IID noise

No trend

no seasonal variations

n different variable from same distribution

d variable observation from the same diseribution

Distributionally, iid ness imply

independent — $f(x_1, x_2, x_3 \dots x_n) = f(x_1) \cdots f(x_n)$

Cannot used for forecasting: $f(x|y) = \frac{f(x,y)}{f(y)}$

so $f(x_{n+1} | x_1, x_2 \dots x_n) = \frac{f(x_1, \dots x_{n+1})}{f(x_1 \dots x_n)} = f(n+1)$

So iid not bared on history

iid mean 0 Gassional called white noise

Random walk — equal prob of up & down

$X_t = x_{t-1} + 1$

$x_t = r_1 + r_2 + \dots r_t \quad t = 1, 2 \dots$

r is step, equil probality

$P(r_t = 1) = \frac{1}{2}$ and $P(r_t = -1) = \frac{1}{2}$

the mean is 0 because up and down equal prob.

$\Downarrow$

$X_t - X_{t-1} = \underbrace{r_t}_{\text{previous step}}$

r_t can be any R.V, means any distribution

add a drife

$x_t = \sigma + X_{t-1} + \overset{r_t}{w_t}$

every time we have fixed part and σ

assume starting point is 0,

$x_0 = 0$

$x_1 = \sigma + w_1$

$x_2 = \sigma + x_1 + w_2 = \sigma + \sigma + w_1 + w_2 = 2\sigma + \sum_{i=1}^{2} w_i$

$x_3 = \sigma + x_2 + w_3 = 3\sigma + w_1 + w_2 + w_3 = 3\sigma + \sum_{i=1}^{3} w_i$

$\therefore X_t = \sigma t + \sum_{i=1}^{t} w_i$

$$x_t = \underline{\delta t} + \sum_{i=1}^{t} w_i$$

time series trend

Auto regression (AR) close to Random walks

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + w_t$$ ↙ AR(p) model

MA :

$$x_t = w_t + \theta_1 w_{t-1} + \cdots + \theta_q w_{t-q}$$ ← moving average MA(q) model

mean of $\{x_t\}$

general case — $\mu_x(t) = E(x_t)$

time series is white noise, is sequence of R.V, it has means

variance of $\{x_t\}$

$$\sigma_x^2(t) = V(x_t) = E[(x_t - \mu_x(t))^2]$$

convariance of $\{x_t\}$

$$\gamma_x(s,t) = cov(x_s, x_t) = E[(x_s - \mu_x(s))(x_t - \mu_x(t))]$$

Mean for Random walk with drift

$$x_t = t\delta + \sum_{i=1}^{t} w_i$$

$$\mu_x(t) = E(x_t) = E[t\delta + \sum_{i=1}^{t} w_i]$$

$E(\alpha + \beta x) = \alpha + \beta E(x)$

$$= t\delta + E[\sum_{i=1}^{t} w_i]$$

$$= t\delta + \sum_{i=1}^{t} E(w_i)$$

$\because w_i$ is Guassian white noise

$\therefore E(w_i) = 0$

$$= t\delta$$

mean of AR(p) :

$$x_t = \phi_1 x_{t-1} \cdots + \phi_p x_{t-p} + X_t$$

Problem of time series : have x variable, each variable has 1 observation
So cannot use the sample of means

means : don't have linear relationship

Auto corelation function

$$\rho(s,t) = \frac{\overset{\text{convariance}}{\gamma(s,t)}}{\sqrt{\underbrace{\gamma(s,s)}_{\text{variance } s}\underbrace{\gamma(t,t)}_{\text{variance } t}}}$$

$$= \rho = \frac{cov(s,t)}{\sigma_s \sigma_t}$$

independent variable $\Rightarrow$ cov = 0

but cov = 0 $\not\Rightarrow$ indenpend variable

Stationarity : when $f(X_{t_1+h}, X_{t_2+h}, \cdots X_{t_k+h}) = f(x_1, x_2 \cdots x_k)$

it means $(X_{t_1+h} \cdots X_{t_k+h})$ and $(x_1, x_2 \cdots x_k)$ has the same Joint distrib

e.g. $x_1 x_2 x_3 x_4 x_5$, $x_2 x_3 x_4 x_5 x_6$, $x_3 x_4 x_5 x_6 x_7$ ← as long as we have 5 samples, we can know the joint distributions if $\{X_t\}$ is week stationary,

Prove this 3 property $\Rightarrow$ say weekly stationary

$$\begin{cases} (1)\ \mu_X(t) \text{ is independent of i.e. } \mu_X(t) = \mu_X \text{ for all } t \text{ and finate} \\ (2)\ \sigma_X^2(t) \text{ is finite} \\ (3)\ \gamma_X(t+h, t) \text{ is independent of } t \text{ for each } h \end{cases}$$

when $Cov(X_1, X_2) = \sigma(1) = Cov(X_2, X_3) = Cov(X_3, X_4)$

Stationary: For Gaussian Time Seriess,

$$\gamma(t+h, t) = E(X_{t+h} - \mu)(X_t - \mu)$$
$$= E(X_h - \mu)(X_0 - \mu) \Rightarrow \gamma(h, 0)$$

Thus for stationary process $\gamma(s,t) = \gamma(s-t)$ and we write:

$$\gamma(h) = E(X_{t+h} - \mu)(X_t - \mu)$$

Same

$$\gamma(0) = \sqrt{\gamma(x_t)\gamma(x_{t+h})}$$
$$\gamma(0) \times \gamma(0)$$

autocorrelation $\rho(h) = \dfrac{\gamma(h)}{\gamma(0)}$

When time series is stationary, $\gamma(h) = \gamma(-h)$

$$\Rightarrow \rho(h) = \rho(-h)$$

For $(h \ll t) \Rightarrow$ can have good estimations

h similar to $t \Rightarrow$ Not good estimates

with stationary, mean is constant $\rightarrow$ estimate mean: $\bar{X} = \dfrac{\sum_{t=1}^{n} X_t}{n}$

For fixed h, $y_t = (X_{t+h} - \bar{X})(X_t - \bar{X})$ have the same distribution.

Then $\dfrac{\sum_{t=1}^{n} y_t}{n} \Rightarrow \gamma(h) = \Sigma(X_h - \bar{X})(X_0 - \bar{X})$, scale by variance: $\hat{\rho}(h) = \dfrac{\hat{\gamma}(h)}{\hat{\gamma}(0)}$

iid : independent variable $\Rightarrow$ covariance $= 0$

if $\{x_t\}$ is iid noise process, $\{x_t\} \sim IID(0, \sigma^2)$

$$\gamma_X(t+h, t) = \begin{cases} \sigma^2, & \text{if } h = 0 \\ 0, & \text{if } h \neq 0 \end{cases}$$

White noise process:

$\{X_t\}$ is a sequence of

uncorrelated Random variable, e.g. $\gamma_X(h) = 0$ for $h \neq 0$

Each variable having zero mean, e.g. $E(X_t) = 0$

Each variable having finite variance, e.g. $V(X_t) = \sigma^2 < \infty$

$\{X_t\} \sim WN(0, \sigma^2)$ $\rightarrow$ covariance function same as iid

random walk process:

$$X_t = w_1 + w_2 + \dots + w_t$$

where $w_t \sim WN(0, \sigma^2)$

First order AR process AR(1)

$$x_t = \varphi x_{t-1} + w_t \qquad t = 0, \pm 1, \pm 2, \ldots$$

where $\begin{cases} \{w_t\} \sim WN(0,\sigma^2) \\ |\varphi| < 1 \\ w_t \text{ is unrelated with } X_s \text{ for each } s<t \end{cases}$

$$\begin{aligned} \text{Variance}(X_t) &= \text{Variance}(\varphi X_{t-1} + w_t) \\ &= V(\varphi X_{t-1}) + V(w_t) \\ &= \varphi^2 \underset{\sigma_X^2}{\underset{\parallel}{V(x_{t-1})}} + \sigma^2 \end{aligned}$$

$w_t \sim WN(0,\sigma^2) \therefore V(w_t) = \sigma^2$

$$\sigma_X^2 = \frac{\sigma^2}{1-\varphi^2}$$

$1-\varphi^2 = 0$

$\varphi = \pm 1$

$\therefore$

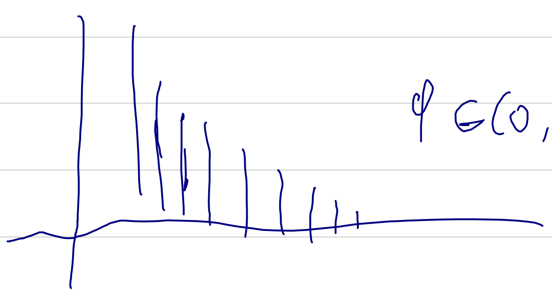
$\boxed{|\varphi| < 1}$

auto covariance function (ACVF)

$$\gamma(h) = \sigma^2 \frac{\varphi^{|h|}}{1-\varphi^2}$$

auto corelation

$$\rho_X(h) = \varphi^{|h|} \qquad (-1 < \varphi < 1)$$



$\varphi \in (0,1)$

$\varphi \in (-1,0)$

First order MA process MA(1)

$$X_t = W_t + \theta W_{t-1} \quad t = 0, \pm 1, \pm 2$$

where

$\{W_t\} \sim WN(0, \sigma^2)$

θ is a real-valued constant

auto covariance function (ACVF)

$$\gamma_X(t+h, t) = \begin{cases} \sigma^2(1+\theta^2), & \text{if } h=0 \\ \sigma^2\theta, & \text{if } h=\pm 1 \\ 0, & \text{if } |h|>1 \end{cases}$$

it is independent with h

So, MA(1) is stationary process

the autocorrelation function (ACF) of an MA(1) is given by

$$\rho_X(t+h, t) = \rho_X(h) = \begin{cases} 1, & \text{if } h=0 \\ \frac{\theta}{(1+\theta^2)}, & \text{if } h=\pm 1 \\ 0, & \text{if } |h|>1 \end{cases}$$

MA(1) is close to 0

test for iid noise using sample (ACF)

For iid noise with finit variance, for $h \neq 0$

$$\hat{\rho}(h) \sim N(0, \tfrac{1}{n})$$

Steps for diagnostic for iid noise

plot lag h vs $\hat{\rho}(h)$

Draw 2 horizontal lines at $\pm \frac{1.96}{\sqrt{n}}$ (can be drawn in R)

should have 95% of the values of $\{\hat{\rho}(h): h = 1, 2, \cdots\}$ within lines if the noise is indeed iid

$r_t = \ln \frac{P_t}{P_{t-1}}$

$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}$

$R_2 = \frac{P_2}{P_1} - 1$ or $\log P_2 - \log P_1$

return from day 1 to day 5

$R_5(4) = \frac{P_5 - P_1}{P_1}$ or $\log P_5 - \log P_1$

libary(fBasics)

> basicStats (y) ⇒ show mean, median, variance, skewness & kurtosis

If $\ln x$ — normal

⇒ X — log normal

likelihood