$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}^{-1} = \begin{pmatrix} d & b \\ c & a \end{pmatrix} \frac{1}{ad-bc}$$

$$(x-\mu)^{T} \Sigma^{-1} (x-\mu) = (x_1\ x_2) \frac{1}{\sigma^2(1-\rho)^2} \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \frac{1}{\sigma-\rho)} \left(x^2 - 2\rho x_1 x + x^2\right)$$

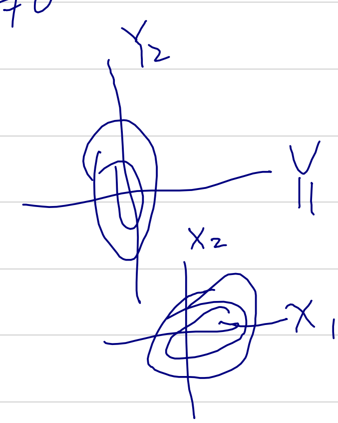
$$\Sigma = \begin{pmatrix} \sigma^2 & \sigma^2\rho \\ \sigma^2\rho & \sigma^2 \end{pmatrix}$$

(1)

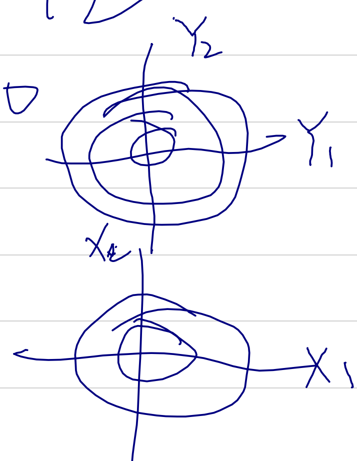
$$\frac{1}{2\sigma^2(1-\rho^2)}\left[(1+\rho)(x_1-x_2)^2 + (1-\rho)(x_1+x_2)^2\right]$$

$$= \underbrace{\frac{\left(\frac{x_1-x_2}{\sqrt{2}}\right)^2}{\sigma^2(1-\rho)}}_{= y_1} + \underbrace{\frac{\left(\frac{x_1+x_2}{\sqrt{2}}\right)^2}{\sigma^2(1+\rho)}}_{= y_2}$$

$\rho \neq 0$



$\rho = 0$



② $\frac{1}{\sigma^2(1-\rho)}\left(\frac{x_1^2-2\rho x_1x_2+\rho^2x_2^2-\rho^2x_2^2+x_2^2}{(x_1-\rho x_2)^2}\right)$

$= \frac{(x_1-\rho x_2)^2}{\sigma^2(1-\rho^2)} + \frac{(1-\rho^2)x_2^2}{\sigma^2(1-\rho^2)}$

$\Rightarrow p(x_1,x_2) = \boxed{p(x_1|x_2)} \times p(x_2)$

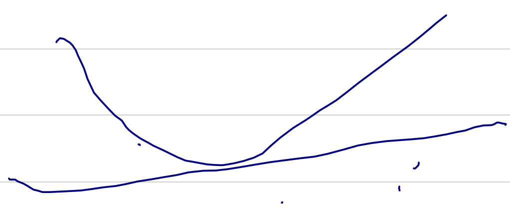
$p(x_1|x_2) \propto e^{-\frac{(x_1-\rho x_2)^2}{2\sigma^2(1-\rho^2)}}$

$KL(p(x)\|q(x)) \geq 0$

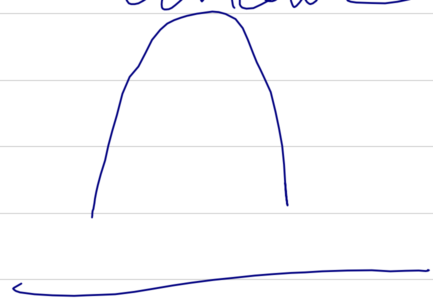
* Jensen's inequality

* convex/concave function

convex



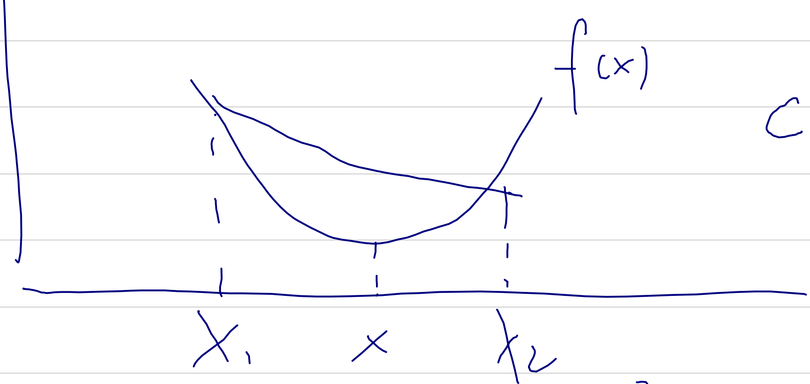
concave



$$\frac{d^2f}{dx^2} > 0$$

always positive

$$\frac{d^2f}{dx^2} < 0$$



$$c y_1 + (1-c) y_2 = c f(x_1) + (1-c) f(x_2)$$

$\forall c \in (0,1)$

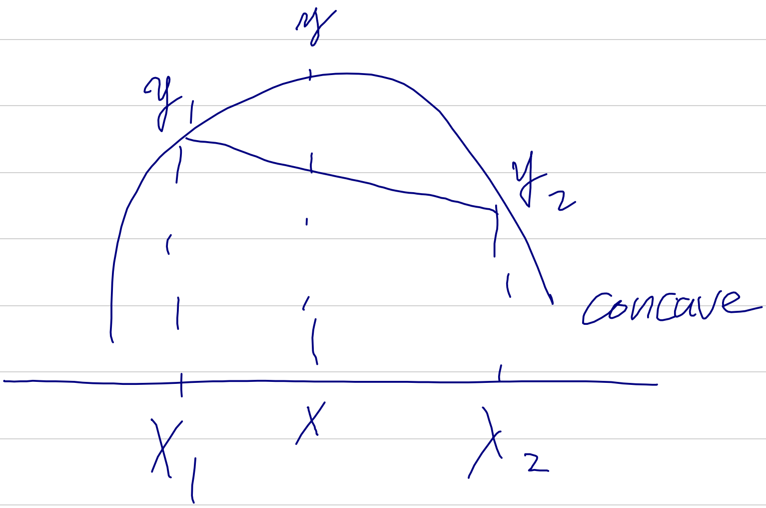
$$x = c x_1 + (1-c) x_2$$

$$\frac{d^2f}{dx^2} > f(c x_1 + (1-c) x_2)$$

Jensen's inequality

$$f(c x_1 + (1-c) x_2) \leq c f(x_1) + (1-c) f(x_2)$$

$$\Rightarrow f\left(\sum_i p_i x_i\right) \leq \sum_i p_i f(x_i) \qquad \sum_i p_i = 1$$



Goal $KL(P(x)\|q(x))$

$$= \sum_i P(x_i)\log\frac{P(x_i)}{q(x_i)}$$

$\log x \rightarrow \frac{1}{x} \rightarrow -\frac{1}{x^2}$ So log is concave function (since $\frac{d^2f}{dx^2}$ always negative)

Concave

switch

$$= -\sum_i P(x_i)\log\frac{q(x_i)}{P(x_i)} \geq \log\sum_i P(x_i)\frac{q(x_i)}{P(x_i)} = \log\left(\sum_i q(x_i)\right)$$

$$= \log 1 = 0$$

Concave:

Jensen's inequation $\sum_i P_i(f(x_i)) \leq \left(\sum_i P_i x_i\right)$