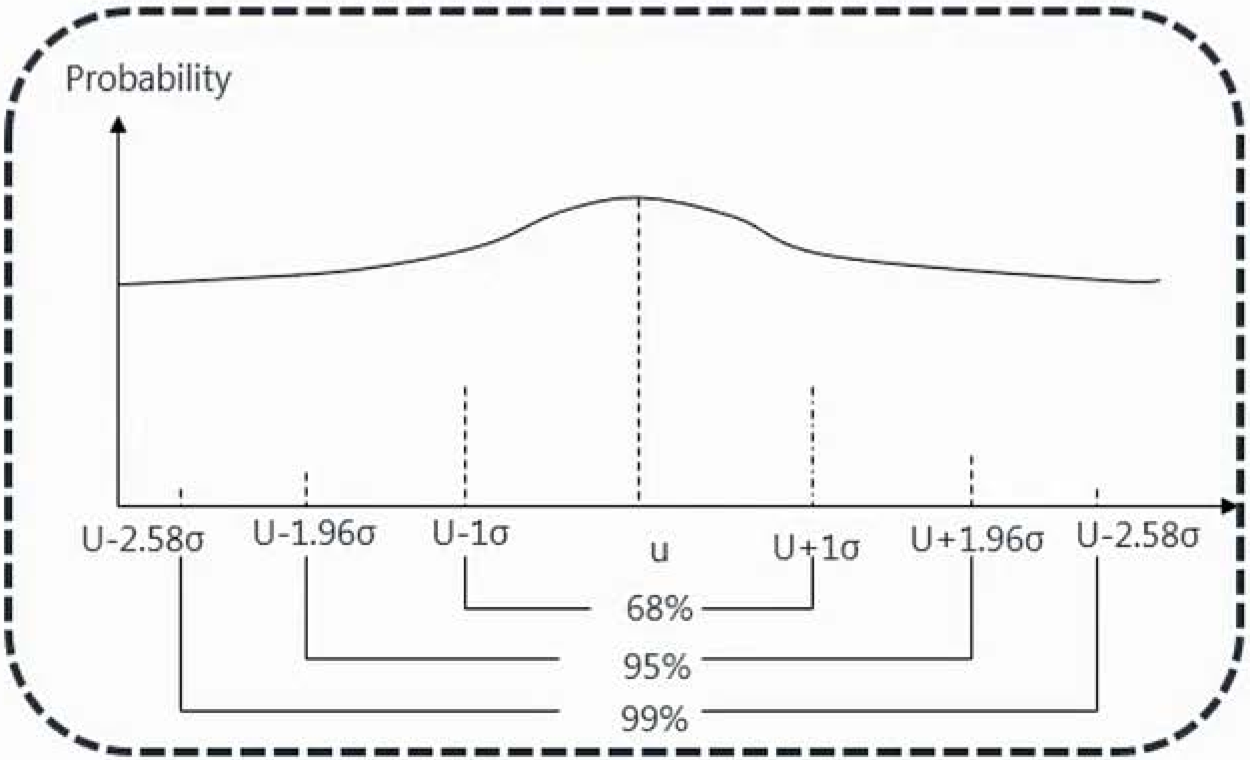


- **Properties:**
  - $X \sim N(\mu, \sigma^2)$
  - Symmetrical distribution: skewness=0; kurtosis=3
  - A linear combination of normally distributed random variables is also normally distributed.
  - As the values of x gets farther from the mean, the probability density get smaller and smaller but are always positive.

Common Probability Distributions

2、置信区间

- **The confidence intervals**
  - 68% confidence interval is $[\mu-\sigma, \mu+\sigma]$
  - 90% confidence interval is $[\mu-1.65\sigma, \mu+1.65\sigma]$
  - 95% confidence interval is $[\mu-1.96\sigma, \mu+1.96\sigma]$
  - 99% confidence interval is $[\mu-2.58\sigma, \mu+2.58\sigma]$



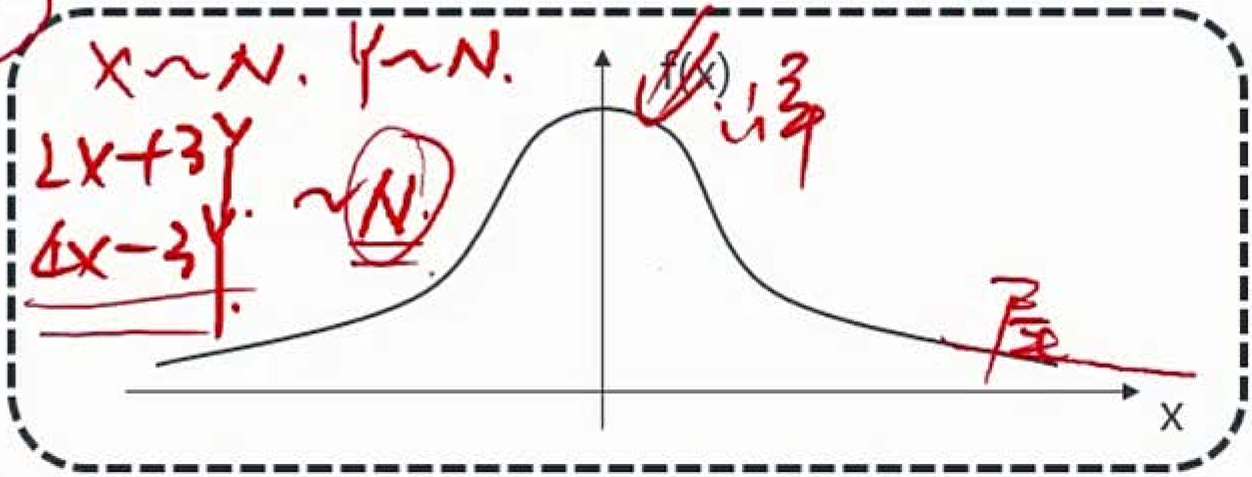
Common Probability Distributions 3. 标准化.

- **Standard normal distribution**
  - N(0,1) or Z
  - Standardization: if X~N(μ , σ^2), then $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$
  - Z-table
- **F(-z)=1-F(z)**
- **P(Z>z) = 1 –F(z)**

Common Probability Distributions

1、性质、

- The shape of the density function



- Properties:

  ① • $X \sim N(\mu, \sigma^2)$ 2个.

  ② • Symmetrical distribution: skewness=0; kurtosis=3

  ③ • A linear combination of normally distributed random variables is also normally distributed.

  ④ • As the values of x gets farther from the mean, the probability density get smaller and smaller but are always positive.

Common Probability Distributions

2、置信区间

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  - 99% confidence interval is $[\mu-2.58\sigma, \mu+2.58\sigma]$

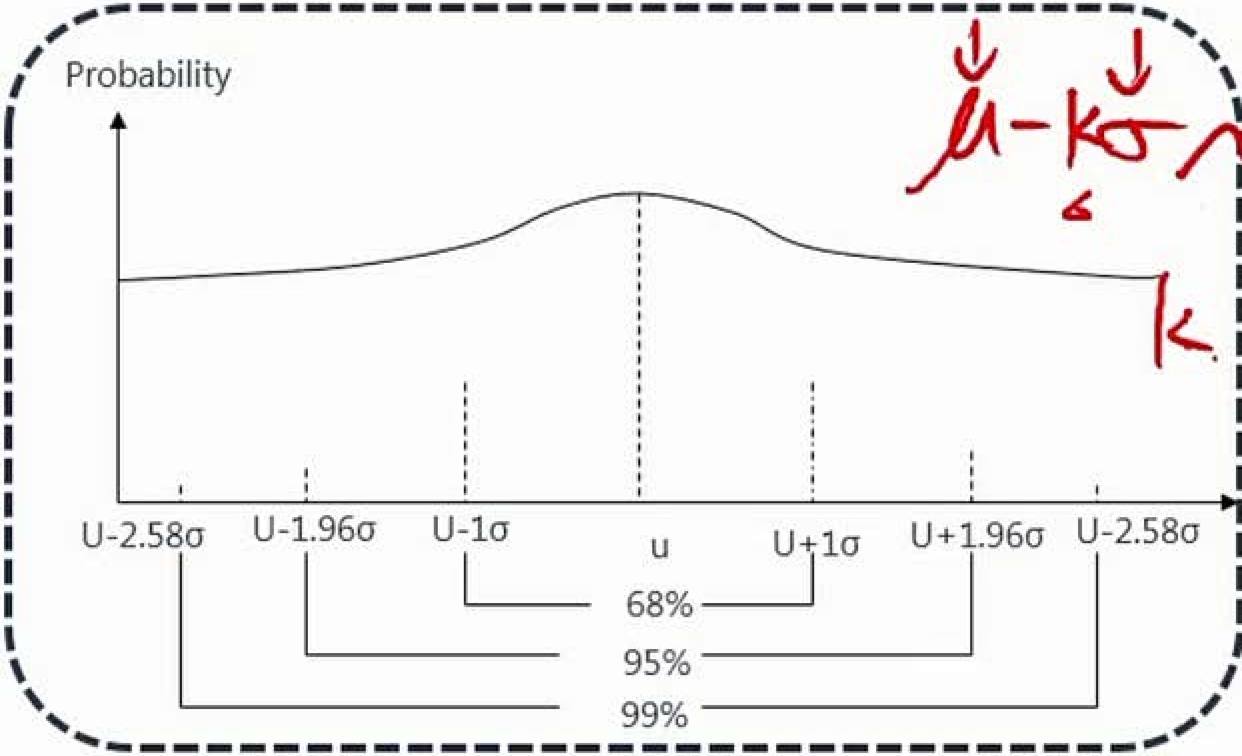
区间

$\Updownarrow$

Prob

$\mu-k\sigma \sim \mu+k\sigma$

$k \Leftrightarrow$ Prob



Prob.	k
68%	1
90%	1.65
95%	1.96
99%	2.58

Common Probability Distributions

2. 置信区间.

- **The confidence intervals**
  - 68% confidence interval is $[\mu-\sigma, \mu+\sigma]$
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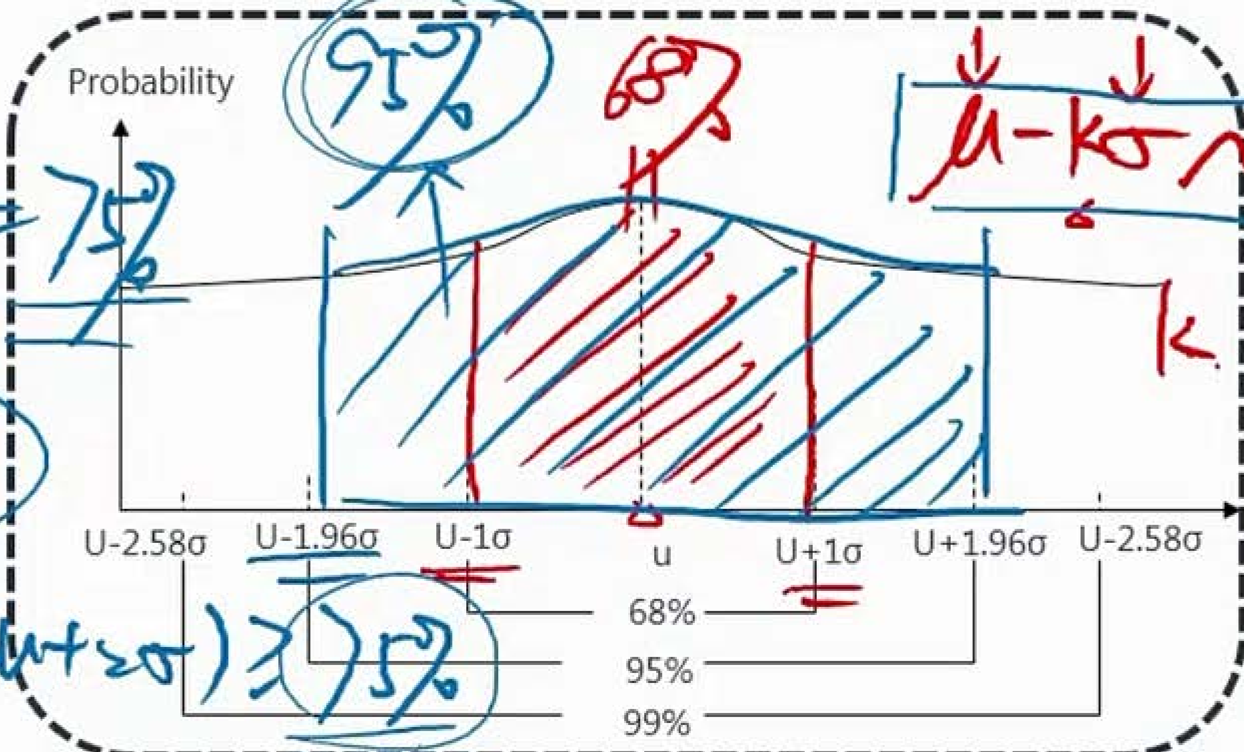
  - 95% confidence interval is $[\mu-1.96\sigma, \mu+1.96\sigma]$
  - 99% confidence interval is $[\mu-2.58\sigma, \mu+2.58\sigma]$

区间 $\Updownarrow$ Prob

Prob $= 1-\frac{1}{k^2} = 75\%$

任何分布

$P(\mu-2\sigma \sim \mu+2\sigma) \geq 75\%$



$\mu-k\sigma \sim \mu+k\sigma$

$k \Leftrightarrow$ Prob

$k=1.96$

≈ 2.

Common Probability Distributions

- The standard deviation of a stock annual returns is 21% and the average return is 9.6% per year. If returns are normally distributed, what is the 95% confidence interval for the stock return next year?

$\sigma = 21\%$ $\mu = 9.6\%$ $K = 1.96$

- **Correct Answer:**

We can infer that μ and σ are 9.6% and 21%. The 95% confidence interval for the return, R, is:

$\mu - K\sigma \sim \mu + K\sigma$.

9.6% ± 1.96*21%= -31.56% to 50.76%

$9.6\% - 1.96 \times 21\% \sim 9.6\% + 1.96 \times 21\%$

So we can conclude that:

$\Rightarrow$ P(-31.56% < R < 50.76%) = 0.95 or 95%

We can say that the annual return is expected to be within this interval 95% of the time, or 95 out of 100 years.

Example

$99\% \Rightarrow k = 2.58$

- An analyst determined that approximately 99 percent of the observations of daily sales for a company were within the interval from $230,000 to $480,000 and that daily sales for the company were normally distributed. The mean daily sales and standard deviation of daily sales, respectively, for the company were closest to:

μ σ

	Mean daily sales	Standard deviation of daily sales
A.	$351,450	$48,450
B.	$351,450	$83,333
C.	$355,000	$48,450

230k ~ 480k

$$\Rightarrow \begin{cases} \mu - k\sigma = 230\,000 \\ \mu + k\sigma = 480\,000 \end{cases}$$

- **Correct Answer: C**

Common Probability Distributions

3、标准化.

- **Standard normal distribution**
  - N(0,1) or Z

    $\mu=0$, $\sigma^2=1$
  - Standardization: if $X\sim N(\mu, \sigma^2)$, then $Z=\dfrac{X-\mu}{\sigma}\sim N(0,1)$
  - Z-table
- **F(-z)=1-F(z)**
- **P(Z>z) = 1 –F(z)**

$$X \sim N(\mu, \underline{\sigma}^2)$$

$$\underline{E}\left(\frac{X-\mu}{\sigma}\right) = \frac{\overset{\mu}{E(X)} - \mu}{\sigma} = \underline{0}$$

$$\underline{Var}\left(\frac{X-\mu}{\sigma}\right) = \frac{\overset{\sigma^2}{Var(X)}}{\sigma^2} = \underline{1}$$

$$Var\left(\frac{X}{\sigma} + \cancel{\frac{\mu}{\sigma}}\right)$$

$$\boxed{\frac{X-\mu}{\sigma}} \sim N\underline{(0, 1)}$$

$$X \sim N(\mu, \sigma^2)$$

$$\boxed{\frac{X-\mu}{\sigma} = z\text{-value}} \sim N(0,1)$$

Common Probability Distributions

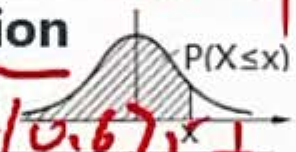
$P(X \le \frac{1}{3}) = F(\frac{1}{3})$

Cumulative Probabilities for a Standard Normal Distribution

$F(0.33)$

$$P(X \le x) = F(x) \text{ for } x \ge 0$$

$F(\frac{2}{3}) = F(0.67)$, $\frac{1}{3}$



x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7582
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767

$X \sim N(2, 9)$

$P(3 \le X \le 4)$

$F(4) - F(3)$

$X \sim N(2, 9)$

$P(3 \le X \le 4)$

$= P\left(\frac{3-2}{3} \le \frac{X-2}{3} \le \frac{4-2}{3}\right)$

$= P\left(\frac{1}{3} \le Z \le \frac{2}{3}\right)$

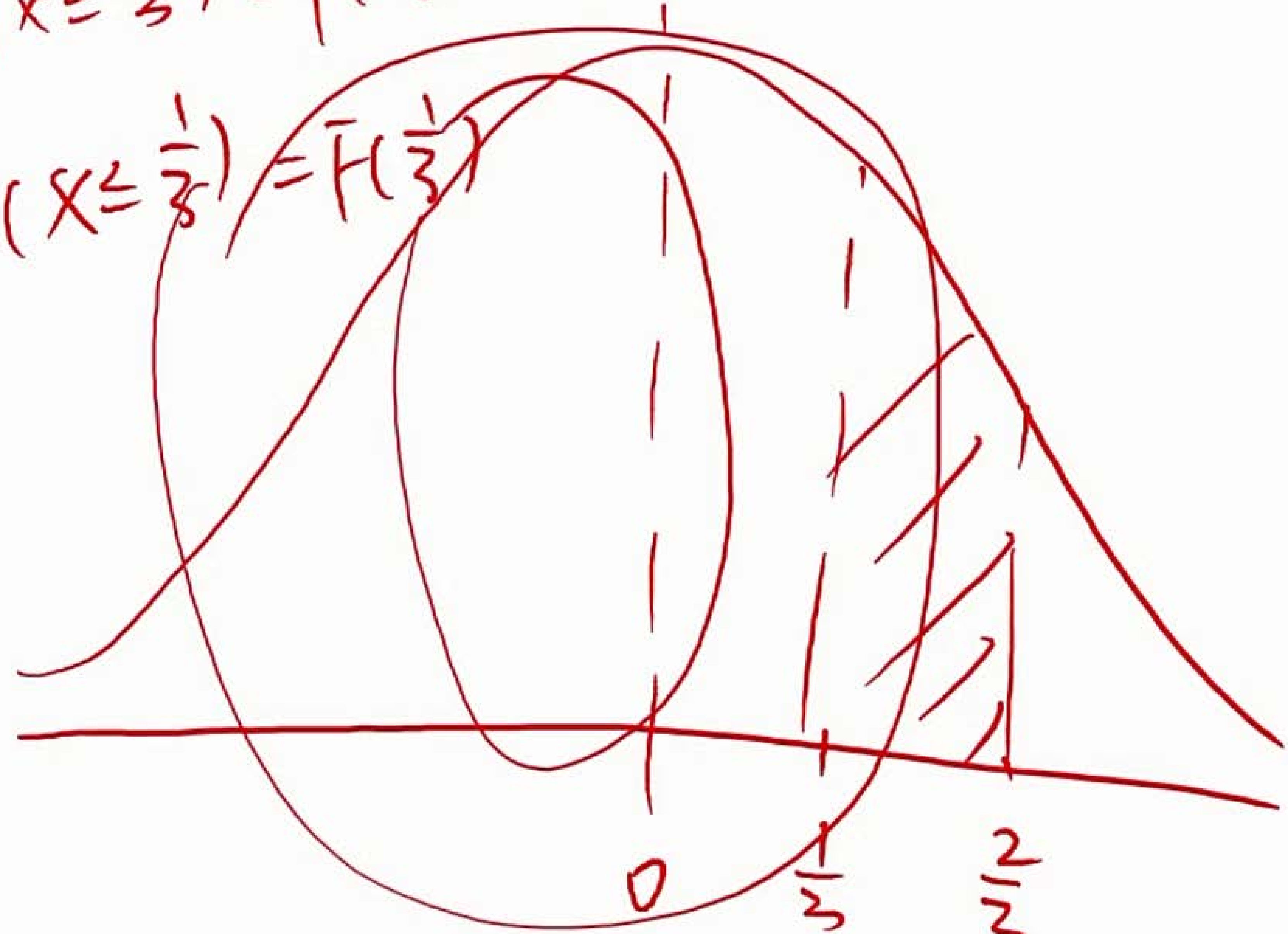
$Z \sim N(0,1)$

① 标准化.

② 计算

$P(X \le \frac{2}{3}) = F(\frac{2}{3})$

$P(X \le \frac{1}{3}) = F(\frac{1}{3})$



① $X \sim N(2, 9)$

$P(3 \le X \le 4)$

$= P\left(\frac{3-2}{3} \le \frac{X-2}{3} \le \frac{4-2}{3}\right)$

$= P\left(\frac{1}{3} \le Z \le \frac{2}{3}\right)$

$= F\left(\frac{2}{3}\right) - F\left(\frac{1}{3}\right)$

$Z \sim N(0,1)$

① 标准化.

② 计算

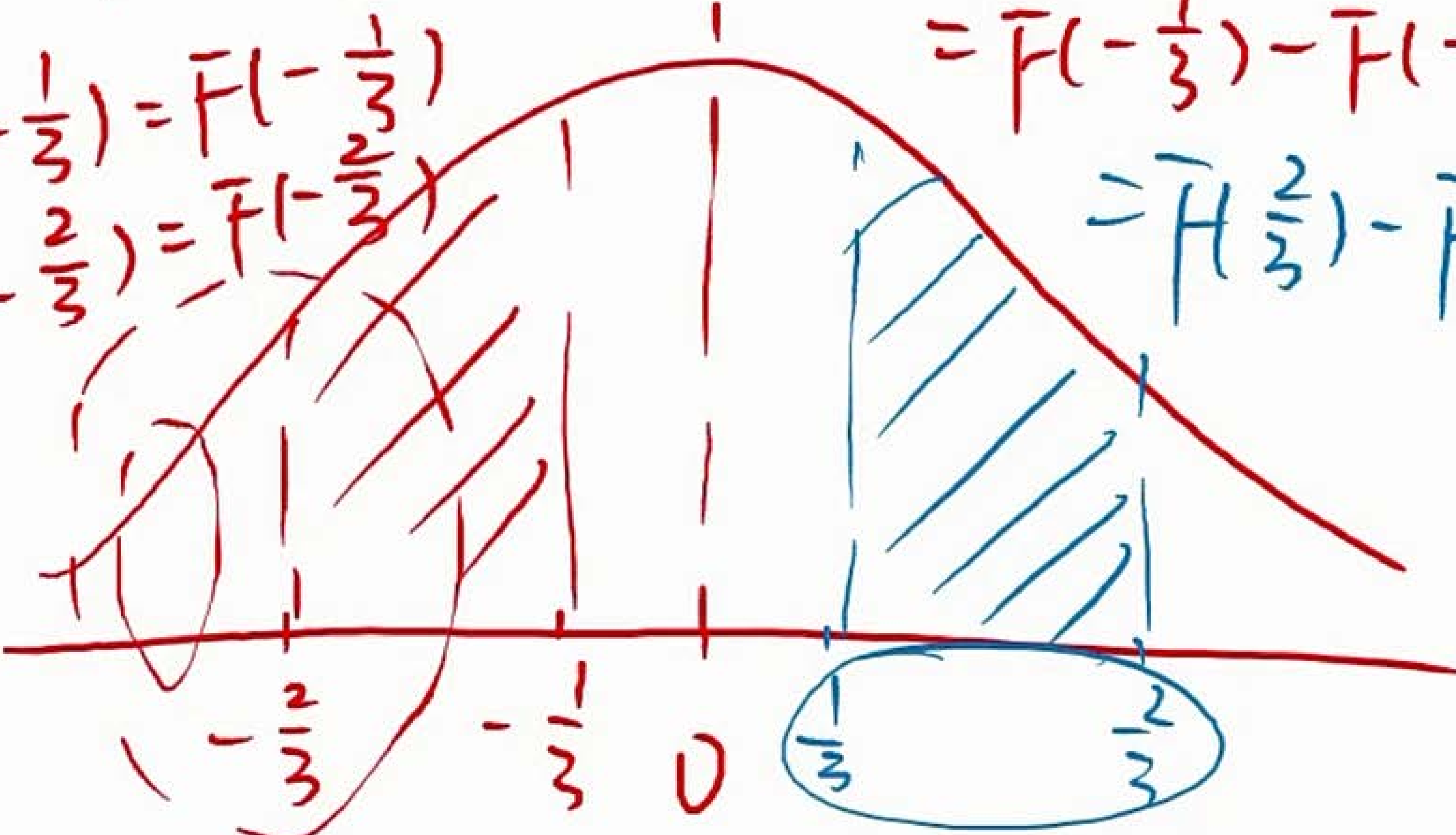
② $P(-\frac{2}{3} \le Z \le -\frac{1}{3})$ $\quad Z \sim N(0,1)$

$= F(-\frac{1}{3}) - F(-\frac{2}{3})$

$= \bar{H}(\frac{2}{3}) - \bar{H}(\frac{1}{3})$

$P(X \le -\frac{1}{3}) = F(-\frac{1}{3})$

$P(X \le -\frac{2}{3}) = F(-\frac{2}{3})$



③ $P(-\frac{1}{3} \le Z \le \frac{2}{3})$ $Z \sim N(0,1)$

$= F(\frac{2}{3}) - F(-\frac{1}{3})$

$= F(\frac{2}{3}) - [1 - F(\frac{1}{3})]$

$P(X \le \frac{2}{3}) = F(\frac{2}{3})$

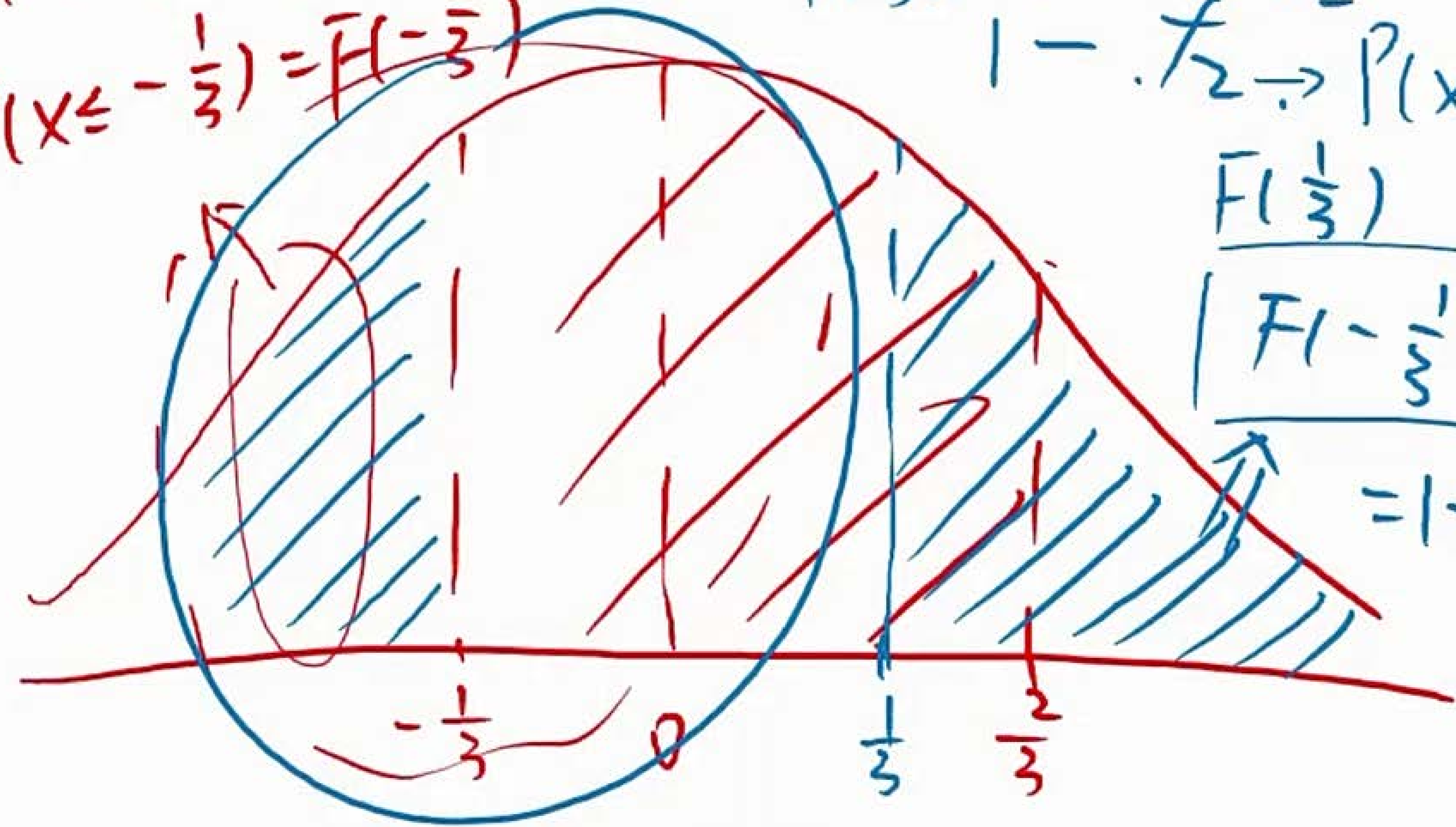
$P(X \le -\frac{1}{3}) = F(-\frac{1}{3})$

$1 - \cdot \frac{1}{2} \to P(X \le \frac{1}{3})$

$F(\frac{1}{3})$

$\boxed{F(-\frac{1}{3})}$

$= 1 - F(\frac{1}{3})$



Common Probability Distributions

3. 标准化.

- **Standard normal distribution**
  - N(0,1) or Z

    $\mu=0,\ \sigma^2=1$

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  - Z-table

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- **F(-z)=1-F(z)**
- **P(Z>z) = 1 –F(z)**

$F(-\frac{1}{3}) = 1 - F(\frac{1}{3})$

Example

$\mu = 12\%$ $\sigma = 22\%$ $R \sim N$

- Assume the portfolio mean return is 12 percent and the standard deviation of return estimate is 22 percent per year. Assuming that a normal distribution describes returns. What is the probability that portfolio return will exceed 20 percent ? $P(R > 20\%)$

$$= P\left(\frac{R-12}{22} > \frac{20-12}{22}\right)$$

- **Correct Answer:**

For X=20%, Z=(20%-12%)/22%=0.3636.

$= P(Z > 0.3636)$

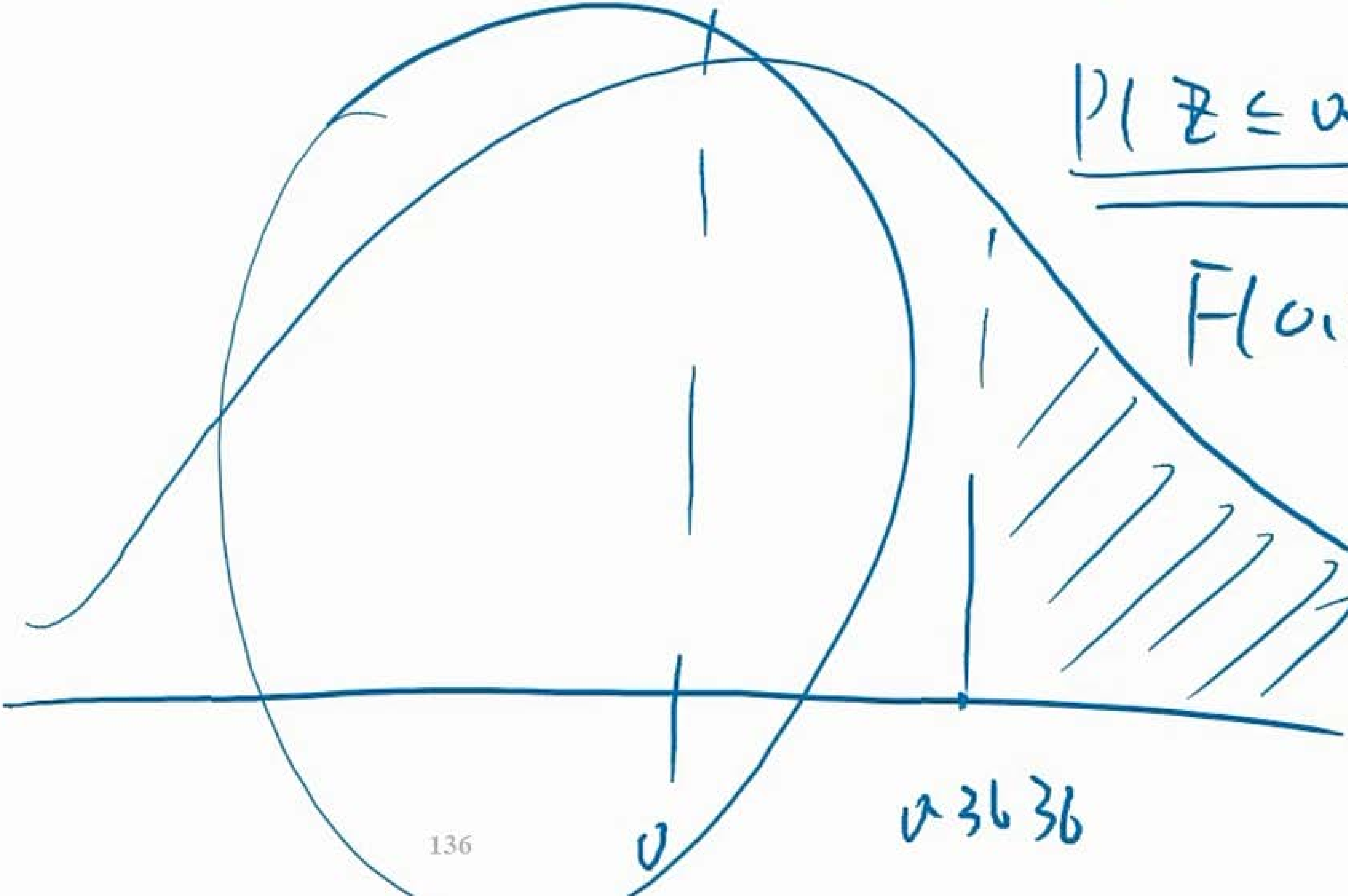
P(Z>x)=1-P($Z \le x$)=1-F(x)

Rounding 0.3636 to 0.36, according to the table, F(0.36)=0.6406.

Thus, P(X>20%)=1-0.6406=0.3594.

The probability that portfolio return will exceed 20 percent is about 36 percent.

1 − 左.



$P(Z \le 0.3636)$

$F(0.3636)$

Example

$\mu = 12\%$ $\sigma = 22\%$ $R \sim N$

- Assume the portfolio mean return is 12 percent and the standard deviation of return estimate is 22 percent per year. Assuming that a normal distribution describes returns. What is the probability that portfolio return will exceed 20 percent ? $P(R > 20\%)$

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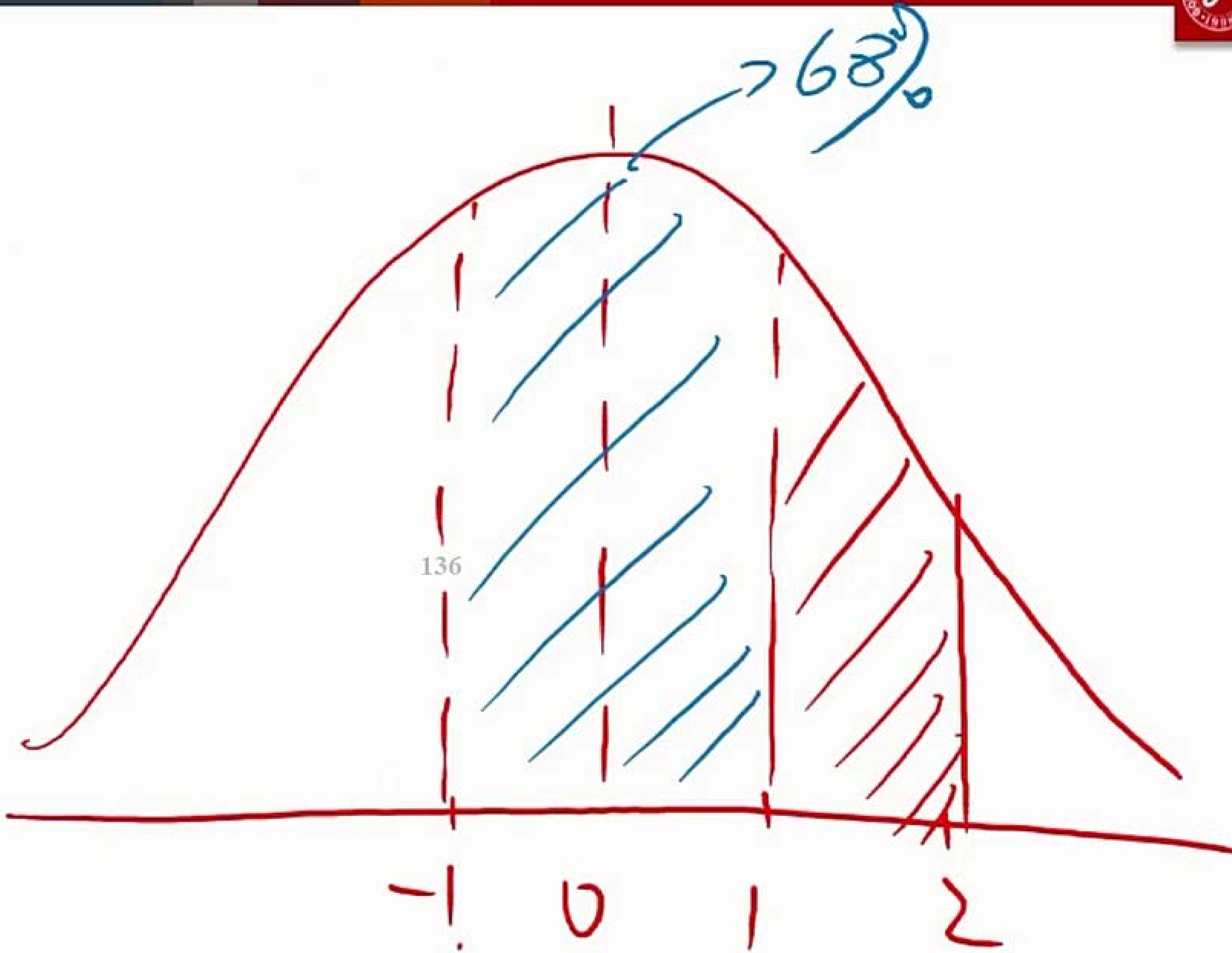

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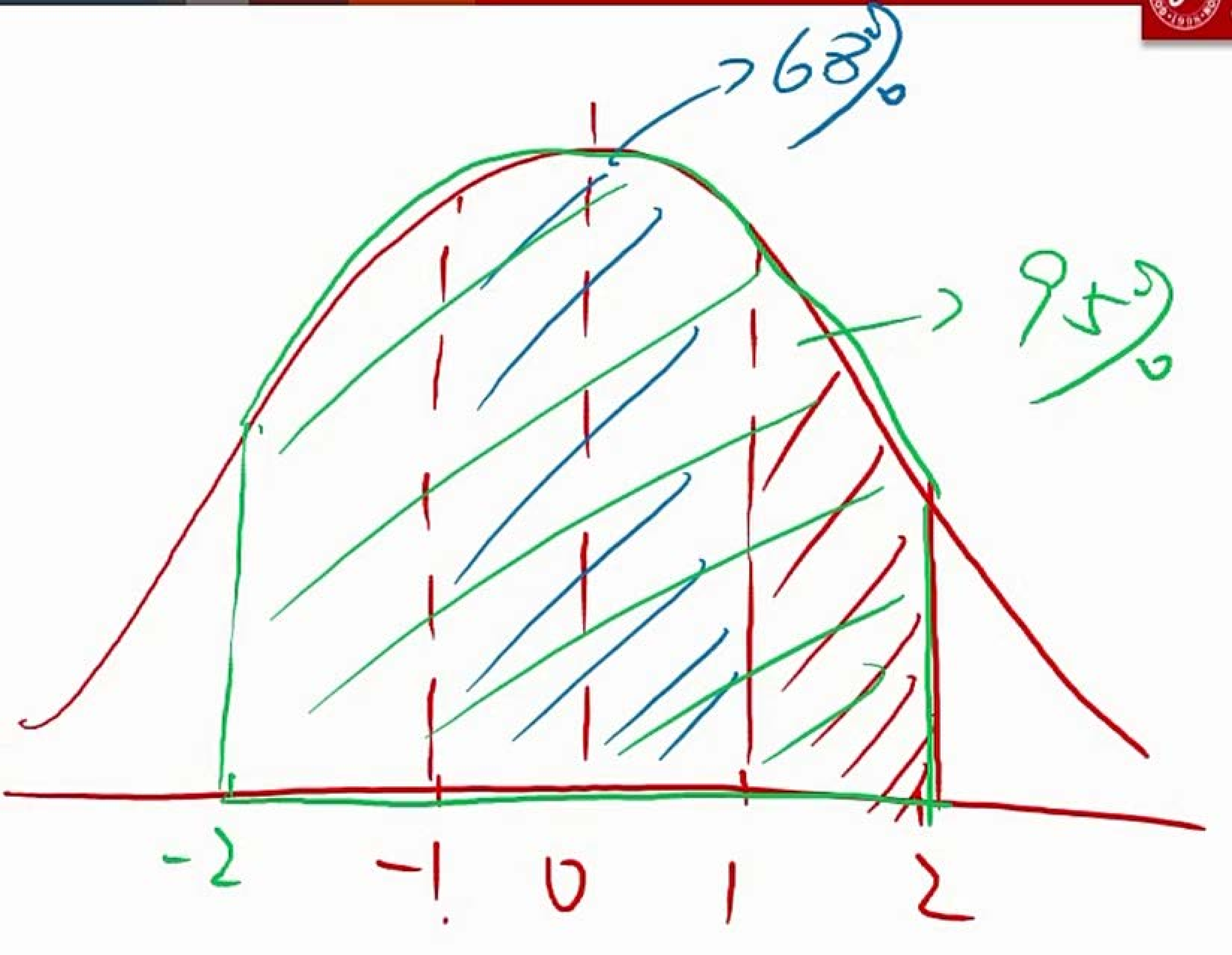
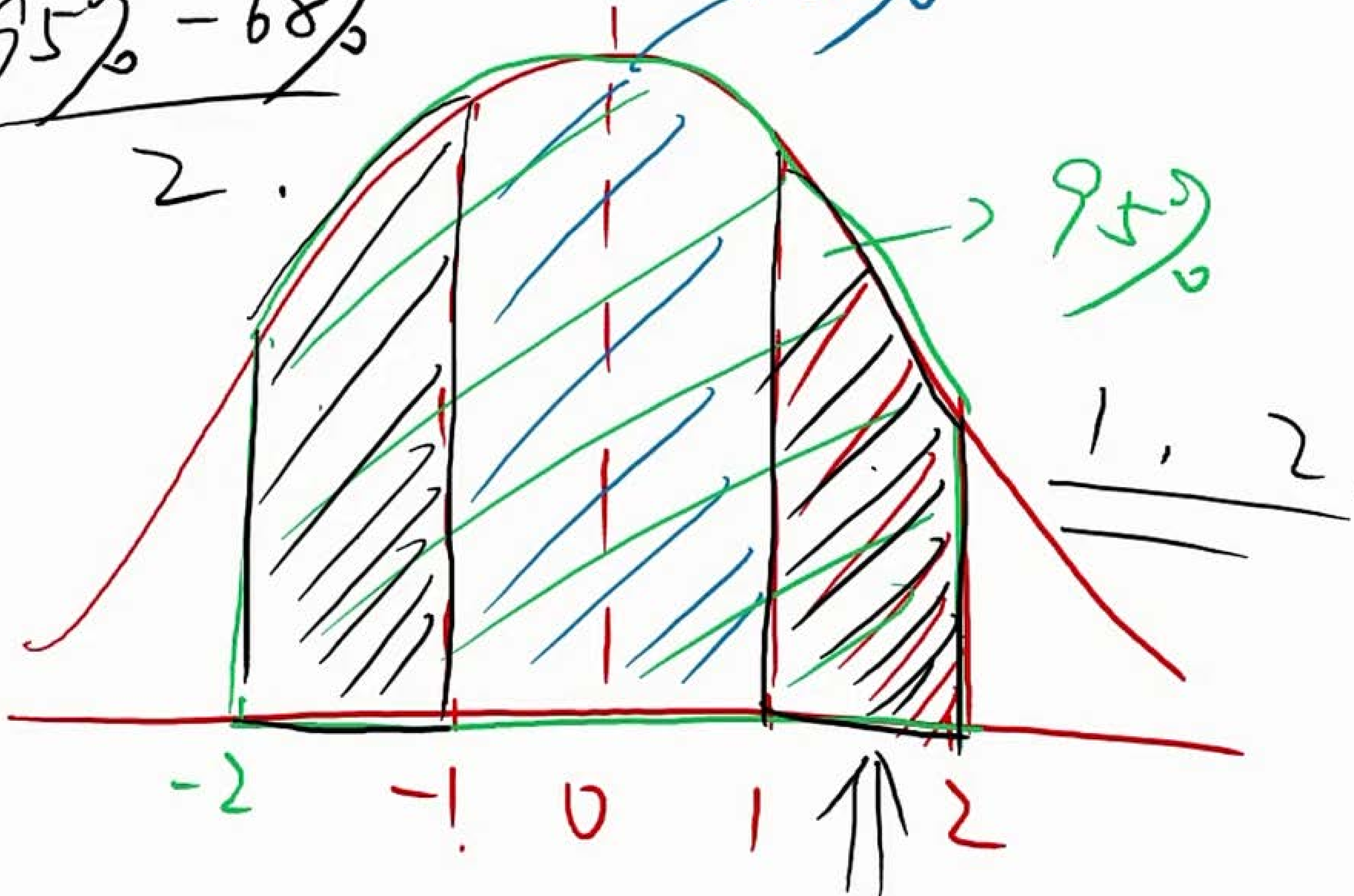

68%
95%
-2
-1
0
1
2

$$\frac{95\% - 68\%}{2}$$



Prob	K.
68%	1
90%	1.65
95%	1.96. ≈ 2.
99%	2.58.

-1.96 ~ +1.96

-2 ~ +2

Common Probability Distributions

1. ➢ **Shortfall risk: R_L = threshold level return, minimum return required** 10%
   - Minimize ($Rp < R_L$)

$P(Rp < R_L)$ ↓↓.

2. ➢ **Roy's safety-first criterion**
   - $[E(R_P) - R_L]/\sigma_P$

第一安全比率.

➢ **Maximize S-F-Ratio**
   - Maximize $SFR = \frac{E(R_P)-R_L}{\sigma_P}$ <=> Minimize $P(Rp < R_L)$

$$SFR = \frac{R_p - R_L}{\sigma_p}. \qquad SR = \frac{R_p - R_f}{\sigma_p}.$$

$$\boxed{\begin{gathered} R_L = R_f. \\ SFR = SR \end{gathered}}$$

Common Probability Distributions

$\downarrow\downarrow$

10%

1. ➢ **Shortfall risk: R_L = threshold level return, minimum return required**
   - Minimize ($Rp < R_L$)

$P(Rp < R_L) \downarrow\downarrow$

2. ➢ **Roy's safety-first criterion**
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第一安全比率

3. ➢ **Maximize S-F-Ratio**
   - Maximize $SFR = \dfrac{E(R_P)-R_L}{\sigma_P}$ <=> Minimize $P(Rp < R_L)$

$$SFR = \frac{Rp - R_L}{\sigma_p} \uparrow\uparrow \longrightarrow P(Rp < R_L) \downarrow\downarrow$$

Example

➢ A portfolio manager gathered the following information about four possible asset allocations:

R_p

Allocation	Expected annual return	Standard deviation of return
A	10%	6%
B	25%	14%
C	18%	17%

$R_L = 8\%$

The manager's client has stated that her minimum acceptable return is 8%. Based on Roy's safety-first criterion, the *most* appropriate allocation is:

A. Allocation A.

B. Allocation B.

C. Allocation C.

$$SFR = \frac{R_p - R_L}{\sigma_p}$$

➢ **Correct Answer: B**

Example

- You are researching asset allocations for a client with an $1,000,000 portfolio. Although her investment objective is long-term growth, at the end of a year she may want to liquidate $40,000 of the portfolio to fund educational expenses. If that need arises, she would like to be able to take out he $40,000 without invading the initial capital of $1000,000. The following table shows three alternative allocations.

	A	B	C
Expected annual return	26	13	15
Standard deviation of return	28	9	21

Address these questions (assume normality for Parts 2 and 3):

1. Given the client's desire not to invade the $1,000,000 principal, what is the shortfall level, R_L? Use this shortfall level to answer Part 2.
2. According to the safety-first criterion, which of the tree allocations is the best?
3. What is the probability that the return on the safety-first optimal portfolio will be less than the shortfall level? (F(1.00)=0.8413)

Example

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$R_L = \frac{4万}{1m} = 4\%$

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GOLDEN FUTURE

Example

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$R_L = \frac{4}{1m} = 4\%$

	A	B	C
Expected annual return	26	13	15
Standard deviation of return	28	9	21

Address these questions (assume normality for Parts 2 and 3):

1. Given the client's desire not to invade the $1,000,000 principal, what is the shortfall level, R_L? Use this shortfall level to answer Part 2.
2. According to the safety-first criterion, which of the tree allocations is the best?

$P(R_B < 4\%)$

3. What is the probability that the return on the safety-first optimal portfolio will be less than the shortfall level? (F(1.00)=0.8413)

$R_L = 4\%$

Example

$SFR = \frac{R_P - R_L}{\sigma_P}$ ↑, E8↑

- **Correct Answer:**

1. R_L=40,000/1,000,000=4.00%
2. A: SFR_A=(26-4.00)/28=0.79; B: SFR_B=(13-4.00)/9=1.00; C: SFR_C=(15-4.00)/21=0.52; B is best.
3. $P(R_B<4.00)=P[(R_B-13)/9 < (4.00-13)/9] = F(-1.00) = 1-F(1.00)=1-0.8413=0.1587$ $R_B \sim N$, $\mu=13\%$ $\sigma=9\%$

The safety-first optimal portfolio has a roughly 16% chance of not meeting a 4.00% return threshold.

$P(R_B<4\%) = P\left(\frac{R_B-13}{9} < \frac{4-13}{9}\right)$

$= P(Z<-1) = F(-1) = 1-F(1) = 1-0.8413$

Common Probability Distributions

定义: lnX ~ N, X ~ log

lognormal

1. ➢ Definition: If lnX is normal, then X is lognormal;

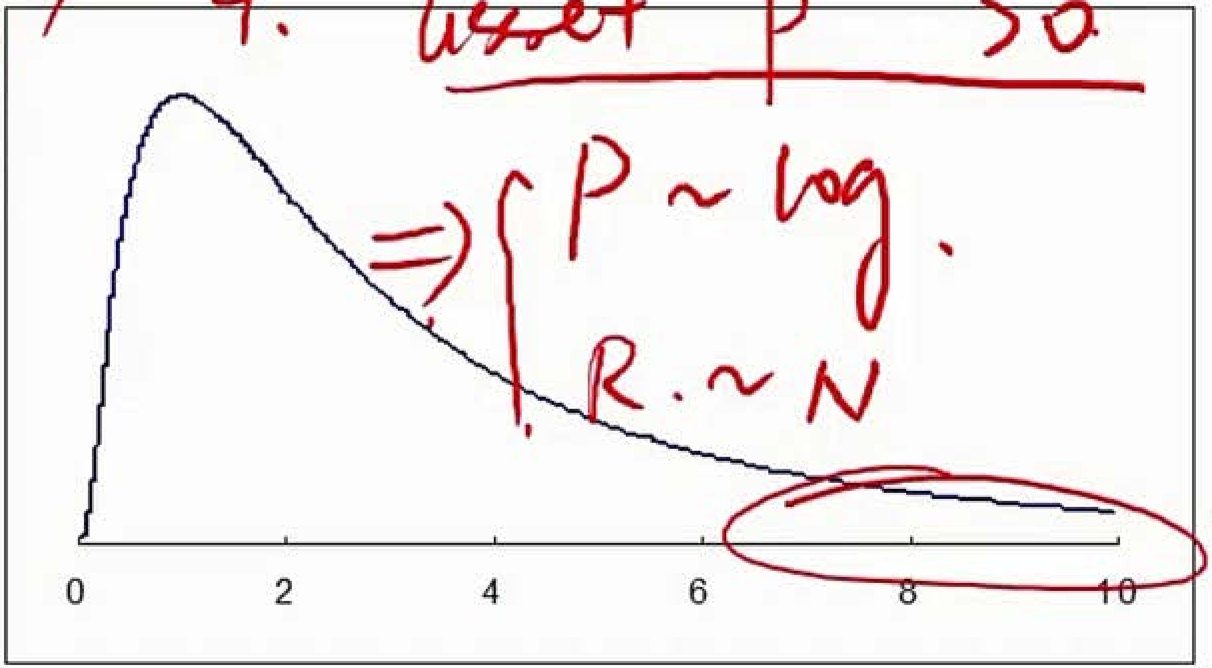
2. ➢ Right skewed; → skewness { tail (long); >0. positive. 对数正态分布

3. ➢ The values of random variables that follow lognormal distribution are always be positive, so it is useful for modeling asset prices

Mode < Median < Mean.

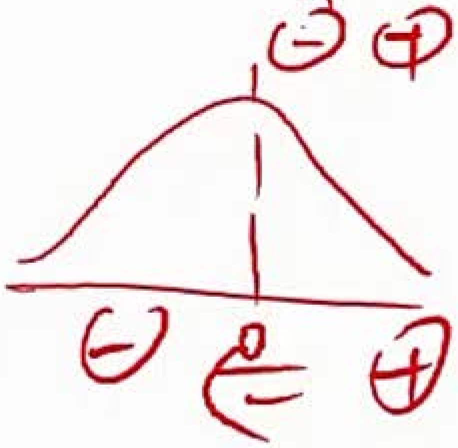
4. asset P > 0

X > 0



⇒ { P ~ log. R ~ N

P & R.



Common Probability Distributions

- Compared to a normal distribution, a lognormal distribution is *least likely* to be:
  A. Skewed to the left.
  B. Skewed to the right.
  C. Useful in describing the distribution of stock prices.
- **Correct Answer: A**

- An analyst stated that lognormal distribution are suitable for describing asset returns and that normal distributions are suitable for describing distributions of asset prices. Is the analyst's statement correct with respect to:

	Lognormal distribution	Normal distribution
A.	No	No
B.	No	Yes
C.	Yes	No

- **Correct Answer: A**

Common Probability Distributions

Stock v.s. Bond.

记：连续复利

- **Discrete:** $\boxed{EAY=(1+\frac{R}{m})^{m}-1}$
- **Continuous:** $\boxed{EAR=\lim_{m\to\infty}(1+\frac{R}{m})^{m}-1=e^{R_c}-1}$
- $\boxed{\frac{S_1}{S_0}=1+HPR=e^{R_c}\text{（持有一年）}}$
- $\boxed{1+HPR_T=e^{R_c\times T}\text{（持有T年）}}$

Common Probability Distributions

S_0 S_1

0 1

- **Discrete:** $\boxed{EAY = (1+\frac{R}{m})^m - 1}$
- **Continuous:** $\boxed{EAR = \lim_{m\to\infty}(1+\frac{R}{m})^m - 1 = e^{R_c} - 1}$
- $\boxed{\frac{S_1}{S_0} = 1 + HPR = e^{R_c}\text{（持有一年）}}$
- $\boxed{1 + HPR_T = e^{R_c \times T}\text{（持有T年）}}$

① $HPR = \frac{S_1 - S_0}{S_0} = \frac{S_1}{S_0} - 1$

$\parallel$

$EAR = e^r - 1$

$\frac{S_1}{S_0} - 1 = HPR = EAR = e^r - 1$

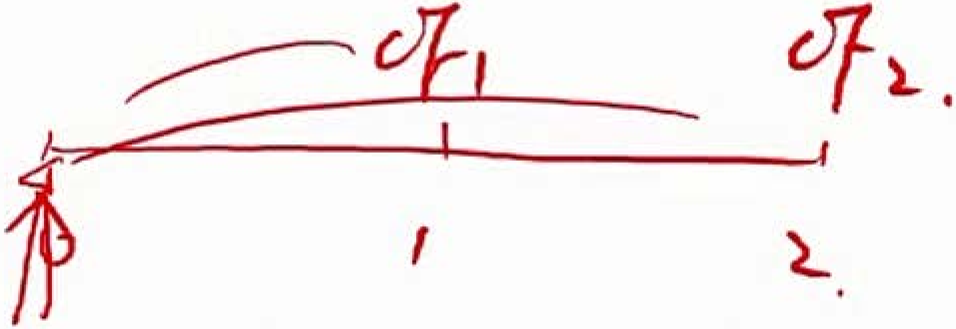
Common Probability Distributions

- **Monte Carlo simulation vs Historical simulation** 模拟
  - Monte Carlo simulation is to generate a large number of random samples from specified probability distribution(s) to represent the operation of risk in the system. It is used in planning, in financial risk management, and in valuing complex securities;
    - ✓ Limitations:
      - ◆ The operating of Monte Carlo simulation is very complex and we must assume a parameter distribution in advance.
      - ◆ Monte Carlo simulation provides only statistical estimates, not exact results.
  - Historical simulation is to repeat sampling from a historical data series. Historical simulation is grounded in actual data but can reflect only risks represented in the sample historical data.
    - ✓ Limitations: Compared with Monte Carlo simulation, historical simulation does not lend itself to "what if " analyses.

Bond

$\Updownarrow$

Loan.



$$P = \sum \frac{CF_t}{(1+r)^t} \quad V \longrightarrow \begin{cases} \text{Int.} \\ \text{PRN} \end{cases}$$

市场r

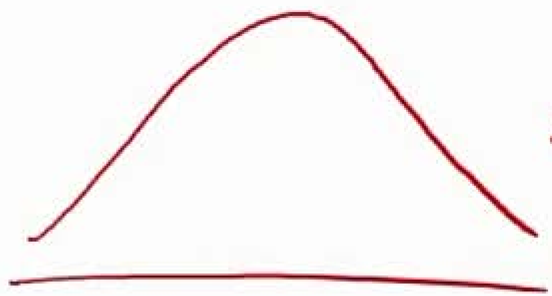
$r_1 \longrightarrow P_1$

$r_2 \longrightarrow P_2$

$\vdots \qquad \vdots$

$r_{120} \qquad P_{120}$

2018.11.25



$\Rightarrow \bar{P}$.

Bond

$\Updownarrow$

Loan.

$$CF_1 \qquad CF_2$$

$$P \qquad 1 \qquad 2$$

$$P = \sum \frac{CF_t}{(1+r)^t}$$

$V \rightarrow \begin{cases} \text{Int.} \\ \text{PRN} \end{cases}$

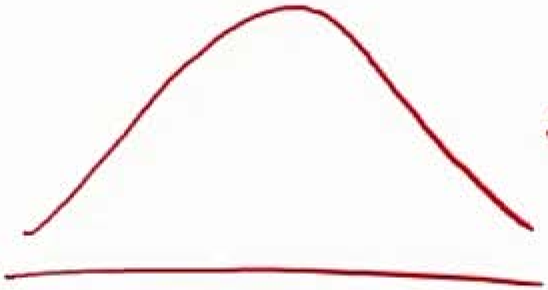
市场r

历史数据

~~校准~~

$$\begin{array}{l} r_1 \rightarrow P_1 \\ r_2 \rightarrow P_2 \\ \vdots \qquad \vdots \\ r_{120} \qquad P_{120} \end{array}$$

2018.11.25



$\Rightarrow \bar{P}.$

Common Probability Distributions

- **Monte Carlo simulation vs Historical simulation** 模拟
  - Monte Carlo simulation is to generate a large number of random samples from specified probability distribution(s) to represent the operation of risk in the system. It is used in planning, in financial risk management, and in valuing complex securities;
    - ✓ Limitations:
      - ◆ The operating of Monte Carlo simulation is very complex and we must assume a parameter distribution in advance.
      - ◆ Monte Carlo simulation provides only statistical estimates, not exact results.

V.S.

  - Historical simulation is to repeat sampling from a historical data series. Historical simulation is grounded in actual data but can reflect only risks represented in the sample historical data.
    - ✓ Limitations: Compared with Monte Carlo simulation, historical simulation does not lend itself to "what if " analyses.

- Monte Carlo simulation is best described as:
  A. An approach to back testing data
  B. A restrictive form of scenario analysis
  C. Providing a distribution of possible solutions to complex functions

- **Correct Answer: C**

一、离散（函数）

1. uniform → 特征

2. Binomial 公式

<1> 贝努里 v.s. 二项式 ⎰相同 ⎱不同

<2> 计算 Prob：n 次 x 次成功 (p)

$$_nC_x\, p^x (1-p)^{n-x}$$

二、连续（不可数）

1. Uniform.

<1>. 特征.

<2> 计算 Prob $a \sim b$.

$$P(X_1 \sim X_2) = \frac{X_2 - X_1}{b - a}.$$

2. Normal ☆☆

<1> 性质

<2> CI.

<3> 标准化 ⟶ 求Prob.

3. Lognormal. ⟶ 特征. 4点.

Sampling and Estimation

一、
- **Sampling and estimation** 方法
  1、 - Simple random sampling. 简单随机抽样法 Prob-一样
  2、 - Stratified random sampling: the population is divided into subpopulations based on some distinguishing characteristics. 分层随机抽样法

    2000 ⇐ 100 × 20 = 2000

- **Sampling error**
  - Sampling error of the mean= sample mean - population mean

    170 - 168 = 2 cm

- The sample statistic itself is a random variable and has a probability distribution. 抽样误差 n↑

  $\bar{x}$ s.

Sampling and Estimation data分类.

一、Time-series data 时间序列数据.

- are a collection of observations at equally spaced intervals of time.

二、Cross-sectional data 横截面数据.

- are a collection of observations at a single point in time.

Time-series data	Cross-sectional data
a collection of data recorded over a period of time	a collection of data taken at a single point of time.

Example

久期

- Greg Goldman, research analyst in the fixed-income area of an investment bank, needs to determine the average duration of a sample of twenty 15-year fixed-coupon investment grade bonds. Goldman first categorizes the bonds by risk class and then randomly selects bonds from each class. After combining the bonds selected (bond ratings and other information taken as of March 31st of the current year), he calculates a sample mean duration of 10.5 years.
- Assuming that the actual population mean is 9.7 years, which of the following statements about Goldman's sampling process and sample is FALSE?

$\bar{X}=10.5$. $\mu=9.7$

A. Goldman used stratified random sampling. E.V.

B. The sampling error of the means equals 0.8 years

C. Goldman is using time-series data. $=\bar{X}-\mu=10.5-9.7$

- **Correct Answer: C**

Sampling and Estimation $\bar{x}$～分布

中心极限定理.

- **Central Limit Theory**
  - For sufficiently large sample sizes n**(n ≥30)**, for any underlying distribution for a random variable with **known population mean and variance**, the sampling distribution
    - ✓ will be approximately normal,
    - ✓ has mean equals to the population mean μ
    - ✓ has variance equal to the population variance of the variable divided by sample size, which equals σ^2/n.

条件：1. $n \geq 30$　　2. 总体均值、方差已知

结论：1.服从正态分布　2. $\mu_{population} = \mu_{sample}$　$s^2 = \sigma^2/n$

- **Standard error of the sample mean**
  - Known population variance $\sigma_{\bar{x}} = \sigma/\sqrt{n}$
  - Unknown population variance $s_{\bar{x}} = s/\sqrt{n}$

条件：① $n \geq 30$ ② 总体 μ，σ^2 已知

结论：$\bar{x} \sim N\left(\mu_{\bar{x}} = \mu_x, \sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{n}\right)$

条件：① $n \geq 30$ ② 总体 μ, σ^2 已知

结论：$\bar{x} \sim N\left(\mu_{\bar{x}} = \mu_x, \sigma_{\bar{x}}^2 = \dfrac{\sigma_x^2}{n}\right)$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$ $\sigma_x \rightarrow S_x$

Standard error $(\sigma_{\bar{x}})$

标准误

$$S_{\bar{x}} = \frac{S_x}{\sqrt{n}}$$

S^2. 样本方差.

1000.
个
100
⇓
方差.

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} \Leftarrow$$

$\sigma_{\bar{x}}^2$. 样本均值的方差

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{n}$$

1000

100. 100. 100 ...

$\bar{x}$ $\bar{x}$ $\bar{x}$ -- $\bar{x}$

Sampling and Estimation

$\bar{X} \sim$ 分布

中心极限定理.

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- **Standard error of the sample mean**
  - Known population variance　$\sigma_{\bar{x}} = \sigma / \sqrt{n}$
  - Unknown population variance　$s_{\bar{x}} = s / \sqrt{n}$

- An analyst gathered the following information:

Sample mean $\bar{x}$	12%
Sample size n.	50
Sample variance s^2.	$30(\%)^2$

The standard error of the sample mean is *closest* to:

$\bar{x}$.

A. 0.47%.

B. 0.64%.

C. 0.77%.

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{s_x}{\sqrt{n}} = \frac{30}{\sqrt{50}}$$

- **Correct Answer: C**

Sampling and Estimation

$E(\bar{X}) = \mu$. $\in$

- **The desirable properties of an estimator:**

1. - **Unbiasedness:** the expected value of the estimator equals the population parameter. 无偏性.
   - **Efficiency:** the unbiased estimator has the smallest variance.
   - **Consistency:** the probability of accurate estimates increases as sample size increases.
     - ✓ the standard deviation of the parameter estimate decreases as the sample size increases
     - ✓ **If the sample size raises, the standard error of the sample mean falls.**

Sampling and Estimation

$E(\overline{X}) = \mu$

- The desirable properties of an estimator:

1、
- **Unbiasedness:** the expected value of the estimator equals the population parameter. 无偏性.

有效性

2、
- **Efficiency:** the unbiased estimator has the smallest variance. Var min.

3.
- **Consistency:** the probability of accurate estimates increases as sample size increases. 一致性 ① $n\uparrow \rightarrow$ 准确 $\uparrow$
  - ✓ the standard deviation of the parameter estimate decreases as the sample size increases
  - ② ✓ **If the sample size raises, the standard error of the sample mean falls.**

$n\uparrow$, $\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}} \downarrow$

Sampling and Estimation

- Shawn Choate is thinking about his graduate thesis. Still in the preliminary stage, he wants to choose a variable of study that has the most desirable statistical properties. The statistic he is presently considering has the following characteristics:
  - ① The expected value of the sample mean is equal to the population mean.

    $E(\bar{X}) = \mu.$
  - ② The variance of the sampling distribution is smaller than that for other estimators of the parameter.
  - ③ As the sample size increases, the standard error of the sample mean rises and the sampling distribution is centered more closely on the mean.

    $n\uparrow.$ $\sigma_{\bar{x}}\uparrow$. X

  Select the best choice. Choate's estimator is:

  A. Unbiased, efficient, and consistent.

  B. Efficient and consistent.

  C. Unbiased and efficient.

- **Correct Answer: C**

Sampling and Estimation

- **Point estimate:** the statistic, computed from sample information, which is used to estimate the population parameter
- **Confidence interval estimate:** a range of values constructed from sample data so the parameter occurs within that range at a specified probability.

  α—the level of significance

- **Interval Estimation（also see Chapter: Hypothesis Testing ）**
  - Level of significance （alpha）
  - Degree of Confidence （1—alpha）
  - Confidence Interval = [Point Estimate +/- (reliability factor) * Standard error]

$$\bar{x} = 170.$$

① $\bar{x} = \mu = 170.$ 点估计

② C.I. 估计

$\bar{X} \rightarrow \mu_X$ $\quad \bar{X} \sim N\left(\mu_{\bar{X}} = \mu_X, \sigma_{\bar{X}}^2 = \frac{\sigma_X^2}{n}\right)$

?

$X \sim N(\mu_X, \sigma_X^2)$

$CI_X: \mu_X \pm k\sigma_X$

$CI_{\bar{X}}: \mu_{\bar{X}} \pm k\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}}$ $\Rightarrow$ $\boxed{\bar{X} \pm k\frac{\sigma}{\sqrt{n}}}$

$\mu_X \leftarrow \bar{X}$

$$\left(\bar{X}+k\frac{\sigma}{\sqrt{n}}\right)-\left(\bar{X}-k\frac{\sigma}{\sqrt{n}}\right)$$

1. 公式：（计算）

$$CI_{\bar{X}}: \quad \bar{X} \pm k\frac{\sigma}{\sqrt{n}}$$

2. $$\text{width} = 2k\frac{\sigma}{\sqrt{n}}$$

$\downarrow$ $\downarrow$ $\downarrow$ $\uparrow$

Sampling and Estimation

- **Point estimate:** the statistic, computed from sample information, which is used to estimate the population parameter
- **Confidence interval estimate:** a range of values constructed from sample data so the parameter occurs within that range at a specified probability.

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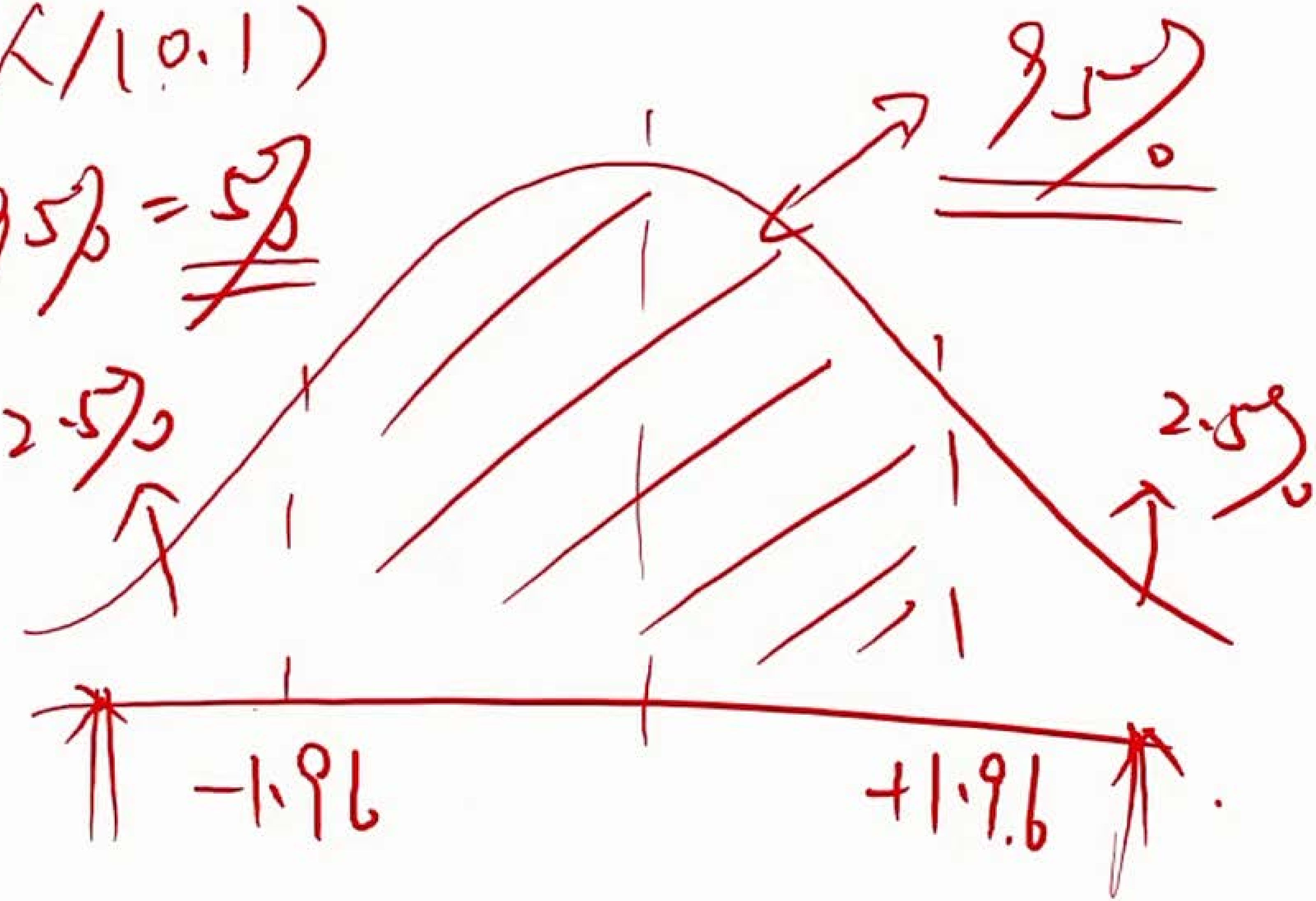
- **Interval Estimation（also see Chapter: Hypothesis Testing ）**
  - Level of significance （alpha）
  - Degree of Confidence （1－alpha）
  - Confidence Interval = [Point Estimate +/- (reliability factor) * Standard error]

Prob.	K.
68%	1
90%	1.65.
95%	1.96.
99%	2.58

$N(0,1)$

$1-95\% = 5\%$



Sampling and Estimation

- **Point estimate:** the statistic, computed from sample information, which is used to estimate the population parameter
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  α—the level of significance

- **Interval Estimation（also see Chapter: Hypothesis Testing ）**
  - Level of significance （alpha） α 显著性水平
  - Degree of Confidence （1—alpha） 置信度 $= 1-\alpha$.
  - Confidence Interval = [Point Estimate +/- (reliability factor) * Standard error]

Sampling and Estimation $CZ_{\bar{x}}$.

- The width of a confidence interval *most likely* will be smaller if the sample variance and number of observations, respectively, are:

	Sample variance	Number of observations
A.	Smaller	Smaller
B.	Smaller	Larger
C.	Larger	Smaller

$$\text{Width} = 2k\frac{\sigma}{\sqrt{n}}$$

S_x

- **Correct Answer: B**

Sampling and Estimation

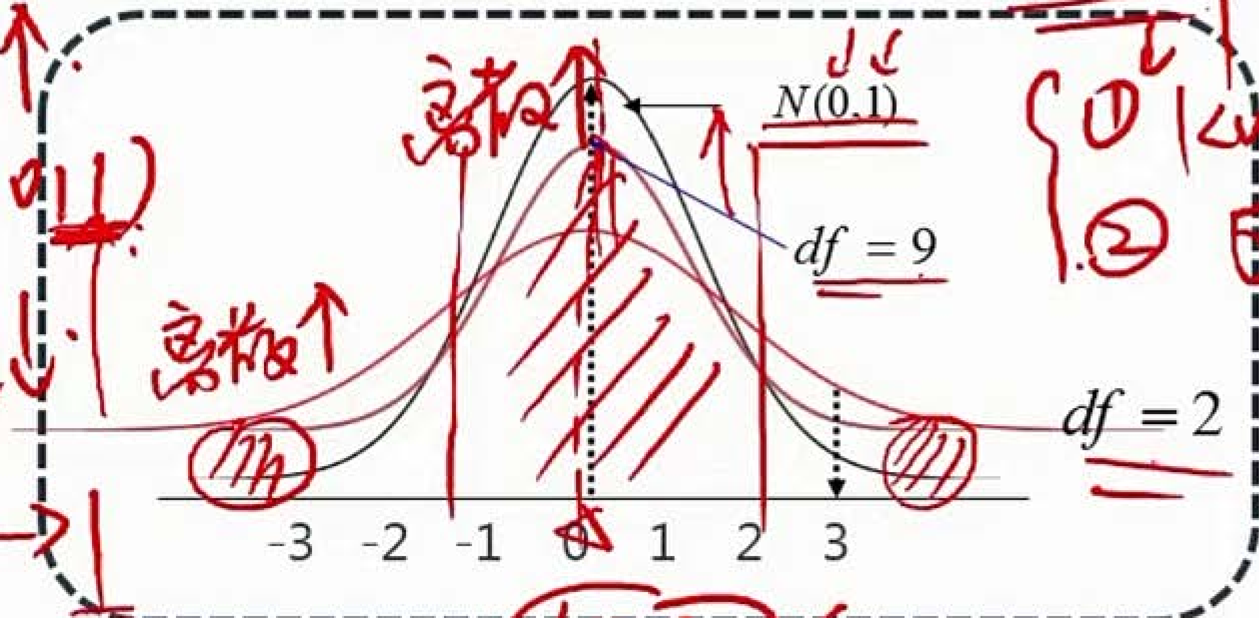
- **Student's t-distribution**
  1. Symmetrical — skewness = 0
  2. Degrees of freedom (df): n-1 — df = n-1
  3. Less peaked than a normal distribution ("fatter tails") — t v.s N.
  4. Student's t-distribution converges to the standard normal distribution as degrees of freedom goes to infinity. — 低峰肥尾.

df↑/n↑

t→N(0,1)

① 峰↑尾↓

② σ_t^2↓ → 1

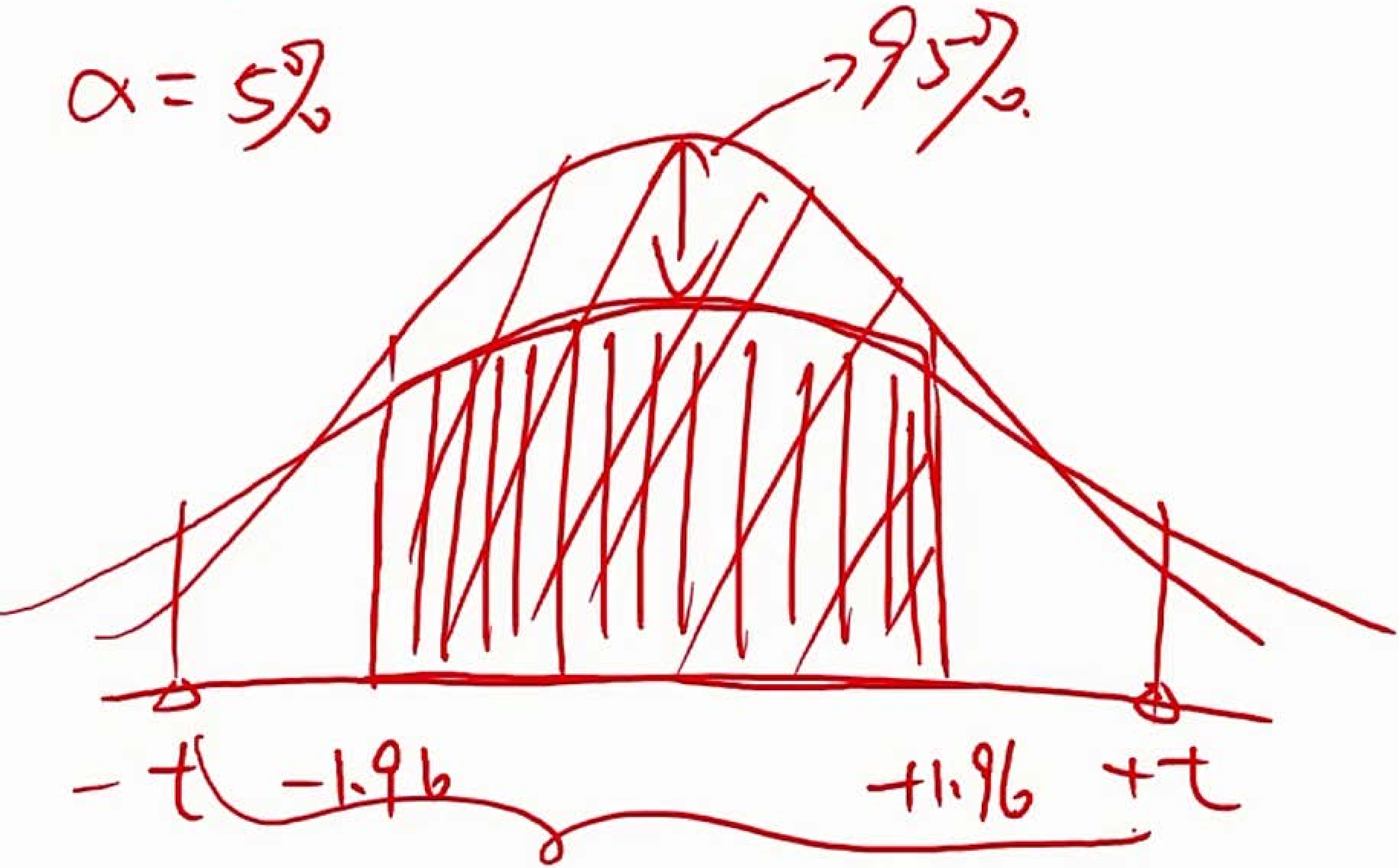


① kurtosis < 3

② σ_t^2 大.

5、相同 α，CI t 更宽.

$\alpha = 5\%$



Sampling and Estimation

2、看表 t vs. z.

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$

Sampling from:	Normal distribution with known variance	Normal distribution with unknown variance	Nonnormal distribution with known variance	Nonnormal distribution with unknown variance
Statistic for small sample size(n<30)	z- Statistic	t- Statistic	not available	not available
Statistic for large sample size(n>=30)	z- Statistic	t- Statistic/z	z- Statistic	t- Statistic/z

Example

峰↓尾↑ df↓ | df↑

- As the t-distribution's degrees of freedom decrease, the t-distribution most likely:

t ~~≈~~ N   t → N(0,1)

A. exhibits tails that become fatter.

峰↑尾↓

B. ~~approaches a standard normal distribution.~~

C. ~~becomes asymmetrically distributed around its mean value.~~

Correct Answers: A.

A standard normal distribution has tails that approach zero faster than the t-distribution. As degrees of the freedom increase, the tails of the t-distribution become less fat and the t-distribution begins to look more like a standard normal distribution. But as degrees of freedom decrease, the tails of the t-distribution become fatter.

方差已知用z、

方差未知用t

非正态总体小样本不可估计.

Sampling and Estimation

α、查表 t v.s. z.

K

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

①

②

K

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$

③

④

Sampling from:	Normal distribution with known variance	Normal distribution with unknown variance	Nonnormal distribution with known variance	Nonnormal distribution with unknown variance
小 Statistic for small sample size(n<30)	z- Statistic	t- Statistic	not available	not available
大 Statistic for large sample size(n>=30)	z- Statistic	t- Statistic/z	z- Statistic	t- Statistic/z

~~总体~~ → 分布

⇒ 条件：① $n \geq 30$ ② 总体 μ, σ^2 已知

结论：$\bar{X} \sim N\left(\mu_{\bar{X}} = \mu_X, \ \sigma_{\bar{X}}^2 = \frac{\sigma_X^2}{n}\right)$

$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}}$ → S_X

Standard error $(\sigma_{\bar{X}})$

标准误

$S_{\bar{X}} = \frac{S_X}{\sqrt{n}}$ ⇐

方差已知用z、

方差未知用t

非正态总体小样本不可估计.

n≥30，任何情况均可用z.

Sampling and Estimation

D ➢ **Data-mining bias** 把偶然当必然.

- Data-mining bias comes from finding models by repeatedly searching through databases for patterns.

➢ **Sample selection bias**

- When data availability leads to certain assets being excluded from the analysis, we call the resulting problem sample selection bias.
- **Survivorship bias**
  - ✓ Survivorship bias occurs if companies are excluded from the analysis because they have gone out of business or because of reasons related to poor performance.

➢ **Look-ahead bias**

- Look-ahead bias exists if the model uses data not available to market participants at the time the market participants act in the model.

➢ **Time-period bias**

- Time-period bias is present if the time period used makes the results time-period specific or if the time period used includes a point of structural change.

S&P 500 → R

10. 12.

$$Y_i = a + b_1 X_1 + b_2 X_2 + \cdots + \varepsilon_i$$

GDP、CPI、I.

a、b.

心情、温度

$> 20^\circ C$

$< 20^\circ C$

120

Sampling and Estimation

把偶然当必然

① **Data-mining bias**

- Data-mining bias comes from finding models by repeatedly searching through databases for patterns. 同一个数据库

② **Sample selection bias** 选择性

- When data availability leads to certain assets being excluded from the analysis, we call the resulting problem sample selection bias.

③ **Survivorship bias** 生存性 (对冲) hedge fund

  - Survivorship bias occurs if companies are excluded from the analysis because they have gone out of business or because of reasons related to poor performance. P/E 2018.1.4

④ **Look-ahead bias** 前视性

- Look-ahead bias exists if the model uses data not available to market participants at the time the market participants act in the model.

1980 - 2010 3.1

⑤ **Time-period bias**

- Time-period bias is present if the time period used makes the results time-period specific or if the time period used includes a point of structural change.

短. 2010.

Example

- Sunil Hameed is a reporter with the weekly periodical The Fun Finance Times. Today, he is scheduled to interview a researcher who claims to have developed a successful technical trading strategy based on trading on the CEO's birthday (sample was taken from the Fortune 500). After the interview, Hameed summarizes his notes (partial transcript as follows). The researcher:
  - Used the same database of data for all his tests and has not tested the trading rule on out-of-sample data.
  - Excluded stocks for which he could not determine the CEO's birthday.
- Select the choice that *best* completes the following: Hameed concludes that the research is flawed because the data and process are biased by:

  A. Data mining and sample selection bias.

  B. Data mining and look-ahead bias.

  C. Time-period bias and survivorship bias.
- **Correct Answer: A**

一、Sampling.

1、方法 — 简单、分层.

2. data分类 — time-series. / cross-sectional.

3. $\bar{x}$~分布：中心极限定理.

(1) 条件，结论：$\bar{x} \sim N\left(\mu_{\bar{x}} = \mu_x, \sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{n}\right)$

(2) $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} \longrightarrow S_x$ Standard error

二、Estimate.

1. 好的估计量的性质

- 无偏性
- 有效性.
- 一致性.

2. 方法

- 点估计 (x)
- CI估计 (√)

$\bar{x} \longrightarrow \mu_x$

(1) 公式. $CI_{\bar{x}}$: $\bar{x} \pm k\frac{\sigma}{\sqrt{n}}$

(2) width $= 2k\frac{\sigma}{\sqrt{n}}$.

三、 t分布.

1. 特征 5点.

2. Z vs. t.

Framework

一、流程.
critical value.
method.
二、P-value method.
三、Type I & II error.

1. The steps of hypothesis testing
2. The null hypothesis and alternative hypothesis, one-tailed and two-tailed test
3. Test statistics
4. Decision rule
5. The Chi-square test and F-test
6. P-value method
7. Type I and type II errors
8. Parameter tests and non-parameter tests

Hypothesis Testing

Step 1: State null and alternative hypotheses

Step 2: Identify the test statistic

检验统计量

Step 3: Select a level of significance

-1.96 +1.96

$\alpha = 5\%$

K

CI

Step 4: Formulate a decision rule

Step 5: Take a sample, arrive at decision

- Do not reject
- Reject

Hypothesis Testing

- **Define Hypothesis** 假设
  - A hypothesis is a statement about one or more <u>population parameters</u>.
    - ✓ For population not sample
  - 原假设 Null hypothesis and Alternative hypothesis (we want to assess) 备择假设
    - ✓ Null hypothesis is the fact we suspect and want to reject

$$H_0 : \mu = \mu_0 \qquad H_a : \mu \neq \mu_0$$

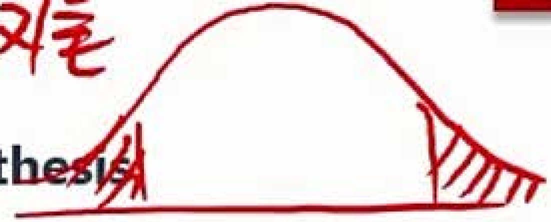
$H_0 : \mu = 170$ $H_a : \mu \neq 170$

H_0: 拒绝

H_a: 接受

Hypothesis Testing

双尾

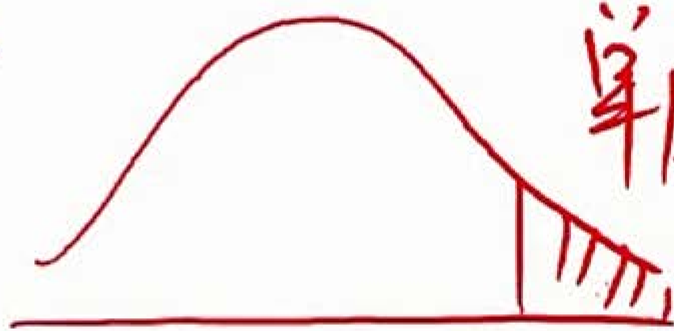


- One-tailed and Two-tailed tests of Hypothesis

Two-tailed	$H_0: \mu = \mu_0$	$H_a: \mu \neq \mu_0$	> < 双.
One-tailed	$H_0: \mu \leq \mu_0$ or, $H_0: \mu \geq \mu_0$	$H_a: \mu > \mu_0$ $H_a: \mu < \mu_0$	单. 右. 左.

拒绝.

接受



单尾

Example

- In the hypothesis testing, assess whether if mean excess the benchmark, how to set the null hypothesis?

  A. $\mu < \mu_0$

  B. $\mu \le \mu_0$

  C. $\mu > \mu_0$

H_0: "="

- **Correct Answer: B**

Example

- Terry believes that the price of his houses is greater than $250,000. the appropriate alternative hypothesis is:

  A. H_a: μ > \$250,000 ⇐. H_0: 拒绝. "="

  ~~B. H_a: μ < \$250,000~~

  C. H_a: $\mu \geq$ \$250,000 H_a: 接受. $\mu > 250000$.

- **Correct Answer: A**

- Mary is an analyst. She want to determine whether the mean time spent on research is different from two hours per day. The appropriate null hypothesis for the test is:

  A. H_0: μ = 2 hours, one-tailed test. H_0: $\mu = 2$.

  ~~B. H_0: $\mu \geq$ 2 hours, two-tailed test.~~ H_a: $\mu \neq 2$. $>$ $<$. 双.

  C. H_0: μ = 2 hours, two-tailed test.

- **Correct Answer: C**

Hypothesis Testing

- **Test statistic** 检验统计量.

$$Test\ Statistic = \frac{Sample\ statistics - Hypothesized\ value}{stanard\ error\ of\ the\ sample\ statistic}$$

- Test Statistic follows Normal, T, Chi Square or F distributions
- Test Statistic has formula. Calculate it with the sample data. We should emphasize Test Statistic is calculated by ourselves not from the table.
- This is the general formula but only for Z and T distribution.

Examples：

$$Test\ Statistic = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \qquad Test\ Statistic = \frac{\overline{X} - \mu_0}{s / \sqrt{n}}$$

$\boxed{估\mu}$

$\bar{x} \rightarrow \mu.$

对象.

查表. $N(0,1)$

$$\bar{x} \sim N\left(\mu_{\bar{x}} = \mu_x,\ \sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{n}\right) \Leftarrow$$

标准化: Z.

$$\frac{\bar{x} - \mu_x}{\sigma_x / \sqrt{n}} = \text{Z-value} \sim N(0,1)$$

$H_0: \mu = 170$

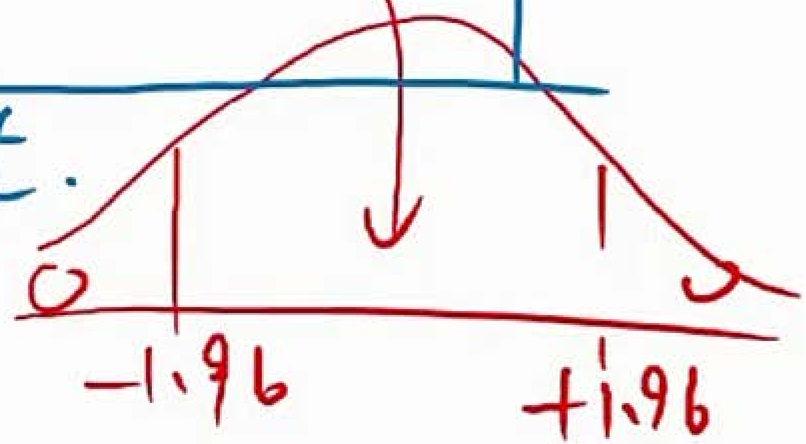
$\sigma_x \rightarrow s_x$

t vs. Z

t ↓ 未知

Z ↓ 已知.

$$\frac{\bar{x} - \mu_x}{s_x / \sqrt{n}}.\quad t.$$



Examples : z t.

$$Test\ Statistic = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \qquad Test\ Statistic = \frac{\overline{X} - \mu_0}{s / \sqrt{n}}$$

Hypothesis Testing

- **Critical value（关键值，实际就是分位数）**
  - Found in the Z, T, Chi Square or F distribution tables not calculated by us
  - Under given one tailed or two tailed assumption, critical value is determined solely by the significance level.

- **Decision rule**
  - **Critical Value Method**

Significance Level?
Two tailed or one tailed test?
Reject region? Critical Value under the condition
Compare the Test Statistic and Critical Value

k 影响因素.

1. 查表.

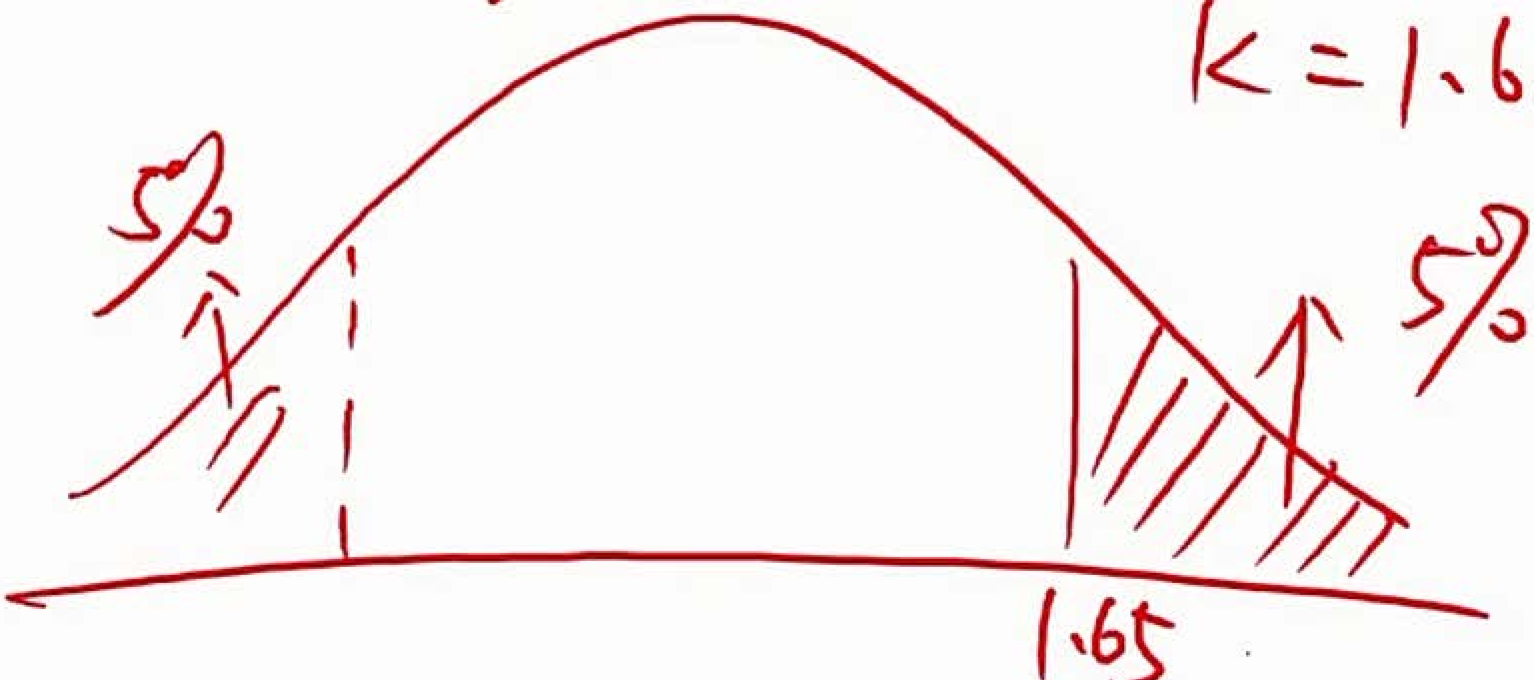
2. α.

3. 单尾 v.s. 双尾.

双尾 $\alpha = 5\%$ $K = 1.96$.

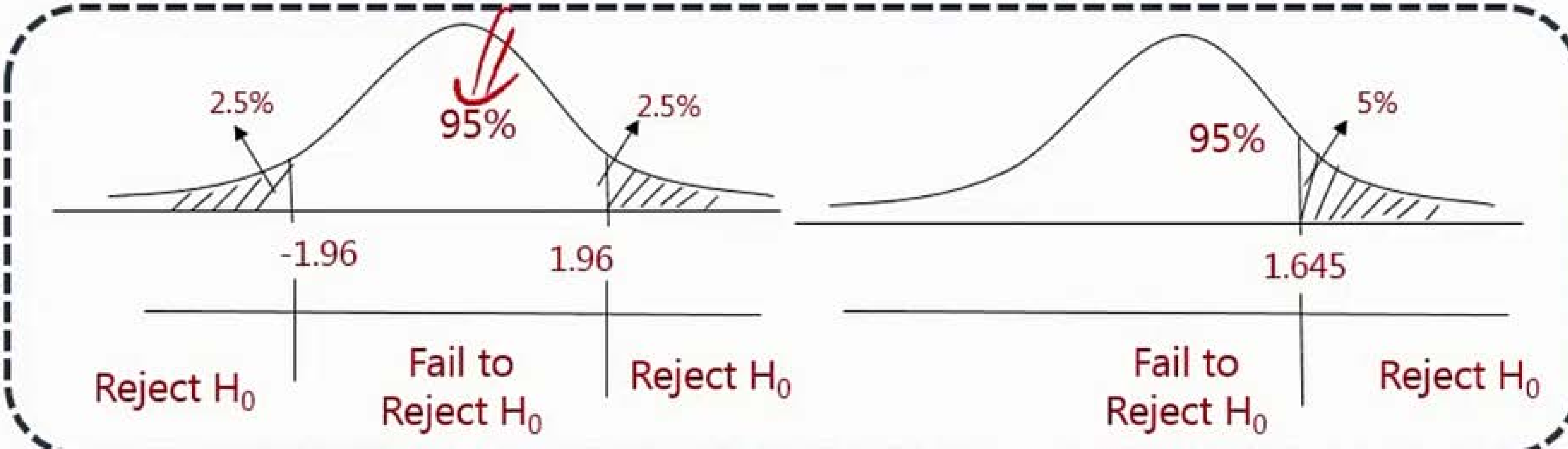
单尾 $\alpha = 5\%$ $\Longleftrightarrow$ 双尾 $\alpha = 10\%$

$K = 1.65$



Hypothesis Testing

- **Critical Value Method**
  - Find reject region with critical value;
  - Reject H_0 if | test statistic | > critical value;
  - Fail to reject H_0 if | test statistic | < critical value.



- **Statement**
  - cannot say "accept the null hypothesis", only can say "cannot reject"
  - ***** is significantly different from ******
  - ***** is not significantly different from ******

1. 设假设. H_0: 拒绝.

H_a: 接受. $\neq$ 双.

$>$ 单.

$<$

2. 画分布 $\rightarrow$ 总体参数 μ, σ^2.

3. 拒绝域: 双尾 or 单尾. (H_a).

4 面积 $\Rightarrow$ $\alpha = 5\%$.

5. k.

6. 计算检验统计量，代 μ.

$\bar{X} \to Z$. $\quad \dfrac{\bar{X}-\mu_x}{\sigma/\sqrt{n}} = z\text{-value}$.

7. z v.s. k.

8. 总结陈词 $\begin{cases} \text{reject } H_0. \\ \text{fail to reject } H_0. \end{cases}$

Example $n=480$ $\bar{x}=0.2\%$ $s_x=0.3\%$

- Example: Two-tailed test
- A financial analyst gathered data about the daily returns of a mutual fund over a recent 480-day period. The mean return is 0.2%, and the sample standard deviation of daily returns is 0.30%. The analyst considers that the mean daily return is unequal to zero. Construct a hypothesis test of the analyst's opinion.

$\mu = 0$?

- **Correct Answer:**
  - **Step 1:** We need to state null and alternative hypotheses. the analyst expects to reject the null hypothesis.

$$H_0 : \mu = 0 \ versus \ H_a : \mu \neq 0$$

  - **Step 2:** The standard error of the sample mean: the standard deviation of the sample is adjusted. IF the sample mean is $\overline{X}$ and the sample size is n , the standard error is calculated as: $S_{\overline{X}} = S / \sqrt{n}$ So the standard error of the sample mean for a sample size of 480 is $0.003 / \sqrt{480}$, and our test statistic is: $\dfrac{0.002}{0.003/\sqrt{480}} = \dfrac{0.002}{0.000137} = 14.61$

Hypothesis Testing

- **Step 3:** Because the null hypothesis is an equality, this is a two-tailed test. The level of significance is 5% , so the critical z-values for the two-railed test are ± 1.96.
- **Step 4:** The decision rule can be stated as:
  Reject H_0 if test statistic < -1.96 or test statistic > +1.96
- **Step 5:** Since 14.61 > 1.96, we reject the null hypothesis, and we accept alternative hypotheses that the mean daily option return is unequal to zero. That is, the mean daily return of 0.002 is statistically different from zero given the sample's standard deviation and size.

1、$H_0: \mu = 0$ $H_a: \mu \neq 0$

2、正分布 μ (t. vs. z)

3、拒绝域：双.

4、两损：$\alpha = 5\%$

5、$K = 1.96$

6、计算检验统计量、 ITM

$$\bar{x} \to z \qquad \frac{\bar{x}-\mu_x}{\sigma/\sqrt{n}} = \frac{0.125-0}{\frac{0.30}{\sqrt{480}}} = 14.61$$

7. K v.s. z.

8. reject H_0.

Hypothesis Testing

- Perform a z-test using the mutual fund data from the previous example to test the opinion that option returns **are greater than zero**.
- **Correct Answer:**
  - **Step 1:** In this case, we use a one-tailed test and state null and alternative hypotheses:

    $$H_0: \mu \leq 0 \text{ versus } H_a: \mu > 0$$

  - **Step 2:** From the previous example, we know that the test statistic for the option return sample is 14.61.
  - **Step 3:** , this is a one-tailed test. The level of significance is 5% , so the critical z-values for the two-railed test are ± 1.645.
  - **Step 4:** The appropriate decision rule is:

    $$\text{Reject } H_0 \text{ if test statistic} > 1.645$$

  - **Step 5:** Because 14.61 > 1.645, we reject the null hypothesis and conclude that mean returns are statistically greater than zero at a 5% level of significance.

1. 设假设： H_0：拒绝　H_a：接受

$\neq$ 双

2. 判断分布 μ, σ^2.　$>$ 单 $<$

Summary of Hypothesis Testing

Test type	Assumptions	H_0	Test-statistic	Critical value
Mean hypothesis testing	Normally distributed population, known population variance	$\mu=0$	$Z=\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$	N(0,1)
	Normally distributed population, unknown population variance	$\mu=0$	$t=\frac{\bar{x}-\mu_0}{s/\sqrt{n}}$	t(n-1)
	Independent populations, unknown population variances assumed equal	$\mu_1-\mu_2=0$	t	$t(n_1+n_2-2)$
	Independent populations, unknown population variances not assumed equal	$\mu_1-\mu_2=0$	t	t
	Samples not independent, **paired comparisons test**	$\mu_d=0$	$t=\bar{d}/s_{\bar{d}}$	t(n-1)
Variance hypothesis testing	Normally distributed population	$\sigma^2=\sigma_0^2$	$\chi^2=\frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2(n-1)$
	Two independent normally distributed populations	$\sigma_1^2=\sigma_2^2$	$F=s_1^2/s_2^2$	$F(n_1-1,n_2-1)$

1. 设假设：H_0：拒绝　H_a：接受.

$\neq$ 双.

$>$ $<$ 单.

× 2. 判断分布 μ, σ^2.

3. K影响因素

① 查表.

② α

③ 单尾 or. 双尾.

4. 计算检验统计量.

1个μ. ($\bar{x} \to z$)　$\dfrac{\bar{x}-\mu_x}{\sigma/\sqrt{n}}$.

5. 综合：$\alpha=5\%$. 计算z. → 判断

Example

$Z = 1.58.$

- An analyst conducts a two-tailed test to determine if earnings estimates are significantly different from reported earnings. The sample size was over 50. The computed Z-statistic is 1.58. At a 5 percent significance level, which of the following statements is TRUE?

$\alpha = 5\%$

N.

A. You cannot determine what to do with the information given.

B. Fail to reject the null hypothesis and conclude that the earnings estimates are not significantly different from reported earnings.

C. Both the null and the alternative are significant.

- **Correct Answer: B.**

-1.96 $\quad$ 1.58 $\quad$ $+1.96$

Summary of Hypothesis Testing

Test type	Assumptions	H_0	Test-statistic	Critical value
1个μ.	Normally distributed population, known population variance	$\mu=0$	$Z=\dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$	N(0,1)
	Normally distributed population, unknown population variance	$\mu=0$	$t=\dfrac{\bar{x}-\mu_0}{s/\sqrt{n}}$	t(n-1)
Mean hypothesis testing 2个μ	Independent populations, unknown population variances assumed equal	$\mu_1-\mu_2=0$	t	$t(n_1+n_2-2)$
μ	Independent populations, unknown population variances not assumed equal	$\mu_1-\mu_2=0$	t	t
	Samples not independent, **paired comparisons test**	$\mu_d=0$	$t=\bar{d}/s_{\bar{d}}$	t(n-1)
Variance hypothesis testing	Normally distributed population 1个σ^2.	$\sigma^2=\sigma_0^2$	$\chi^2=\dfrac{(n-1)s^2}{\sigma_0^2}$	$\chi^2(n-1)$
σ^2.	Two independent normally distributed populations 2个σ^2.	$\sigma_1^2=\sigma_2^2$	$F=s_1^2/s_2^2$	$F(n_1-1,n_2-1)$

ℓ.

- μ.
  - 1个μ. — Z. vs. t.
  - 2个μ. — t分布.
    - 独立. → t.
    - 不独立 → 成对数检验. t.

X		Y	d
X_1	$-$	Y_1	d_1
X_2	$-$	Y_2	d_2
⋮		⋮	⋮
X_{100}	$-$	Y_{100}	d_{100}

$\Downarrow$ t

H_0: $\Updownarrow$ $\boxed{\mu_d = 0} \Leftarrow$

$\bar{X} \to \mu_X$ $\qquad$ $\bar{d} \to \mu_d$

$$\frac{\bar{X}-\mu_X}{\sigma_X / \sqrt{n}} \qquad \frac{\bar{d}-\underbrace{\mu_d}_{=0}}{\cancel{\sigma_d} / \sqrt{n}}, \quad \sigma_d \to s_d$$

$\boxed{1个\mu} \Leftarrow$

$\mu_d = 0$

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$$X \qquad Y \qquad d$$

$$X_1 - Y_1 \qquad d_1$$

$$X_2 - Y_2 \qquad d_2$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$X_{100} - Y_{100} \qquad d_{100}$$

$$t.$$

$\boxed{1个\mu} \Leftarrow$

$\mu_d = 0$

H_0:

$\Updownarrow$

$\boxed{\mu_d = 0} \Leftarrow$

$\bar{X} \to \mu_X$

$\bar{d} \to \mu_d$

$$\frac{\bar{X} - \mu_X}{\cancel{\sigma_X}/\sqrt{n}} \Rightarrow \frac{\bar{d} - \overset{=0}{\mu_d}}{\cancel{\sigma_d} \to s_d / \sqrt{n}}$$

$$\frac{\bar{d}}{s_d/\sqrt{n}}$$

Summary of Hypothesis Testing

Test type	Assumptions	H_0	Test-statistic	Critical value
Mean hypothesis testing	Normally distributed population, known population variance	$\mu=0$	$Z=\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$	N(0,1)
	Normally distributed population, unknown population variance	$\mu=0$	$t=\frac{\bar{x}-\mu_0}{s/\sqrt{n}}$	t(n-1)
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	Two independent normally distributed populations	$\sigma_1^2=\sigma_2^2$	$F=s_1^2/s_2^2$	$F(n_1-1,n_2-1)$

$$
\begin{array}{ccc}
X & Y & d \\
X_1 - Y_1 & & d_1 \\
X_2 - Y_2 & & d_2 \\
\vdots & \vdots & \vdots \\
X_{100} - Y_{100} & & d_{100} \\
& & t
\end{array}
$$

$\boxed{1个\mu} \Leftarrow$

$\mu_d = 0$

H_0:

$\Updownarrow$

$\boxed{\mu_d = 0} \Leftarrow$

$\bar{x} \to \mu_x \qquad \bar{d} \to \mu_d$

$$\frac{\bar{x} - \mu_x}{\cancel{\sigma_x / \sqrt{n}}} \Rightarrow \frac{\bar{d} - \mu_d^{=0}}{\cancel{\sigma_d / \sqrt{n}}}, \quad \sigma_d \to s_d$$

$$s_{\bar{d}} = \frac{\sigma}{\sqrt{n}} = \frac{s_d}{\sqrt{n}}, \quad \sigma \to s_d \Rightarrow \frac{\bar{d}}{s_d / \sqrt{n}}$$

Hypothesis Testing

- An analyst collects the following data related to paired observations for Sample A and Sample B. Assume that both samples are drawn from normally distributed populations and that the population variances are not known.

Paired Observation	Sample A Value	Sample B Value
1	25	18
2	12	9
3	-5	-8
4	6	3
5	-8	1

The *t*-statistic to test the hypothesis that the mean difference is equal to zero is *closest* to:

A. 0.23
B. 0.27
C. 0.52

- **Correct Answer: C**

Hypothesis Testing

1个$\mu \Leftarrow \bar{x} \to \mu_x$.

- An analyst collects the following data related to paired observations for Sample A and Sample B. Assume that both samples are drawn from normally distributed populations and that the population variances are not known.

DATA STAT 2ND 7 [8]

$\bar{d}$, S_d

$\bar{x} \to \bar{d}$ $S_x \to S_d$

Paired Observation	Sample A Value	Sample B Value		d
1	25	18	X01	7
2	12	9	X02	3
3	-5	-8	X03	3
4	6	3	X04	3
5	-8	1	X05	-9

2ND 8 清零.

The *t*-statistic to test the hypothesis that the mean difference is equal to zero is *closest* to:

A. 0.23
B. 0.27
C. 0.52

- **Correct Answer: C**

$H_0: \mu_d = 0$

$$\frac{\bar{x}-\mu_x}{\sigma_x/\sqrt{n}} \qquad \frac{\bar{d}-\mu_d}{\sigma_d/\sqrt{n}} = \frac{\bar{d}}{S_d/\sqrt{n}}$$

Variance hypothesis testing	Normally distributed population 1个σ^2	$\sigma^2=\sigma_0^2$	$\chi^2=\frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2(n-1)$
	Two independent normally distributed populations 2个σ^2	$\sigma_1^2=\sigma_2^2$	$F={s_1^2}/{s_2^2}$	$F(n_1-1,n_2-1)$

σ^2

1个 σ^2 χ^2 卡方.

2个 σ^2.   F.

Example

- Which of the following is *most appropriate* to test the equality of the variances of two normally distributed populations?
  - A. F-test
  - B. T-test
  - C. χ^2-test

  σ

  2个 σ^2 =>

- **Correct Answer: A**

  μ_1 μ_2.

- Smith wants to know whether the mean returns of two stocks are the same. If the two normally distributed stock returns are dependent, the appropriate type of test and test statistic are:
  - A. Difference in means test, t-statistic
  - B. Paired comparisons test, t-statistic
  - C. ~~Difference in means test, F-statistic~~

  2个μ → t

  不独立. ⇐

- **Correct Answer: B.**

Hypothesis Testing

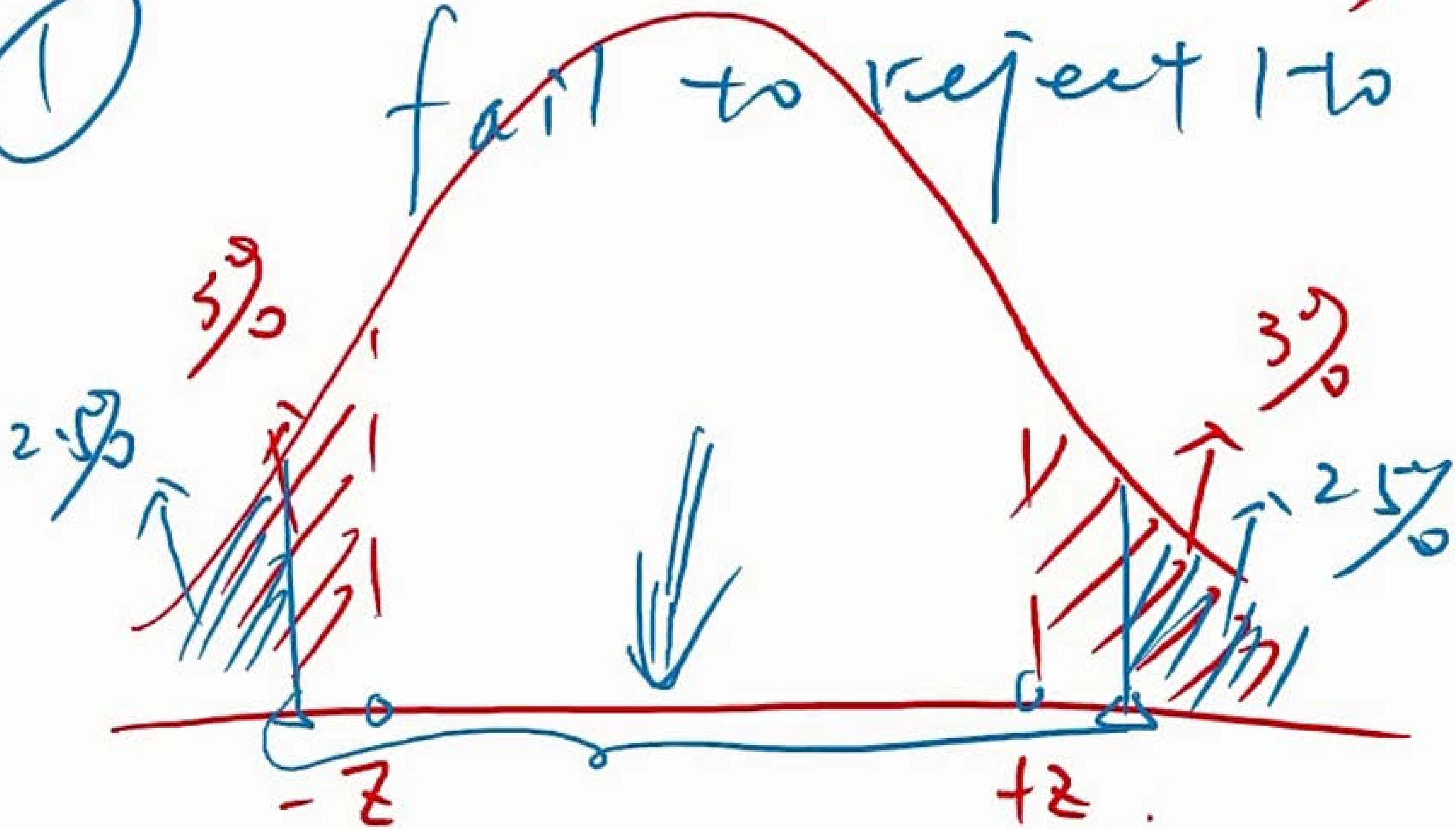
- **P-value Method** 定义：拒绝H_0的最小α → Prob
  1、 - The **p-value** is the smallest level of significance at which the null hypothesis can be reject (0~1) 不.
  2. - p-value < α: reject H_0; p-value > α: do not reject H_0.
  - P↓, easier to reject H_0 判断准则

- **Example:**
- The p-value for a two-tailed test of sample mean is 1.68%. Which of the following is true?
  A. We can reject the null with 95% confidence
  B. We can reject the null with 99% confidence
  C. the largest probability of rejecting the null hypothesis is 1.68%

- **Correct Answer: A**

$Z \Leftrightarrow$ P-value $= 6\% > \alpha = 5\%$

① fail to reject H_0



② p-value = 2% < α = 5%

reject H_0.

Hypothesis Testing

➢ **P-value Method** 定义：拒绝H_0的最小α → Prob

1、 • The **p-value** is the smallest level of significance at which the null hypothesis can be reject (0~1)

2、 • p-value < α: reject H_0; p-value > α: do not reject H_0.

• P↓, easier to reject H_0 判断准则 越小越拒绝

v.s. α.

➢ **Example:**

➢ The p-value for a two-tailed test of sample mean is 1.68%. Which of the following is true?

p-value = 1.68% < α = 5%

A. We can reject the null with 95% confidence

B. We can reject the null with 99% confidence

C. the largest probability of rejecting the null hypothesis is 1.68%

α = 1%

➢ **Correct Answer: A**

p-value.

A +1 X

B -1

C. 0 ⇐

reject H_0

prob : 0 ~ 1

Type I error and Type II error

- **Type I error**: 拒真，reject the null hypothesis when it's actually true
  - ✓ Significance level (α): the probability of making a Type I error
  - ✓ Significance level $=P(\text{Type I error})=P(H_0 \times | H_0 \surd)$
- **Type II error**: 取伪，fail to reject the null hypothesis when it's actually false
  - ✓ $P(\text{Type II error})=P(H_1 \times | H_1 \surd)$
  - ✓ Power of a test: the probability of correctly rejecting the null hypothesis when it is false
  - ✓ Power of a test $= 1-P(\text{Type II error})= P(H_1 \surd | H_1 \surd)$

Decision	True condition	
	H_0 is true	**H_0 is false**
Do not reject H_0	Correct Decision	Incorrect Decision Type Ⅱ error
Reject H_0	Incorrect Decision Significance level =P (Type I error)	Correct Decision Power of test = 1- P (Type Ⅱ error)

- With other conditions unchanged, either error probability arises at the cost of the other error probability decreasing.
- How to reduce both errors? Increase the Sample Size.

1. 定义: I. $X \mid H_0 \checkmark$, 拒真, 错杀好人.

II. $\checkmark \mid H_0 X$, 取伪, 放走坏人.

2. $P(\mathrm{I}) = \alpha$.

3. Power of test: $P(X \mid H_0 X) = 1 - P(\mathrm{II})$

4. $P(\mathrm{I}) \uparrow \quad P(\mathrm{II}) \downarrow$.

5. $n \uparrow \quad P(\mathrm{I})\ \&\ P(\mathrm{II}) \downarrow$.

- Kyra Mosby, M.D., has a patient who is complaining of severe abdominal pain. Based on an examination and the results from laboratory tests, Mosby states the following diagnosis hypothesis: H_0: Appendicitis, H_a: Not Appendicitis. Dr. Mosby removes the patient's appendix and the patient still complains of pain. Subsequent tests show that the gall bladder was causing the problem. By taking out the patient's appendix, Dr. Mosby:
  A. Made a Type I error.
  B. Is correct.
  C. Made a Type II error.

- **Correct Answer: C**

- Kyra Mosby, M.D., has a patient who is complaining of severe abdominal pain. Based on an examination and the results from laboratory tests, Mosby states the following diagnosis hypothesis: H_0: Appendicitis, H_a: Not Appendicitis. Dr. Mosby removes the patient's appendix and the patient still complains of pain. Subsequent tests show that the gall bladder was causing the problem. By taking out the patient's appendix, Dr. Mosby:
  A. Made a Type I error.
  B. Is correct.
  C. Made a Type II error.

- **Correct Answer: C**

$n\uparrow$

- If the sample size increases, the probability of get the Type I and Type II error will

	Type I	Type II
A.	increase	increase
B.	not change	not change
C.	decrease	decrease

- **Correct Answer: C**

Example

$\alpha \uparrow = P(\mathrm{I}) \uparrow \rightarrow P(\mathrm{II}) \downarrow.$

- All else equal, is specifying a larger significance level in a hypothesis test likely to increase the probability of a:

	Type I error?	Type II error?
A.	No	No
B.	No	Yes
C.	Yes	No

- **Correct Answer: C**

- What is the definition of the power of test? Power of test is the probability to:

$X \mid H_0 \; X$

  A. Reject the true null hypothesis while it is true
  B. Reject the false null hypothesis while it is indeed false
  C. Can not reject the true hypothesis

- **Correct Answer: B**

Hypothesis Testing

μ, σ^2 参数检验

- **Parametric tests**
  - based on specific distributional assumptions for the population
  - concerning a parameter of population.
  - For example, t-test.

非参数检验

- **Nonparametric tests**
  - a nonparametric test either is not concerned with a parameter or makes minimal assumptions about the population from which the sample comes.
  - Nonparametric tests are used:
    - ① ✓ when data do not meet distributional assumptions
      - ◆ Example: hypothesis test of the mean value for a variable, but the distribution of the variable is not normal and the sample size is small so that neither the t-test nor the z-test are appropriate.
    - ② ✓ when data are given in ranks.
    - ③ ✓ when the hypothesis we are addressing does not concern a parameter

1. 不满足分布的假设.

2. 序数排列.

3. 检验的不是参数.

$+ + - - + -$

Technical Analysis

- **Principles:**
  - Prices are the result of the interaction of supply and demand in the real time.
  - The greater the volume of trades, the more impact that market participants will have on price.
  - Trades determine volume and price.
- **Assumptions:**
  - Market prices reflect both rational and irrational investor behavior.
    - ✓ Market trends and patterns reflect the irrational human behavior.
    - ✓ Market trends and patterns repeat themselves and are somewhat predictable.
    - ✓ Efficient markets hypothesis dose not hold.

Technical Analysis

- **Principles:** Mkt P. ⇐ D&S ⇐ 交易行为
  - Prices are the result of the interaction of supply and demand in the real time.

    买 D↑ → P↑

  - The greater the volume of trades, the more impact that market participants will have on price.

    心理

  - Trades determine volume and price.

    卖 S↑ → P↓

- **Assumptions:**
  - ① Market prices reflect both rational and irrational investor behavior.
    - ✓ Market trends and patterns reflect the irrational human behavior.
    - ✓ Market trends and patterns repeat themselves and are somewhat predictable.
    - ② ✓ Efficient markets hypothesis dose not hold.

EMH. 有效市场假说.

弱式：历史

半强式：历史+公开

强式：历史+公开+私募

Technical Analysis

基本面分析

- **The differences among technicians, fundamentalists and Efficient market followers.**

技术分析

  - Fundamental analysis of a firm seeks to **determine the underlying long-term(intrinsic) value** of an asset by using the financial statements and other information.
  - While technical analysis uses more concrete data, primarily **price and volume data**, and seek to project the level at which a financial instrument will trade.
  - Fundamentalists believe that prices react quickly to changing stock values, while technicians believe that the reaction is slow.
  - **Technicians look for changes in supply and demand**, while fundamentalists look for changes in value.

Model.

$$\boxed{P = \sum \frac{CF_t}{(1+r)^t}}$$

$\underline{\underline{V}}$.

⇓.

合理.

内在价值 V.　　　市场价格 P.

$V > P$.　　$P\uparrow$.　　buy.

$V < P$.　　$P\downarrow$.　　sell.

- **The differences among technicians, fundamentalists and Efficient market followers.**

技术分析

  - Fundamental analysis of a firm seeks to **determine the underlying long-term(intrinsic) value** of an asset by using the financial statements and other information. 内在价值 V
  - While technical analysis uses more concrete data, primarily **price and volume data**, and seek to project the level at which a financial instrument will trade.
  - Fundamentalists believe that prices react quickly to changing stock values, while technicians believe that the reaction is slow. DOS
  - **Technicians look for changes in supply and demand**, while fundamentalists look for changes in value.

Technical Analysis

- **Advantages of technical analysis:**
  1. Actual price and volume data is easy to access.
  2. Technical analysis is objective (although require subjective judgment), while much of the data used in fundamental analysis is subject to assumptions or restatements.
  3. It can be applied to the prices of assets that do not produce future cash flows, such as commodities.

     K线

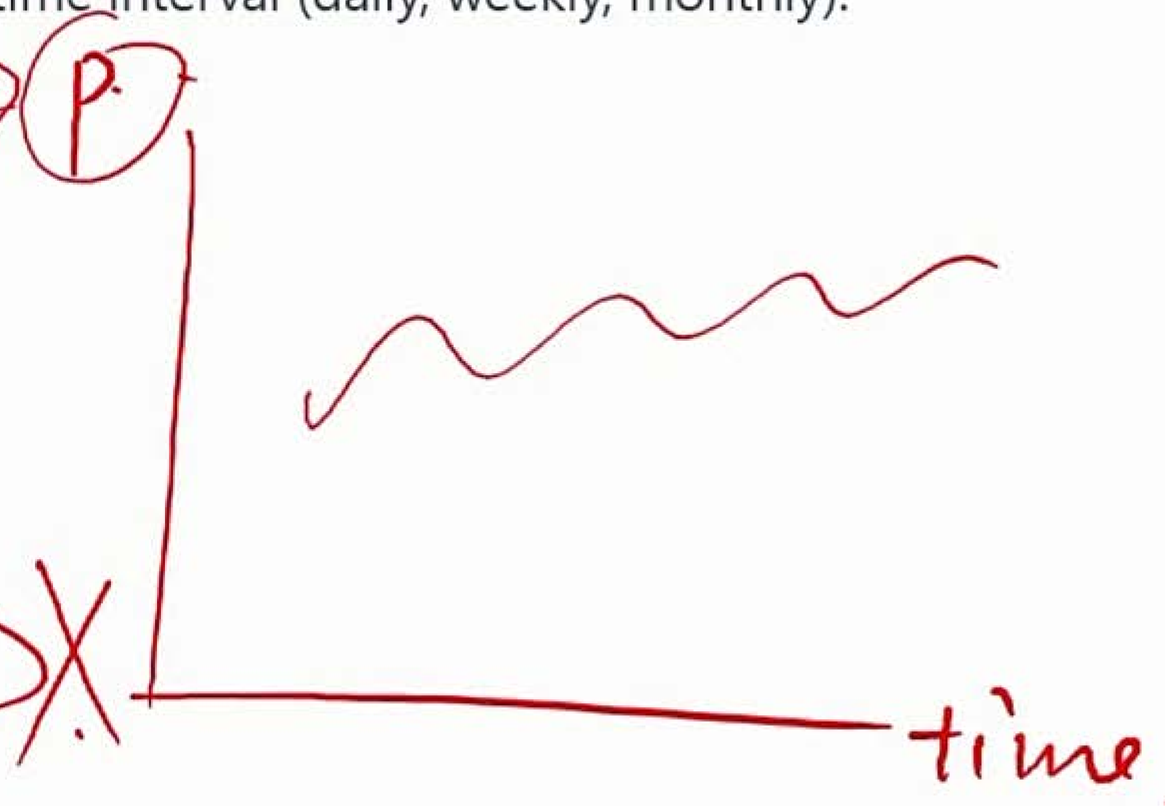
  4. Fundamental analysis may have the risk of financial statement fraud, while technical analysis doesn't have.

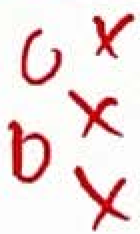
合理 P ⇐ DOS 合理.

- **Disadvantage:**
  1. In markets that are subject to large outside manipulation, the application of technical analysis is limited.
  2. Technical analysis is also limited in illiquid markets, where even modestly sized trades can have an inordinate impact on prices.

Technical Analysis

- Charts are the graphical display of **price and volume data**.
  - Horizontal axis: usually time interval (daily, weekly, monthly).
  - Vertical axis: price level.
- **Types of charts**
  - Line charts.
  - Bar charts.
  - Candlestick charts.
  - Point and figure charts.

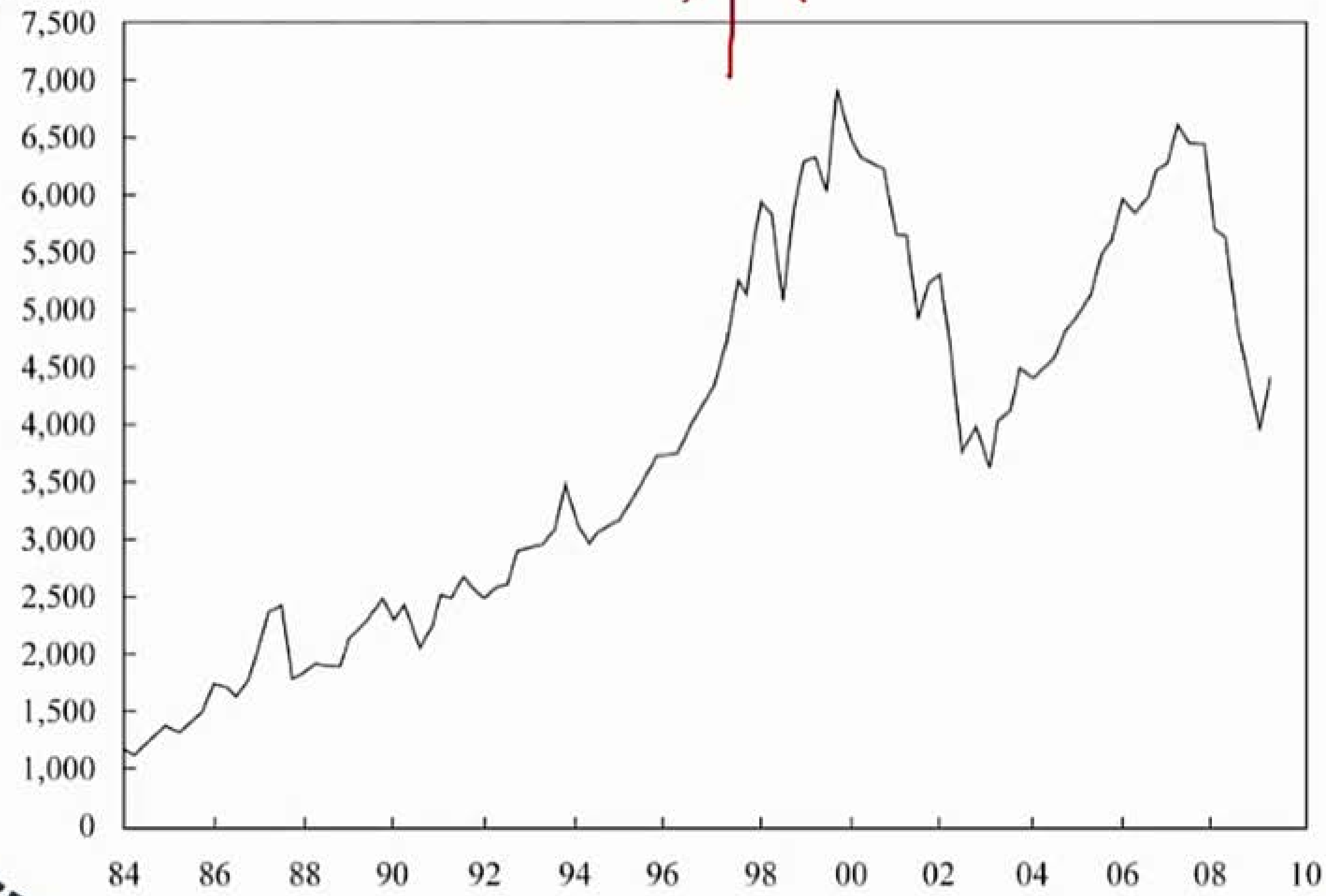




Technical Analysis

收盘价

- **Line Charts** are a simple graphic display of price trends over time. Line charts are typically drawn with **closing prices** as the data points.



- **Bar charts** have four bits of data in each entry — **the high and low price** encountered during the time interval plus the **opening and closing prices**.

Technical Analysis K线

- **Candlestick charts** provides four prices per data point entry: the opening and closing prices and the high and low prices during the period.
  - Box is clear: closing price>opening price;
  - Box is filled: closing price<opening price

白

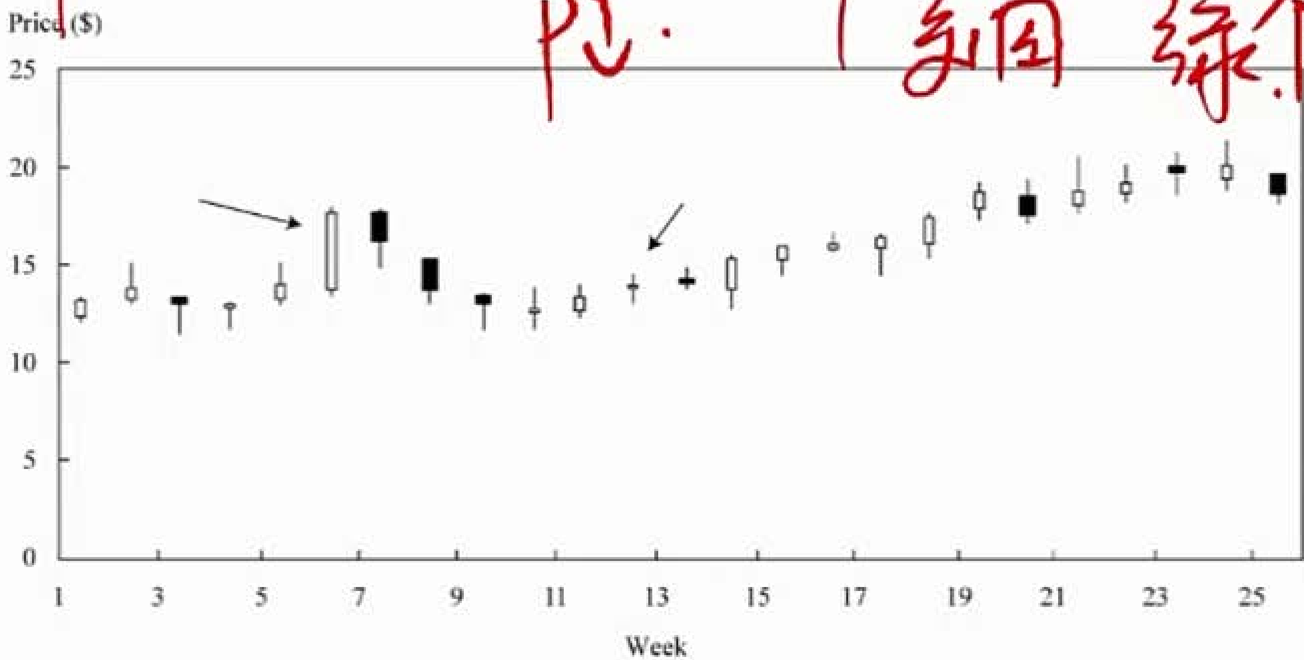
黑.

P↑.

P↓.

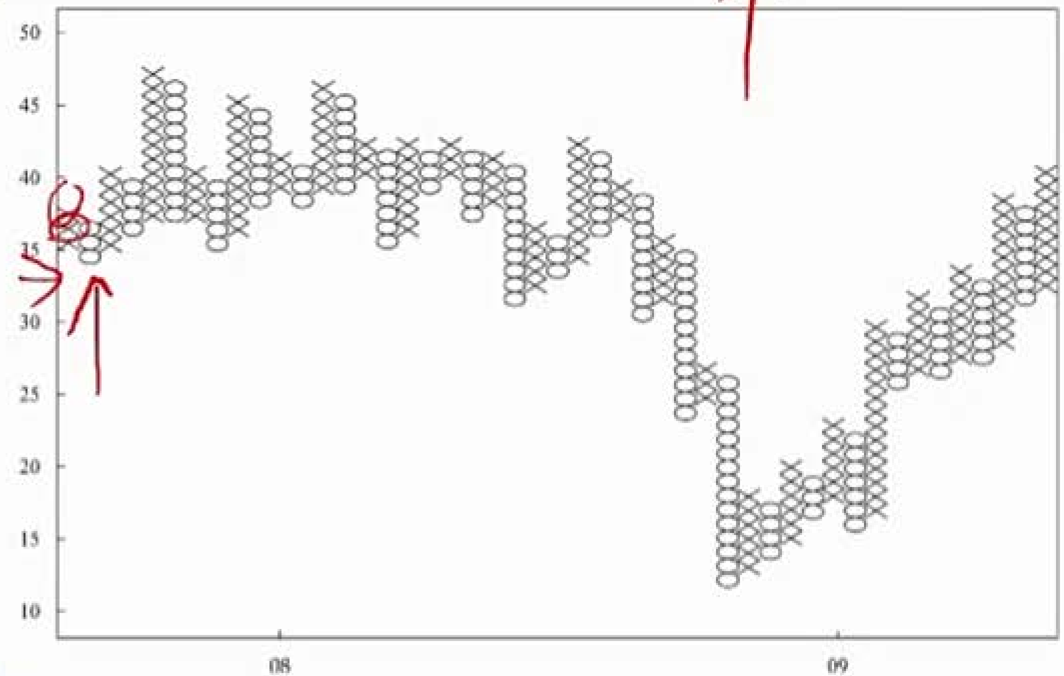
中国. 绿↓. 红↑

美国 绿↑ 红↓



Technical Analysis

- **Point and figure charts** are helpful in identifying changes in the direction of price movements.
  - Starting form opening price;
  - X: increase of one box size, O: indicate a decrease.
  - Analyst will begin the next column when the price changes in the opposite direction by at least the reversal size (3 times the box size).



Technical Analysis

相对强度分析

- **Relative strength analysis:** compare the performance of a particular asset, such as a common stock, with that of some benchmark.
  - Positive relative strength: an increasing trend indicates that the asset is outperforming the benchmark
  - Negative relative strength: an decreasing trend indicates that the asset is underperforming the benchmark

v.s.

$$\frac{P_{医药}=10}{P_{行业}}$$

$$\frac{P_{银行}=30}{P_{行业}}$$

>1

<1

Technical Analysis

- **Trend:** is the most important aspect of technical analysis.
  - **Uptrend**: An uptrend for a security is when the price goes to higher highs and higher lows. (Demand>Supply)
  - **Downtrend**: is when a security makes lower lows and lower highs. (Demand<Supply)
- **Trend line:** can help to identify whether a trend is continuing or reversing.
  - **Uptrend line**: connecting the low of the price chart.
  - **Downtrend line**: connecting the highs of the price chart.
  - When price drops through and below the trend line by a significant amount, indicate that the uptrend is over and may signal a further deadline in the price.

Technical Analysis

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    P↑ D>S 买↑

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    P↓ D<S 卖↑

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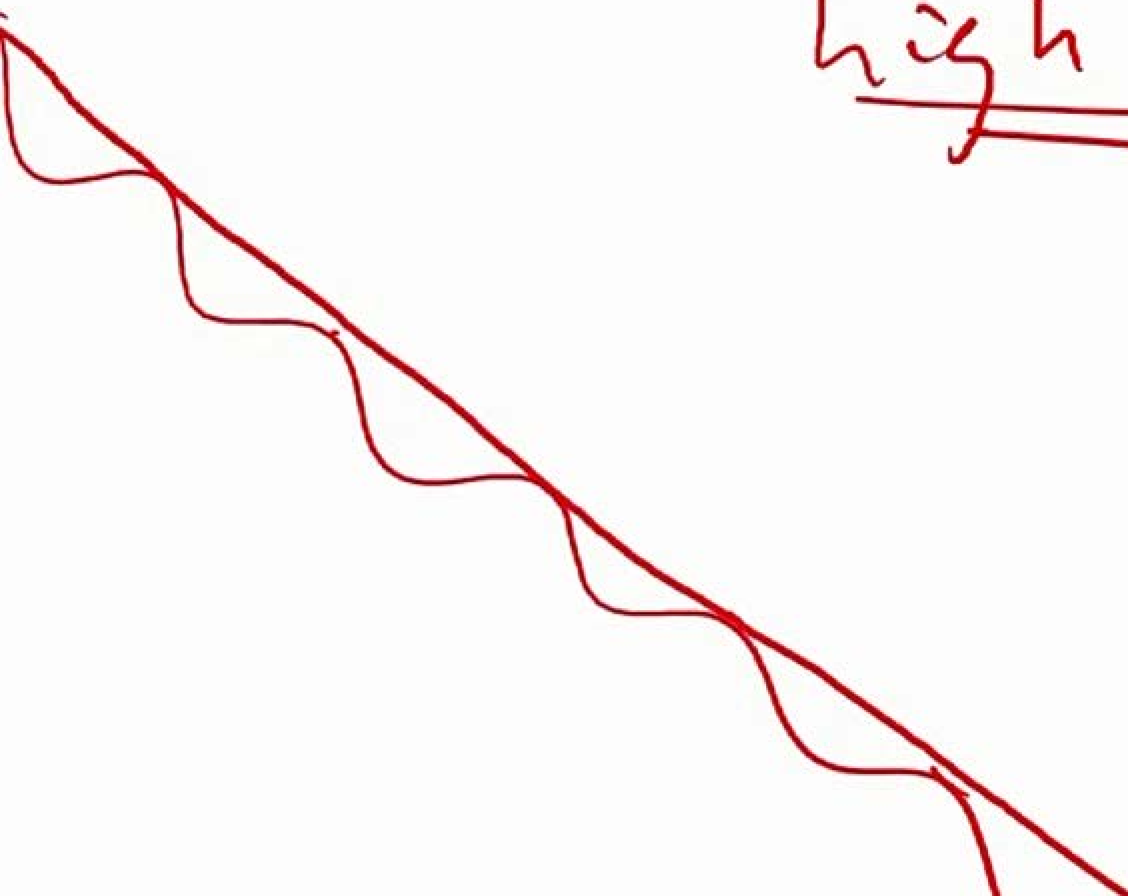
Low price.

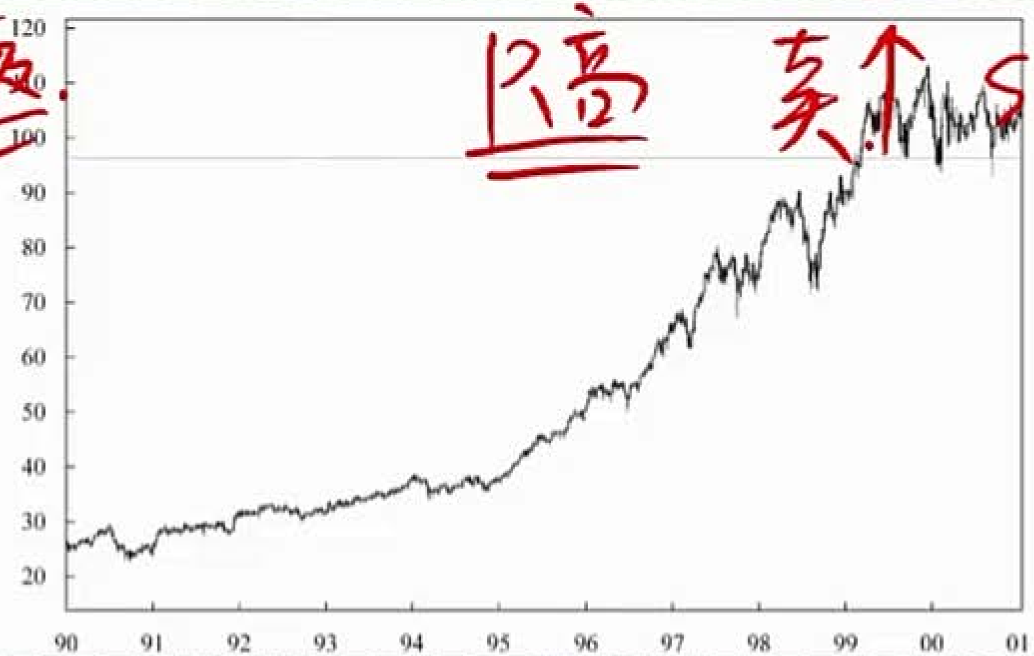
high price.



Technical Analysis

- **Support level:** a low price range in which buying activity is sufficient to stop the decline in price. 支撑位 P低
- **Resistance level:** a price range in which selling is sufficient to stop the rise in price. 阻力位 D>S
- **Change in polarity:**
  - once a support level is breached, it becomes a resistance level.
  - once a resistance levels breached, it becomes a support level.

P高 卖↑ S>D P↓



- **Common chart patterns:**
  - **Reversal patterns**
    - ✓ For uptrend: Head-and shoulders pattern, Double top and triple top
    - ✓ For downtrend: inverse head-and shoulders pattern, Double bottom, and triple bottom
  - **Continuation patterns**
    - ✓ Triangles 保持
    - ✓ Rectangles

Technical Analysis

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头肩底

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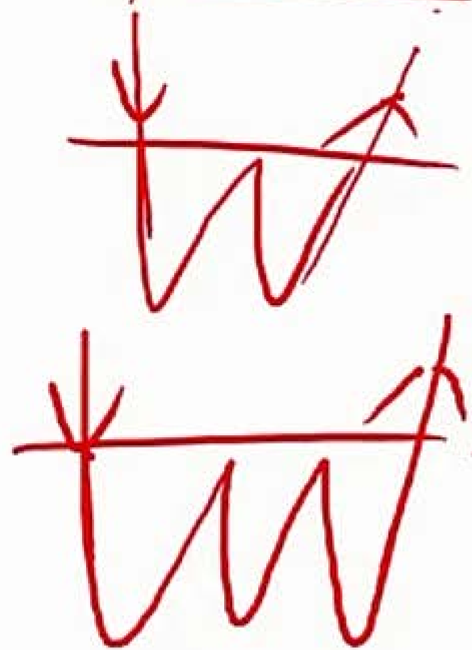

Technical Analysis

- **Head-and-shoulders pattern**
  - The size of the head-and-shoulders pattern: the difference in price between the head and the neckline.
  - **Price target** = Neckline – (Head – Neckline)
- **Inverse head and shoulders pattern:**
  - price target = neckline + (neckline – head)

181-201

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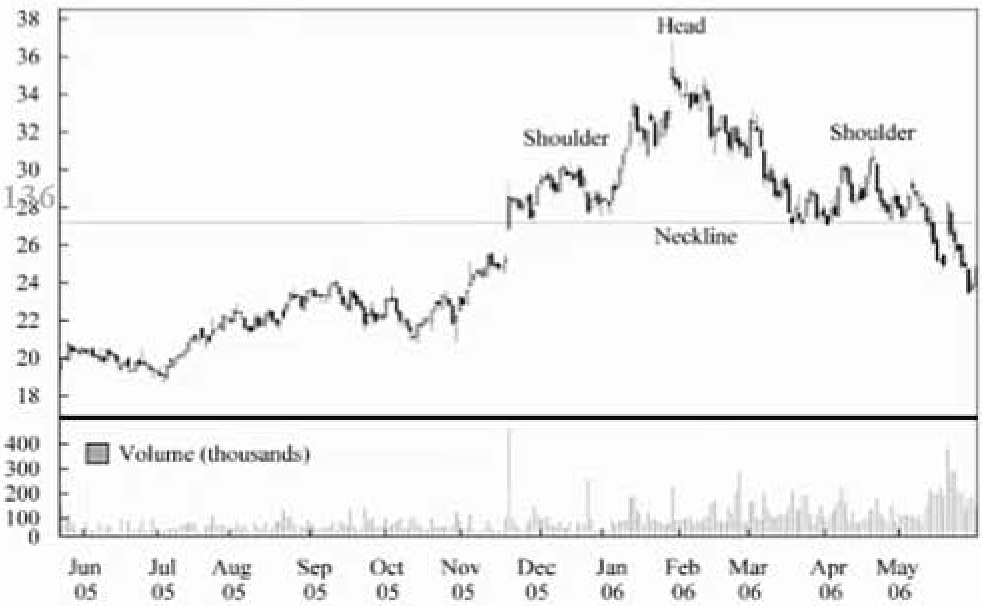

136

Technical Analysis

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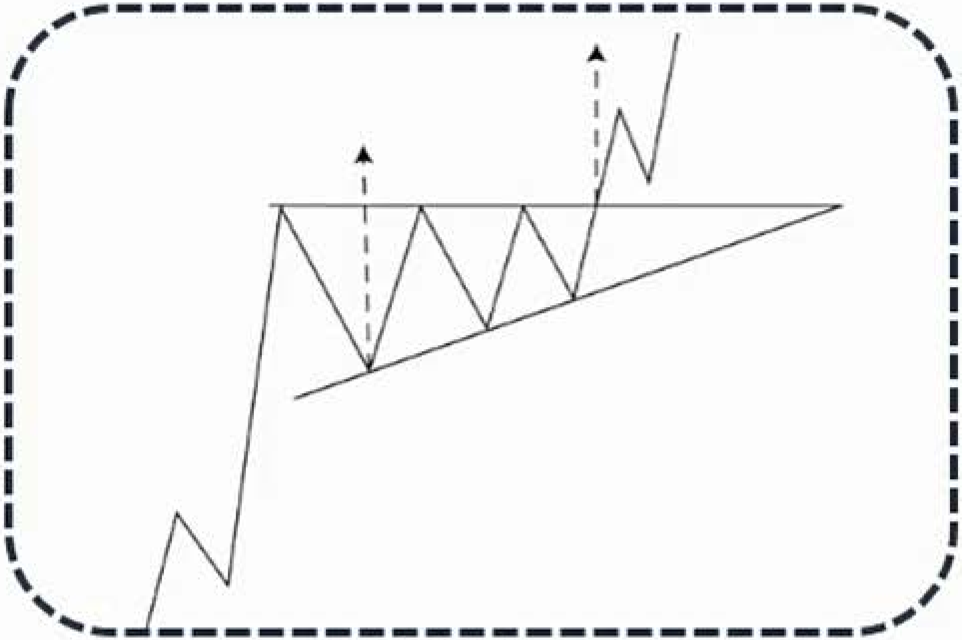
头

- **Triangles:** A triangle pattern forms as the range between high and low prices narrows, visually forming a triangle.
- **Rectangles:** A rectangle pattern is a continuation pattern formed by two parallel trend lines, one formed by connecting the high prices during the pattern, and the other formed by the lows.
- **Flags and pennants:** the form over short periods of time-on a daily price chart, typically over a week.



- **Technical Analysis Indicators**
  - Price-based
    - ✓ Moving average lines
    - ✓ Bollinger bands
  - Momentum oscillators
    - ✓ Rate of change oscillator
    - ✓ Relative Strength Index
    - ✓ Moving average convergence/divergence
    - ✓ Stochastic oscillator

- **Technical Analysis Indicators**
  - Sentiment
    - ✓ Put/call ratio
    - ✓ Volatility Index
    - ✓ Margin debt
    - ✓ Short interest ratio
  - Flow of funds
    - ✓ Short-term trading index
    - ✓ Margin debt
    - ✓ Mutual fund cash position
    - ✓ New equity issuance

Technical Analysis

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一看酒.

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汗.

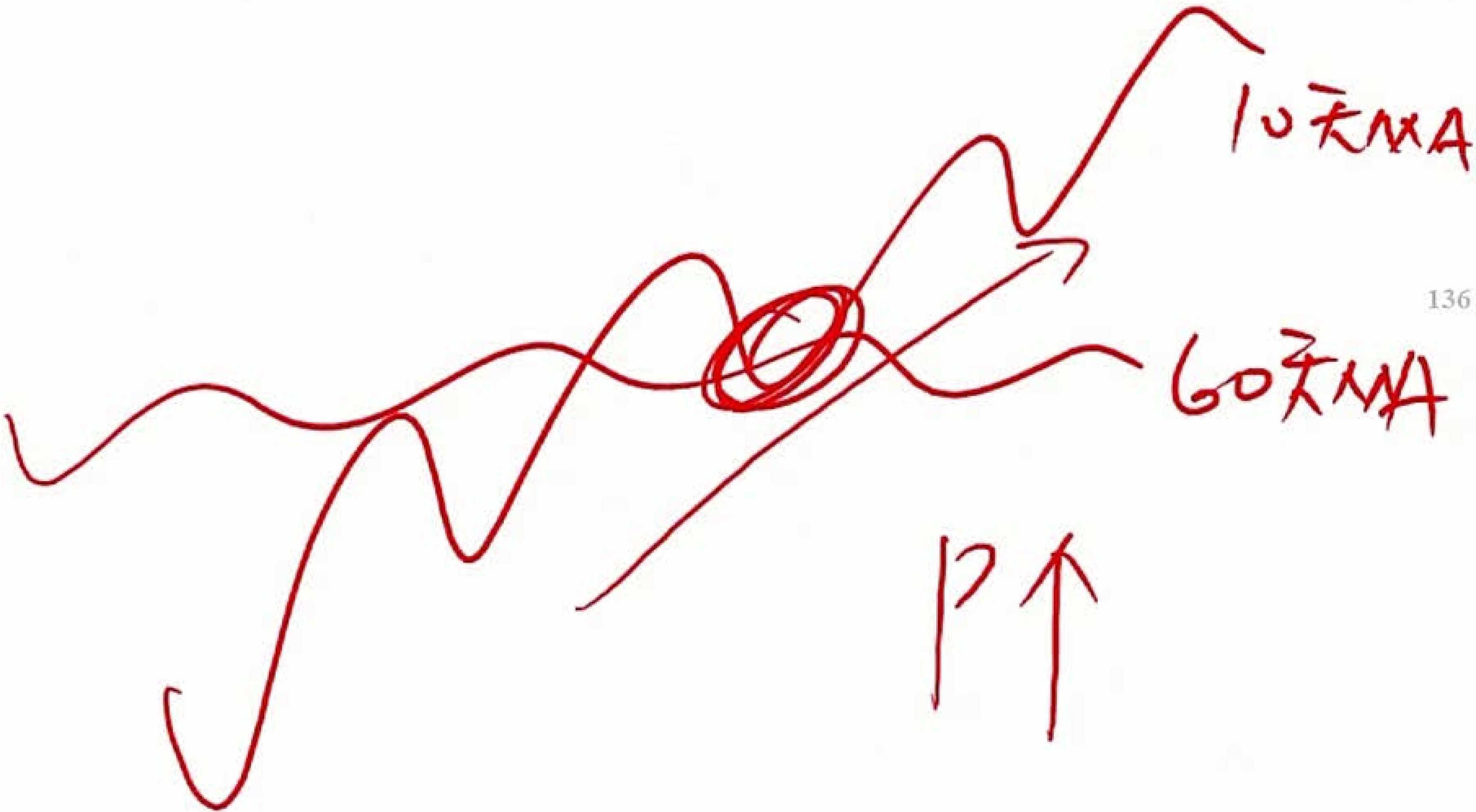
Technical Analysis 移动平均线 MA

- **Moving Average:** is the average of the closing price of a security over a specified number of periods.
  - Technicians commonly use a simple moving average, which weights each price equally in the calculation of the average price.
  - Some technicians prefer to use an exponential moving average (also called an exponentially smoothed moving average), which gives the greatest weight to recent prices while giving exponentially less weight to older prices.
  - Trading strategies
    - ✓ First, whether price is above or below its moving average is important.
      - ◆ A security that has been trending down in price will trade below its moving average, and a security that has been trending up will trade above its moving average.
    - ✓ Second, the distance between the moving-average line and price is also significant.
      - ◆ Once price begins to move back up toward its moving-average line, this line can serve as a resistance level.

Technical Analysis

黄金交叉

- When a short-term moving average crosses from underneath a longer-term average, this movement is considered **bullish** and is termed **a golden cross**.
- Conversely, when a short-term moving average crosses from above a longer-term moving average, this movement is considered **bearish** and is called a **dead cross**.


10天MA
60天MA
P↑


DL↓
60天MA
10天MA

Technical Analysis

黄金交叉

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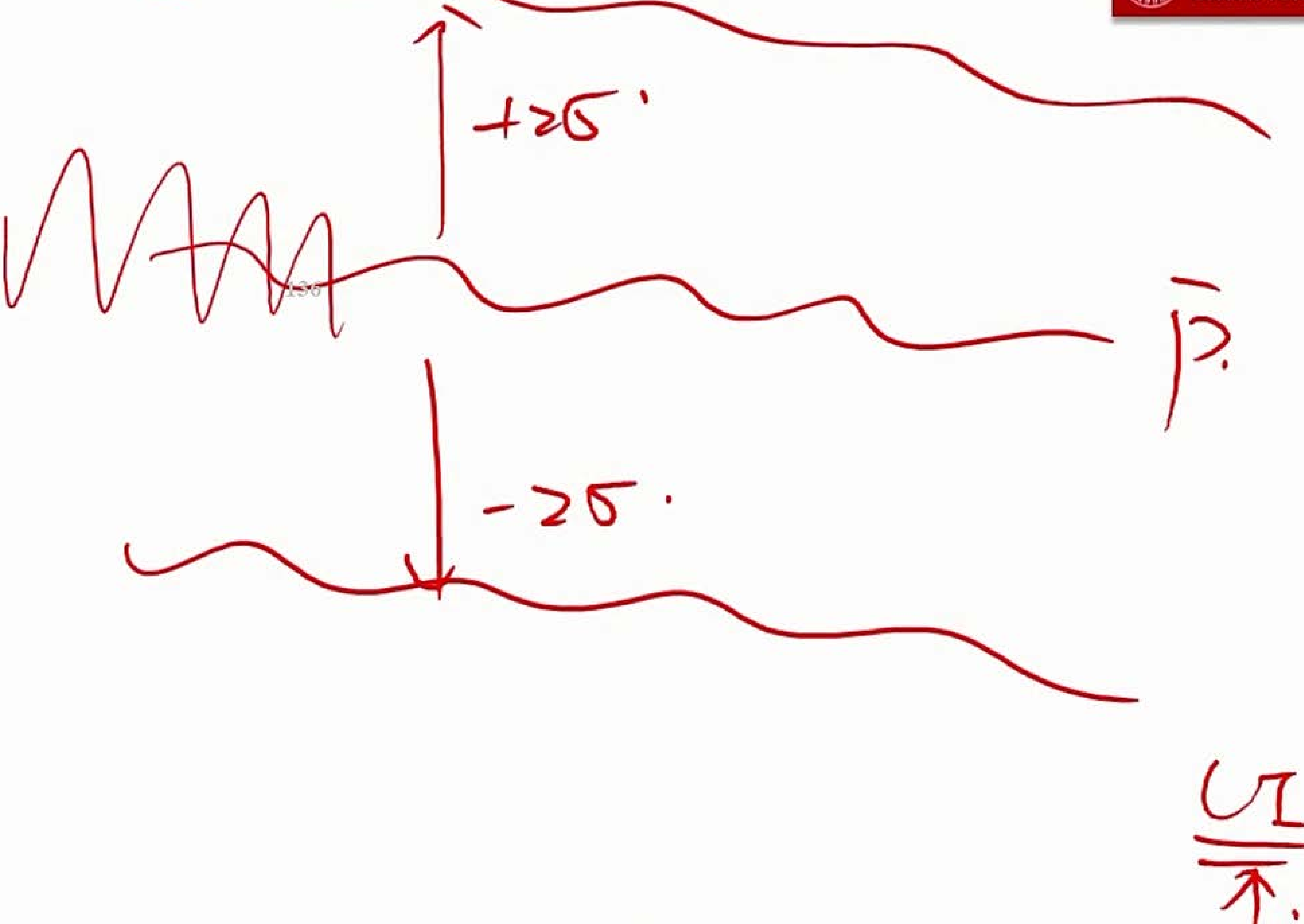
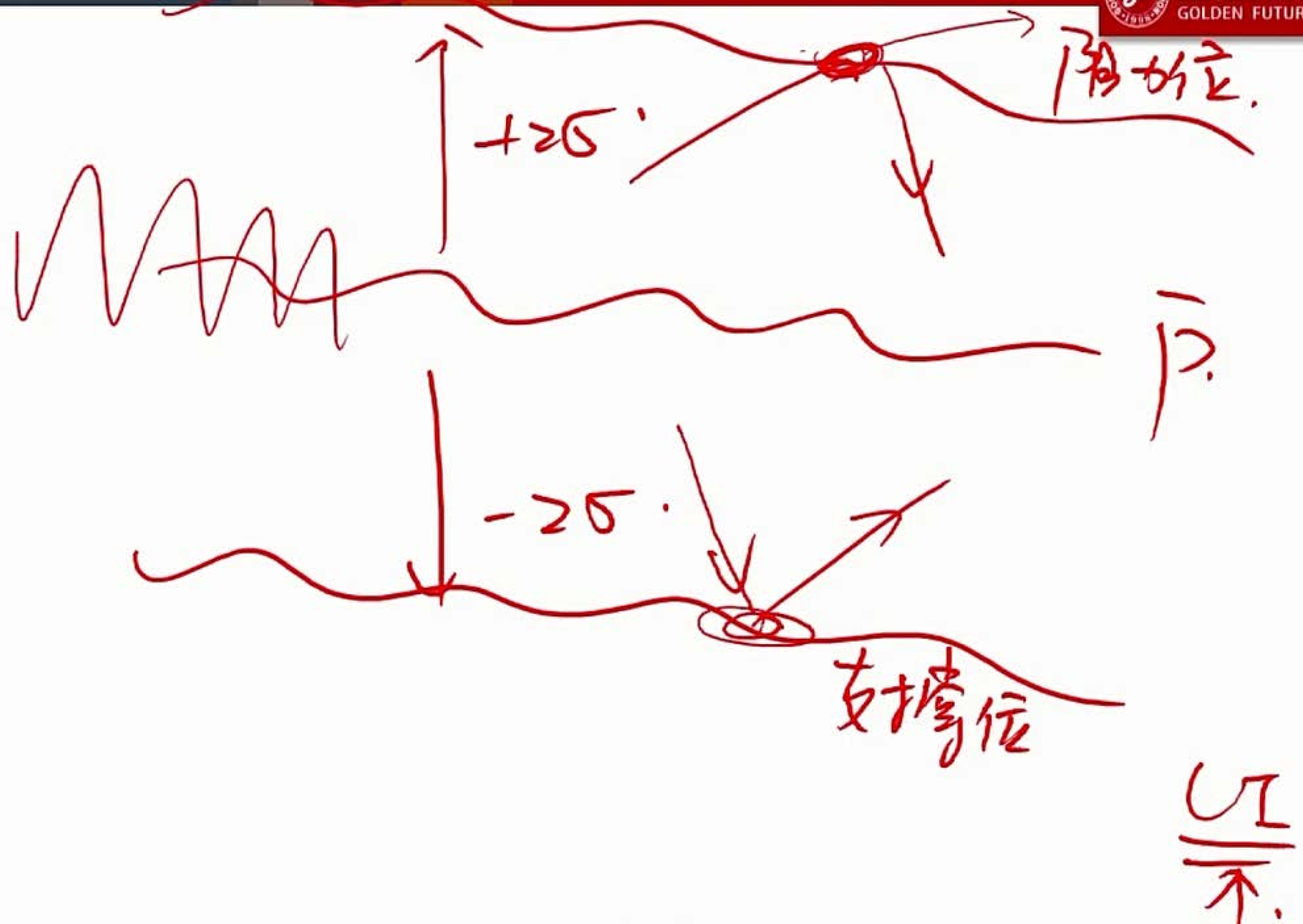
死亡交叉

Technical Analysis

- **Bollinger bands**
  - Moving average +/- 2σ
  - **Trading strategies**
    - ✓ Investor sells when a security price reaches the upper band and buys when it reaches the lower band. (This strategy assumes that the security price will stay within the bands.)
    - ✓ The long-term investors might actually **buy** on a significant breakout above the upper boundary band.
    - ✓ The long-term investor would **sell** on a significant breakout below the lower band.



+2σ
$\bar{p}$
-2σ
UI


阻力位
+2σ
P
-2σ
支撑位

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- **Rate of Change Oscillator (ROC)**

$$M = (V - V_x) \times 100 \quad \text{or} \quad M = \frac{V}{V_x} \times 100$$

<0

$V < V_x$

where

M = momentum oscillator value

V = last closing price

$V > V_x$

V_x = closing price x days ago, typically 10 days

- **Strategy**

$\ominus \rightarrow \oplus$

① ✓ If the ROC oscillator crosses into positive territory during an uptrend, it is a **buy signal.**

✓ If it enters into negative territory during a downtrend, it is considered a **sell signal**.

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Technical Analysis

100天

$$\frac{\uparrow 天}{\downarrow 天}$$

➢ **Relative Strength Index (RSI)**

$$\boxed{RSI = 100 - \frac{100}{1+RS}}$$

$$where\ RS = \frac{\Sigma(\text{Up changes for the period under consideration})}{\Sigma(\text{Down changes for the period under consideration})}$$

- An RSI is based on the ratio of total price increases to total price decreases over a selected number of periods.
- The index construction forces that RSI to lie within 0 to 100.
- **Strategy**
  - ✓ A value above 70 represents an **overbought** situation.
  - ✓ A Value below 30 suggests the asset is **oversold**.

$$RSI = 100 - \frac{100}{1+RS} \qquad RS = \frac{\uparrow}{\downarrow}$$

$$= \frac{100(1+RS) - 100}{1+RS} = \frac{\cancel{100} + 100RS \cancel{-100}}{1+RS}$$

$$= 100 \times \frac{RS}{1+RS} = \cancel{100 \times \frac{\frac{\Sigma\uparrow}{\Sigma\downarrow}}{1+\frac{\Sigma\uparrow}{\Sigma\downarrow}}}$$

$$= 100 \times \frac{\Sigma\uparrow}{\underline{\underline{\Sigma\downarrow + \Sigma\uparrow}}}$$

$$RSI = 100 \times \frac{\Sigma \uparrow}{\Sigma 天数}$$

100天

① 0 ~ 100 ↑

(1) 30 ~ 70 ⇐

① > 50 P↑

② < 50 P↓

Technical Analysis

100天

$\frac{\Sigma \uparrow 天}{\Sigma \downarrow 天}$

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56 55 P↑ P↑↑

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②
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P↓↓ 卖 S↑↑

Technical Analysis

- **Stochastic Oscillator**

$$\%K = 100\left(\frac{C - L14}{H14 - L14}\right)$$

where

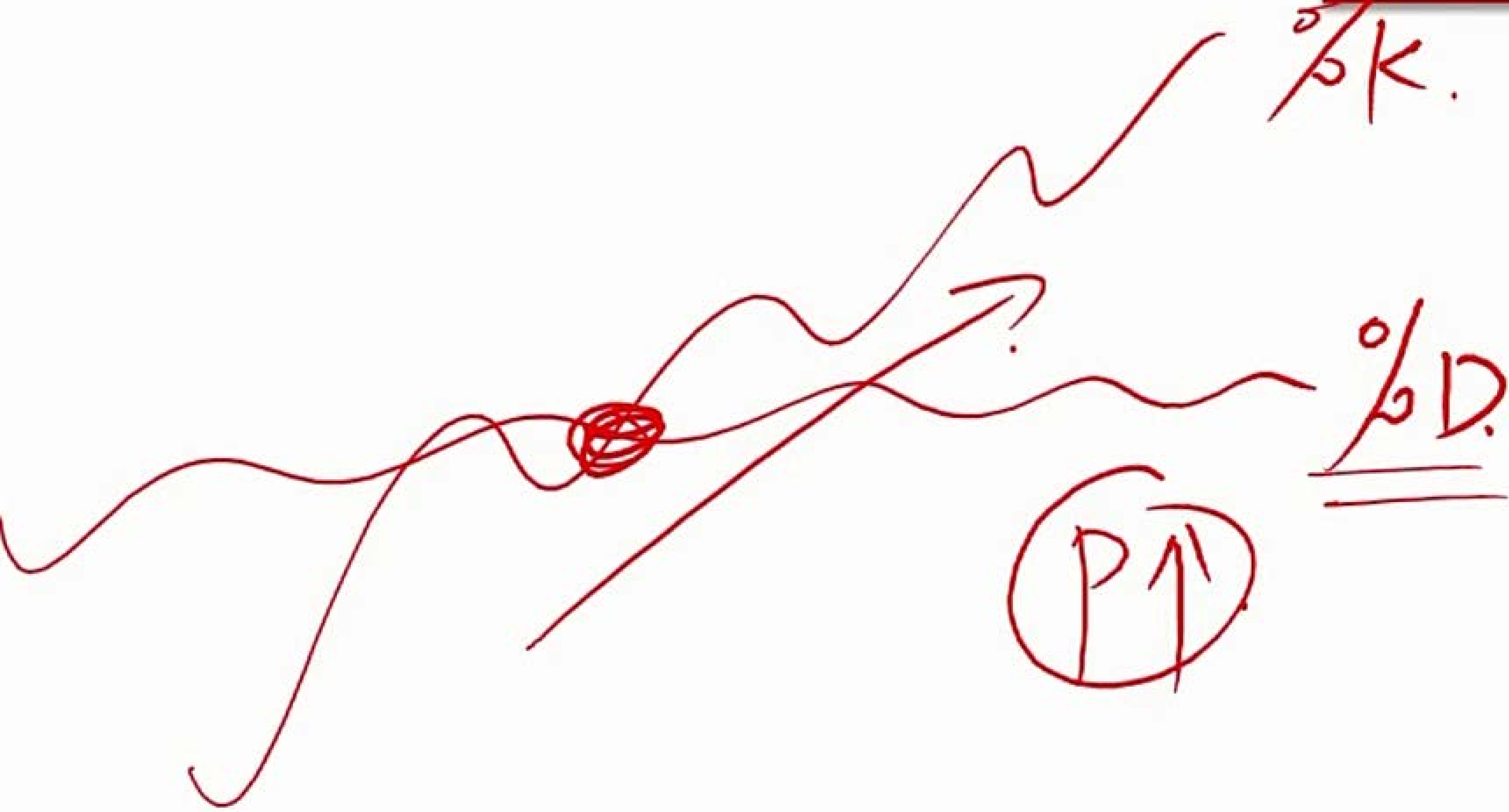
C = latest closing price

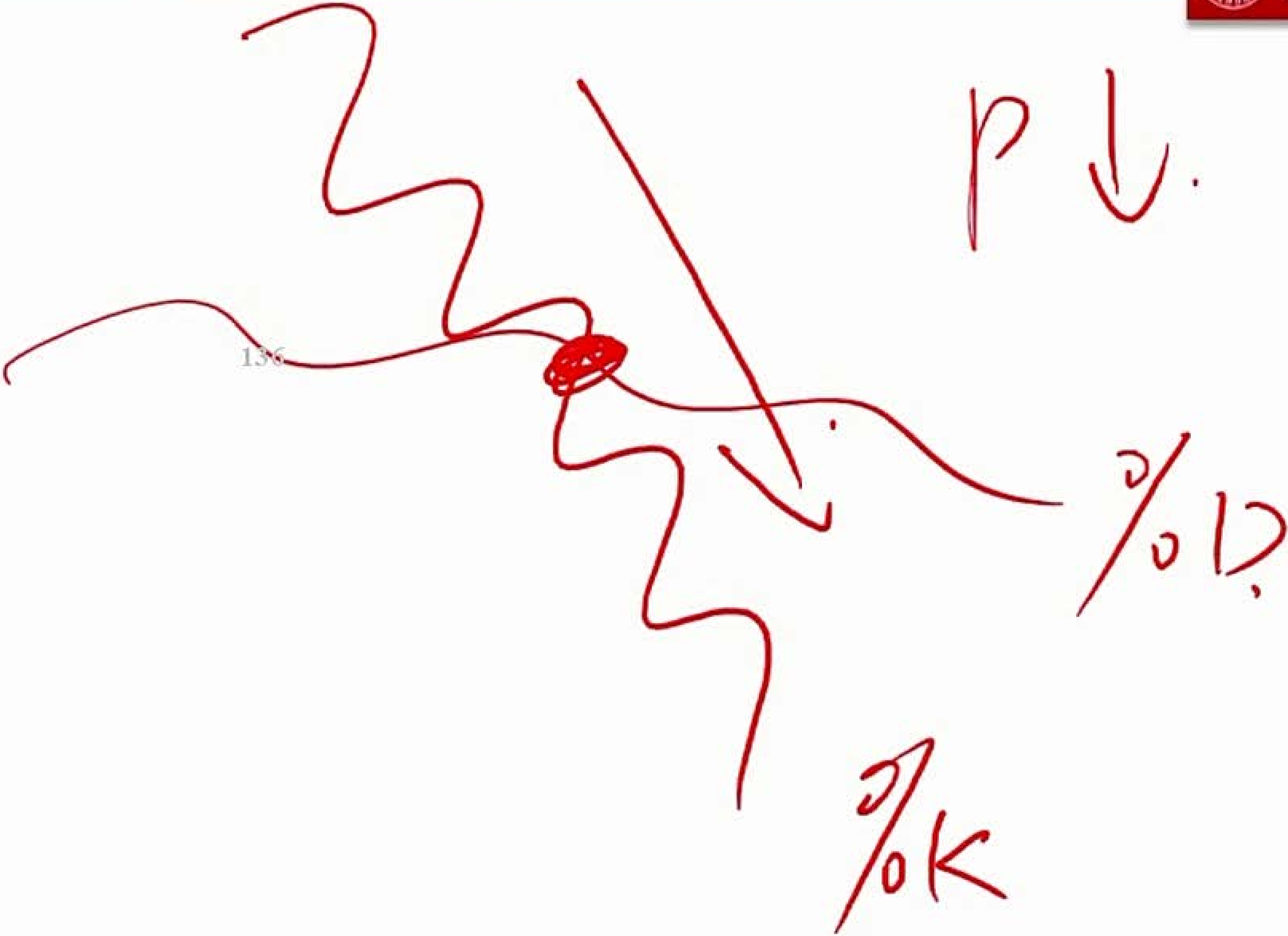
$L14$ = lowest price in past 14 days

$H14$ = highest price in past 14 days

$\%D$ = average of the last three %K values calculated daily

- The absolute level of the two lines should be considered in light of their normal range.
- **Strategy**
  - ✓ **Movements above** this range indicate to a technician an overbought security and are considered **bearish;**
  - ✓ **Movements below** this range indicate an oversold security and are considered **bullish.**


%K.
%D.
P↑


P↓.
%D.
%K

Technical Analysis

- **Stochastic Oscillator**

2

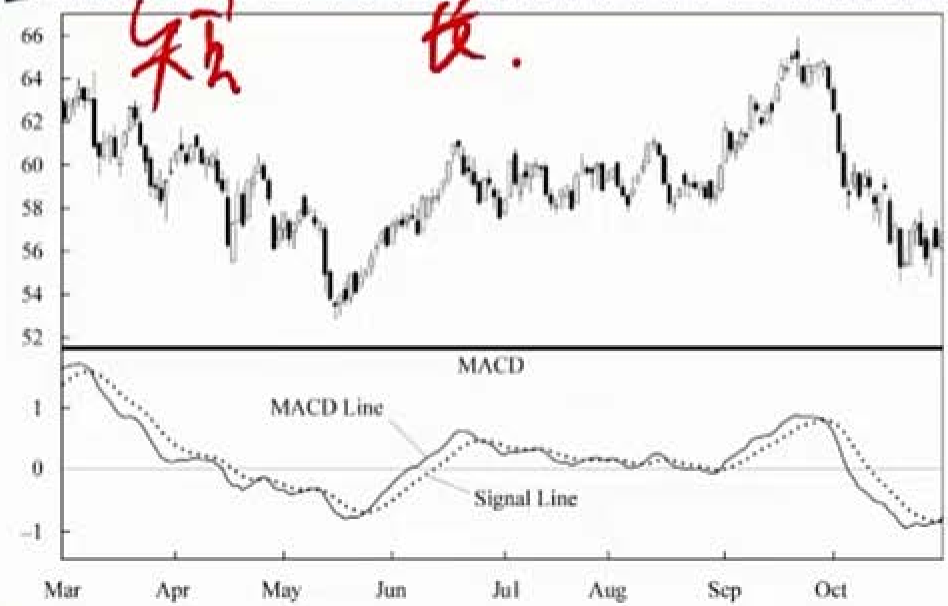
- Crossovers of the two lines can also give trading signals the same way crossovers of two moving averages give signals.
  - ✓ When the %K moves from below the %D line to above it, this move is considered a bullish short-term trading signal;
  - ✓ Conversely, when %K moves from above the %D line to below it, this pattern is considered bearish.

Technical Analysis 平滑异同移动平均线

Moving Average Convergence/Divergence (MACD)

- The MACD is the difference between a short-term and a long-term 之差 moving average of the security's price. The MACD is constructed by calculating two lines, **the MACD line and the signal line:**
- **MACD line**: difference between two exponentially smoothed moving averages, generally 12 and 26 days.
- **Signal line**: exponentially smoothed average of MACD line, generally 9 days.

短 长.



Technical Analysis

12天MA $\bar{P}$ $\bar{P}^1_{12}$ $\bar{P}^2_{12}$

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26天MA $\bar{P}$ $\bar{P}^1_{26}$ $\bar{P}^2_{26}$

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d_1 d_2 d_3

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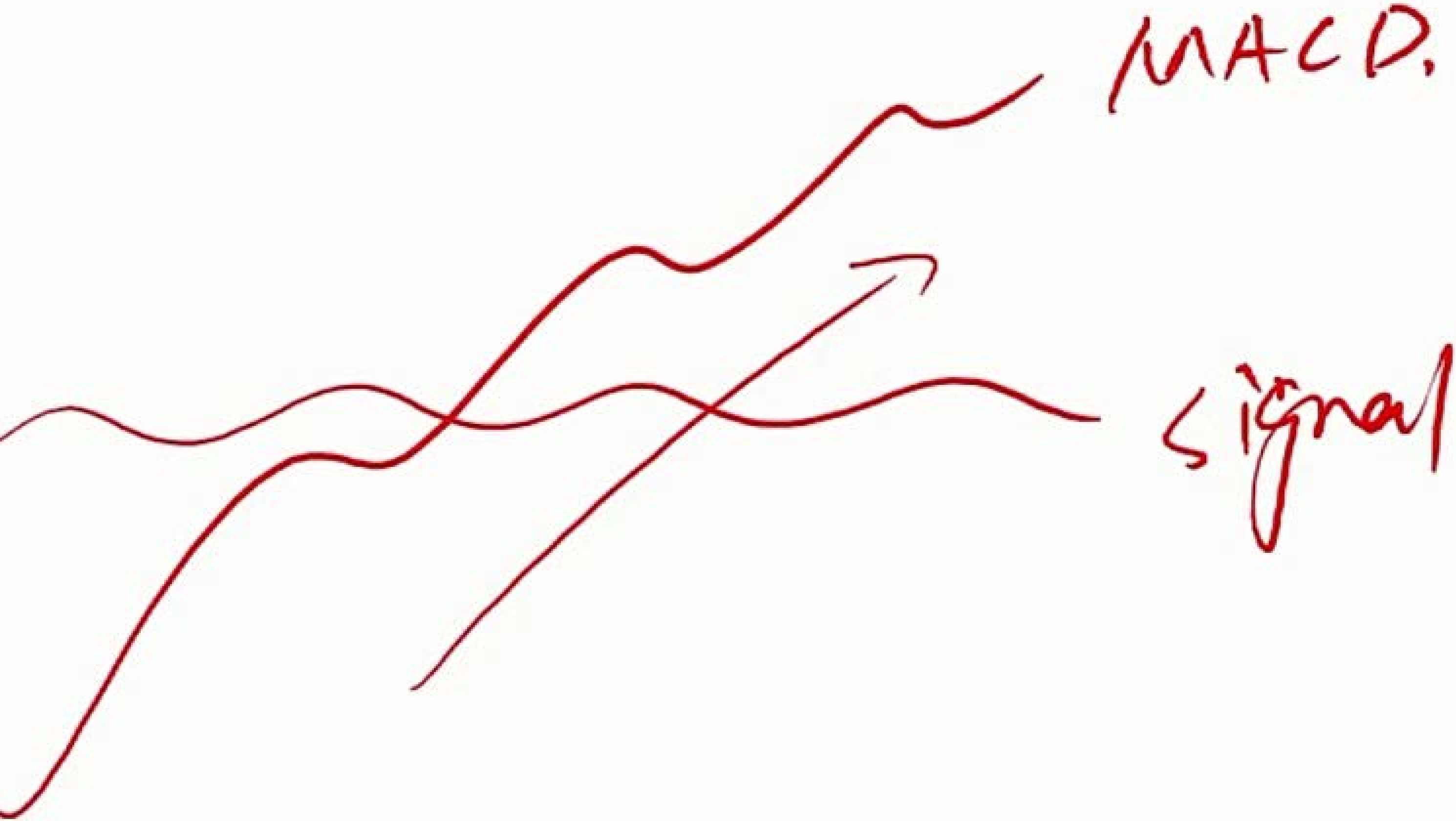
MACD.
signal

- **Put/call ratio**
  - The put /call ratio is the volume of put options traded divided by the volume of call option traded.
  - A **high** put/ call ratio usually indicates **bearish market**
  - At extreme highs in the put/call ratio, market sentiment is said to be so extremely negative that an increase in price is likely.
- **Volatility index (VIX)**
  - The VIX is a measure of near-term market volatility calculated by the Chicago Board Options Exchange.
    - ✓ The VIX rises when market participants become fearful of an impending market decline. These participants then bid up the price of puts, and the result is an **increase** in the VIX level.
    - ✓ When other indicators suggest that the market is oversold and the VIX is at an extreme high, this combination is considered **bullish**.

$$\frac{\text{put. \#}}{\text{Call \#.}}$$

看跌期权

看涨期权

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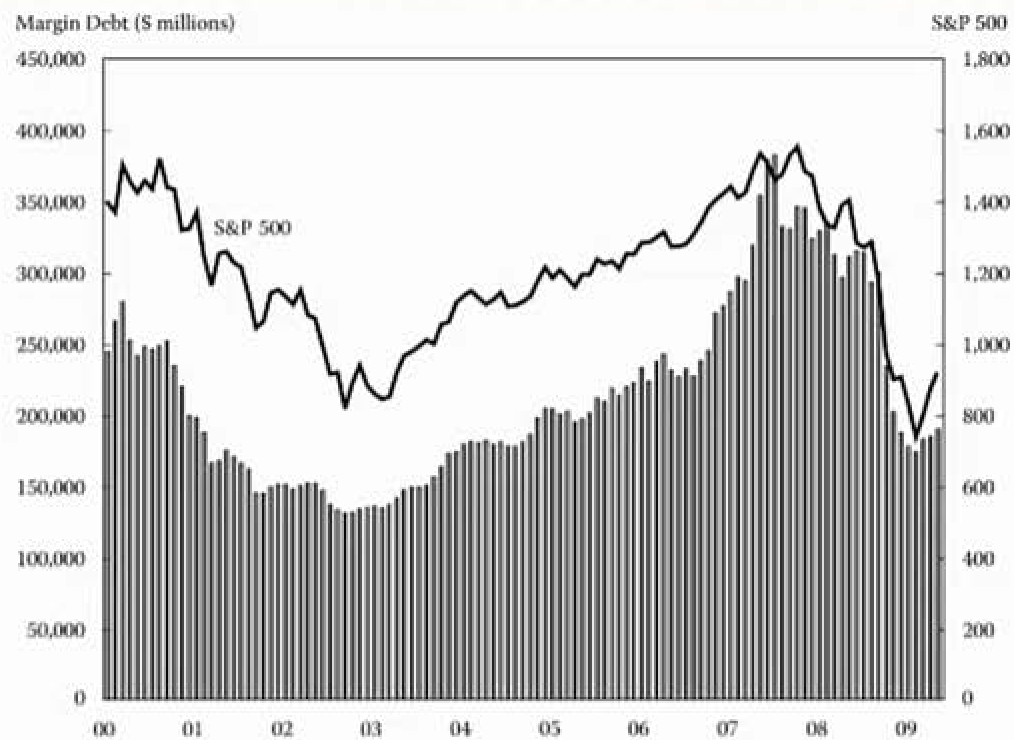
Technical Analysis

- **Margin debt** = margin buy 借钱买股票
  - When stock margin debt is **increasing**, investors are aggressively buying and stock prices will **move higher** because of increased demand.
  - **Falling** prices may trigger margin calls and forced selling, thereby driving prices **even lower**.



➤ **Short interest ratio** ⇒ short sale 做空.

- Investors sell shares short when they believe the share prices will decline. The number of shares of a particular security that are currently sold short is called "short interest."
- **Short interest ratio = Short interest / Average daily trading volume**
- If a large number of shares are sold short and the short interest ratio is high, the market should expect a falling price for the shares because of so much negative sentiment about them.
- Therefore, the short interest ratio constitutes future demand for the shares.

short sale . 做空

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P ↓ ↓

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Arms index or Short-term trading index (TRIN)

- The TRIN is a measure of funds flowing into advancing and declining stocks. The index is calculated as:

$$\text{Arms Index} = \frac{\text{Number of advancing issues} \div \text{Number of declining issues}}{\text{Volume of advancing issues} \div \text{Volume of declining issues}}$$

- When the index value close to 1, the market is in balance.
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$$\text{Arms Index} = \frac{V\downarrow / N\downarrow}{V\uparrow / N\uparrow}$$

>1 P↓

<1 P↑

Technical Analysis

cash ↑ [illegible]

- **Mutual fund cash position**
  - Technical analysts typically view mutual fund cash as a **contrarian indicator**. When mutual funds accumulate cash, this represents future ⇒ buying power in the market. A high mutual fund cash ratio therefore suggests market prices are likely to increase.
- **New equity issuance**
  - The owners want to offer the shares when they can sell them at a premium price.
  - Premium prices occur near market tops.
  - The new equity issuance indicator suggests that as the number of initial public offerings increases, the upward price trend may be about to turn down.

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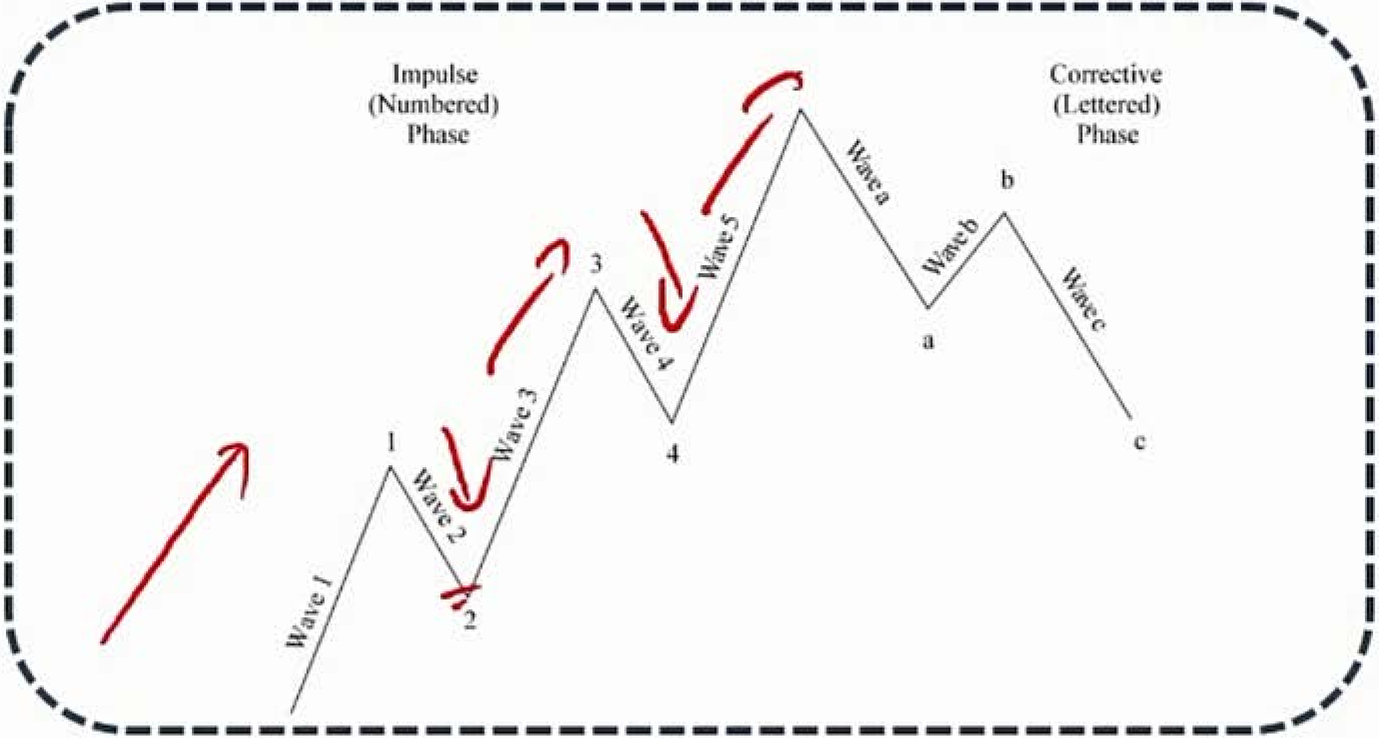
Technical Analysis

- **Cycle theory: the study of cycles in the markets is part of broader cycle studies that exist in numerous fields of study.**
  - 4-year presidential cycles: related to election years in the USA
  - Decennial patterns: 10-year cycles
  - Kondratieff wave: 18-year cycles, 54-year cycles

- **Elliott wave theory:** the market moves in regular, repeated waves or cycles.
- **Waves:** how the market moved in a pattern of five waves moving up.
  - **up trend**: consist of 5 upward waves and 3 downward waves
  - **down trend**: consist of 5 downward waves and 3 upward waves
  - **Fibonacci ratios**: the ratio of the size of subsequent waves was generally a Fibonacci ratio.



199-201

- **Intermarket analysis:** is a field within technical analysis that combines analysis of major categories of securities-namely, equities, bonds, currencies, and commodities-to identify market trends and possible inflections in a trend.
- Intermarket analysis can also be used to **identify sectors** of the equity market to invest in-often in connection with technical observations of the business cycle at any time.

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$r \uparrow$ $\quad$ $\$ \uparrow$