Framework

Risk Management and Investment Management

CAPM / APT

- Factor Investing
- Portfolio Construction
- Portfolio Risk Measures
- Portfolio Risk Management
- Performance Measurement and Evaluation (T^2)
- Hedge Funds

$$E_{R_i} = r_f + \beta (E_{r_m} - r_f)$$

Factors Theory

- **There are many similarities between food and assets, for example:**
  - Five macronutrients—water, carbohydrates, protein, fiber, and fat—for an "average" male, female, and child.
  - Three major risk factors are the driving force behind assets' risk premiums —market risk, credit risk and operational risk factors.
  - All Food contains different types of nutrients and the more expensive the nutrients, the higher the price of the food. 单价/贵重
  - All assets have different exposures to the market factor and the greater the exposure, the higher the risk premium.
  - Nutrients can be cheap or expensive under certain circumstance.
  - Assets have different payoffs during high or low inflation periods or during economic recessions and expansions, in other words, risk premiums can be cheap or expensive under certain circumstance.

Factor Risk and CAPM

- The **factor theory** is trying to explain the phenomenon:
  - Investors exposed to losses during bad times are compensated by risk premiums in good times.
- **The basic intuition of the CAPM still holds true:**
  - that the factors underlying the assets determine asset risk premiums and that these risk premiums are **compensation for investors' losses during bad times.**

2

$$E(R_i) = r_f + \beta(E_m - r_f)$$

<0 → market risk factor

<1

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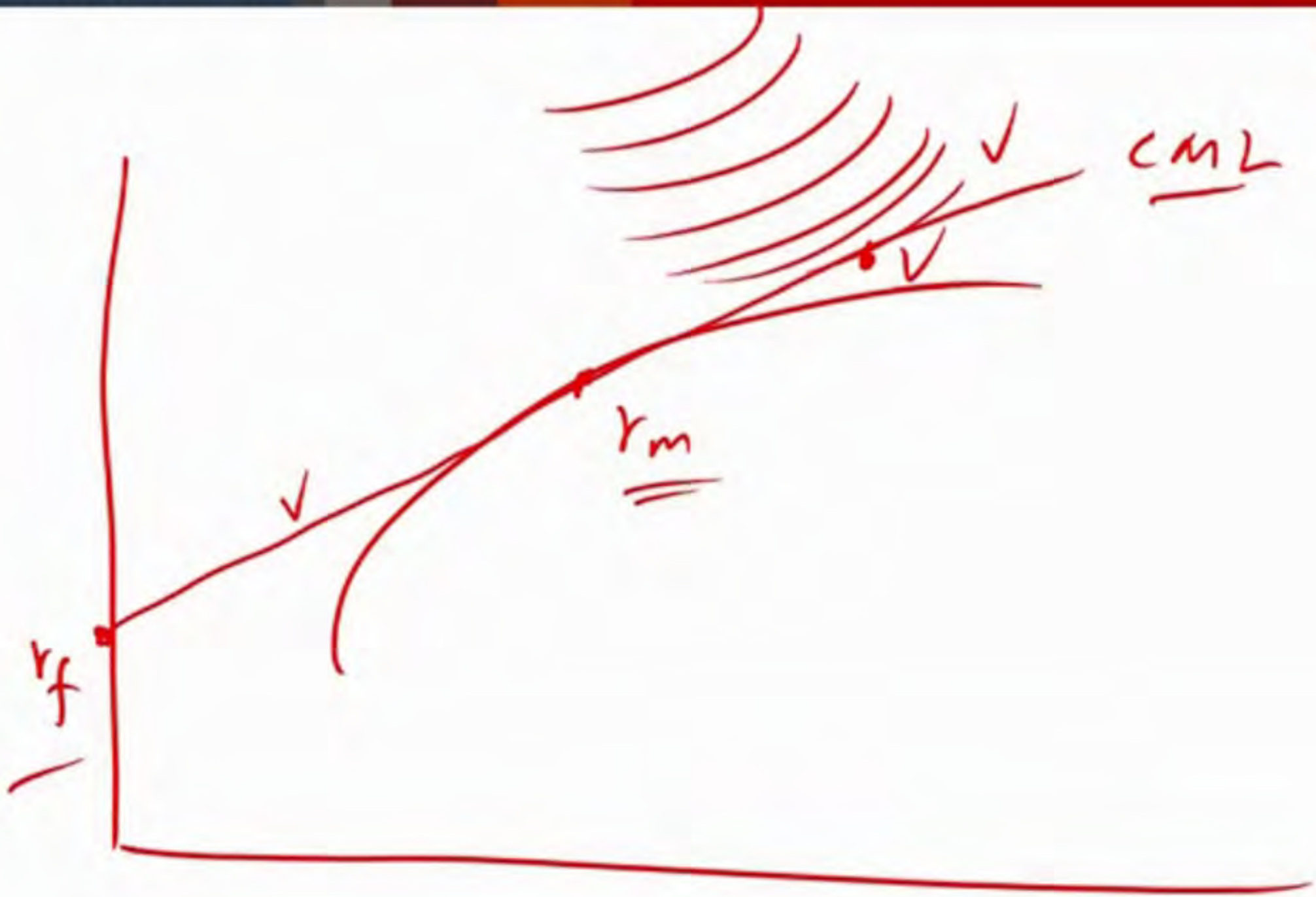
$\bar{E}R_i = r_f + \beta(\bar{E}r_m - r_f)$

25%

<0 → market risk factor

<1

long index, short r_f asset



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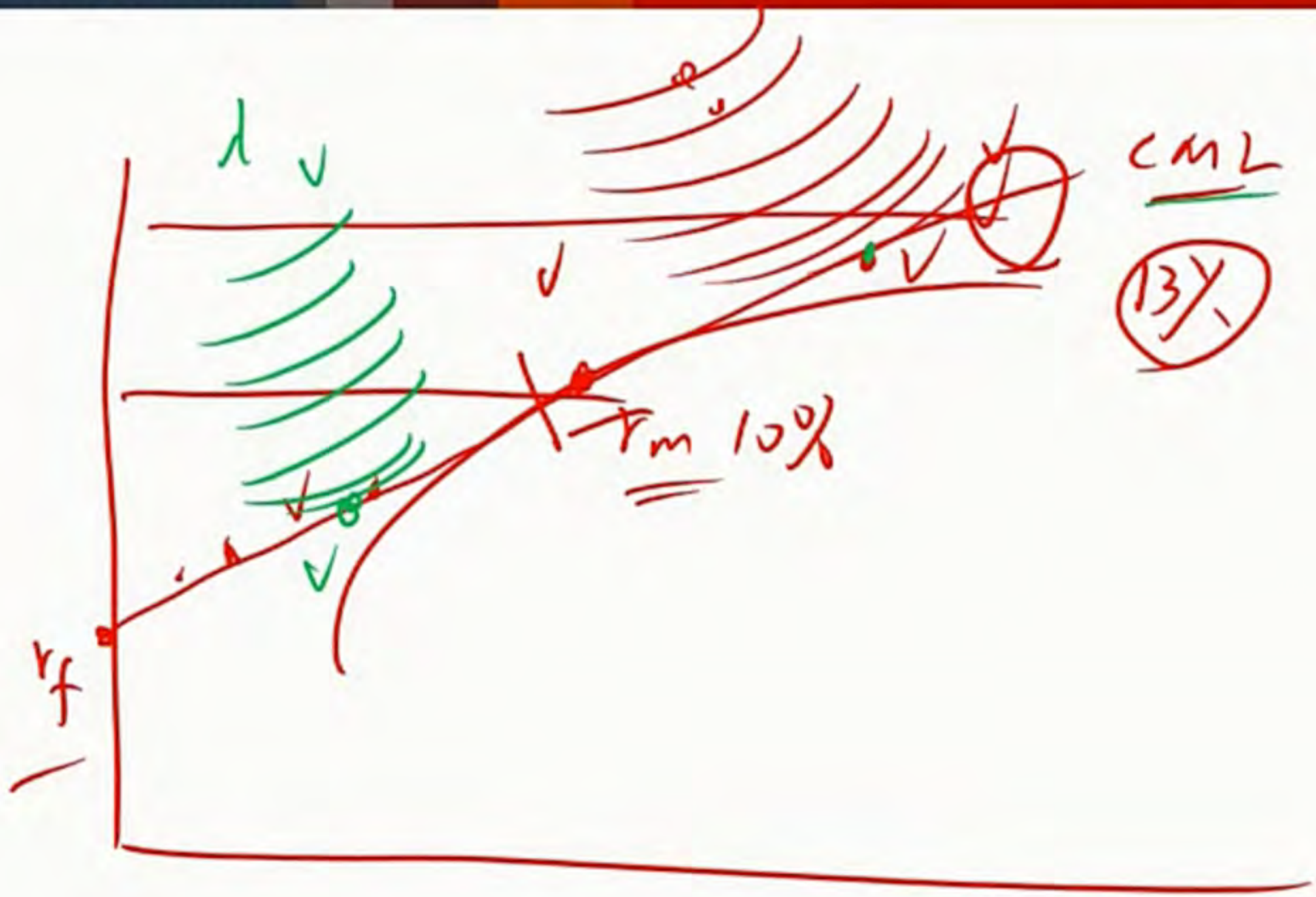
Implications of CAPM

$\beta(r_m - r_f)$

- Lessons learnt form CAPM:
  - Don't hold an individual asset, hold the factor(the market risk premium)
  - Each investor has his own optimal exposure of factor risk(CAL and CML)
  - The average investor holds the market(MVE=market portfolio)
  - The factor risk premium has an economic story
    - ✓ $E(R_m) - R_f = \bar{\lambda}\sigma_m^2$, which indicated that risk premium should be compensated by average risk aversion or the variance of the market
  - Risk is factor exposure(beta matters how much the risk is to be taken)
  - Assets paying off in bad times have low risk premiums, in some circumstance, they are also defined as valuable assets such as gold.



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Gain

黄金

Good (70%) +10%

bad (30%) −5%

$\bar{E}(R) = 0.7 \times 10\% + 0.3 \times (-5\%) = 5.5\%$

Good/bad 5.5%

Shortcomings of CAPM

- **There are seven major shortcomings of CAPM**
  - Investors only have financial wealth.
  - Investors have mean-variance utility.
  - Investors have single period investment horizon.
  - Investors have homogeneous (identical) expectations.
  - Market are frictionless.
  - All investors are price takers.
  - Information is free and available to everyone.

Shortcomings of CAPM

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  - Investors only have financial wealth. human capital
  - Investors have mean-variance utility.
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$$E(R_i) = r_f + \beta\left(E(R_m) - r_f\right)$$

Multifactor Model

- Multifactor models recognize that bad times can be defined more broadly than just bad returns on the market portfolio.
  - The first multifactor model was the arbitrage pricing theory (APT), developed by Stephen Ross (1976).
- It uses the word "**arbitrage**" because the factors cannot be arbitraged or diversified away—just like the single market factor in the CAPM.
- In equilibrium, investors must be compensated for bearing these multiple sources of factor risk.
  - While the CAPM captures the notion of bad times solely by means of low returns of the market portfolio, each factor in a multifactor model provides its own definition of bad times.

Stochastic Discount Factors

- To capture the composite bad times over multiple factors, the new asset pricing approach uses the notion of a pricing kernel. This is also called a ***stochastic discount factor*** *(SDF).*
- Just as the CAPM gives rise to assets having betas with respect to the market, multiple factors in the SDF gives rise to a multi-beta relation for an asset's risk premium:

$$E(r_i) = r_f + \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2 + \cdots + \beta_{i,k}\lambda_k$$

- where $\beta_{i,k}$ is the beta of asset i with respect to factor k and $E(f_k)$ is the risk premium of factor k.
  - ✓ For macro factors, f_1 could be inflation and f_2 could be economic growth, for example. Bad times are characterized by times of high inflation, low economic growth, or both.

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② CAPM (EF, CML SML...)

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$$E(R_i) = r_f + \beta\left(E_{rm} - r_f\right)$$

↑

系统性风险

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Efficient Market Theory

- The "classical" notions of weak, semi-strong, and strong efficiency were laid out by **Fama** (1970) and are obsolete. Fama was awarded the Nobel Prize in 2013.
- In that year, the Nobel Prize committee also gave **Robert Shiller** the prize, representing the opposite viewpoint (nearly efficient) of behavioral, or non-rational, influences on financial markets.
- Grossman and Stiglitz develop a model in which markets are **near-efficient**. Active managers search for pockets of inefficiency, and in doing so cause the market to be almost efficient.
  - In these pockets of inefficiency, active managers earn excess returns **as a reward** for gathering and acting on costly information.

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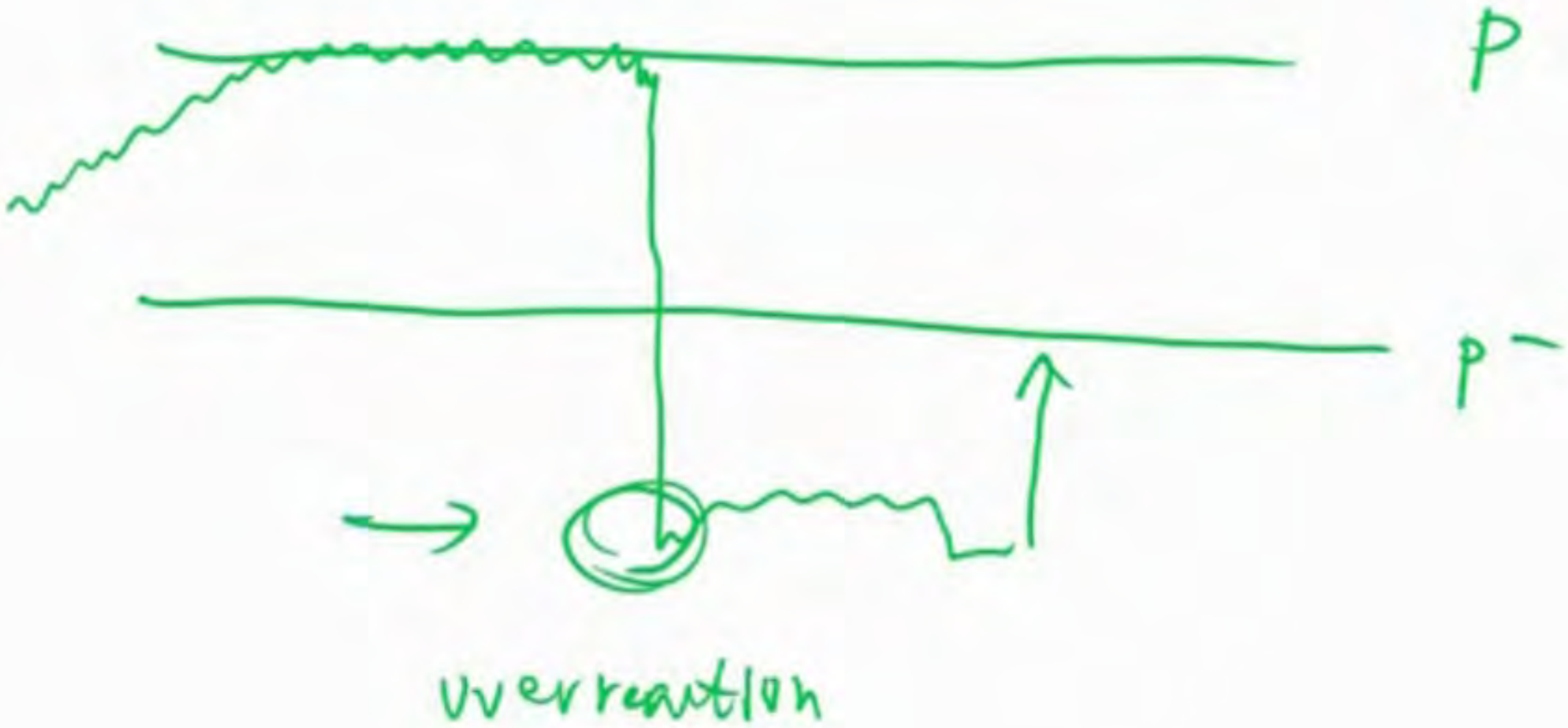
Cost (gather info)
Analysis cost
Wrong conclusion cost ←

Why Inefficient

info — price

- **In a rational explanation**, high returns compensate for losses during bad times. This is the pricing kernel approach to asset pricing.
  - The key is defining those bad times and deciding whether these are actually bad times for an individual investor.

    inflation
  - Certain investors, for example, benefit from low economic growth even while the majority of investors find these to be bad periods.
- **In a behavioral explanation**, high expected returns result from agents' under-or overreaction to news or events. Behavioral biases can also result from the inefficient updating of beliefs or ignoring some information.
- For some risk premiums, the most compelling explanations are rational (as with the volatility risk premium), for some behavioral (e.g., momentum), and for some others **a combination of rational and behavioral** stories prevails (like value/growth investing).



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Factors

Factor Investing

- factor
  - macro fundamental
    - Eco growth
    - inflation
    - volatility
  - investment style
    - static
      - market factor ($\bar{E}r_m - r_f$)
    - Dynamic
      - SMB
      - HML
      - WML

Macroeconomic Risk Factors

- Economic growth: 1
  - Risky assets generally perform poorly and are much more volatile during periods of low economic growth. However, **government bonds** tend to do well during these times. (bad time)
- Inflation: 2
  - High inflation tends to be bad for both stocks and bonds. During periods of high inflation, all assets tend to do poorly. Part of the long-run risk premiums for both equities and bonds represents compensation for doing badly when inflation is high. The **exception** exists when it refers to commodities.

Real estate

↓

抵消的

Macroeconomic Risk Factors

3 Volatility: P↑ → Equity↑ → $L = \frac{A}{E}$ ↓ → risk ↓ → σ ↓

- The negative relation between volatility and returns is called the **leverage effect**. When stock returns drop, the financial leverage of firms increases since debt is approximately constant while the market value of equity has fallen. This makes equities riskier and increases their volatilities.

A

- There is another channel where high volatilities lead to low stock returns: **an increase in volatility raises the required return on equity demanded by investors, also leading to a decline in stock prices.**

B

- Bonds offer some but not much respite during periods of high volatility, as the correlation between bond returns and VIX changes is only 0.12. Thus, bonds are not always a safe haven when volatility shocks hit.

$P\downarrow \rightarrow E\downarrow \rightarrow L\uparrow \rightarrow risk\uparrow \rightarrow \sigma\uparrow$

Expected return

Realized return

$P\downarrow \rightarrow E\downarrow \rightarrow L\uparrow \rightarrow risk\uparrow \rightarrow \sigma\uparrow$

Expected return

$E(R)\uparrow$

Realized return

$R(r)\downarrow$

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$\rho(\sigma, E(R)) > 0$ $\rho(\sigma, R_i) < 0$

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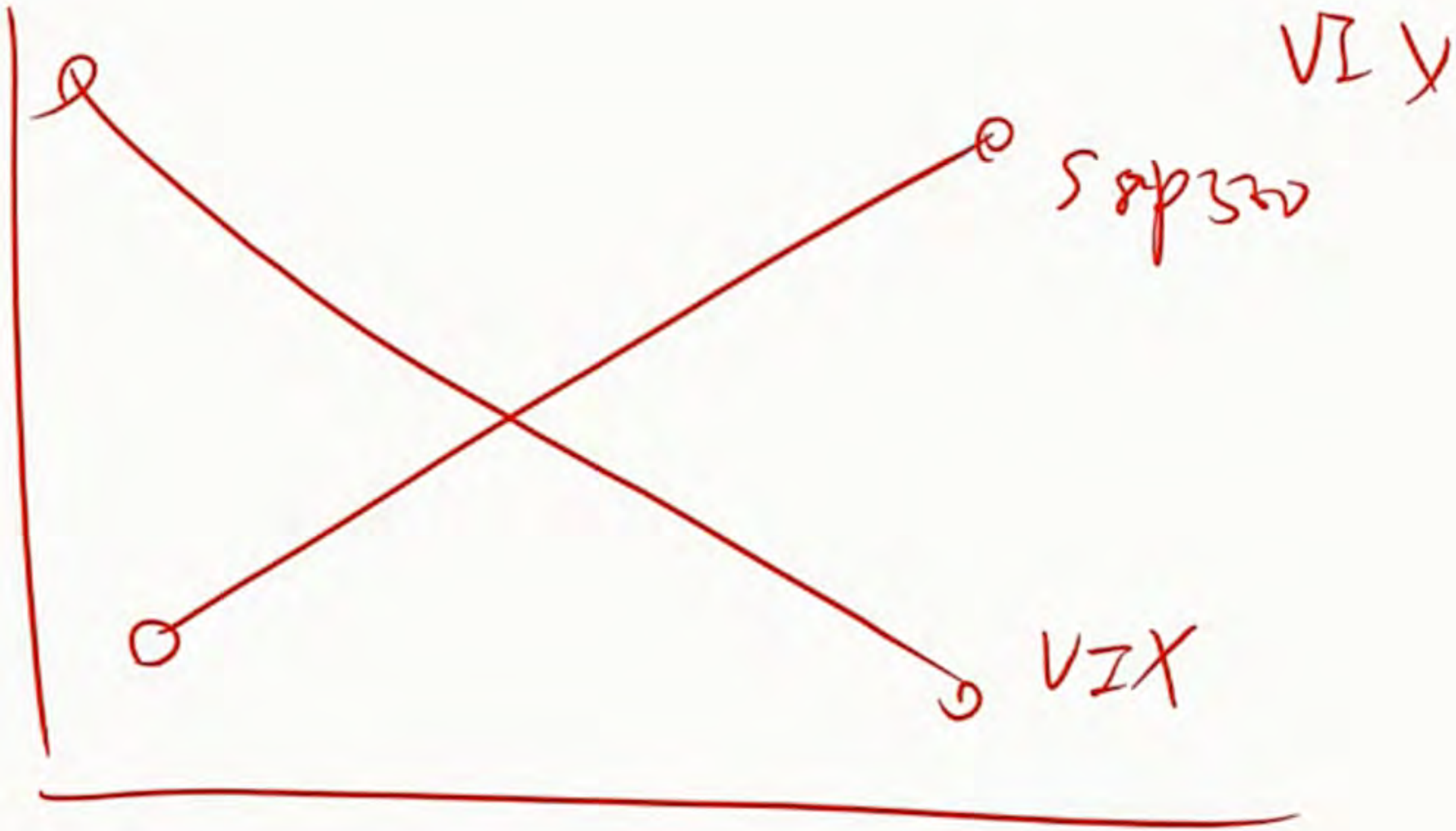

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Macroeconomic Risk Factors

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Mitigating Volatility Risk

- Investors who dislike volatility risk can buy volatility protection
  - **Buying volatility protection can benefit from increasing market volatility.**
- There are many ways to buy volatility
  - Buying or selling volatility protection can be done in option markets, but traders can also use other derivatives contracts, such as **volatility swaps**.
  - assets that pay off during high volatility periods, like **out-of-the-money puts**, provide hedges against volatility risk. S + P
  - **Fixed income, currency, and commodity markets**, like the aggregate equity market, have a negative price of volatility risk.

Challenges to Manage Volatility Risk

- Investors who love volatility risk can sell volatility protection
  - Although selling volatility produces high and steady payoffs **during stable times**, once every decade or so, there is a **huge crash** where sellers of volatility experience large, negative payoffs.
  - Unfortunately, some investors who **sold volatility prior to the financial crisis** failed to anticipate that a crash like the one of 2008 would materialize.
  - Constructing valuation models with volatility risk can be tricky because the **relation between volatility and expected returns is time varying** and **switches signs** and is thus very hard to pin down.

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Dynamic Risk Factors

- The best-known example of a tradeable multifactor model applied to present dynamic factors is introduced by **Fama and French.**
- The Fama–French (1993) model explains asset returns with three factors. There is the traditional CAPM market factor and there are two additional factors to capture a size effect and a value/growth effect:
  - ✓ $E(r_i) = r_f + \beta_{i,MKT}E(r_m - r_f) + \beta_{i,SMB}E(SMB) + \beta_{i,HML}E(HML)$

1 • **SMB factor**, which refers to the differential returns of small stocks minus big stocks

2 • **HML *factor***, which stands for the returns of a portfolio of high book-to-market stocks minus a portfolio of low book to market stocks

$R_{small} - R_{big}$

Long small cap stock

Short big cap stock

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R

净

市↑

↓

Growth

Value and Size Investment Strategies

- **Size strategy is to go long small cap stocks and short large cap stocks**
  - The risk is that some small cap companies can be large cap companies eventually.

SMB

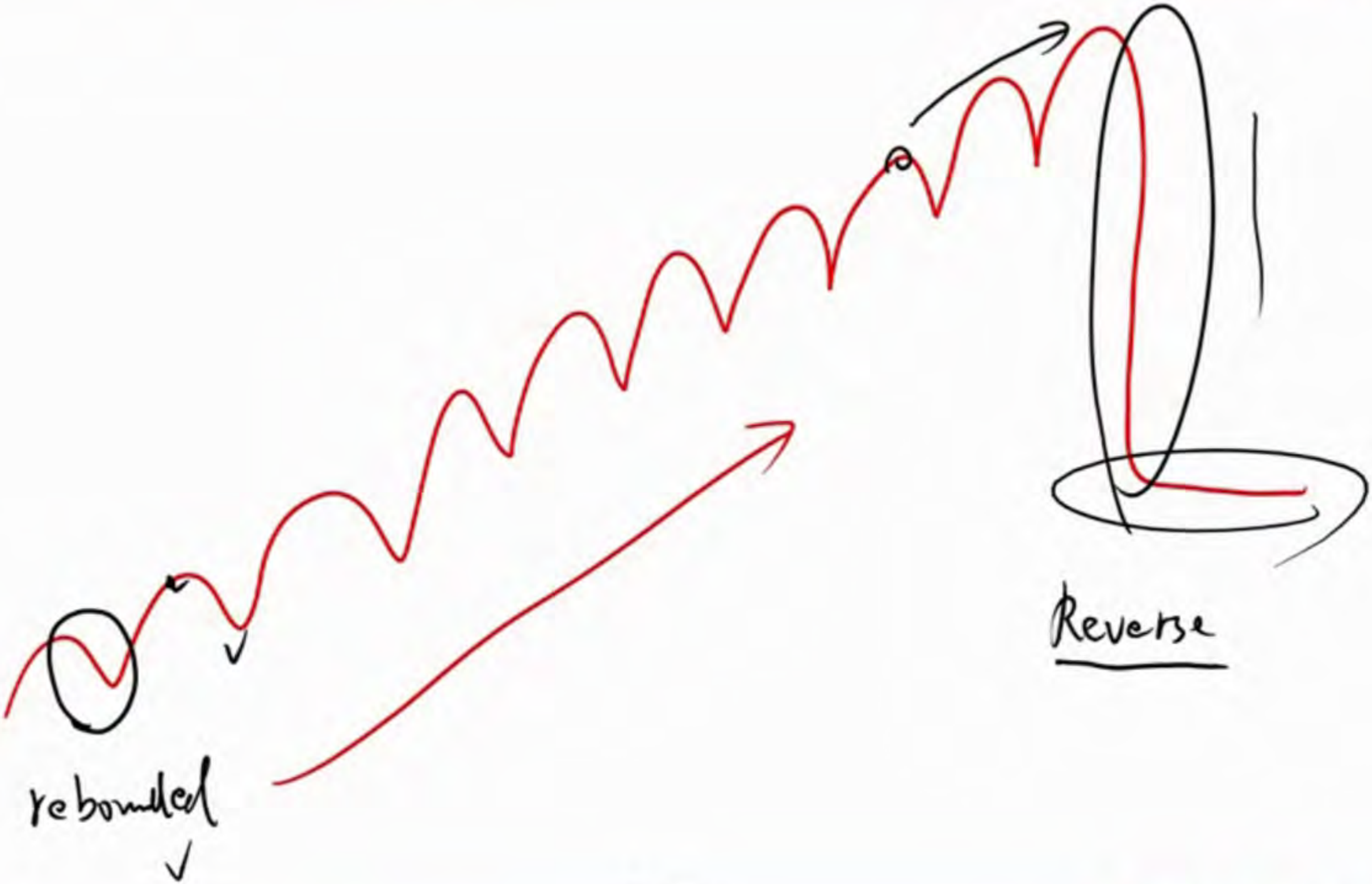
- **Value strategy is to go long value stocks and short growth stocks**
  - The risk of the value strategy is that although value outperforms over the long run, value stocks can underperform growth stocks during certain periods.

HML >0

2008

Momentum Investment Strategies

- **In the same year of 1993, Jagadeesh and Titman** identified a momentum effect, which is financial introduced by **Carhart in his Four-Factor model.**
- **Momentum** is the strategy of buying stocks that have gone up over the past six (or so) months (winners) and shorting stocks with the lowest returns over the same period (losers).
  - The momentum effect refers to the phenomenon that winner stocks continue to win and losers continue to lose, just like "Matthew Effect".
  - Since momentum strategies were short these stocks, momentum investors experienced large losses when these stocks **rebounded.**
  - In both the underreaction and overreaction models, prices **eventually reverse** when they revert to fundamentals in the long run.



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Value and Momentum Investment Strategies

- **Same**:
  - The momentum strategy, like size and value, is a **cross-sectional strategy**, meaning that it compares one group of stocks against another group of stocks in the cross section, rather than looking at a single stock over time.

buy and hold

- **Difference**:

Size

  - Value is a **negative feedback strategy**, where stocks with declining prices eventually fall far enough that they become value stocks.
  - Momentum is a **positive feedback strategy**. Stocks with high past returns are attractive, momentum investors continue buying them, and they continue to go up!

Size

P↑ → big cap → Sell

Value

P↑ → growth → Sell

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Alpha (and the Low-Risk Anomaly)

$\alpha\uparrow$ risk$\downarrow$

Factor Investing

$\alpha\downarrow$ risk$\uparrow$

Ideal Benchmark

- Well defined
- Tradeable
- Replicable
- **Adjusted for risk**

GOLDEN FUTURE

R_P — 30% r_f (0%)

70% portfolio (10%)

$\downarrow$

20%

R_B

$\downarrow$

S&P 500

$\downarrow$

8%

$\downarrow$

$20\% \times 10\% + 30\% \times 0$

$= 2\%$

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Fundamental Law of Active Management

$$IR \approx IC \times \sqrt{BR}$$

- IR is the information ratio;
- IC is the information coefficient, which is the correlation of the manager's forecast with the actual returns (how good the forecasts are);
- BR is the **breadth** of the strategy (how many bets are taken). Breadth is the number of securities that can be traded and how frequently they can be traded.

➢ **Assumptions:** Ignore transactions costs, restrictions on trading, and other real-world considerations

➢ **Limitation：** The fundamental law ignores downside risk and other higher moment risk while assuming that all information is used optimally.

- A crucial assumption is that the forecasts are independent of each other.

Impact of Benchmark Choice on Alpha

- **Failing to adjust the benchmark for risk can make a huge difference in the alpha!**
  - Example:
    - ✓ Take the R1000 index as a benchmark to get an alpha=0.0150

$$r_t = 0.0150 + r_t^{R1000} + \varepsilon_t$$

    - ✓ Take the combination of R1000 and risk free asset to get an alpha=0.0344

$$r_t = 0.0344 + 0.2728 r_t^f + 0.7272 r_t^{R1000} + \varepsilon_t$$

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$$r_f = \alpha + \underbrace{\left[(1-\beta) r_f + \beta \cdot R^{R1000}\right]}_{\text{Benchmark}}$$

R_P

30% r_f (0%)

70% portfolio (10%)

R_B

S&P 500

8%

-1%

20%

R_B^*

30% r_f 0%

70% S&P 500 8%

20% × 10% + 30% × 0

= 2%

α = 1.4%

$R_B^* = 30\% \times 0 + 70\% \times 8\%$

= 5.6%

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Alpha (and the Low-Risk Anomaly)

$\alpha\uparrow$ risk$\downarrow$

Factor Investing

$\alpha\downarrow$ risk$\uparrow$

Further Study: Factor Regression

- **The first approach:** Estimate the risk-adjusted factor benchmark
  - CAPM Benchmark

$(1-\beta) r_f + \beta R_m$

$$r_{it} - r_{ft} = \alpha + \beta(r_{mt} - r_{ft}) + \varepsilon_{it}$$

  - Size and Value-Growth Benchmarks

$$r_{it} - r_{ft} = \alpha + \beta(r_{mt} - r_{ft}) + sSMB_t + hHML_t + \varepsilon_{it}$$

  - Adding Momentum

V $$r_{it} - r_{ft} = \alpha + \beta(r_{mt} - r_{ft}) + sSMB_t + hHML_t + uUMD_t + \varepsilon_{it}$$

$$r_{it} = \alpha + (1-\beta) r_{ft} + \beta r_{mt} + s(R_s - R_B) + h(R_v - R_g) + u(R_w - R_l$$

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  - Adding Momentum

$$r_{it} - r_{ft} = \alpha + \beta(r_{mt} - r_{ft}) + sSMB_t + hHML_t + uUMD_t + \varepsilon_{it}$$

$\beta + s + h + u = 100\%$

$$r_{it} = \alpha + (1-\beta)r_{ft} + \beta r_{mt} + s(R_S - R_B) + h(R_V - R_G) + u(R_W - R_L)$$

$1-\beta+\beta+s-s+h-h+u-u=100\%$

Further Study: Factor Regression

- **The second approach:** Mimic portfolio
  - Without risk free asset: A benchmark is a passive portfolio of index funds in stocks and bonds

$$r_{it} = \alpha + \beta_s r_{st} + \beta_b r_{bt} + \varepsilon_{it}$$

$\beta_s + \beta_b = 100\%$

  - Adding real estate

$$r_{it} = \alpha + \beta_{REIT} REIT_t + \beta_s r_{st} + \beta_b r_{bt} + \varepsilon_{it}$$

$$r_{it} = \alpha + \beta_1 r_f + \beta_2 r_s + \beta_3 r_B + \varepsilon$$

100%

$$r_i = 0.3 + 0.1 r_f + 0.9 R_m + \varepsilon$$

$$r_i = 0.3 + 0.2 r_f + 0.8 R_m + 0.3 \, SMB + \varepsilon \quad \checkmark$$

$$\begin{cases} r_i = 0.4 + -0.2 r_f + 1.2 R_m + 0.3 \, SMB + \varepsilon & \text{——答案} \\ r_i = 0.4 \, SMB + \varepsilon & \text{——都对} \end{cases}$$

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Style Analysis

- **To illustrate, let's take the following index ETFs**
  - SPY: SPDR S&P 500 ETF, which is designed to mimic the S&P 500;
  - SPYV: SPDR S&P 500 Value ETF, which tracks the S&P 500 value index; and
  - SPYG: SPDR SP& 500 Growth ETF, which replicates the S&P 500 growth index.
- **The style analysis can be demonstrated through two ways**
  - Style Analysis with No Shorting
  - Style Analysis with Shorting

Style Analysis

- **Style Analysis with No Shorting**

$$r_{t+1} = \alpha_t + \beta_{SPY,t} SPY_{t+1} + + \beta_{SPYV,t} SPYV_{t+1} + \beta_{SPYG,t} SPYG_{t+1} + \varepsilon_{t+1}$$

$$\beta_{SPY,t} + \beta_{SPYV,t} + \beta_{SPYG,t} = 1$$

$$\beta_{SPY,t}, \beta_{SPYV,t}, \beta_{SPYG,t} > 0$$

- **Style Analysis with Shorting**

$$r_{t+1} = \alpha_{i,t} + \beta_{SPY,t} SPY_{t+1} + (1 - \beta_{SPY,t}) r_{f,t+1} + h_t (SPYV_{t+1} - SPYG_{t+1}) + \varepsilon_{t+1}$$

  - The SPYV-SPYG is an investment that goes long the value SPYV ETF and simultaneously shorts the growth SPYG ETF**. Thus, it is analogous to the HML factor**.
  - The strong value bias is shown with a positive h loading on the SPYV-SPYG factor.

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HML

$R_m - r_f$

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slope

Style Analysis

- **Style Analysis with No Shorting**

A

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0.7 → 0.4

$$\beta_{SPY,t} + \beta_{SPYV,t} + \beta_{SPYG,t} = 1$$

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- **Style Analysis with Shorting**

0.8 → -0.12

HML

B

$$r_{t+1} = \alpha_{i,t} + \beta_{SPY,t} SPY_{t+1} + (1 - \beta_{SPY,t}) r_{f,t+1} + h_t (SPYV_{t+1} - SPYG_{t+1}) + \varepsilon_{t+1}$$

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0.12 → 1.2

0.7 → -0.12

$$= \alpha + r_{f_{t+1}} + \beta_1 (SPY_{t+1} - r_{f_{t+1}}) + \beta_2 HML$$

Fundamental Law of Active Management

$$IR \approx IC \times \sqrt{BR}$$

- IR is the information ratio;
- IC is the information coefficient, which is the correlation of the manager's forecast with the actual returns (how good the forecasts are);
- BR is the **breadth** of the strategy (how many bets are taken). Breadth is the number of securities that can be traded and how frequently they can be traded.

➢ **Assumptions:** Ignore transactions costs, restrictions on trading, and other real-world considerations

➢ **Limitation：**The fundamental law ignores downside risk and other higher moment risk while assuming that all information is used optimally.

- A crucial assumption is that the forecasts are independent of each other.

$$\Sigma R = \frac{\alpha}{TEV} \quad O_{(\alpha)}$$

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GOLDEN FUTURE

$\rho = 0.8$

10支股票　每季度预测一次

$IR = 0.8 \times \sqrt{40}$

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Alphas for Nonlinear Strategies

- Alphas are computed in a linear framework. There are many nonlinear strategies, especially those involving **dynamic option strategies,** that can masquerade as alpha.
- **There are two ways to account for nonlinear payoffs.**
  - Include Tradeable Nonlinear Factors
    - ✓ Nonlinear factors(volatility) can also be included in factor benchmarks. By doing so, the asset owner is assuming that she can **trade these nonlinear factors** by himself/herself.
    - ✓ Common specifications include quadratic terms, like r^2_t, or option-like terms like $\max(r_t, 0)$.
  - Examine Nontradeable Nonlinearities
    - ✓ These will not be alphas, but they can still be used to rank managers and evaluate skill. <u>One state-of-the-art measure has been introduced by Goetzmann *et.al.*</u>

$$r_t = \alpha + \beta_1 (R_m - r_f) + \beta_2 SMB + \beta_3 HML$$

$$+ \beta_4 \sigma_m^2 + \beta_5 \underline{\max\{S - X, 0\}}$$

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$$+ \underbrace{\beta_4 \sigma_m^2}_{\text{VIX}} + \beta_5 \underbrace{\max\{S-X, 0\}}_{\text{option}}$$

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Alpha 结构：

- Benchmark 结构
- Alpha 没有考虑非线性
  - trade
  - non-tradable

Alpha 是动的. —— 没有考虑 TEV

↓

$IR \approx IC \cdot \sqrt{BR}$

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Low-risk Anomaly

要证明

- **The risky anomaly**—that stocks with low betas and low volatilities have high returns—appears to be a strong source of alpha relative to standard market-weighted benchmarks and value-growth, momentum, and other dynamic factors.
  - Low volatility seemed too good to be true
    - ✓ Historical data may not fit future
    - ✓ High tracking error with low benchmark volatility
  - High return seemed too good to be true
    - ✓ High alpha compared with benchmark
    - ✓ Leverage effect

Volatility Anomaly

- **Data evidence:** the returns of high-volatility stocks were "abysmally low." So low that they had zero average returns.
  - Lagged Volatility and Future Returns(negative correlation) $\rho(\sigma_{t-1}, r_t) < 0$
  - Contemporaneous Volatility and Returns(negative coorelation) $\rho(\sigma_t, r_t) < 0$
- **Conclusion:** there is still a negative relation between contemporaneous volatility and returns.
  - Thus, the most volatile stocks currently lose money (which we cannot forecast), and they also tend to lose money in the future as well (which is predictable).

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σ, realized return

Beta Anomaly

- **Data evidence:** the relation between beta and returns to be **"too flat"** compared with what the CAPM predicted, but at least the relation was positive. In fact, their estimated relation between beta and returns was statistically **insignificant.**
  - Lagged Beta and Future Returns(betas are noisy) $\rho = 0.1$
  - Contemporaneous Beta and Returns(positive) $(\beta_t, r_t) > 0$
- **Conclusion: the real mystery in the low-beta anomaly is actually not so much that beta does not work**
  - it is that we have such difficulty in predicting future betas, especially with past betas.

Risk Anomaly Explanations

- Data Mining
- Leverage Constraints (杠杆约束) < built in leverage >
  - They can't borrow money and instead choosing to invest in high beta stocks
    高贝塔
- Agency Problems
  - Many institutional managers have constraints against short selling, which allows mispricing to be existing.
- Preferences
  - If asset owners simply have a preference for low-volatility and low-beta stocks.
    大妈买黄金

Jan effect

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Application of Factor Regression

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slope

Illiquid Assets

Factor Investing

Characteristics of Illiquid Markets

- Characteristics
  - Most Asset Classes Are Illiquid
  - Illiquid Asset Markets Are Large
  - Investors Hold Lots of Illiquid Assets
  - Liquidity Dries Up
- Illiquid asset classes as a whole are larger than the traditional liquid, public markets of stocks and bonds. **Even normally liquid markets periodically become illiquid.**
- Most investors' wealth is tied up in illiquid assets. Thus asset owners must consider illiquidity risk in the construction of their portfolios.
  - Doing this requires estimating risk-return **trade-offs** of illiquid assets, but measuring illiquid asset returns is not straightforward.

The Source of Illiquidity Market Imperfections

- **Clientele effects or Participation Cost**
  - Entering markets can be costly; investors often must spend money, time or energy to learn their way and gain the necessary skills.
- **Transaction Cost**
  - Commissions, taxes, costs of due diligence, fees paid to lawyers, accountants and investment bankers.
- **Search Frictions**
  - For many illiquid assets, you need to search to find an appropriate buyer or seller. Therefore, need to wait a long time to transact.

The Source of Illiquidity Market Imperfections

- **Asymmetric Information**
  - Markets can be illiquid because one investor has superior knowledge compared with other investors. Fearing they'll be fleeced, investors become reluctant to trade.
- **Price Impact**
  - Large trades will move markets.
- **Funding Constraints**
  - Many of the investment vehicles used to invest in illiquid assets are **highly leveraged**. If access to credit is impaired, investors cannot transact in illiquid asset markets.

$$RP = R_p - r_f$$

market RP

illiquidity RP

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Biases on Returns for Illiquid Assets

- **Illiquid asset reported returns are not returns**
  - Three key biases cause people to overstate expected returns and under state the risk of illiquid asset:
    - ✓ Survivorship bias
    - ✓ Selection bias
    - ✓ Infrequent sampling
- **Survivorship bias**
  - Survivorship bias results from the tendency of poorly performing funds to stop reporting. Many of these funds ultimately fail—but we only rarely count their failures. This makes true illiquid asset returns worse than the reported data.

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(历史 return)

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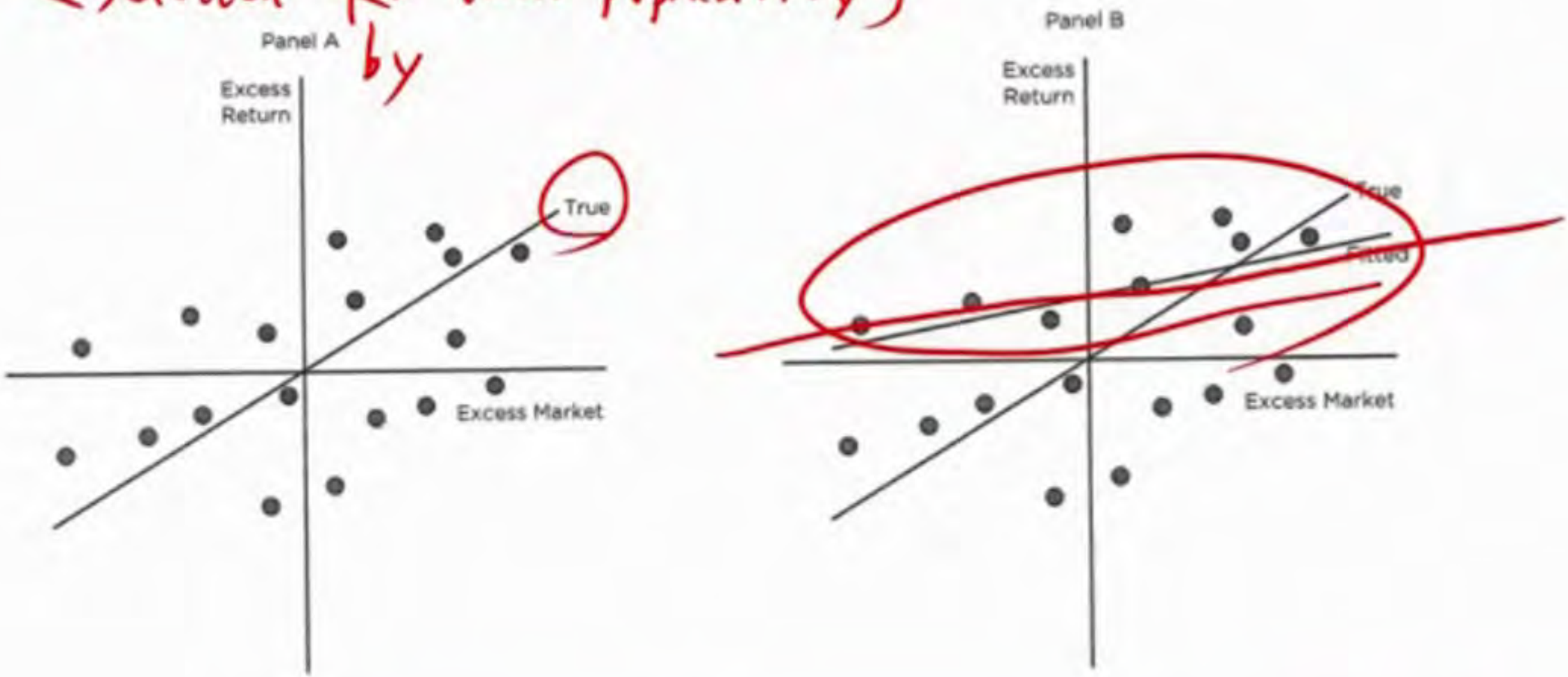
Biases on Returns for Illiquid Assets

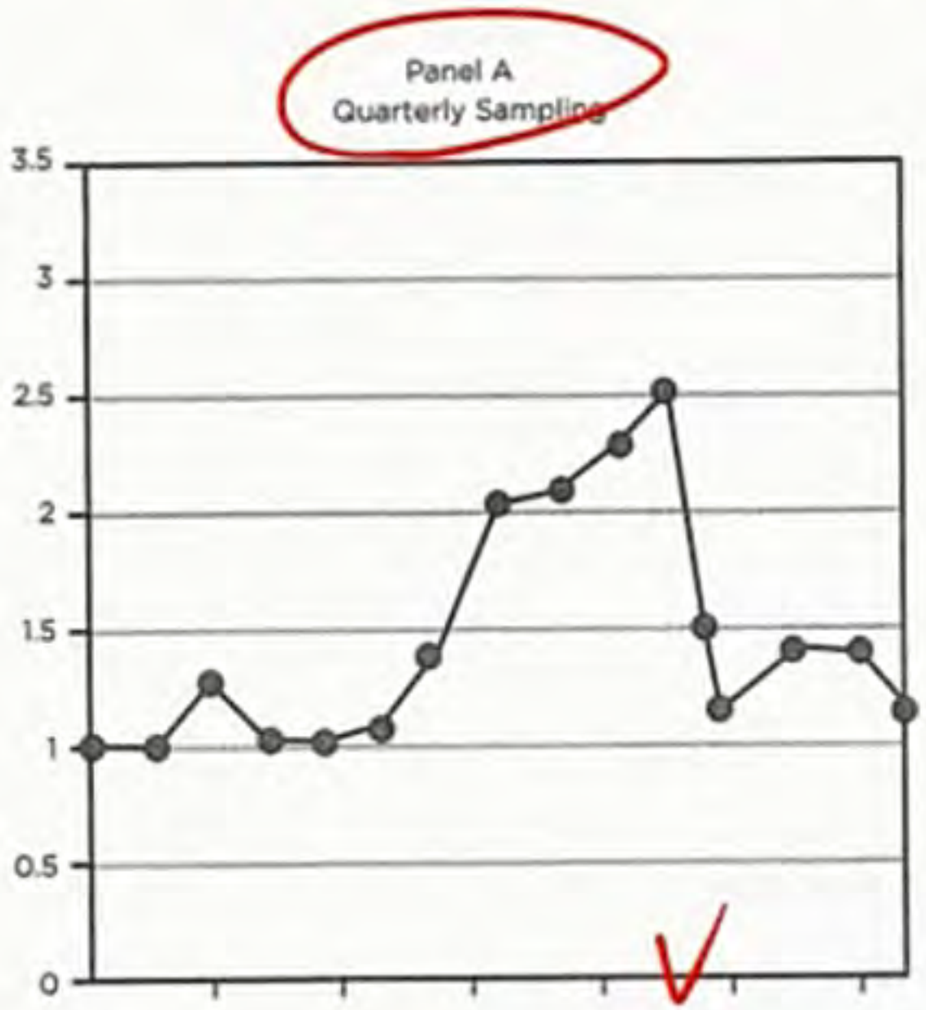
- **Selection bias.** 高估 return
  - Sample selection bias results from the tendency of returns only to be observed when underlying asset values are high, and result in underestimated risk as measured buy beta and volatility.

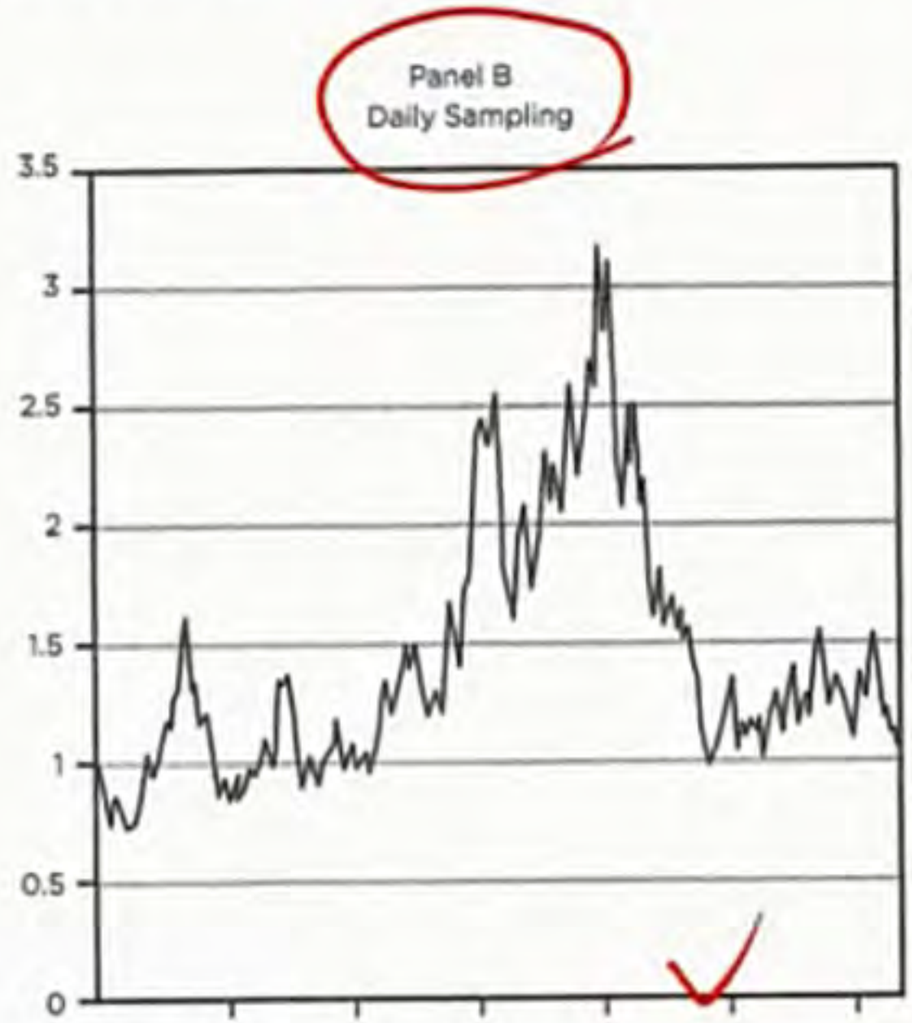
<(selected ~~for~~ by their popularity)



Biases on Returns for Illiquid Assets

- **Infrequent sampling bias**
  - With infrequent trading, estimates of risk—volatilities, correlations, and betas—are too low when computed using reported returns. Thus risk estimates are biased downward by infrequent sampling.





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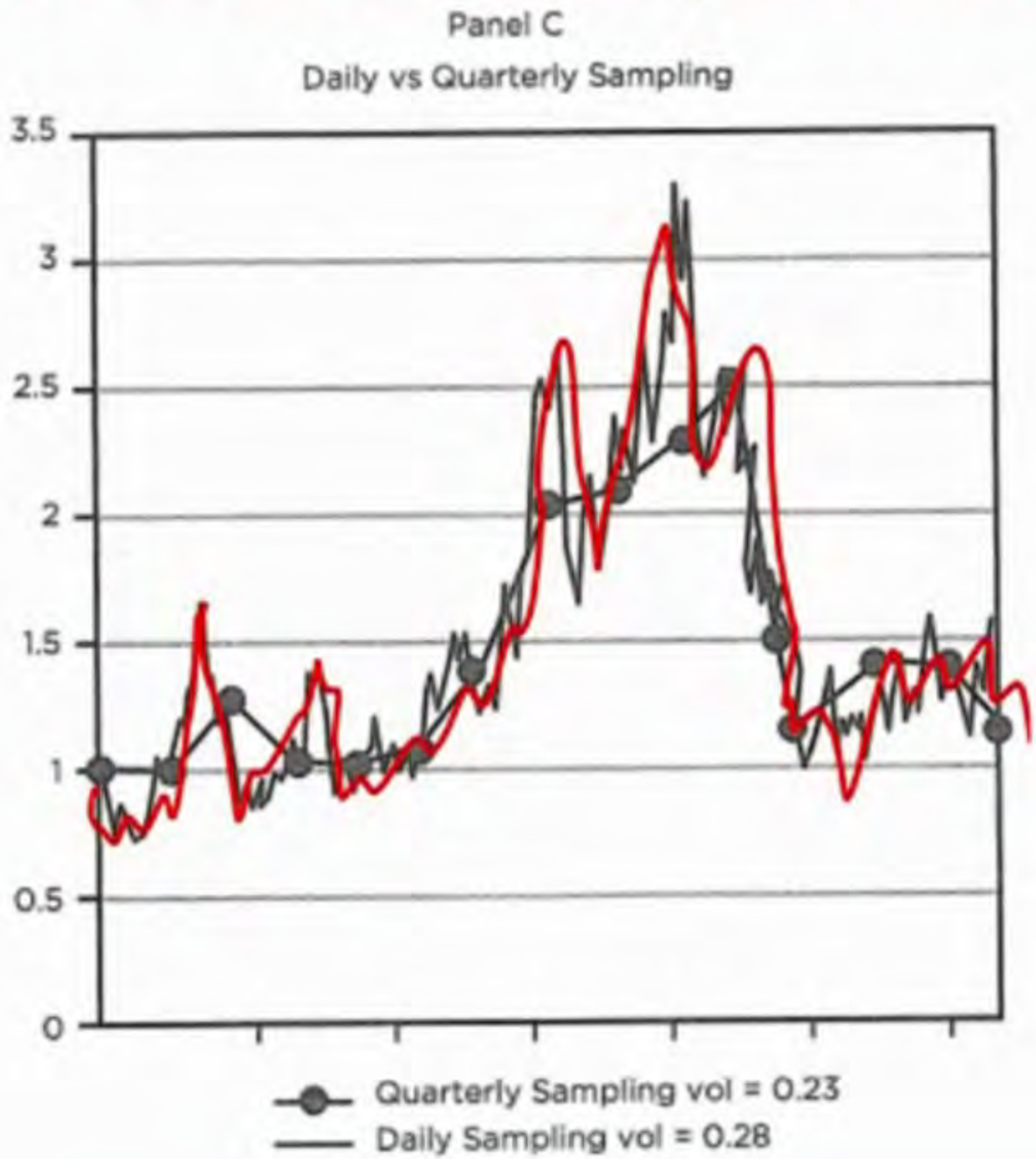

Biases on Returns for Illiquid Assets

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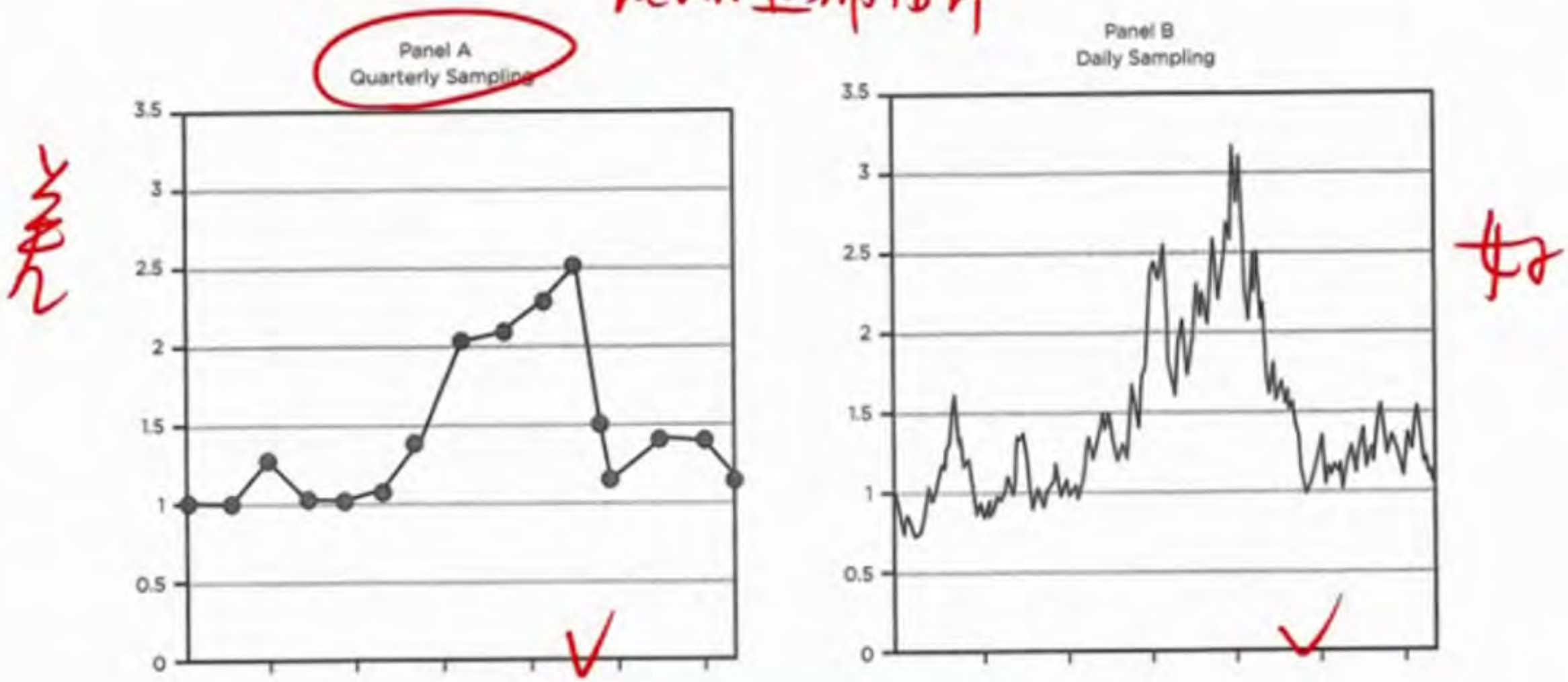
return 正确估计



$$Rp = R_p - r_f$$

market RP

illiquidity RP

$$\uparrow SR = \frac{R_p - r_f}{\sigma_p} \downarrow$$

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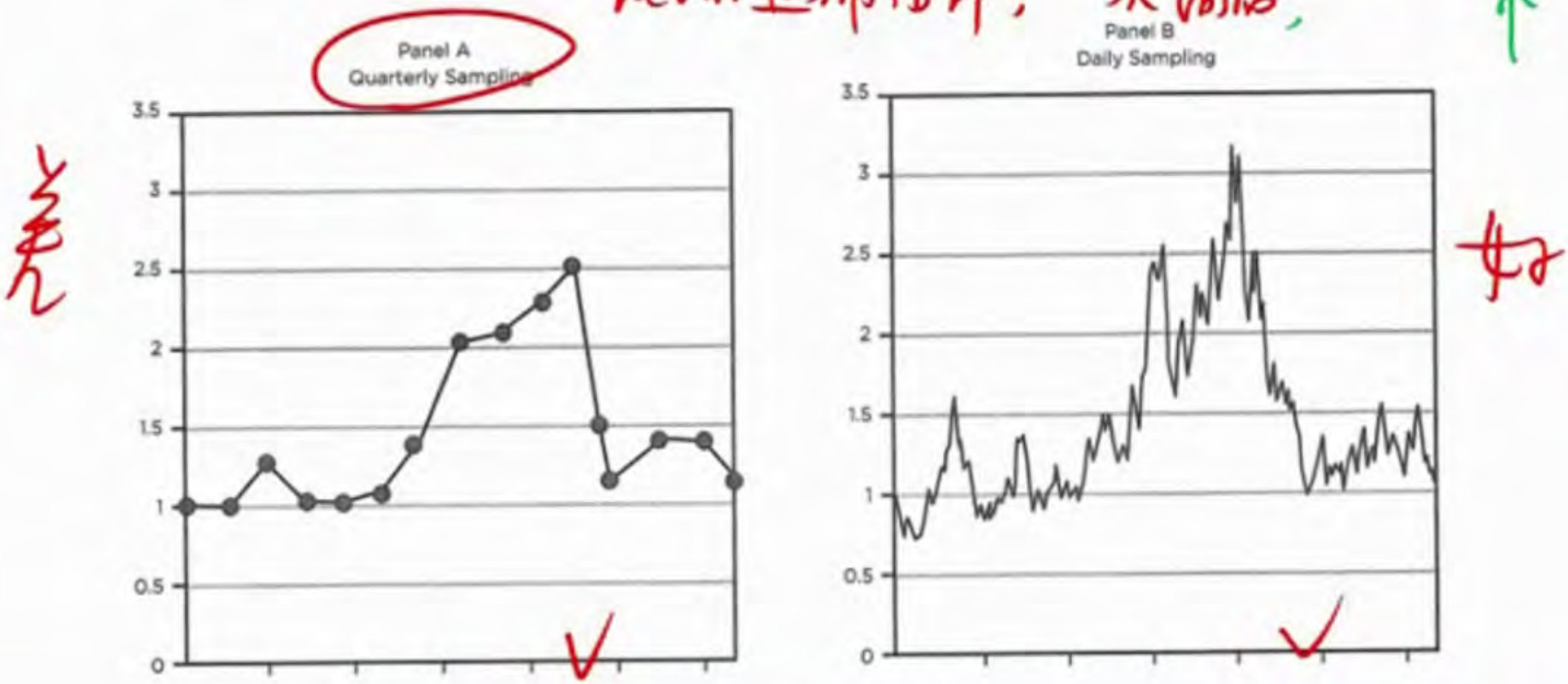
Autocorrelation ↑

return 正确估计，SR 高估，



差

好

Exercise

- For a portfolio of illiquid assets, hedge fund managers often have considerable discretion in portfolio valuation at the end of each month and may have incentives to smooth returns by marking values below actual in high-return months and above actual in low-return months. Which of the following is not a consequence of return smoothing over time?

A. Higher Sharpe ratio

B. Lower volatility

C. Higher serial correlation

D. Higher market beta

Correct Answer: D

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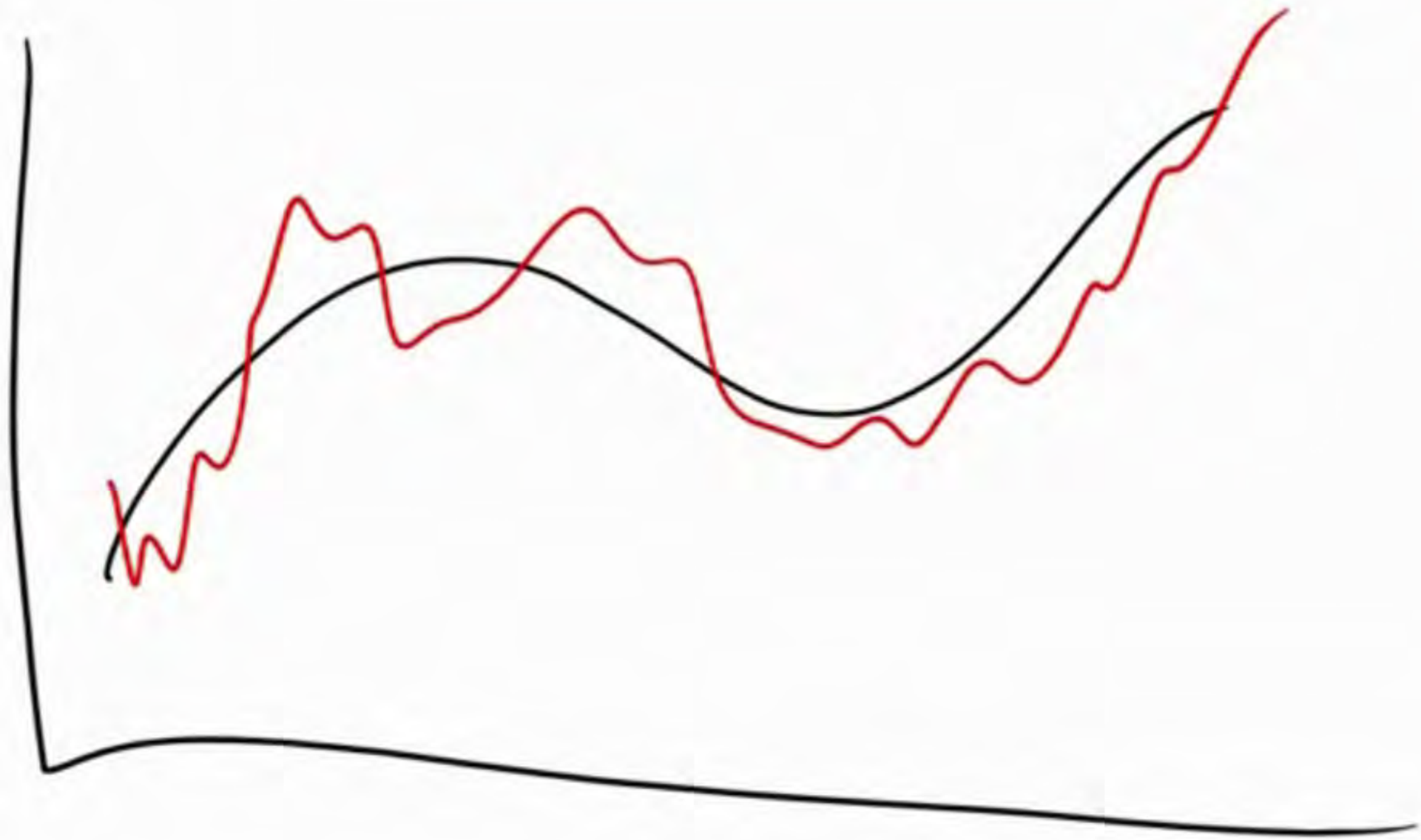

Unsmoothing Returns——Filter Problem

- To account for the infrequent trading bias, we need to go from the samples which are too smooth to noisier datas. This process is called unsmoothing or de-smoothing. **Unsmoothing is a filter problem**.
- **Properties**
  - Unsmoothing affects only risk estimates and not expected returns.
    - ✓ Intuitively, estimates of the mean require only the first and last price observation
  - Unsmoothing has no effect if the observed returns are uncorrelated.
  - Unsmoothing is an art.

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autocorrelation = 0

Collect Illiquidity Risk Premiums

- **There are four ways to collect Illiquidity Risk Premiums**
  - holding passive allocations to illiquid asset classes 买房子 (across asset class)
  - holding less liquid securities within asset classes stock
  - market making at the individual security level bid - ask spread
  - dynamic rebalancing at the aggregate level: **the simplest** to implement and has the greatest impact on portfolio returns.

60% stock 40% bond

70% ↑↑ 30%

Collect Illiquidity Risk Premiums

- **Across asset classes**
  - there is a reward to bearing illiquidity across asset classes.
    - ✓ Illiquidity biases.(overestimated asset returns)
    - ✓ Much ignored risk.(Illiquid asset classes contain far more than just illiquidity risk) fraud risk
    - ✓ There is no "market index" for illiquid asset classes. REIT
    - ✓ You cannot separate factor risk from manager skill.(investing in illiquid markets is always a bet on management talent.)

Collect Illiquidity Risk Premiums

- **Within asset classes**
  - Within all the major asset classes, these illiquidity premiums can be accessed by dynamic factor strategies which take long positions in illiquid assets and short positions in liquid ones.
    - ✓ **U.S. Treasury bond**: on-the-run/off-the-run
    - ✓ **Corporate bond: one that** trades less frequently or has larger **bid–ask** spreads has higher returns
    - ✓ **Equity**: bid–ask spreads, volume, turnover)
    - ✓ **Illiquidity asset:** Hedge funds that place more restrictions on the withdrawal of capital

Collect Illiquidity Risk Premiums

- large illiquidity risk premiums exist **within asset classes** but not across asset classes
  - limited integration across asset classes
    - ✓ Switching capital and investment strategies is significantly constrained.
  - Illiquidity across asset class might be smaller
    - ✓ investors overpay for illiquid asset across asset classes and thus drives the illiquidity risk premium down.
  - Illiquidity within asset class might be larger
    - ✓ Illiquidity-shy investor overpay for liquidity asset within asset class and thus drives the risk premium up.

✓ on

off

illiq risk premium = $\underline{R_{off}} - R_{on}$

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illiq risk premium = $\underline{R_{off}}$ − R_{on}

P_{on} ↑

YTM ↓

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✓ on

off

Portfolio Choice Decisions

- **Asset allocation with transactions costs**
  - Illiquid asset investors should expect to rebalance very infrequently.
  - Paying a cost can not solve illiquidity.
- **Asset allocation with infrequent trading**
  - Illiquid assets are illiquid and cannot be traded, unless the liquidity event arrives.
  - Illiquidity risk causes the investor to behave in a more **risk-averse** fashion toward both liquid and illiquid assets.
    - ✓ Illiquidity markedly reduces optimal holdings
    - ✓ Rebalance illiquid assets to positions below the long-run average holding
    - ✓ Consume less with illiquid assets
    - ✓ There are no illiquidity "arbitrages"
    - ✓ Investors must demand high illiquidity hurdle rates(premiums)

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(历史 return)

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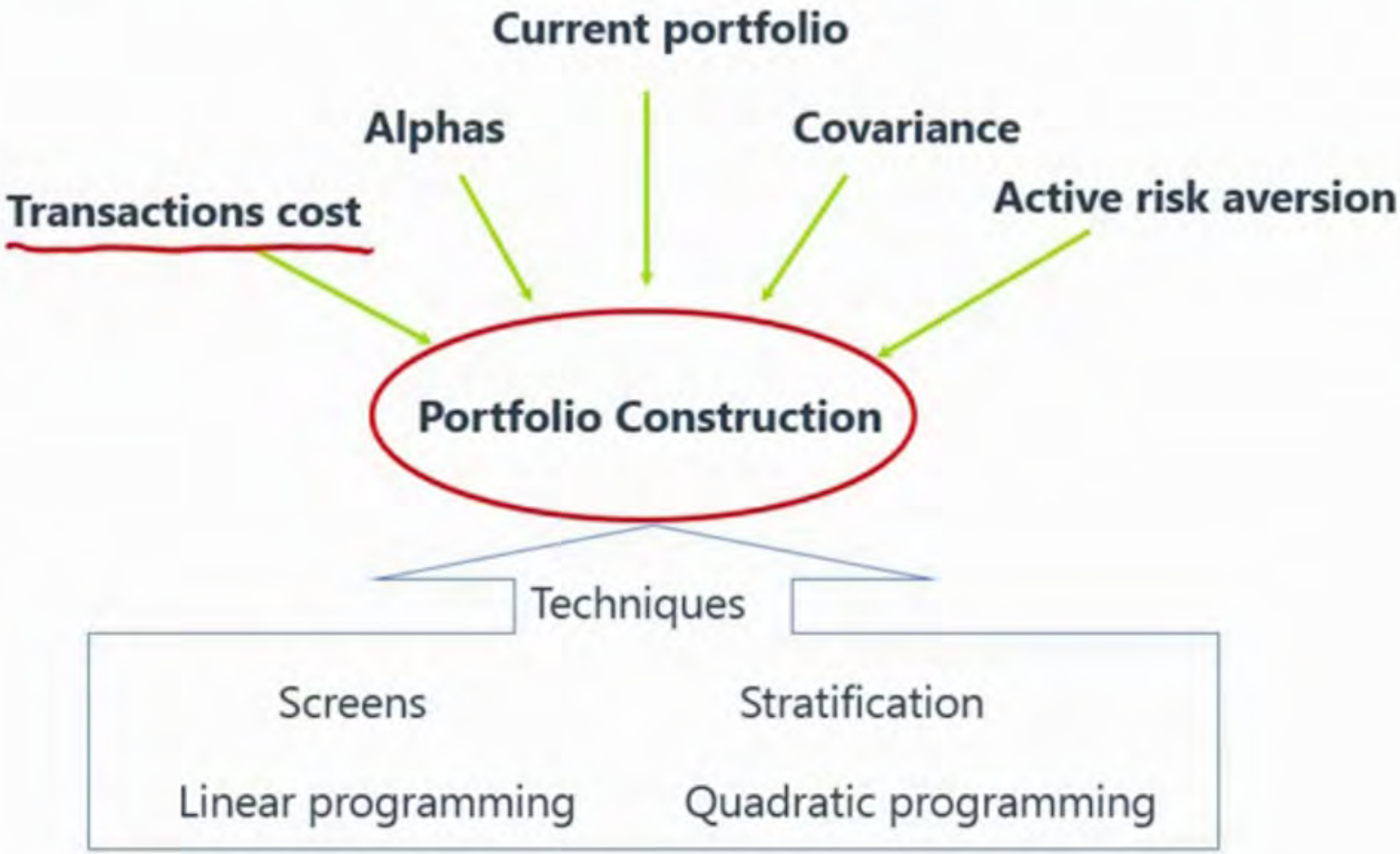
Reading 2

Portfolio Construction

Inputs to the Portfolio Construction Process



Inputs to the Portfolio Construction Process

- **Portfolio construction requires several inputs:**
  - The current portfolio: of these inputs, we can measure only the current portfolio **with near certainty**.
  - Alphas
  - Covariance estimates
  - Transactions cost estimates
  - Active risk aversion: Most active managers will have a target level of active risk that we must make consistent with an active risk aversion.

Refining Alphas

Data cleaning

- With alpha analysis, the alphas can be adjusted so that they are in line with the manager's desires for risk control and anticipated sources of value added.
  - Scale the alphas
  - Trim alpha outliers
  - Neutralization
    - ✓ Benchmark- and Cash-Neutral Alphas
    - ✓ Risk-Factor-Neutral Alphas

Scale the alphas

IR ≈ IC · √BR

- **Alpha has a natural structure called Grinold Rule:**

$$\alpha = \sigma \times IC \times Score$$

TEV

  - We expect the information coefficient (IC) and residual risk (volatility) for a set of alphas to be constant.

Score ~N(0,1²)

  - Score follows a standard normal distribution, hence the alphas should have mean of zero and standard deviation IC* residual risk.
  - If the original unconstrainted alphas have volatility that is different with IC* residual risk, the alphas are scaled.

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$$\alpha \sim N(0, \sigma^2 \cdot IC^2)$$

GOLDEN FUTURE

α (真实) $\mu_\alpha = 0$. $\sigma^2 = 0.8$

$\alpha / \sqrt{2}$

Grmd $\alpha \sim N(0, 1.2\sigma^2)$

$\alpha \sim N(0, 0.4)$

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Scale the alphas 标准化

- **Alpha has a natural structure called Grinold Rule:**

$$\alpha = \sigma \times IC \times Score$$

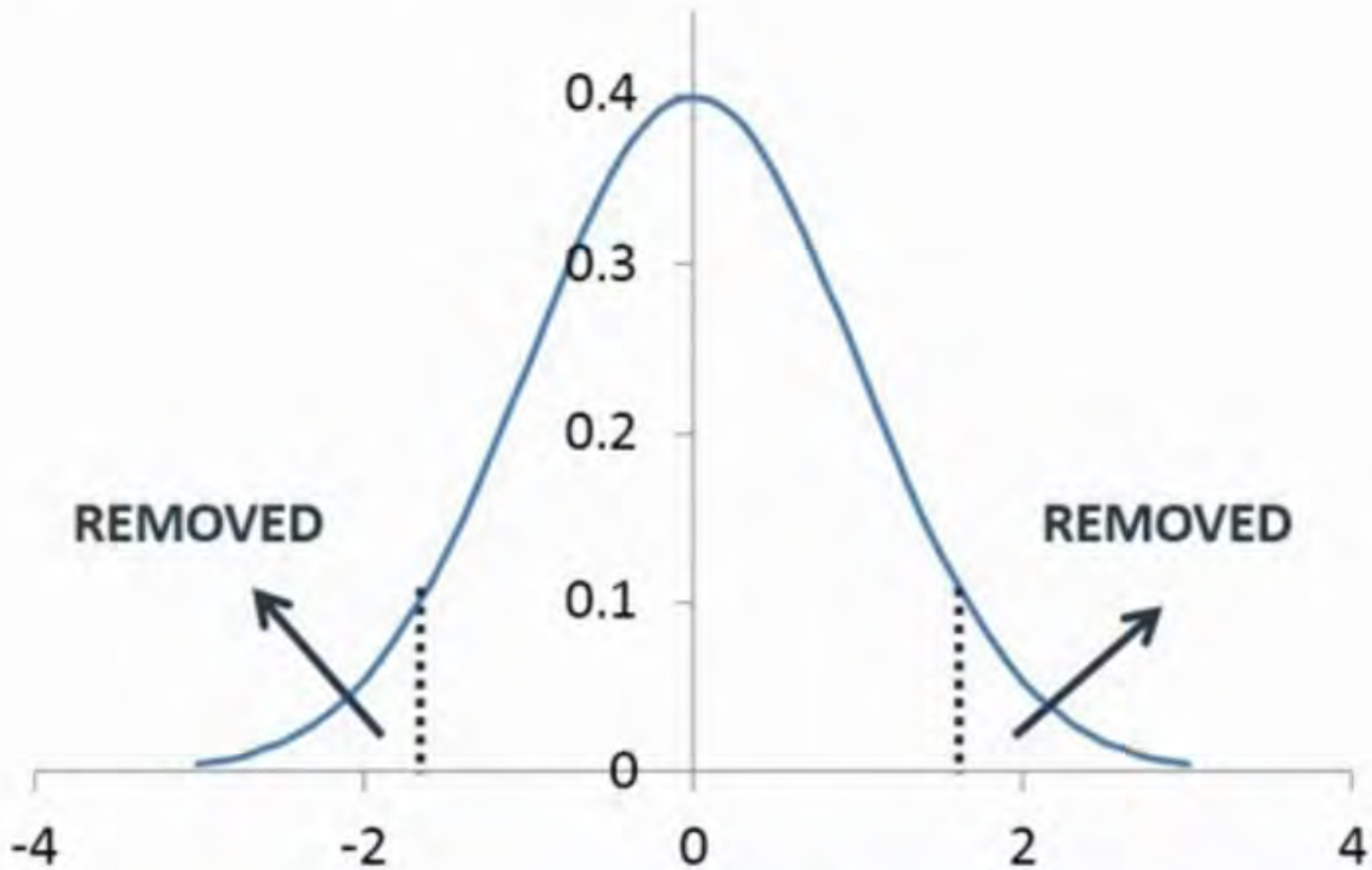
  - We expect the information coefficient (IC) and residual risk (volatility) for a set of alphas to be constant.
  - Score follows a standard normal distribution, hence the alphas should have mean of zero and standard deviation IC* residual risk.
  - If the original unconstrainted alphas have volatility that is different with IC* residual risk, the alphas are scaled.

$\alpha \sim N(0, \sigma^2 \cdot IC^2)$

实操中
alpha都是
缩小

Trim alpha outliers

- **The definition of the trim:**
  - Trim means removing the extreme outlier values.
  - The outliers can cause some difficulties in predicting the alphas.
  - An example can be described as follow:



Refining Alphas

- **Neutralization is to remove biases or undesirable bets from our alphas.**
  - Benchmark-Neutral Alphas

    R_m

    - ✓ Means that the benchmark has **0 alpha**. If our initial alphas imply an alpha for the benchmark, the neutralization process recenters the alphas to remove the benchmark alpha.
    - ✓ **For example**, the alpha of benchmark and portfolio are 1% and 5%, respectively. The benchmark-neutral alpha for a portfolio with beta equals to 1.5 is 5%-1.5*1%=3.5%.
  - Cash-Neutral Alphas: the alphas will not lead to any active cash position.
  - Risk-Factor-Neutral Alphas

    $\beta_1 (R_m - r_f) + \beta_2 SMB + \beta_3 HML$

    - ✓ If the manager does not have any ability to forecast the risk factors, we can use risk-factor-neutral alphas. **Once neutralized, the alphas of the risk factors will be 0.**

$$\frac{R_P - R_B}{R_P - \left[r_f + \beta\left(R_B - r_f\right)\right]} = 4\%$$

$$\frac{\beta}{} \quad R_B \quad \frac{(1-\beta)}{} \quad r_f$$

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Refining Alphas

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α (neutralized)

$$= \alpha_{(p)} - \beta \times \alpha_{(\beta)}$$

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Transactions Cost

- **Implications Transaction Costs have on Portfolio Construction**
  - Transactions costs force greater precision on our estimates of alpha.
  - Convert a one-dimensional problem (trade-off between alpha and active risk) into a two-dimensional problem (alpha, active risk, transaction costs).
  - Constructing, rebalancing and liquidating incur transaction costs.

Practical Issues

1. **Determination of Risk Aversion**

- If we have better intuition about our information ratio and our desired amount of active risk than our risk aversion
  - ✓ Utility = excess return – (risk aversion × variance)

$$\lambda_A = \frac{IR}{2 \times TEV}$$

- For example, if our information ratio is 0.5, and we desire 5% active risk, we should choose an active risk aversion of 0.05(ignore percentage). Furthermore, if the excess return is 2%, our maximum utility will be 2-0.05*5^2=0.75(percent)

筛选客户

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$$U = \underline{\alpha} - \lambda \cdot TEV^2 \qquad IR = \frac{\alpha}{TEV}$$

$$= \underline{IR} \cdot TEV - \lambda TEV^2$$

$$= \underline{-\lambda} \cdot TEV^2 + \underline{IR} \cdot TEV$$

$$\frac{1}{2} TEV = -\frac{b}{2a} = -\frac{IR}{2(-\lambda)} = \frac{IR}{2\lambda}$$

$$\frac{1}{2} \lambda = \frac{IR}{2TEV}$$

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$$\lambda = \frac{0.5}{2 \times 5} = 0.05$$

Practical Issues

2. **Incorporation of Specific Risk Aversion**
   - Aversion to specific as opposed to common-factor risk.
   - Commercial optimizers utilize this decomposition of risk to allow **differing aversions to these different sources of risk**.
   - Since specific risk arises from bets on specific assets, **a high aversion to specific risk** reduces bets on any one stock.

$$U = \alpha - (\lambda_{CF} \times TEV_{CF}^2 + \lambda_{SF} \times TEV_{SF}^2)$$

Practical Issues

3. **Proper Alpha Coverage**
   - If we forecast returns on stocks that are not in the benchmark, we can always handle that by expanding the benchmark to include those stocks, albeit with zero weight.
   - If there is a lack of forecast returns in the benchmark, adjust alphas to make benchmark neutral.

$w_1 R_1 + w_2 R_2 + \cdots \quad w_n R_n$

$- R_1 - \cdots \quad + w_{n+1} R_{n+1}$

$= 0$

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$\alpha(R_B) = 0$

α 无法计算 $\alpha = R_P - R_B$

Portfolio Construction Techniques

Portfolio Construction

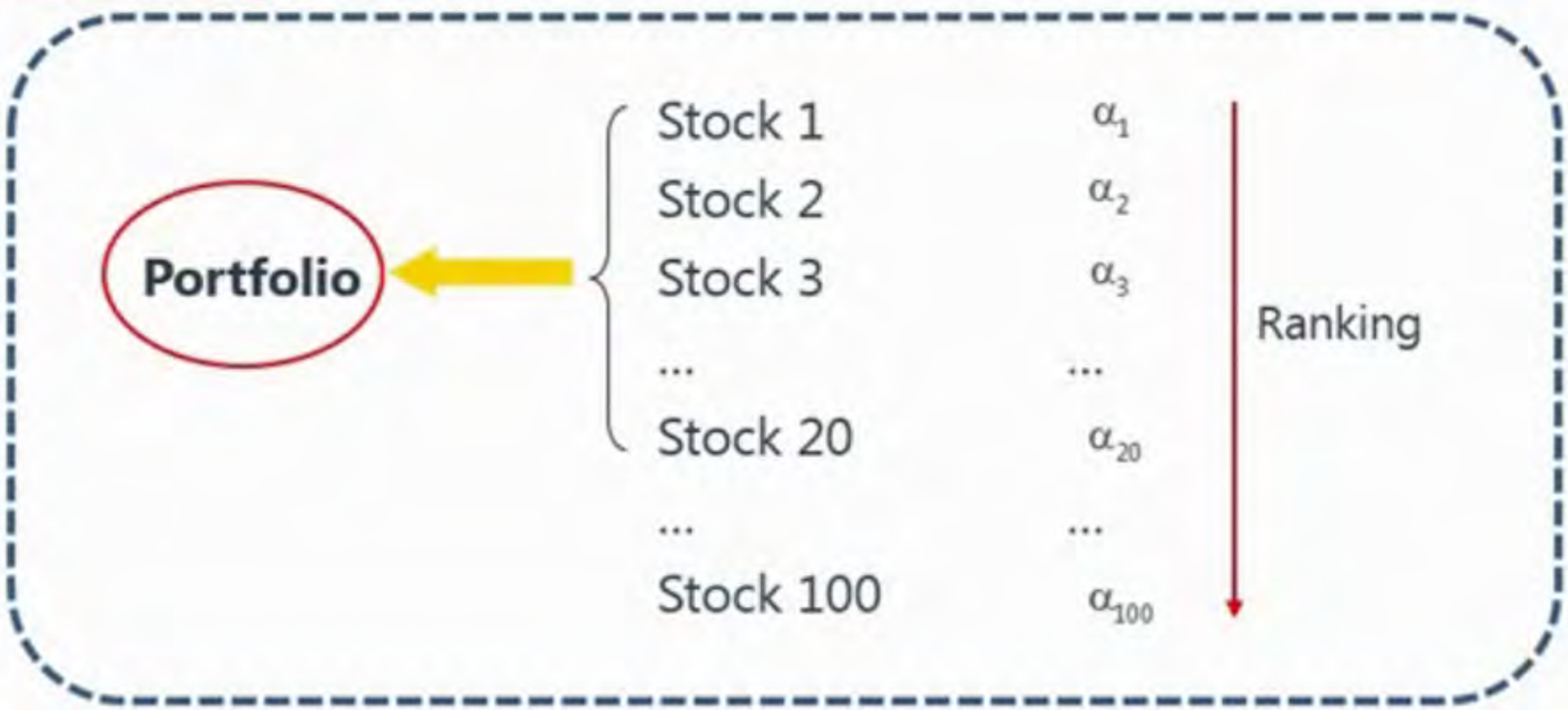
Portfolio Construction Techniques

- **Four portfolio construction techniques**
  - Screen
  - Stratification
  - Linear programming
  - Quadratic programming

Portfolio Construction Techniques

- **Screens**
  - Rank the original 100 stocks by alpha.
  - Choose the first 50 stocks (for example).
  - Equal-weight (or capitalization-weight) the stocks

2%



Portfolio Construction Techniques

- **Strength**
  - Easy to implement and understand.
  - It **enhances return** by selecting high-alpha assets and controls risk by having a sufficient number of assets for **diversification.**
  - Clear link between cause (membership on a buy/sell list) and effect (portfolio membership). outlier
  - Robust in that **extreme estimates** of alpha will not bias the outcome.
- **Weakness**
  - **Ignores** all information in alphas aside from ranking. Do not protect against biases in alpha
  - **Excluding those categories** of assets that tend to have low alphas.

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Portfolio Construction Techniques

- **Stratification** 分层抽样
  - Stratification is "glorified" screening.
  - Divide the stocks in categories (e.g., economic sectors, big/medium/small) and mimic the screening exercise.
- **Major strength over screens**
  - ignoring biases in the alphas across categories.

10个行业 选5个

Portfolio Construction Techniques

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Portfolio Construction Techniques

- **Linear Programming**
  - The linear programming approach characterizes stocks along dimensions of risk, e.g., **industry, size, volatility, and beta**.
  - The linear program will then attempt to build portfolios that are reasonably close to the benchmark portfolio in all of the dimensions used for risk control.
  - **Strength**
    - ✓ Takes all the information about alpha into account and controls tracking risk by keeping the characteristics of the portfolio close to the characteristics of the benchmark.
  - **Weakness**
    - ✓ Has difficulty producing portfolios with a pre-specified number of stocks.
    - ✓ The risk-control characteristics may conflict with the alphas.

Portfolio Construction Techniques

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分层的依据从行业变为

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P

β

industry, beta - — —

β>5, 且 size > 100m

① ①

② ② β<5 且 size < 50m

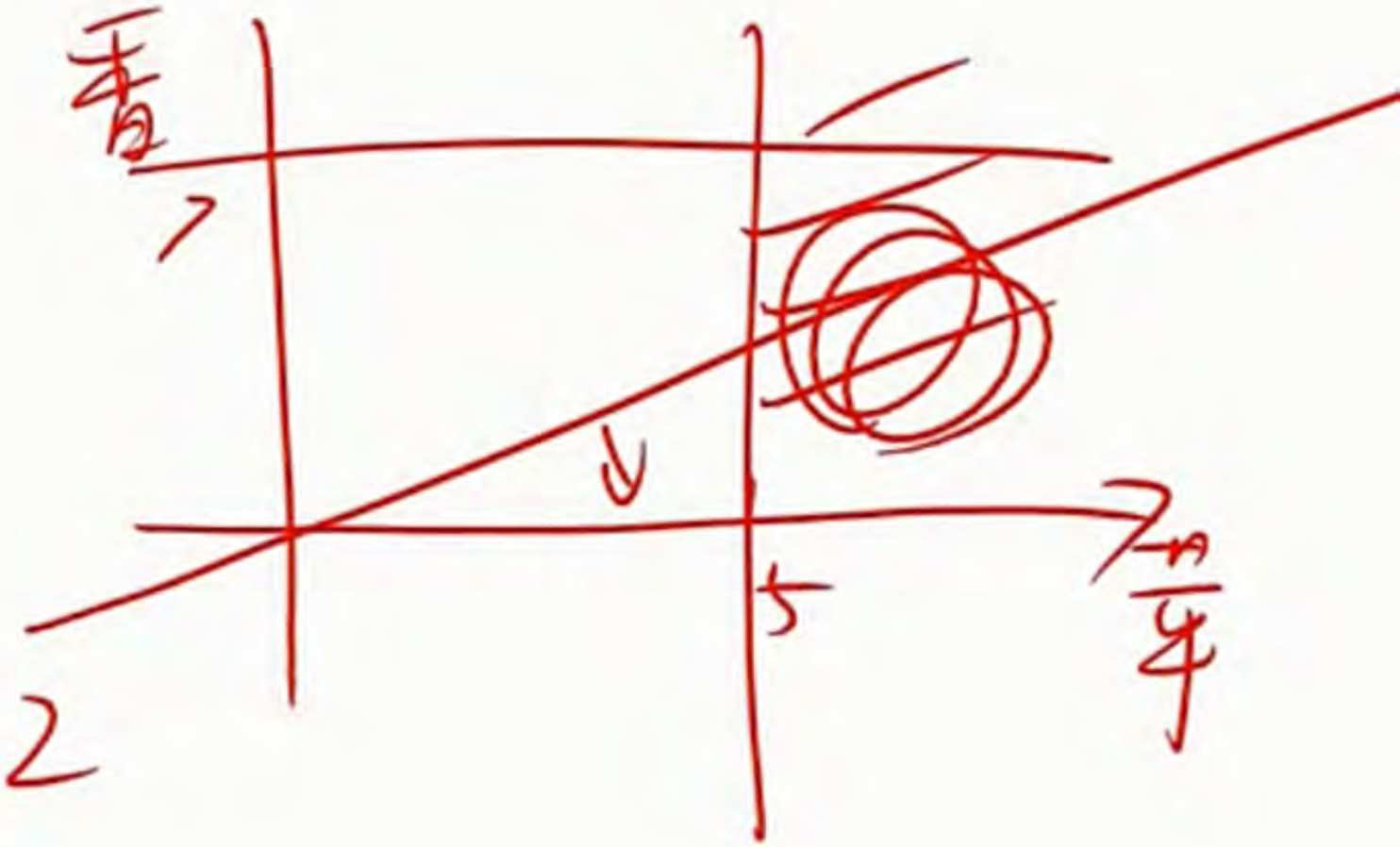
③ ③

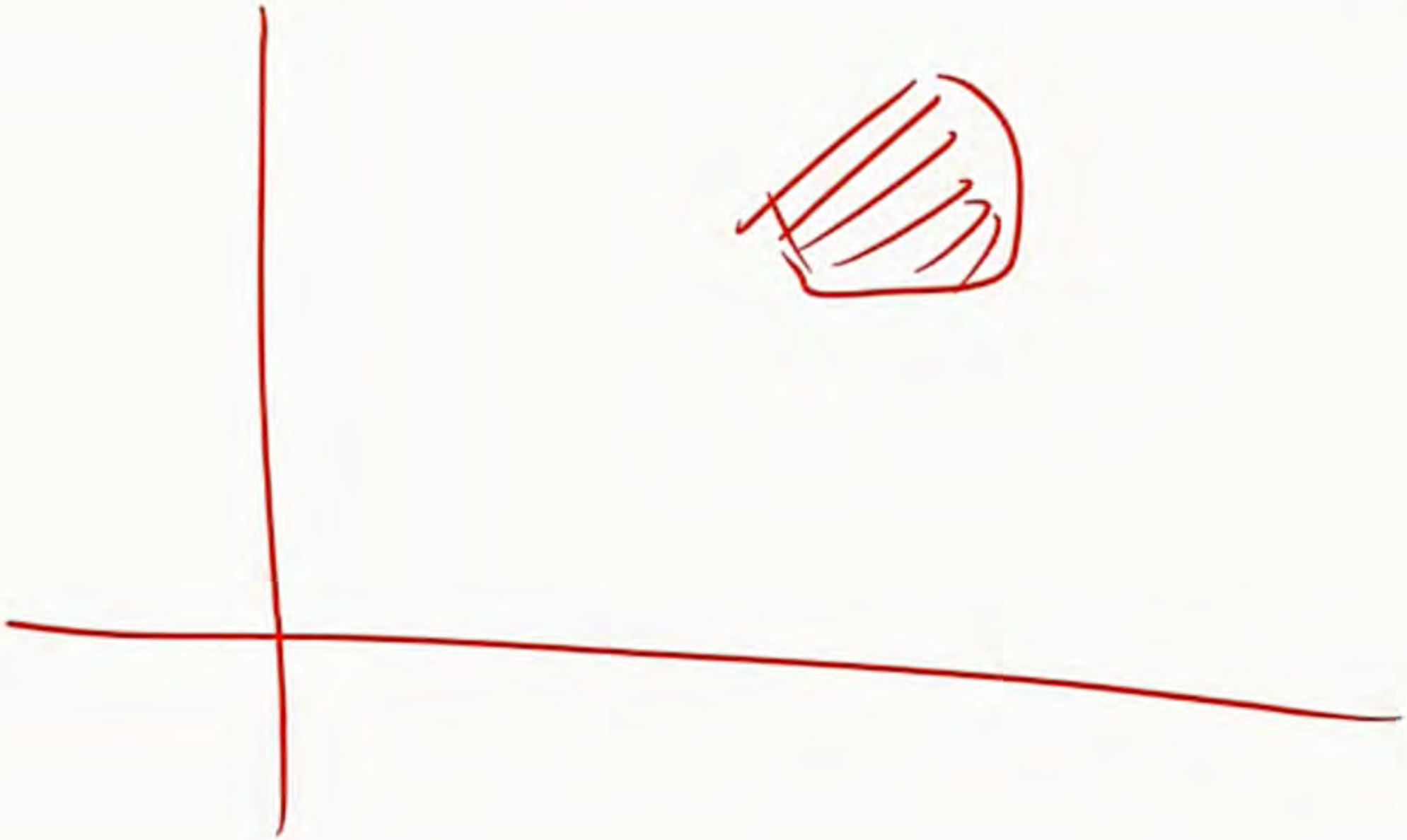
100 幸 看

幸 > 5

看 ≤ 7

幸 > 看 × 2





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Portfolio Construction Techniques

- **Quadratic Programming**
  - **Ultimate** in portfolio construction
  - Explicitly considers all three elements: **alpha, risk, and transactions costs**.
  - Requires a great many more inputs than the other portfolio construction techniques. **More inputs mean more noise**.

Portfolio Revisions and Rebalancing

Portfolio Construction

Portfolio Revisions

- **Frequent revision or less frequent revision** (耳熟能)
  - If a manager knows how to make the correct trade-off between expected active return, active risk, and transactions costs, frequent revision will not present a problem
  - If manager is unsure of ability to correctly specify alphas, active risk, and transactions costs, then may resort to less frequent revision as safeguard
  - Manager who underestimates transactions costs, makes large changes in alpha estimates very frequently, will result in the situation that has a lower expected alpha and higher transaction cost. **A crude but effective cure is to revise the portfolio less frequently.**

第1年	S	B
→	40%	60%
	45%	55%

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Portfolio Revisions

- **Frequent revision or less frequent revision** (取舍)
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Portfolio Rebalancing

- **The optimal alpha band**
  - In practice, there is a band around the alpha for each stock, as long as the alpha stays within a particular band, we should not make any changes.

$$2 \times \lambda_A \times \varphi \times MCAR_n - SC_n \leq \alpha_n \leq 2 \times \lambda_A \times \varphi \times MCAR_n + PC_n$$

    - ✓ It is simplified when the problem only involves estimating alphas, risks, and transaction costs over time.

$$U = \alpha - \lambda \cdot TEV^2$$

$$\frac{\partial U}{\partial TEV} = -2\lambda \cdot TEV$$

$$(-2\lambda \cdot TEV) \times \varphi$$

个单位的价格

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Portfolio Rebalancing

每增加一单位风险，会带来变化多少

- **Decide whether to trade**

$$MCVA_n = \alpha_n - \boxed{2 \times \lambda_A \times \varphi} \times MCAR_n$$

  - $MCVA_n$: Marginal contribution to value added for stock *n*
  - $MCAR_n$: Marginal contribution to active risk of asset n. （加入 $asset_n$ 之后风险主动加了 $MCAR_n$
  - λ_A: risk aversion of investor
  - φ: active risk

- **Optimal No-Trade Region with transaction costs**
  - As long as the $MCVA_n$ stays within the negative cost of selling and the cost of purchase, the portfolio will remain optimal, and we should not react to new information.

$$-SC_n \leq MCVA_n \leq PC_n$$

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GOLDEN FUTURE

$Asset_n$

① 使utility ↑ α_n

② 使utility ↓ $2\lambda \cdot \varphi \times MCAR_n$ PC_n

希望采入 $Asset_n$

$Asset_n$

$\alpha_n > 2\lambda \cdot \varphi \cdot MCAR_n + PC_n \longrightarrow$ 买、

$\Longleftrightarrow MCUA_n < PC_n$

$\alpha_n < 2\lambda \cdot \varphi \cdot MCAR_n + PC_n$ 不买

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Portfolio Rebalancing

每增加一单位风险，会增加变化额

- **Decide whether to trade**

>0

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GOLDEN FUTURE

$\boxed{\text{Asst}_n}$ $\longrightarrow$ 卖入

$-SC_n < MCVA_n$

$u\uparrow$ $\quad 2\lambda\cdot\varphi\cdot MCAR_n$

$u\downarrow$ $\quad \alpha_n \quad SC_n$

$2\lambda\cdot\varphi\cdot MCAR_n > \alpha_n + SC_n$ 卖

$2\lambda\cdot\varphi\cdot MCAR_n < \alpha_n + SC_n$ 不卖

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Portfolio Rebalancing

每增加一单位风险，会增加收益

- **Decide whether to trade**

$$>0 \quad MCVA_n = \alpha_n - 2 \times \lambda_A \times \varphi \times MCAR_n$$

  - $MCVA_n$: Marginal contribution to value added for stock *n*
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Dispersion

Portfolio Construction

Dispersion

- **Introduction**
  - The **difference between the maximum return and minimum return** for separate account portfolios. It is a measure of how an individual client's portfolio may differ from the manager's reported composite returns.
  - If the holdings in each account are identical, dispersion will disappear. If transactions costs were zero, dispersion would disappear.
- **Sources of Dispersion**
  - Client-driven: clients impose different constraints.
  - Lack of attention to separate accounts
- **Controlling Forms of Dispersion**
  - If transactions costs were zero, rebalancing all the separate accounts so that they hold exactly the same assets in the same proportions would have no cost. Dispersion would disappear, at no cost to investors.
  - With transactions costs, managers should reduce dispersion only until further reduction would substantially lower returns on average because much higher transactions costs would be incurred.

↓frequency

Portfolio Rebalancing

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Reading 3

Portfolio Risk Measures

Portfolio VaR

$|\mu - z \cdot \sigma| \cdot V$

- **Individual VaR**
  - This is the VaR of an individual position in isolation.
  - Where W is portfolio value and ω(i) is the weight assigned to individual position, individual VaR is given by:

$$VaR_i = z_c \sigma_i |V_i| = z_c \sigma_i |\omega_i| V$$

- **Diversified VaR and Undiversified VaR**
  - Diversified VaR accounts for diversification effects. The two-asset variance is given by:

$$\sigma_p^2 = \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + 2\rho\omega_1\omega_2\sigma_1\sigma_2$$

  - Undiversified VaR is simply the sum of the individual VaRs. If correlation <1, diversified VaR is less than Undiversified VaR.

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$$\begin{cases} VaR_p^2 = VaR_1^2 + VaR_2^2 + 2\rho VaR_1 VaR_2 \\ VaR_p = z \cdot \sigma_p \cdot V \end{cases}$$

Portfolio VaR

- **Example**
  - An analyst computes the VaR for the two position in her portfolio. The VaRs: VaR_1 = \$2.4million and VaR_2 = \$1.6million. Compute the VaR_P if the returns of the two assets are uncorrelated.
  - **Answer: 2.88**

$VaR_p^2 = VaR_1^2 + VaR_2^2 + 0$

  - An analyst computes the VaR for the two position in her portfolio. The VaRs: VaR_1 = \$2.4million and VaR_2 = \$1.6million. Compute the VaR_P if the returns of the two assets are perfectly positive correlated.
  - **Answer: 4**

$\rho = 1$

$VaR_p = VaR_1 + VaR_2$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho w_1 w_2 \sigma_1 \sigma_2 \quad (\%)$$

$$\sigma_p^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2 \quad (\$)$$

$$R_p = w_1 R_1 + w_2 R_2 \quad (\%)$$

$$R_p = R_1 + R_2 \quad (\$)$$

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Portfolio VaR

- **Marginal VaR**
  - The change in portfolio VaR resulting from taking an additional dollar of exposure to a given component. It is also the partial derivative with respect to the component position.

$$
\begin{aligned}
MVaR_A &= \frac{\partial VaR_P}{\partial V_A} \\
&= z_\alpha \times \frac{Cov(R_A, R_P)}{\sigma_P} \\
&= z_\alpha \times \rho_{A,P} \times \sigma_A \\
&= z_\alpha \times \beta_{A,P} \times \sigma_P \\
&= \frac{VaR_P}{V_P} \times \beta_{A,P}
\end{aligned}
$$

$mVaR_2$

$mVaR_1$

Asset 1 ↑ 1\$

VaR_p ↑ ?

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&= z_\alpha \times \beta_{A,P} \times \sigma_P \\
&= \frac{VaR_P}{V_P} \times \beta_{A,P}
\end{aligned}
$$

$$\frac{\partial VaR_p}{\partial V_A} = \frac{\partial (Z \cdot \sigma_p \times \cancel{V_p})}{\partial (w_A \cdot \cancel{V_p})} = Z \cdot \frac{\partial \sigma_p}{\partial w_A}$$

$$= Z \cdot \frac{\partial \left[\sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + \cdots + 2 w_A w_B \sigma_A \sigma_B (R_A, R_B) + \cdots} \right]}{\partial w_A}$$

$$\frac{\partial \sigma_p^2}{\partial w_A} = \boxed{\frac{\partial \sigma_p^2}{\partial \sigma_p}} \cdot \frac{\partial \sigma_p}{\partial w_A} = 2\sigma_p \cdot \frac{\partial \sigma_p}{\partial w_A}$$

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$$\frac{\partial \sigma_p^2}{\partial w_A} = \frac{\partial \left[\boxed{w_A^2 \sigma_A^2} + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + \cdots + \boxed{2 w_A w_B \mathrm{Cov}(R_A R_B) + 2 w_A w_C \mathrm{Cov}(R_A R_C) \cdots} + 2 w_B w_C \mathrm{Cov}(R_B R_C) \right]}{\partial w_A}$$

$$= 2 w_A \overset{\mathrm{Cov}(R_A, R_A)}{\downarrow} + 2 w_B \underline{\mathrm{Cov}(R_A R_B)} + 2 w_C \underline{\mathrm{Cov}(R_A R_C)} + \cdots$$

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$$2\,w_B\,\mathrm{cov}(R_A, R_B)$$

$$= \underline{2\,\mathrm{cov}(R_A, w_B R_B)} + \underline{2\,\mathrm{cov}(R_A, w_C R_C)}$$

$$\underline{2\,\mathrm{cov}(R_A, w_B R_B + w_C R_C)}$$

GOLDEN FUTURE

$$\frac{\partial \sigma_p^2}{\partial w_A} = \frac{\partial \left[\boxed{w_A^2 \sigma_A^2} + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + \cdots + \boxed{2 w_A w_B \mathrm{Cov}(R_A R_B) + 2 w_A w_C \mathrm{Cov}(R_A R_C) \cdots} + 2 w_B w_C \mathrm{Cov}(R_B R_C) \right]}{\partial w_A}$$

$$= 2 w_A \overset{\mathrm{Cov}(R_A, R_A)}{\sigma_A^2} + 2 w_B \mathrm{Cov}(R_A R_B) + 2 w_C \mathrm{Cov}(R_A R_C) + \cdots$$

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$$2\, w_B \operatorname{cov}(R_A, R_B)$$

$$= 2 \operatorname{cov}(R_A, w_B R_B) + 2 \operatorname{cov}(R_A, w_C R_C)$$

$$2 \operatorname{cov}(R_A, w_B R_B + w_C R_C + w_A R_A)$$

$$w_A R_A$$

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$$\frac{\partial \sigma_p^2}{\partial w_A} = \frac{\partial \left[\boxed{w_A^2 \sigma_A^2} + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + \cdots + \boxed{2 w_A w_B \mathrm{Cov}(R_A R_B) + 2 w_A w_C \mathrm{Cov}(R_A R_C) \cdots} + 2 w_B w_C \mathrm{Cov}(R_B R_C) \right]}{\partial w_A}$$

$$= 2 w_A \overset{\mathrm{Cov}(R_A, R_A)}{\sigma_A^2} + 2 w_B \mathrm{Cov}(R_A R_B) + 2 w_C \mathrm{Cov}(R_A R_C) + \cdots$$

$$= 2 \mathrm{Cov}(R_A, R_p)$$

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$$\frac{\partial VaR_p}{\partial V_A} = \frac{\partial (Z \cdot \sigma_p \times \cancel{V_p})}{\partial (w_A \cdot \cancel{V_p})} = Z \cdot \frac{\partial \sigma_p}{\partial w_A}$$

$$= Z \cdot \frac{\partial \left[\sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + \cdots + 2 w_A w_B \sigma_A \sigma_B (R_A, R_B) + \cdots} \right]}{\partial w_A}$$

$$\frac{\partial \sigma_p^2}{\partial w_A} = \boxed{\frac{\partial \sigma_p^2}{\partial \sigma_p}} \cdot \frac{\partial \sigma_p}{\partial w_A} = 2\sigma_p \cdot \frac{\partial \sigma_p}{\partial w_A}$$

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$$\frac{\partial \sigma_p}{\partial w_A} = \frac{\partial \sigma_p^2}{\partial w_A} \Big/ 2\sigma_p = \frac{2\,\mathrm{cov}(R_A, R_p)}{2\sigma_p}$$

$$= Z\,\frac{\mathrm{cov}(R_A, R_p)}{\sigma_p}$$

$$\frac{\partial VaR_p}{\partial V_A} = \frac{\partial (Z \cdot \sigma_p \times \cancel{V_p})}{\partial (w_A \cdot \cancel{V_p})} = Z \cdot \frac{\partial \sigma_p}{\partial w_A}$$

$$= Z \cdot \frac{\partial \left[\sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + \cdots + 2 w_A w_B \, Cov(R_A, R_B) + \cdots} \right]}{\partial w_A}$$

$$\frac{\partial \sigma_p^2}{\partial w_A} = \boxed{\frac{\partial \sigma_p^2}{\partial \sigma_p}} \cdot \frac{\partial \sigma_p}{\partial w_A} = 2\sigma_p \cdot \frac{\partial \sigma_p}{\partial w_A}$$

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Portfolio VaR

- **Marginal VaR**
  - The change in portfolio VaR resulting from taking an additional dollar of exposure to a given component. It is also the partial derivative with respect to the component position.

$$MVaR_A = \frac{\partial VaR_P}{\partial V_A}$$

$$= z_\alpha \times \frac{Cov(R_A, R_P)}{\sigma_P}$$

$\rho \cdot \sigma_A \cdot \sigma_P$

$$= z_\alpha \times \rho_{A,P} \times \sigma_A$$

$$= z_\alpha \times \beta_{A,P} \times \sigma_P$$

$$= \frac{VaR_P}{V_P} \times \beta_{A,P}$$

Portfolio VaR

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$\rho \cdot \sigma_A \cdot \cancel{\sigma_P}$

$$= z_\alpha \times \rho_{A,P} \times \sigma_A$$

$$= z_\alpha \times \beta_{A,P} \times \sigma_P$$

$$= \frac{VaR_P}{V_P} \times \beta_{A,P}$$

$\rho = \beta \cdot \frac{\sigma_P}{\sigma_A}$

Portfolio VaR

- **Marginal VaR**
  - The change in portfolio VaR resulting from taking an additional dollar of exposure to a given component. It is also the partial derivative with respect to the component position.

$$MVaR_A = \frac{\partial VaR_P}{\partial V_A}$$

$$= z_\alpha \times \frac{Cov(R_A, R_P)}{\sigma_P}$$

$$= z_\alpha \times \rho_{A,P} \times \sigma_A$$

$$= z_\alpha \times \beta_{A,P} \times \sigma_P$$

$$= \frac{VaR_P}{V_P} \times \beta_{A,P}$$

$$WWR_A = Z \cdot \frac{Cov(A,P)}{\sigma_P}$$

$$= Z \cdot \frac{\rho_A \cdot \sigma_A \cdot \cancel{\sigma_P}}{\cancel{\sigma_P}}$$

$$= Z \, \beta \cdot \sigma_P^{\cancel{2}} / \cancel{\sigma_P}$$

$$= Z \cdot \beta \cdot \sigma_P \times V_P \div V_P$$

$$= \beta \cdot V_{WR_P} / V_P$$

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Portfolio VaR

- **Incremental VaR**
  - Change in VaR owing to a new position.
  - Differs from marginal VaR: amount added or subtracted can be large.
  - Incremental VaR requires a **full revaluation** of the portfolio VaR with the new trade:

$$Incremental\ VaR = VaR_{P+a} - VaR_P$$

  - Approximate computation:

$$Incremental\ VaR_A \approx MVaR_A \times W_A(any\ amount)$$

Portfolio VaR

- **Component VaR**
  - In order to manage risk, it would be extremely useful to have a risk **decomposition** of the current portfolio.
  - A partition of the portfolio VaR that indicates how much the portfolio VaR would change **approximately** if the given component was deleted.
  - By construction, component VaRs sum to portfolio VaR.

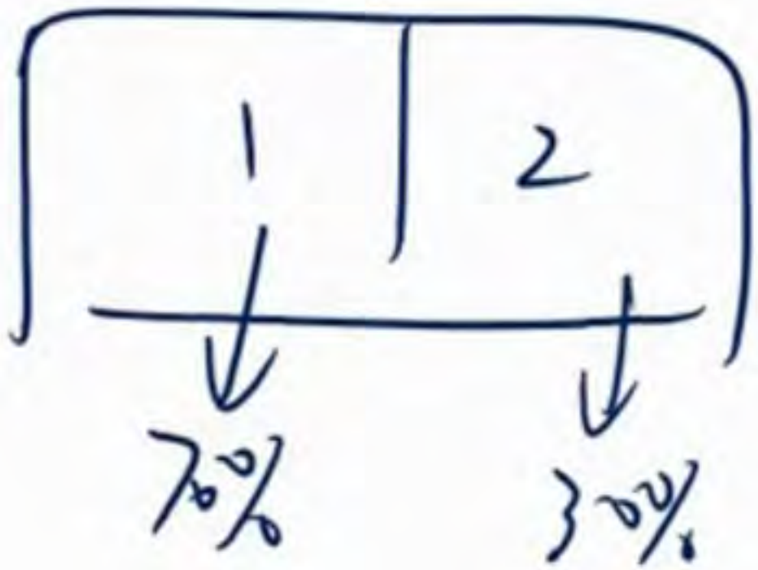
分解

$$CVaRA = MVaRA \times V_A$$

$$\frac{CVaR_A}{VaR_p} = \omega_A \times \beta_{A,P}$$

$$VaR_p = \sum_{i=1}^{N} CVaR_i = VaRp(\sum_{i=1}^{N} \omega_i \beta_{i,p})$$

$$\sum_{i=1}^{N} \omega_i \beta_{i,p} = 1$$



$VaR_p = 50m$

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GOLDEN FUTURE

$$\frac{CVaR_A}{VaR_p} = \frac{mVaR_A \times V_A}{VaR_p}$$

$$= \frac{\frac{VaR_p}{V_p} \times \beta \times V_A}{VaR_p}$$

专业 · 创新 · 增值

Portfolio VaR

- **Marginal VaR**

  1$

  - The change in portfolio VaR resulting from taking an additional dollar of exposure to a given component. It is also the partial derivative with respect to the component position.

$$
\begin{aligned}
MVaR_A &= \frac{\partial VaR_P}{\partial V_A} \\
&= z_\alpha \times \frac{Cov(R_A, R_P)}{\sigma_P} \\
&= z_\alpha \times \rho_{A,P} \times \sigma_A \\
&= z_\alpha \times \beta_{A,P} \times \sigma_P \\
&= \frac{VaR_P}{V_P} \times \beta_{A,P}
\end{aligned}
$$

$$\frac{CVaR_A}{VaR_P} = \frac{mVaR_A \times V_A}{VaR_P}$$

$$= \frac{\frac{\cancel{VaR_P}}{V_P} \times \beta \times \cancel{V_A}}{\cancel{VaR_P}}$$

$$= \beta \cdot w_A$$

Portfolio VaR

- **Component VaR**
  - In order to manage risk, it would be extremely useful to have a risk **decomposition** of the current portfolio.

分解

  - A partition of the portfolio VaR that indicates how much the portfolio VaR would change **approximately** if the given component was deleted.
  - By construction, component VaRs sum to portfolio VaR.

$$CVaRA = MVaRA \times V_A$$

$$\frac{CVaR_A}{VaR_p} = \omega_A \times \beta_{A,P}$$

% Contribution

$$VaR_p = \sum_{i=1}^{N} CVaR_i = VaRp(\sum_{i=1}^{N} \omega_i \beta_{i,p})$$

$$\sum_{i=1}^{N} \omega_i \beta_{i,p} = 1$$

GOLDEN FUTURE

	w_i	β_i	$w_i \cdot \beta_i$
A	10%	2.5	25%
B	20%	1.25	
C	30%	0.83	
D	40%	0.625	

Risk parity

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Portfolio VaR

- **Example 1**
  - A portfolio consists of assets A and B. There are $4 million and $2 million invested in them respectively. The covariance between A and this portfolio is 0.0144, and 0.0392 is the covariance between B and the portfolio. If we assume they are uncorrelated with each other, compute the **marginal VaR** of assets A and B; Assume a Z-score of 1.65, the variance of the portfolio is 0.136.
  - **Answer:**

$$MVaR_{\mathrm{A}} = Z_{\alpha} \times \frac{\mathrm{cov}(R_A, R_P)}{\sigma_P} = 1.65 \times \frac{0.0144}{\sqrt{0.136}} = 0.0644$$

$$MVaR_{B} = Z_{\alpha} \times \frac{\mathrm{cov}(R_B, R_P)}{\sigma_P} = 1.65 \times \frac{0.0392}{\sqrt{0.136}} = 0.1754$$

Portfolio VaR

- **Marginal VaR**

1$

  - The change in portfolio VaR resulting from taking an additional dollar of exposure to a given component. It is also the partial derivative with respect to the component position.

$$
\begin{aligned}
MVaR_A &= \frac{\partial VaR_P}{\partial V_A} \\
&= z_\alpha \times \frac{Cov(R_A, R_P)}{\sigma_P} \\
&= z_\alpha \times \rho_{A,P} \times \sigma_A \\
&= z_\alpha \times \beta_{A,P} \times \sigma_P \\
&= \frac{VaR_P}{V_P} \times \beta_{A,P}
\end{aligned}
$$

$$\begin{cases} V_A = 4m \\ V_B = 2m \\ Cov(A,p) = 0.0144 \\ Cov(B,p) = 0.0392 \\ \rho(A,B) = 0 \\ \sigma_p = 0.136 \\ Z = 1.65 \end{cases}$$

Portfolio VaR

➢ **Example 1**

4 2

- A portfolio consists of assets A and B. There are $4 million and $2 million invested in them respectively. The covariance between A and this portfolio is 0.0144, and 0.0392 is the covariance between B and the portfolio. If we assume they are uncorrelated with each other, compute the **marginal VaR** of assets A and B; Assume a Z-score of 1.65, the variance of the portfolio is 0.136.

Cov

- **Answer:**

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$$MVaR_B = Z_\alpha \times \frac{\mathrm{cov}(R_B, R_P)}{\sigma_P} = 1.65 \times \frac{0.0392}{\sqrt{0.136}} = 0.1754$$

Portfolio VaR

- **Example 2**
  - Recall the example above of a portfolio invested \$4 million in A and \$2million in B. Using their respective marginal VaRs, 0.0644 and \$0.1754, compute the component VaRs.
  - **Answer:**

$$CVaR_A = MVaR_A \times (w_A \times P) = 0.0644 \times \$4million = \$257{,}713$$
$$CVaR_B = MVaR_B \times (w_B \times P) = 0.1754 \times \$2million = \$350{,}777$$

Portfolio VaR

- **Component VaR**
  - In order to manage risk, it would be extremely useful to have a risk **decomposition** of the current portfolio.

分解

  - A partition of the portfolio VaR that indicates how much the portfolio VaR would change **approximately** if the given component was deleted.
  - By construction, component VaRs sum to portfolio VaR.

$$CVaRA = MVaRA \times V_A$$

$$\frac{CVaR_A}{VaR_p} = \omega_A \times \beta_{A,P}$$

% Contribution

$$VaR_p = \sum_{i=1}^{N} CVaR_i = VaRp(\sum_{i=1}^{N} \omega_i \beta_{i,p})$$

$$\sum_{i=1}^{N} \omega_i \beta_{i,p} = 1$$

Portfolio VaR

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  - Recall the example above of a portfolio invested \$4 million in A and \$2million in B. Using their respective marginal VaRs, 0.0644 and \$0.1754, compute the component VaRs.
  - **Answer:**

$$CVaR_A = MVaR_A \times (w_A \times P) = 0.0644 \times \$4million = \$257{,}713$$
$$CVaR_B = MVaR_B \times (w_B \times P) = 0.1754 \times \$2million = \$350{,}777$$

Applications of Portfolio VaR

- **How to minimize portfolio risk**
  - The portfolio manager can decrease portfolio risk by reducing positions with the highest marginal VaR.
  - Repeat process until the portfolio risk has reached a **global minimum**.
  - At this global risk minimum, all of the marginal VaRs, or portfolio betas, must be equal.

Applications of Portfolio VaR

- **How to minimize portfolio risk**
  - The portfolio manager can decrease portfolio risk by reducing positions with the highest marginal VaR.
  - Repeat process until the portfolio risk has reached a **global minimum**.
  - At this global risk minimum, all of the marginal VaRs, or portfolio betas, must be equal.

$$mVaR_A = \frac{VaR_p}{V_p} \times \beta_{A,P}$$

$$mVaR_B = \frac{VaR_p}{V_p} \times \beta_{B,P}$$

Case Study

- **Case information:**
  - Consider a portfolio with two foreign currencies, 2 million Canadian dollar (CAD) and 1 million Euro (EUR).
  - Assume that these two currencies are uncorrelated and have volatility of 5% and 12% respectively.
  - In addition, the confidence level is 95%.
- **Question:**
  - A: Calculate the Marginal VaR, component VaR for each position.
  - B: If the portfolio manager wants to decrease portfolio risk by rebalance between the two asset, calculate the optimal weights assigned to each position.
  - C: If the expected returns for the two currencies are 8% and 5% respectively, how to adjust weights so that the portfolio will represent the best combination of expected risk and return.

GOLDEN FUTURE

$V_{CAD} = 2m$ $\sigma_{CAD} = 5\%$ ✓ $\rho_{CAD,EWR} = 0$

$V_{EWR} = 1m$ $\sigma_{EWR} = 12\%$

$Z = 1.65$

CAD → 1 EWR → 2

1. $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho_{12} w_1 w_2 \sigma_1 \sigma_2$

$= (2/3)^2 \times (5\%)^2 + (1/3)^2 \times (12\%)^2 + 0$ $\Rightarrow \sigma_p = 5.207\%$ ✓

2. $\begin{cases} \rho_{1,p} = \dfrac{Cov(R_1, R_p)}{\sigma_1 \cdot \sigma_p} = \dfrac{Cov(R_1, w_1R_1 + w_2R_2)}{\sigma_1 \cdot \sigma_p} \\ \rho_{2,p} \end{cases}$

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$$\left(Cov(R_1, w_1R_1 + w_2R_2)\right)$$

$$= Cov(R_1, w_1R_1) + Cov(R_1, w_2R_2)$$

$$= w_1\sigma_1^2 + w_2 \times 0$$

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GOLDEN FUTURE

$V_{CAD} = 2m$ $\quad \sigma_{CAD} = 5\%$ ✓ $\quad \rho_{CAD,ENR} = 0$

$V_{ENR} = 1m$ $\quad \sigma_{ENR} = 12\%$

$Z = 1.65$

CAD → 1 ENR → 2

1. $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho_{12} w_1 w_2 \sigma_1 \sigma_2$

$= (2/3)^2 \times (5\%)^2 + (1/3)^2 \times (12\%)^2 + 0$ $\Rightarrow \sigma_p = 5.207\%$ ✓

2. $\rho_{1,p} = \dfrac{Cov(R_1, R_p)}{\sigma_1 \cdot \sigma_p} = \dfrac{Cov(R_1, w_1R_1 + w_2R_2)}{\sigma_1 \cdot \sigma_p} = \dfrac{w_1\sigma_1^2}{\sigma_1 \cdot \sigma_p} = w_1 \dfrac{\sigma_1}{\sigma_p}$

$= 0.6402$

$\rho_{2,p} = 0.7682$

$Cov(1,p) = \rho_{1,p} \cdot \sigma_1 \cdot \sigma_p$ $\quad \beta_{(1,p)} = \rho_{1,p} \cdot \dfrac{\sigma_1}{\sigma_p}$

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Case Study

➢ **Calculation of Question A:**

Currency	Currency Position	Volatility	Correlation Coefficient	Beta
CAD	2million	$\sigma_1 = 5\%$	$\rho_{1p} = \frac{Cov(R_1, R_p)}{\sigma_1 \sigma_p} = w_1 \frac{\sigma_1}{\sigma_p} = 0.64$	$\beta_1 = \frac{Cov(R_1, R_p)}{\sigma_p^2} = 0.61$
EUR	1million	$\sigma_2 = 12\%$	$\rho_{2p} = \frac{Cov(R_2, R_p)}{\sigma_2 \sigma_p} = w_2 \frac{\sigma_2}{\sigma_p} = 0.768$	$\beta_2 = \frac{Cov(R_2, R_p)}{\sigma_p^2} = 1.77$
Total	3million	$\sigma_p = 5.21\%$	Nil	Nil

Case Study

➢ **Calculation of Question A:**

Currency	Individual VaR $VAR_i = \alpha_i \sigma_i W_i$	Marginal VaR $\Delta VAR_i = \frac{VAR_p}{W}\beta_i$	Component VaR $CVAR_i = VAR_i \omega_i$	Percent Contribution $\beta_i \omega_i$
CAD	$165,000	0.0528	$105,630	41%
EUR	$198,000	0.1521	$152,108	59%
Undiversified VaR	$363,000	Nil	Nil	Nil
Diversified VaR	Nil	Nil	$257,738	100%

Portfolio VaR

- **Marginal VaR**
  - The change in portfolio VaR resulting from taking an additional dollar of exposure to a given component. It is also the partial derivative with respect to the component position.

$$MVaR_A = \frac{\partial VaR_P}{\partial V_A}$$

$$= z_\alpha \times \frac{Cov(R_A, R_P)}{\sigma_P}$$

$$= z_\alpha \times \rho_{A,P} \times \sigma_A$$

$$= z_\alpha \times \beta_{A,P} \times \sigma_P$$

$$= \frac{VaR_P}{V_P} \times \beta_{A,P}$$

GOLDEN FUTURE

1. CAD $\longrightarrow w_1^*$

EHR $\longrightarrow w_2^*$

$$\begin{cases} w_1^* + w_2^* = 100\% & \text{——— ①} \\ mVaR_1^* = mVaR_2^* \\ \Updownarrow \\ z \cdot \rho_{1,p} \cdot \sigma_1 = z \cdot \rho_{2,p} \cdot \sigma_2 \\ \Updownarrow \\ z \cdot \boxed{w_1^* \cdot \frac{\sigma_1}{\sigma_p}} \cdot \sigma_1 = z \cdot \boxed{w_2^* \cdot \frac{\sigma_2}{\sigma_p}} \sigma_2 \\ \Updownarrow \\ w_1^* \cdot \sigma_1^2 = w_2^* \cdot \sigma_2^2 & \text{——— ②} \end{cases}$$

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Case Study

➢ **Calculation of Question B:**

Asset	Original Position	Marginal VaR	Final Position	Marginal VaR	Beta
CAD	66.67%	0.0528	85.21%	0.0762	1.000
EUR	33.33%	0.1521	14.79%	0.0762	1.000
Total	100.00%		100.00%		
Diversified VaR	$257,738		$228,462		
Standard deviation	5.207%		4.615%		

GOLDEN FUTURE

$$\begin{cases} w_1 + w_2 = 100\% \\ w_1 \cdot \beta_1 + w_2 \beta_2 = 100\% = \beta(w_1 + w_2) \\ \beta_1 = \beta_2 \end{cases}$$

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Case Study

➢ **Calculation of Question B:**

步骤

Asset	Original Position	Marginal VaR	Final Position	Marginal VaR	Beta
CAD	66.67%	0.0528	85.21%	0.0762	1.000
EUR	33.33%	0.1521	14.79%	0.0762	1.000
Total	100.00%		100.00%		
Diversified VaR	$257,738		$228,462		
Standard deviation	5.207%		4.615%		

66.7%

33.3%

Applications of Portfolio VaR

- **How to minimize portfolio risk**
  - The portfolio manager can decrease portfolio risk by reducing positions with the highest marginal VaR.
  - Repeat process until the portfolio risk has reached a **global minimum**.
  - At this global risk minimum, all of the marginal VaRs, or portfolio betas, must be equal.

$$\begin{cases} mVaR_A = \dfrac{VaR_p}{V_p} \times \beta_{A,P} \\ mVaR_B = \dfrac{VaR_p}{V_p} \times \beta_{B,P} \end{cases}$$

Applications of Portfolio VaR

- **How to balance between risk and return**
  - The role of a portfolio manager is to choose a portfolio that represents the best combination of expected risk and return.
  - The optimal portfolio has the highest Sharpe ratio:

$$SR = \frac{E(R_P) - R_f}{\sigma_P} \Rightarrow \frac{E(R_P) - R_f}{VaR_P}$$

  - In order to make this ratio maximized, we have:

$$\frac{Position\ i\ return\ -\ risk\ free\ rate}{MVaR_i} = \frac{Position\ j\ return\ -\ risk\ free\ rate}{MVaR_j}$$

Case Study

- **Case information:**
  - Consider a portfolio with two foreign currencies, 2 million Canadian dollar (CAD) and 1 million Euro (EUR).
  - Assume that these two currencies are uncorrelated and have volatility of 5% and 12% respectively. $\rho_{CAD, EUR} = 0$
  - In addition, the confidence level is 95%. $Z = 1.65$
- **Question:**
  - A: Calculate the Marginal VaR, component VaR for each position.
  - B: If the portfolio manager wants to decrease portfolio risk by rebalance between the two asset, calculate the optimal weights assigned to each position.
  - C: If the expected returns for the two currencies are 8% and 5% respectively, how to adjust weights so that the portfolio will represent the best combination of expected risk and return.

Case Study

$r_f = 0$

Calculation of Question C:

Risk and Return-Optimizing Position							
Asset	Expected Return E_i	Original Position w_i	Beta β_i	Ratio E_i/β_i	Final Position w_i	Beta β_i	Ratio E_i/β_i
CAD	8.00%	66.67%	0.615	0.1301	90.21%	1.038	0.0771
EUR	5.00%	33.33%	1.770	0.0282	9.79%	0.649	0.0771
Total	-	100.00%	-	-	100.00%	-	-
Diversified VaR	-	$257.738	-	-	$230.720	-	-
Standard Deviation	-	15.62%	-	-	13.98%	-	-
Expected Return	-	7.00%	-	-	7.71%	-	-
Sharpe Ratio	-	0.448	-	-	0.551	-	-

Applications of Portfolio VaR

- **How to balance between risk and return**
  - The role of a portfolio manager is to choose a portfolio that represents the best combination of expected risk and return.
  - The optimal portfolio has the highest Sharpe ratio:

$$SR = \frac{E(R_P) - R_f}{\sigma_P} \Rightarrow \frac{E(R_P) - R_f}{VaR_P}$$

  - In order to make this ratio maximized, we have:

$$\frac{Position\ i\ return\ -\ risk\ free\ rate}{MVaR_i} = \frac{Position\ j\ return\ -\ risk\ free\ rate}{MVaR_j}$$

excess return to MVaR Ratio

1. CAD $\longrightarrow w_1^*$

EhR $\longrightarrow w_2^*$

$$w_1^* + w_2^* = 100\% \quad \text{——— ①}$$

$$\frac{mVaR_1^*}{} = \frac{mVaR_2^*}{} \quad \times \quad \frac{R_1 - r_f}{mVaR_1^*} = \frac{R_2 - r_f}{mVaR_2^*}$$

$$\Updownarrow$$

$$\frac{Z \cdot \rho_{1,p} \cdot \sigma_1}{} = Z \cdot \rho_{2,p} \cdot \sigma_2$$

$$\Updownarrow$$

$$Z \cdot \boxed{w_1^* \cdot \frac{\sigma_1}{\sigma_p}} \cdot \sigma_1 = Z \cdot \boxed{w_2^* \cdot \frac{\sigma_2}{\sigma_p}} \sigma_2$$

$$\Updownarrow$$

$$w_1^* \cdot \sigma_1^2 = w_2^* \cdot \sigma_2^2 \quad \text{——— ②}$$

mVaR

IVaR

Component VaR

① $\frac{CVaR_A}{VaR_p}$ = % Contribution

$\frac{CVaR_1}{VaR_p} = \frac{CVaR_2}{VaR_p} \cdots\cdots$

Risk parity

② $mVaR_1 = mVaR_2$

$\cdots\cdots mVaR_n$

$VaR_p \downarrow$ (组合)

Global minimum

③ $\frac{R_i - r_f}{mVaR_i} = \frac{R_j - r_f}{mVaR_j}$

$SR_p \uparrow$ (最大)

Risk budget

Exercise

- Consider the following two asset portfolios:

Asset	Position Value (in thousands of USD)	Return Standard Deviation (%)	Beta
A	400	3.60	0.5
B	600	8.63	1.2
Portfolio	1,000	5.92	1

Calculate the component VaR of asset A and marginal VaR of asset B, respectively, at the 95% confidence level.

A. USD 21,773 and 0.1306

B. USD 21,773 and 0.1169

C. USD 19,477 and 0.1169

D. USD 19,477 and 0.1306

Correct Answer：C

Exercise

- Consider the following two asset portfolios:

Asset	Position Value (in thousands of USD) V	Return Standard Deviation (%) σ	Beta β
A	400	3.60	0.5
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A. USD 21,773 and 0.1306

B. USD 21,773 and 0.1169

C. USD 19,477 and 0.1169

D. USD 19,477 and 0.1306

Correct Answer：C

$$mVaR_A = z \cdot \frac{Cov(A,P)}{\sigma_P}$$

$$= z \cdot \rho_{A,P} \cdot \sigma_A$$

$$= z \cdot \beta_{A,P} \cdot \sigma_P$$

$$= \frac{VaR_P}{V_P} \times \beta_{A,P}$$

Exercise

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Calculate the component VaR of asset A and marginal VaR of asset B, respectively, at the 95% confidence level.

1.65

A. USD 21,773 and 0.1306

B. USD 21,773 and 0.1169

C. USD 19,477 and 0.1169

D. USD 19,477 and 0.1306

Correct Answer : C

$$mVaR_A = z \cdot \frac{Cov(A,P)}{\sigma_P}$$

$$= z \cdot \rho_{A,P} \cdot \sigma_A \quad \checkmark$$

$$= z \cdot \beta_{A,P} \cdot \sigma_P \quad \checkmark$$

$$= \frac{VaR_P}{V_P} \times \beta_{A,P} \quad \checkmark$$

Exercise

- A risk manager assumes that the joint distribution of returns is multivariate normal and calculates the following risk measures for a two-asset portfolio:

Component VaR

Asset	Position V	Individual VaR VaR_i	Marginal VaR	VaR Contribution
1	USD 100	USD 23.3	0.176	USD 17.6
2	USD 100	USD 46.6	0.44	USD 44.0
Portfolio	USD 200	USD 61.6		USD 61.6

What is β_2?

A. 0.714

B. 1.429

C. 1.513

D. Cannot determine from information provided

Correct Answer：B

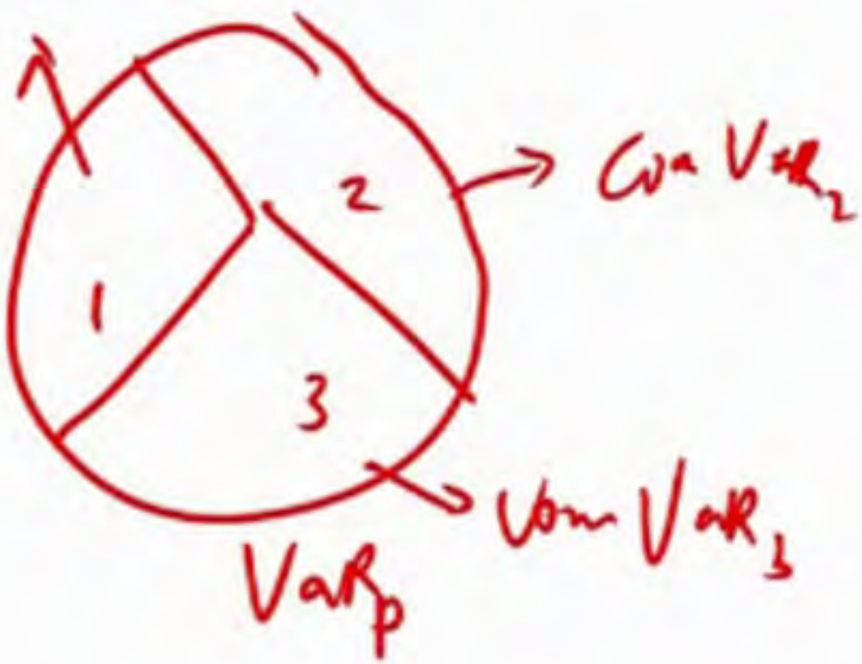
% Contribution = $\beta_i \cdot w_i$

$w_2 \cdot \beta_2 = \frac{44}{17.6144}$

$$mVaR = z \cdot \frac{cov}{\sigma_p} \cdots$$

$$\underline{IVaR}_x = VaR_{\underline{p+x}} - VaR_p$$

$ConVaR_1$



$$\sum_{i=1}^{3} ConVaR_i = VaR_p$$

$$\sum_{i=1}^{1} \frac{ConVaR_i}{VaR_p} = 100\%$$

% Contribution $= w_i \cdot \beta_i$

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Portfolio VaR

- **Incremental VaR**
  - Change in VaR owing to a new position.
  - Differs from marginal VaR: amount added or subtracted can be large.
  - Incremental VaR requires a **full revaluation** of the portfolio VaR with the new trade:

$$Incremental\ VaR = VaR_{P+a} - VaR_P$$

  - Approximate computation:

$$Incremental\ VaR_A \approx MVaR_A \times W_A(any\ amount)$$

Exercise

$VaR_B = z \cdot \sigma_B \cdot V_B = 1.65 \times 8.63\% \times 600$

- Consider the following two asset portfolios:

Asset	Position Value (in thousands of USD)	Return Standard Deviation (%)	Beta
A	400	3.60	0.5
B	600	8.63	1.2
Portfolio	1,000	5.92	1

$VaR_P = z \cdot \sigma_P \times V_P = 1.65 \times 5.92\% \times 1000$

Calculate the component VaR of asset A and marginal VaR of asset B, respectively, at the 95% confidence level.

A. USD 21,773 and 0.1306

B. USD 21,773 and 0.1169

C. USD 19,477 and 0.1169 $\neq IVaR_A$?

D. USD 19,477 and 0.1306

Correct Answer : C

Exercise

$mVaR_2 = z \cdot \beta \times \sigma_p = \frac{VaR_p}{V_p} \times \beta$

- A risk manager assumes that the joint distribution of returns is multivariate normal and calculates the following risk measures for a two-asset portfolio:

0.44 1.61

Asset	Position	Individual VaR	Marginal VaR	VaR Contribution
1	USD 100	USD 23.3	0.176	USD 17.6
2	USD 100	USD 46.6	0.44	USD 44.0
Portfolio	USD 200	USD 61.6		USD 61.6

$= z \cdot \sigma_p \times V_p$

What is β_2?

A. 0.714

B. 1.429

C. 1.513

D. Cannot determine from information provided

Correct Answer : B

% Contribution $= w_i \beta_i$

$= w_2 \times \beta_2 = \frac{44.0}{61.6}$

50%

Exercise

- A risk manager assumes that the joint distribution of returns is multivariate normal and calculates the following risk measures for a two-asset portfolio:

Asset	Position	Individual VaR	Marginal VaR	VaR Contribution
1	USD 100	USD 23.3	0.176	USD 17.6
2	USD 100	USD 46.6	0.44	USD 44.0
Portfolio	USD 200	USD 61.6		USD 61.6

What is β_2?

A. 0.714

B. 1.429

C. 1.513

D. Cannot determine from information provided

Correct Answer：B

$IVaR_2 = 61.6 - 23.3$
$= 38.3$

Exercise

- Suppose a portfolio consists of four assets. The risk contribution of each asset is as follows: UK large cap, 3.9%; UK small cap, 4.2%; UK bonds, 0.9%; non-UK bonds, 1.1%. Which of the following would not be a possible explanation for the relatively high risk contribution values for UK equities?

  A. High expected returns on UK equities.

  B. High weights on UK equities.

  C. High volatilities of UK equities.

  D. High correlation of UK equities with all other assets in the portfolio.

$\% = w \times \beta$

$\text{Con VaR} = \text{mVaR} \times V$

$z \cdot \sigma_p \times \beta$

Correct Answer : A $z \cdot \rho \times \sigma_A$

Exercise

- A portfolio has an equal amount invested in two positions, X and Y. The expected excess return of X is 9% and that of Y is 12%. Their marginal VaRs are 0.06 and 0.075 respectively. To move toward the optimal portfolio, what the manager would probably do:

SR↑

  A. Increase the allocation in Y and/or lower that in X.
  B. Increase the allocation in X and/or lower that in Y.
  C. Do nothing because the information is insufficient.
  D. Not change the portfolio because it is already optimal.

Y

Correct Answer : A

$$\frac{9\%}{0.06} = \frac{R_1 - r_f}{mVaR_1} = \frac{R_2 - r_f}{mVaR_2} = \frac{12\%}{0.075} = 1.6$$

$= 1.5$

Reading 4

Portfolio Risk Management

The three legged risk management stool

- **The risk plan** 1
  - The risk plan should be incorporated as a separate section of the organization's strategic planning document.
  - The risk plan should:
    - ✓ Set expected return and volatility (e.g., VaR and tracking error) goals for the relevant time period.
    - ✓ Define points of success or failure.
    - ✓ Paint a vision of how risk capital will be deployed to meet the organization's objectives.
    - ✓ help organizations define the bright line between those events that are merely disappointing and those that inflict serious damage.
    - ✓ Identify critical dependencies that exist inside and outside the organization.

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The three legged risk management stool

- **The risk budgeting** 2
  - The risk budget—often called asset allocation—should **quantify** the vision of the plan.
  - The budget helps the organization stay on course with respect to its risk plan.

top

1. budgetting across asset class

α manager

down

3 portfolios

Example

- A manager has a portfolio with only one position: a $500 million investment in W. The manager is considering adding a $500 million position X or Y to the portfolio. The current volatility of W is 10%. The manager wants to limit portfolio VaR to $200 million at the 99% confidence level. Position X has a return volatility of 9% and a correlation with W equal to 0.7. Position Y has a return volatility of 12% and a correlation with W equal to zero. Determine which of the two proposed additions, X or Y, will keep the manager within his risk budget.

$$VaR_W = 2.33 \times 10\% \times 500 = 116.50 million$$

$$VaR_X = 2.33 \times 9\% \times 500 = 104.85 million$$

$$VaR_Y = 2.33 \times 12\% \times 500 = 139.80 million$$

$$VaR_{W+X} = \sqrt{(116.5)^2 + (104.85)^2 + 2 \times 0.7 \times 116.5 \times 104.85} = 204 million$$

$$VaR_{W+Y} = \sqrt{(116.5)^2 + (139.8)^2} = 182 million$$

- Y keeps the total portfolio within the risk budget.

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$\rho = 0.7$

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$$VaR_{W+Y} = \sqrt{(116.5)^2 + (139.8)^2} = 182 million$$

- Y keeps the total portfolio within the risk budget.

Risk Budgeting

- **Budgeting across Asset Classes**
  - Process of allocating and managing risk using a top-down approach to different aspects of the investment process.
  - Process intended to systematically allocate return volatility across portfolio components (asset class, managers, and/or securities) to maximize return at a targeted level of risk.
    - ✓ First, determine the total Value at Risk (VaR) which can be "budgeted" to the firm.
    - ✓ Second, choose the optimal allocation of assets given the total risk profile.

Risk Budgeting

- **Budgeting across Active Managers**
  - The traditional method for evaluating active managers is by measuring their tracking error and using it to derive a measure known as information ratio. And the tracking error is the active return minus the benchmark return.

$$IR_i = \frac{\text{expected tracking error of the manager}}{\text{volatility of the manager's tracking error}}$$

$$IR_P = \frac{\text{expected tracking error of the portfolio}}{\text{volatility of the portfolio's tracking error}}$$

  - The optimal allocation across managers is:

$$\text{weight of portfolio managed by manager } i$$

$$= \frac{IR_i \times (\text{Portfolio's tracking error volatility})}{IR_P \times (\text{manager's tracking error volatility})}$$

GOLDEN FUTURE

$\alpha = 0$

passive

diversify

active $\longrightarrow \uparrow\uparrow w_i = \dfrac{\uparrow 2R_i \times TEV_p}{2R_p \times TEV_i \downarrow}$

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$\alpha = 0$

passive

diversify

active $\longrightarrow$ $W_i = \dfrac{IR_i \times TEV_p}{IR_p \times TEV_i}$

$IR = \dfrac{\alpha}{TEV}$

$= \dfrac{IR_i / TEV_i}{IR_p / TEV_p}$

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$\alpha = 0$

passive $\quad w = 100\% - \sum w_i$

diversify

active $\longrightarrow$ $w_i = \dfrac{IR_i \times TEV_p}{IR_p \times TEV_i}$

$= \dfrac{IR_i / TEV_i}{IR_p / TEV_p}$

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$\alpha = 0$

passive

diversify

$w = 100\% - \sum w_i$

(benchmark portfolio)

$\begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$

active $\longrightarrow$ $w_i = \dfrac{IR_i \times TEV_p}{IR_p \times TEV_i}$

$= \dfrac{IR_i / TEV_i}{IR_p / TEV_p}$

专业 · 创新 · 增值

Example

- (Cont.)The fund has an allocation of $60 million devoted to U.S. equities. Now the fund wants to allocate this $60 million to two managers so as to maximize the information ratio of the fund subject to an overall TEV of 4%. This is equivalent to a risk budget of $3.948 million. Each manager has a TEV of 6%. The fund managers have different capabilities, their IRs are 0.6 and 0.4, respectively. To achieve an exact TEV of 4%, the fund also need some residual investment in the benchmark, which has a TEV of zero.

	Inputs			Outputs	
	TEV	Information Ratio	Weight	Allocated Principal	Relative Risk Budget
Manager1	w_1=6%	IR_1=0.6	X_1=55%	60×0.55=33	1.645×6%×33 =3.2571
Manager2	w_2=6%	IR_2=0.4	X_2=37%	60×0.37=22.2	1.645×6%×22.2 =2.19114
Index	w_B=0%	IR_0=0	X_B=8%	60×0.08=4.8	0
Portfolio	w_P=4%	IR_P=0.72	100%	60	3.948

Example

- (Cont.)The fund has an allocation of $60 million devoted to U.S. equities. Now the fund wants to allocate this $60 million to two managers so as to maximize the information ratio of the fund subject to an overall TEV of 4%. This is equivalent to a risk budget of $3.948 million. Each manager has a TEV of 6%. The fund managers have different capabilities, their IRs are 0.6 and 0.4, respectively. To achieve an exact TEV of 4%, the fund also need some residual investment in the benchmark, which has a TEV of zero.

$TEV_p = 4\%$

$TEV_1 = TEV_2 = 6\%$

$IR_1 = 0.6$

$IR_2 = 0.4$

	Inputs			Outputs	
	TEV	Information Ratio	Weight	Allocated Principal	Relative Risk Budget
Manager1	w_1=6%	IR_1=0.6	X_1=55%	60×0.55=33	1.645×6%×33 =3.2571
Manager2	w_2=6%	IR_2=0.4	X_2=37%	60×0.37=22.2	1.645×6%×22.2 =2.19114
Index	w_B=0%	IR_0=0	X_B=8%	60×0.08=4.8	0
Portfolio	w_P=4%	IR_P=0.72	100%	60	3.948

$$w_1 = \frac{IR_1 / TEV_1}{IR_p / TEV_p} = \frac{6\% / 6\%}{IR_p / 4\%}$$

$$w_2 = \frac{IR_2 / TEV_2}{IR_p / TEV_p}$$

$$w_3 = 1 - w_1 - w_2$$

$$IR_p = \frac{\alpha_p}{TEV_p} = \frac{IR_1 \times TEV_1 + IR_2 \times TEV_2 + 0}{TEV_p}$$

专业 · 创新 · 增值

Risk Budgeting

- **Calculation Process:**

$$X_1 = \frac{IR_1/\omega_1}{IR_P/\omega_P} = \frac{0.6}{IR_P} \times \frac{4\%}{6\%}$$

$$X_2 = \frac{IR_2/\omega_2}{IR_P/\omega_P} = \frac{0.4}{IR_P} \times \frac{4\%}{6\%}$$

$$IR_P = \frac{E(e_P)}{\omega_P} = \frac{X_1E(e_1) + X_2E(e_2)}{\omega_P} = \frac{X_1E(e_1) + X_2E(e_2)}{4\%}$$

$$IR_1 = \frac{E(e_1)}{\omega_1} = 0.6 \rightarrow E(e_1) = 3.6\%$$

$$IR_2 = \frac{E(e_2)}{\omega_2} = 0.4 \rightarrow E(e_2) = 2.4\%$$

$$\downarrow$$

$$X_1 = 55\%, X_2 = 37\%$$

$X_3 = 8\%$

$$E(e_P) = X_1E(e_1) + X_2E(e_2) = 3.6\% \times 55\% + 2.4\% \times 37\% = 2.9\%$$

$$IR_P = \frac{E(e_P)}{\omega_P} = \frac{2.9\%}{4\%} = 0.72$$

$$Cov(x, by) = b\,Cov(x, y)$$

$$Cov(x, y+z) = Cov(x, y) + Cov(x, z)$$

$$IR_p = \frac{\alpha_p}{TEV_p} = \frac{\sum IR_i \cdot TEV_i}{TEV_p}$$

专业 · 创新 · 增值

Exercise

- The AT&T pension fund has 68%, or about $13 billion invested in equities. Assume a normal distribution and volatility of 15% per annum. The fund measures absolute risk with a 95%, one-year VaR, which gives $3.2 billion. The pension plan wants to allocate this risk to two equity managers, each with the same VaR budget. Given that the correlation between managers is 0.5, the VaR budget for each should be

A. $3.2 billion

B. $2.4 billion

C. $1.9 billion

D. $1.6 billion

Correct Answer: C

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$VaR_p^2 = VaR_1^2 + VaR_2^2 + 2\rho\, VaR_1\, VaR_2$

$VaR_1 = VaR_2$

Correct Answer: C

3.2

0.5

Exercise

- The pension management analysts at Bing Inc. use a two-step process to manage the assets and risk in the pension portfolio. First, they use a VaR-based risk budgeting process to determine the asset allocation across for broad asset classes. Then, within each asset class, they set a maximum tracking error allowance from a benchmark index and determine an active risk budget to distribute among individual managers. Assume the returns are normally distributed. From the first step in the process, the following information is available.

	Expected Return	Volatility	Asset Allocation	Individual VaR	Marginal VaR
Small cap	0.2%	2.66%	35.0%	6,491	0.055
Large cap	0.15%	2.33%	40.0%	6,497	0.044
Commodities	0.10%	1.91%	16.7%	2,216	0.020
Emerging market	0.15%	2.70%	8.3%	1,570	0.047
	Total VaR:13,322				

Exercise

➢ Which of the following statements is/are correct?

I. Using VaR as the risk budgeting measure, the emerging markets class has the smallest risk budget.

II. If an additional dollar were added to the portfolio, the marginal impact on portfolio VaR would be greatest if it were invested in small caps.

III. As the maximum tracking error allowance is lowered, the individual managers have more freedom to achieve greater excess returns.

IV. Setting well-defined risk limits and closely monitoring risk levels guarantee that risk limits will not be exceeded.

A. I and II only

B. I,II,III and IV

C. II and III

D. I only

Correct Answer: A

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Correct Answer: A

The three legged risk management stool

- **The risk monitoring** 3
  - Risk monitoring is required to ensure that material deviations from risk budget are detected and addressed in a timely fashion

VaR Application to Monitor risk

- **Passive and active allocations**
  - With a VaR system, investors can use this system to monitor their market risk better. This applied to both **passive and active allocations**.
  - **Passive allocations**
    - ✓ We can also call it benchmarking, however, it does not keep risk constant because the composition of the indices can change substantially.
  - **Active allocations**
    - ✓ Active portfolio management can change the risk profile of the fund.
    - ✓ In that case, VaR monitoring is a process to capture the reason why a sudden jump increase in the reported VaR of the fund. There are several explanations as follow:
      - ◆A manager taking more risk.
      - ◆Different managers taking similar bets.
      - ◆More volatile markets.

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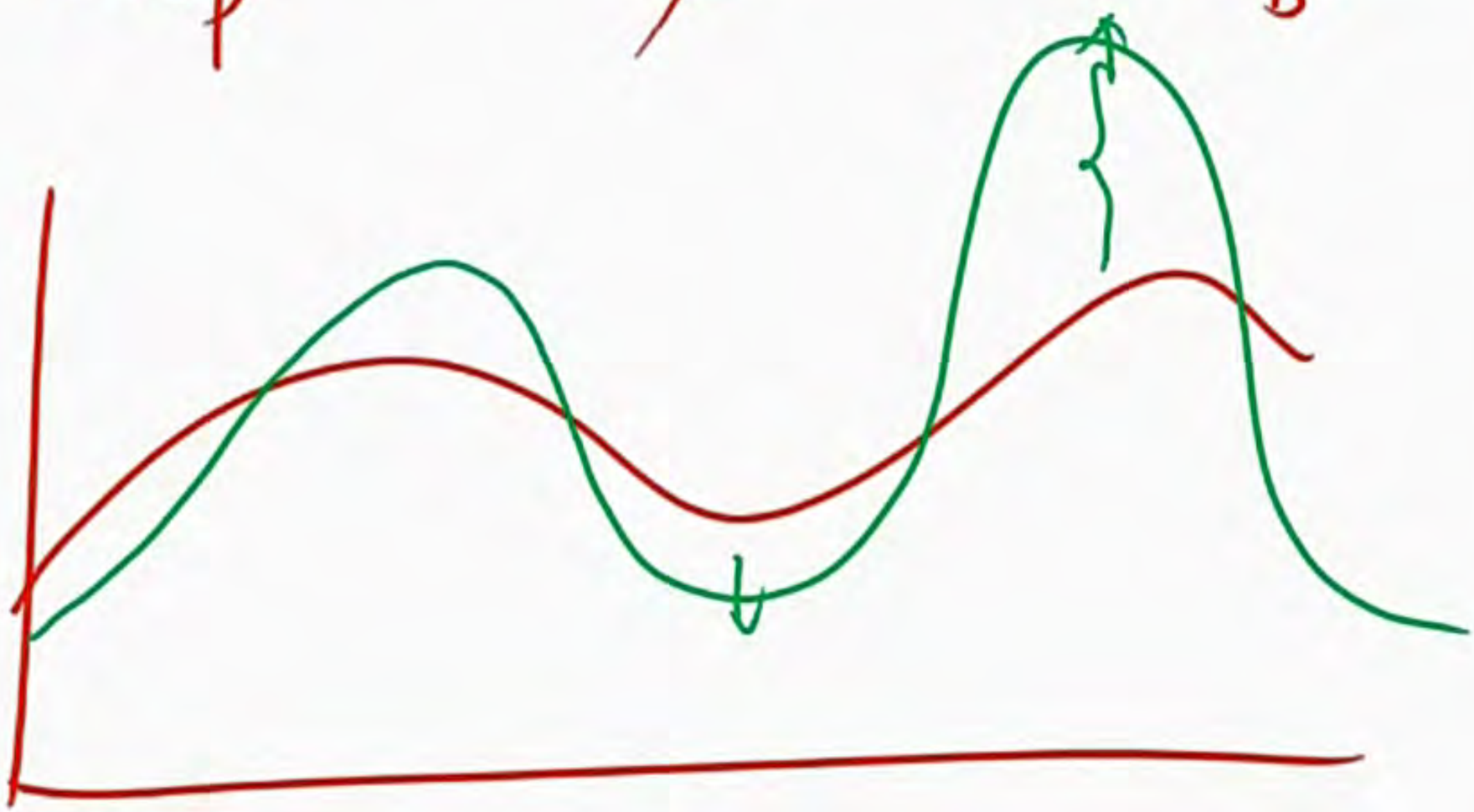
$\overline{\quad}$

$VaR_P \not\geq VaR_B$



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      - ✓ ◆A manager taking more risk.
      - ◆Different managers taking similar bets. ✓
      - ◆More volatile markets. (+ leverage)

VaR Application to Monitor risk

Conditional VaR / ES

- **VaR application**
  - Measures of marginal and component VaR can be used to identify where position changes will have the greatest effect on the total portfolio risk.
- **Limitation of the system**
  - Risk cannot be measured easily for some important asset classes such as real estate, venture capital, and some categories of hedge funds owing to illiquidity.
  - Other series may have very short histories, such as emerging markets, or none at all, such as initial public offerings.

GOLDEN FUTURE

plan

budget

① Asset class

$VaR_{(p+x)} = \sqrt{VaR_p^2 + VaR_x^2 + 2\rho VaR_p VaR_x}$

$\leq$ risk budget

② manager

Active

$w_i = \frac{IR_i / TE_i}{IR_p / TE_p}$

passive

$1 - \sum w_i$

③ portfolio

$\frac{R_X - r_f}{mVaR_X} = \frac{R_Y - r_f}{mVaR_Y} = \cdots$

monitor

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Pension plan

VaR Applications to different risks

Portfolio Risk Management

VaR Applications to Different Risks

- **VaR applications to different risks**
  - Absolute Risk vs. Relative Risk
  - Policy mix risk vs. active management risk
  - Funding Risk
  - Sponsor Risk

Measure Different Risks

- **Absolute Risk vs. Relative Risk**
  - The difference is whether the loss is measured relative to zero or a benchmark.
  - Absolute Risk: risk of a dollar loss over the horizon.
  - Relative Risk: the risk of a dollar loss relative to benchmark. Shortfall measured as dollar difference between the fund return and benchmark return. Relevant return is the tracking error (TE), which is excess return of asset over benchmark.

$$TE = R_P - R_B$$

$$TEV = \sigma(\mathrm{e}) = \omega = \sqrt{\sigma_P^2 - 2\rho\sigma_P\sigma_B + \sigma_B^2}$$

$TEV = \sigma_{(R_P - R_B)}$

Measure Different Risks

benchmark risk

- **Policy mix risk vs. active management risk**
  - The absolute risk can be broken down into two components, one is the **policy mix risk,** the other is **active management risk.**
    - ✓ **Policy mix risk:** the policy mix risk is the risk of a dollar loss owing to the policy mix selected by the fund.
    - ✓ **Active management risk:** the active management risk is the risk of a dollar loss owing to the total deviations from the policy mix.
- **Policy mix return vs. active management return**
  - The asset return can be decomposed into:

$$R_{asset} = R_{policy\ mix} + R_{active\ mgt} = \sum_{i} \omega_i^b R_i^b + \sum_{i} \left(\omega_i R_i - \omega_i^b R_i^b\right)$$

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$$VaR_p = \sqrt{VaR_{\text{portfolio}}^2 + VaR_{\text{active mgt}}^2 + 2\rho V. \quad V.}$$

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Example

- The Ontario Teachers' Pension Plan Board (OTPPB) estimates that its annual VaR at the 99 percent level of confidence can be decomposed as follows (in percent of the initial fund value):

Source of Risk	VaR
Policy-mix VaR	19.6%
Active-mgt. VaR	1.6%
Asset VaR	19.3%

19.3 < 19.6 + 1.6

- **From the table, we can conclude that:**
  - Most of the risk is due to the policy mix.
  - Active management VaR is rather small, there may be two reasons:
    - ✓ the fund diversified with the benchmark or other managers.
    - ✓ the assets are invested in indexed or closely indexed funds.
  - Policy-mix VaR and active management VaR do not add up to the total-asset VaR, owing to diversification effect.

$$w_1 = \frac{IR_1 / TEV_1}{IR_p / TEV_p} = \frac{6\% / 6\%}{IR_p / 4\%}$$

$$w_2 = \frac{IR_2 / TEV_2}{IR_p / TEV_p}$$

$$w_3 = 1 - w_1 - w_2$$

$$IR_p = \frac{\alpha_p}{TEV_p} = \frac{w_1 \times IR_1 \times TEV_1 + \overset{w_2 \times}{IR_2} \times TEV_2 + 0 \times w_3}{TEV_p}$$

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Risk Types

- **Funding Risk**
  - The risk that assets values will not be sufficient to fund the liabilities.
  - **Surplus** (S) is the difference between the value of the assets (A) and the liabilities (L). The change in the surplus (△S) is equal to the change in assets (△A) minus the change in liabilities (△L).
  - Funding risk should be measured as the potential shortfall in surplus over the horizon, this is sometimes called **surplus at risk**.

$$Expected\ surplus = A \times (1 + R_A) - L \times (1 + R_L)$$

$$\sigma_{Surplus} = \sqrt{A^2 {\sigma_A}^2 + L^2 {\sigma_L}^2 - 2\sigma_A \sigma_L \rho}$$

$$Surplus\ at\ risk = Expected\ surplus - z_\alpha \times \sigma_{Surplus}$$

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我们交纳 回报	老人养老金

82-300

Risk Types

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$S = A - L$

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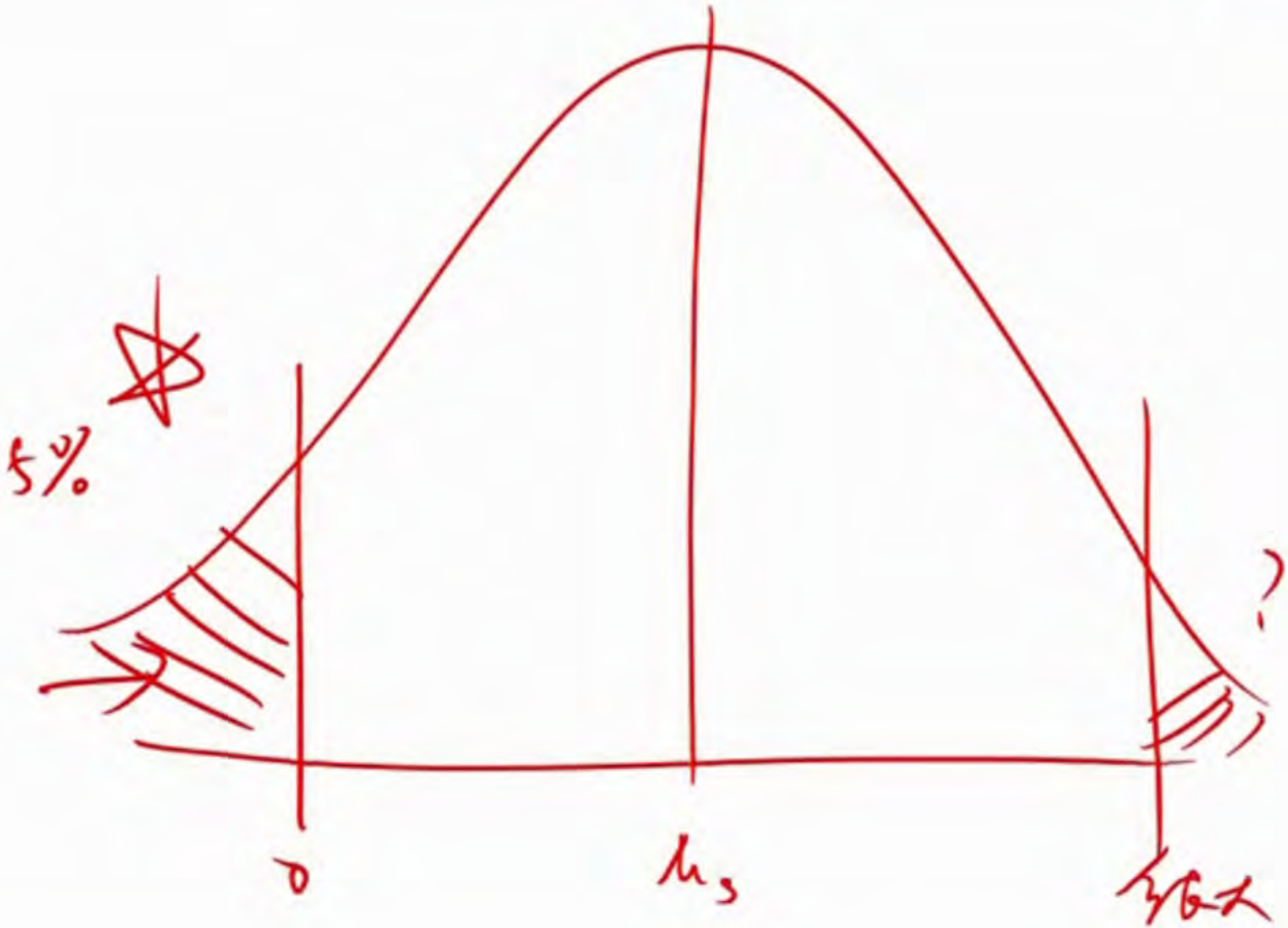

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Risk Types

VaR(surplus)

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S = A − L

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SaR

$$Expected\ surplus = A \times (1 + R_A) - L \times (1 + R_L)$$

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$$Surplus\ at\ risk = Expected\ surplus - z_\alpha \times \sigma_{Surplus}$$

$$VaR = |\mu - z \cdot \sigma| \cdot V$$

$$SaR = |\mu_s - z \cdot \sigma_s| \, V_s$$

$$= \overset{(\$)}{\mu_s} - z \cdot \overset{(\$)}{\sigma_s}$$

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Risk Types

- **Funding Risk**

$S = A - L$

VaR(surplus)

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SaR

$$\text{Expected surplus} = A \times (1 + R_A) - L \times (1 + R_L)$$

$$\sigma_{Surplus} = \sqrt{A^2\sigma_A{}^2 + L^2\sigma_L{}^2 - 2\sigma_A\sigma_L\rho}$$

$$\text{Surplus at risk} = \text{Expected surplus} - z_\alpha \times \sigma_{Surplus}$$

Example

- An analyst reports the following fund information to the advisor of a pension fund that currently invests in government and corporate bonds and carries a surplus of USD 10 million:

Pension	Assets	Liabilities
Amount (USD million)	180	140
Expected annual growth	6%	10%
Modified duration	14	8
Annual volatility of growth	25%	12%

Example

$180 \times 6\% - 140 \times 10\%$

- An analyst reports the following fund information to the advisor of a pension fund that currently invests in government and corporate bonds and carries a surplus of USD 10 million:

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Modified duration	14	8
Annual volatility of growth	25%	12%

40

$E(S) = M_S = 180 \times (1+6\%) - 140 \times (1+10\%)$

Risk Types

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S_0

$$\text{Expected surplus} = A \times (1 + R_A) - L \times (1 + R_L)$$

ΔS

$\sigma_{(A-L)}$

$$\sigma_{Surplus} = \sqrt{A^2\sigma_A{}^2 + L^2\sigma_L{}^2 - 2\sigma_A\sigma_L\rho}$$

$$\text{Surplus at risk} = \text{Expected surplus} - z_\alpha \times \sigma_{Surplus}$$

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Annual volatility of growth	25%	12%

$$\sigma_{(S)} = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A \cdot \sigma_L}$$

$$= (25\% \times 180)^2 + (12\% \times 140)^2 - 2$$

Example

- To evaluate the sufficiency of the fund's surplus, the advisor estimates the possible surplus values at the end of one year. The advisor assumes that annual returns on assets and the annual growth of the liabilities are jointly normally distributed and their correlation coefficient is 0.68. The advisor can report that, with a confidence level of 95%, the surplus value will be greater than or equal to:

  A. USD -58.2 million

  B. USD -22.0 million

  C. USD 1.0 million

  D. USD 21.0 million

Answer: B

Example

- An analyst reports the following fund information to the advisor of a pension fund that currently invests in government and corporate bonds and carries a surplus of USD 10 million:

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$$\sigma_{(S)} = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A \cdot \sigma_L}$$

$$= \sqrt{(25\% \times 180)^2 + (12\% \times 140)^2 - 2 \times .68 \times (\quad)(\quad)}$$

Example

- **Explanation:**
  - The lower bound of the 95% confidence interval is equal to: Expected Surplus – (95% confidence factor * Volatility of Surplus).
  - The required variables can be calculated as follows:

$$V_S = \sqrt{V_A^2 \times \sigma_A^2 + V_L^2 \times \sigma_L^2 - 2V_A \times V_L \times \sigma_A \times \sigma_L \times \rho_{AL}}$$

$$= \sqrt{180^2 \times 0.25^2 + 140^2 \times 0.12^2 - 2 \times 180 \times 140 \times 0.25 \times 0.12 \times 0.68}$$

$$= 35.764 \text{ (m\$)}$$

$$Expected\ Surplus = V_A \times (1 + R_A) - V_L \times (1 + R_L)$$

$$= 180 \times 1.06 - 140 \times 1.10 = USD\ 36.80 million$$

  - Therefore, the lower bound of the 95% confidence interval = 36.80 – 1.645*35.764 = USD -22.032 million.

SCR =

1,65 ✓

1,645

2,33

2,326

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Example

- To evaluate the sufficiency of the fund's surplus, the advisor estimates the possible surplus values at the end of one year. The advisor assumes that annual returns on assets and the annual growth of the liabilities are jointly normally distributed and their correlation coefficient is 0.68. The advisor can report that, with a confidence level of 95%, the surplus value will be greater than or equal to:

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Answer: B

Measure Different Risks

- **Sponsor risk**
  - **The sponsor is the owner of the fund,** who is ultimately responsible for the pension fund.
  - **The sponsor risk** is measured both the movements in the assets, or even the surplus, and the ultimate effect on the economic value of the firm.
    - ✓ Cash-flow risk: is the risk of year-to-year fluctuations in contributions to the pension fund.
    - ✓ Economic risk: is the risk of variation in total economic earnings of the plan sponsor.
  - The pension-plan management should be integrated with the overall financial goals of the plan sponsor.

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DB plan

2000k

DC plan

3000 400

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Measure Different Risks

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    surplus < 0

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Reading 5

Performance Measurement Evaluation

127-201

Performance Measurement Framework

- Tool 1: The Green Zone
- Tool 2: The Sharpe and Information Ratios
- Tool 3: Alpha versus the Benchmark
- Tool 4: Alpha versus the Peer Group
- Tool 5: Attribution of Returns

Performance Measurement Framework

- Tool 1: The Green Zone
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$$\begin{cases} \frac{1-4}{5\cdot 9} & \text{——} \quad 3 \\ & \text{——} \quad 3-4 \\ \geq 10 & \text{——} \quad 4 \end{cases}$$

Performance Measurement Framework

- **Tool 1: The Green Zone**
  - For prior week, month, year: calculate normalized returns (excess returns/tracking error) and tracking error.
  - Compare actual to target.
  - Policy decisions about **deviations**. (green/yellow/red)
    - ✓ **Green zone**: usual event with insignificant deviations.
    - ✓ **Yellow zone**: unusual event, but still is expected to occur with some regularity.
    - ✓ **Red zone**: truly unusual events and required immediate follow-up.

Performance Measurement Framework

- **Tool 2: The Sharpe and Information Ratios**
  - Can be used to measure relative performance vis à vis the competition; e.g., peer group comparisons.
  - They test whether the manager has generated sufficient excess returns to compensate for the risk assumed.

Risk adjusted Ratio

$$SR = \frac{R_p - r_f}{\sigma_p}$$

Performance Measurement Framework

- **Tool 3: Alpha versus the Benchmark**
  - This tool regresses the excess returns of the fund against the excess returns of the benchmark.
  - The outputs of this regression are:
    - ✓ An intercept, often referred to as "**alpha**", or skill.
    - ✓ A slope coefficient against the excess returns of the benchmark, often referred to as "**beta**", or leverage.
- **Tool 4: Alpha versus the Peer Group**
  - This tool regresses the manager's excess returns against the excess returns of the manager's peer group.
  - This tool is used to determine whether the manager demonstrates skill over and above what is found in the peer group.

$$\alpha = \underline{R_p - R_B} = \text{excess return}$$

α (阿尔法)

$$R_p = \alpha + \beta R_B + \varepsilon$$

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Performance Measurement Framework

- **Tool 3: Alpha versus the Benchmark**
  - This tool regresses the excess returns of the fund against the excess returns of the benchmark.
  - The outputs of this regression are: $R_p = \alpha + \beta R_B + \varepsilon$
    - ✓ An intercept, often referred to as "**alpha**", or skill.
    - ✓ A slope coefficient against the excess returns of the benchmark, often referred to as "**beta**", or leverage.
- **Tool 4: Alpha versus the Peer Group** $R_p = \alpha + \beta R_{peer} + \varepsilon$
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  - This tool is used to determine whether the manager demonstrates skill over and above what is found in the peer group.

被动的收益

Performance Measurement Framework

- **Tool 5: Attribution of Returns** 业绩归因
  - A commonly used tool to measure the quality of returns is performance attribution. This technique attributes the source of returns to individual securities and/or common factors.

Return Calculation

$SR = \frac{R_p - r_f}{\sigma_p}$

- **Holding Period Return**

$$r_i = \frac{Total\ proceeds}{Initial\ investment} = \frac{Int(Div)Income + Capital\ gain}{Initial\ investment}$$

- **Time-Weighted Return**
  - Also known as the geometric average
  - It is not affected by cash flows during the investment periods

$$1 + r_G = [(1 + r_1)(1 + r_2)\ldots(1 + r_n)]^{1/n}$$

- **Dollar-Weighted Return**
  - Also Known as Internal rate of Return (IRR).
  - The rate of return at which the present value of cash inflows equals the present value of cash outflows.
  - Most suitable to show clients the performance of their own funds.

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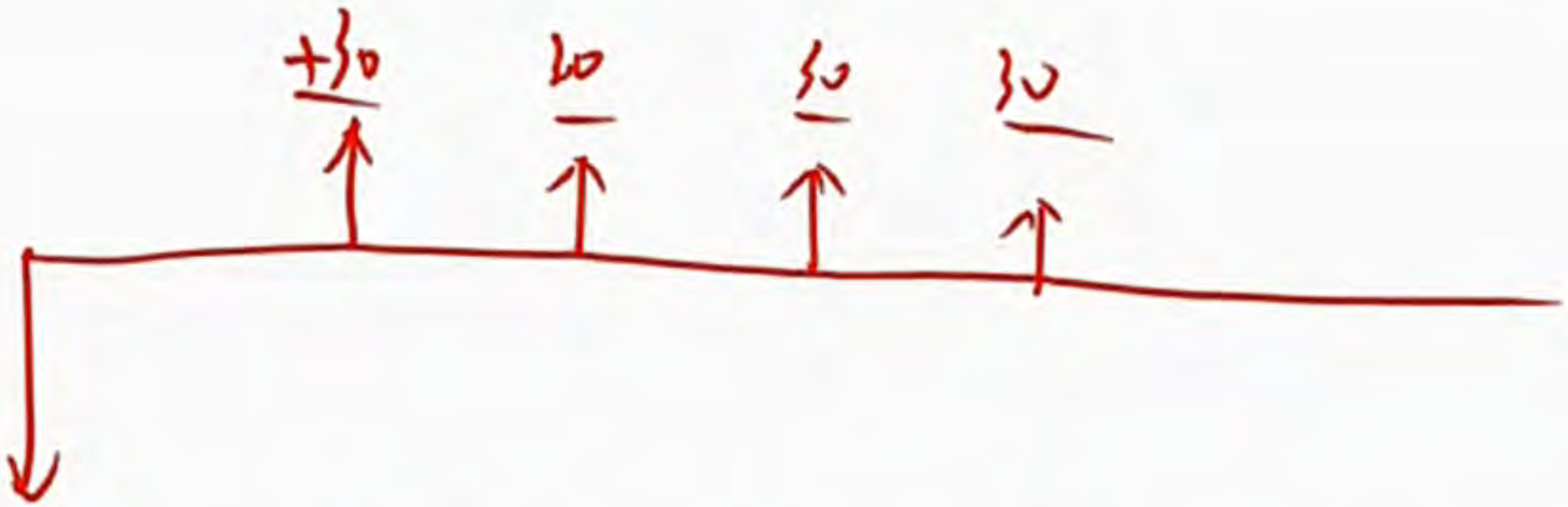

-100

$$\frac{30}{1+x}+\frac{30}{(1+x)^2}+\frac{30}{(1+x)^3}+\frac{30}{(1+x)^4}=100$$

YTM

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Return Calculation

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$$r_i = \frac{Total\ proceeds}{Initial\ investment} = \frac{Int(Div)Income + Capital\ gain}{Initial\ investment}$$

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  - Also known as the geometric average $\sqrt{(1+10\%)(1+20\%)} - 1$
  - It is not affected by cash flows during the investment periods

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- **Dollar-Weighted Return** (计算器) IRR
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Time-Weighted and Dollar-Weighted Returns

➢ **Example**

- At T_0, a client invests $100,000 in a fund. By the end of the year, the value has risen to $105,000. Both time-weighted return and dollar-weighted return for the first year will be 5%.
- At T_1, the client invests $95,000 more in the fund. So, the portfolio manager has $200,000 to invest. By the end of second year (T_2), the portfolio value rises to $220,000. The TWR and DWR for the second year will be 10%. What is the annualized TWR and DWR for the 2-year period?

$$TWR = \sqrt{1.05 \times 1.10} - 1 = 7.47\%$$

$$DWR = \$100{,}000 + \frac{\$95{,}000}{(1+r)} = \frac{\$220{,}000}{(1+r)^2} \Rightarrow r = 8.24\%$$

- The annualized DWR is higher that TWR because the fund manager had more money to invest in the second year in which it earned a higher return.

20 —— 22W

-95000

$$TWR = \sqrt{(5\%+1)(10\%+1)} - 1$$

$$\frac{+220000}{(HX)^2} - \frac{95000}{HX} = 100000$$

$$DWR = X \qquad 100000 \times (HX)^2 + 95000 \times (HX) = 220000$$

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Time-Weighted and Dollar-Weighted Returns

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20 —— 22W

-100000 105000 220000

0 1 2

(若观)

-95000

$$TWR = \sqrt{(5\%+1)(10\%+1)} - 1$$

(实现)

DWR = X

$$\frac{+220000}{(HX)^2} - \frac{95000}{HX} = 100000$$

$$100000 \times (HX)^2 + 95000 \times (HX) = 220000$$

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Risk adjusted Return

- Evaluating performance based on average return alone is not very useful. Returns must be adjusted for risk before they can be compared meaningfully.
  - The simplest and most popular way is to compare rates of return with those of other investment funds with **similar risk characteristics**, which is called the **comparison universe**.
- **Adjusting returns for risk**
  - Total risk adjusted return
    - ✓ M^2 measure of performance
  - Systematic risk adjusted return
    - ✓ T^2 measure of performance
  - Unsystematic risk adjusted return
    - ✓ IR measure of performance

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    - ✓ IR measure of performance

SR TR IR

total systematic

Risk-Adjusted Performance Measures

- **Sharpe Ratio** measures the excess return per unit of total risk.

$$SR = \frac{R_P - R_F}{\sigma_P}$$

- **Modigliani-Squared (M^2)**
  - Adjusting the portfolio such that its standard deviation is identical to that of the market portfolio.
  - The M^2 will then be the excess returns earned by the adjusted portfolio.

$$M^2 = \frac{\sigma_M}{\sigma_P}(R_p - R_f) - (R_M - R_f) = \sigma_M \times (SR_P - SR_M)$$

- M^2 is similar to Sharpe Ratio, but it is **a return and easy to understand**. It can be used to rank portfolios. The resultant rankings will be the same as with Sharpe Ratio.

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投资 portfolio 与 投资 benchmark, 1 return 高多少

Risk-Adjusted Performance Measures

- **Example**
  - Calculate the M^2 measure for Portfolio P:

✓ Portfolio P mean return	10%
✓ Portfolio P standard deviation	40%
✓ Market portfolio mean return	12%
✓ Market portfolio standard deviation	20%
✓ Risk-free rate	4%

- **Answer:**

$(SR_p - SR_B) \cdot \sigma_B$

$$M^2 = \frac{\sigma_M}{\sigma_P}(E_{R_P} - R_f) - (E_{R_M} - R_f)$$

$$= \frac{20\%}{40\%}(10\% - 4\%) - (12\% - 4\%) = -5\%$$

$$(SR_p - SR_B) \times \sigma_B$$

$$= \frac{R_p - r_f}{\sigma_p} \times \sigma_B - \frac{R_B - r_f}{\sigma_B} \times \sigma_B$$

$$(SR_p - SR_B) \times \sigma_B$$

$$= \frac{R_p - r_f}{\sigma_p} \times \sigma_B - (R_B - r_f)$$

✓

Risk-Adjusted Performance Measures

- **Example**
  - Calculate the M^2 measure for Portfolio P:
    - ✓ Portfolio P mean return 10%
    - ✓ Portfolio P standard deviation 40%
    - ✓ Market portfolio mean return 12%
    - ✓ Market portfolio standard deviation 20%
    - ✓ Risk-free rate 4%
  - **Answer:**

$(SR_p - SR_B) \cdot \sigma_B$

$$M^2 = \frac{\sigma_M}{\sigma_P}(E_{R_P} - R_f) - (E_{R_M} - R_f)$$

$$= \frac{20\%}{40\%}(10\% - 4\%) - (12\% - 4\%) = -5\%$$

Risk-Adjusted Performance Measures

- **Treynor Ratio** measures the excess return per unit of systematic risk.

$$TR = \frac{R_P - R_F}{\beta_P}$$

- T^2
  - Adjusting the portfolio such that its beta is identical to that of the market portfolio.
  - The T^2 is calculated If you subtract the market excess return from Treynor's measure

$$T^2 = \beta_M \times (TR_P - TR_M) = TR_P - (R_M - R_f)$$

- T^2 is similar to Treynor Ratio, but it is **a return and easy to understand**. It can be used to rank portfolios. The resultant rankings will be the same as with Treynor Ratio but may be different from that with Sharpe Ratio.

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total risk — SR — $M^2 = (SR_p - SR_B) \times \sigma_B$

systematic risk — TR — $T^2 = (TR_p - TR_B) \times \beta_B$

$$\left(\frac{R_p - r_f}{\beta_p} - \frac{R_B - r_f}{1} \right) \times 1$$

$$= TR_p - (R_B - r_f)$$

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Risk-Adjusted Performance Measures

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$$TR = \frac{R_P - R_F}{\beta_P}$$

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market risk premium

$$T^2 = \beta_M \times (TR_P - TR_M) = TR_P - (R_M - R_f)$$

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total risk	SR	$M^2 = (SR_p - SR_B) \times \sigma_B$
systematic risk	TR	$T^2 = (TR_p - TR_B) \times \beta_B$
unsystematic risk	IR	$(IR_p - IR_B) \times TEV_B$

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total risk	SR	$M^2 = (SR_p - SR_B) \times \sigma_B$
systematic risk	TR	$T^2 = (TR_p - TR_B) \times \beta_B$
unsystematic risk	IR	$\cancel{\sigma (IR_p - IR_B) \times IEV_B}$ $= 0$

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Risk-Adjusted Performance Measures

- **Excess Return, Alpha and Jensen's Alpha**
  - Excess return is the difference between the portfolio and benchmark

$$\text{Excess return} = E(R_P) - E(R_B)$$

  - Alpha is the intercept term of a regression model between the excess return of the portfolio and the excess return of the benchmark
    - ✓ Beta measures the leverage towards benchmark

$$R_P - R_F = \alpha - \beta[R_M - R_F] + \varepsilon$$

  - Jensen's alpha calculates the returns of a portfolio in excess of the theoretical expected returns such as CAPM or APT

$$\alpha_P = E(R_P) - \{R_F + \beta_P[E(R_M) - R_F]\}$$

$$\alpha_P = E(R_P) - \left(R_F + \sum \beta_i \lambda_i\right)$$

Risk-Adjusted Performance Measures

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risk premium

$$\text{Excess return} = E(R_P) - E(R_B)$$

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$= R_p - r_f$

    - ✓ Beta measures the leverage towards benchmark

$$R_P - R_F = \alpha - \beta[R_M - R_F] + \varepsilon$$

  - Jensen's alpha calculates the returns of a portfolio in excess of the theoretical expected returns such as CAPM or APT

$$\alpha_P = E(R_P) - \{R_F + \beta_P[E(R_M) - R_F]\}$$

$$\alpha_P = E(R_P) - \left(R_F + \sum \beta_i \lambda_i\right)$$

$$R_p = R_f + \alpha - \beta R_m + \beta R_f + \varepsilon = \alpha + \left[(1+\beta) r_f - \beta R_m\right] + \varepsilon$$

Risk-Adjusted Performance Measures

- **Excess Return, Alpha and Jensen's Alpha**
  - Excess return is the difference between the portfolio and benchmark

$$\text{Excess return} = E(R_P) - E(R_B)$$

  - Alpha is the intercept term of a regression model between the excess return of the portfolio and the excess return of the benchmark
    - ✓ Beta measures the leverage towards benchmark

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Jen'α = R(产) − R(理论) < CAPM / APT

Exercise

- Portfolio Q has a beta of 0.7, an expected return of 12.8%, and an market risk premium of 5.25%. The risk-free rate is 4.85%. Calculate Jensen's alpha measure for portfolio Q.

A. 7.67%

B. 2.70%

C. 5.73%

D. 4.27%

Correct Answer: D

R(市) — R(无)

4.85% + 0.7×5.25%

12.8%

Exercise

- Assume that a hedge fund provides a large positive alpha. The fund can take leveraged long and short positions in stocks. The market went up over the period. Based on this information,

  A. If the fund has net positive beta, all of the alpha must come from the market.

  B. If the fund has net negative beta, part of the alpha comes from the market.

  C. If the fund has net positive beta, part of the alpha comes from the market.

  D. If the fund has net negative beta, all of the alpha must come from the market.

  Correct Answer: C

Exercise

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  A. If the fund has net positive beta, all of the alpha must come from the market.

  None

  B. If the fund has net negative beta, part of the alpha comes from the market.

  C. If the fund has net positive beta, part of the alpha comes from the market.

  D. If the fund has net negative beta, all of the alpha must come from the ~~market~~. manager

  Correct Answer: C

Exercise

- Risk Masler is considering the performance of the managers of two funds, the HCM Fund and the GRT Fund. He uses a linear regression of each manager's excess return (ri) against the excess returns of a peer group (rB):

$$r_i = a_i + b_i \times r_B + \varepsilon_i$$

Fund	Initial Equity	Borrowed Funds	Total Investment Pool	a_i	b_i
HCM	USD 100	USD 0	USD 100	0.0150 (t = 4.40)	0.9500 (t = 12.1)
GRT	USD 500	USD 3,000	USD 3,500	0.0025 (t = 0.02)	3.4500 (t = 10.20)

- Based on this information, which of the following statements is correct?
  A. The regression suggests that both managers have greater skill than the peer group.
  B. The αi term measures the extent to which the manager employs greater or lesser amounts of leverage than do his/her peers.
  C. If the GRT Fund were to loss 10% in the next period, the return on equity (ROE) would be -60%
  D. The sensitivity of the GRT fund to the benchmark return is much higher than that of the HCM fund.

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  - **D. The sensitivity of the GRT fund to the benchmark return is much higher than that of the HCM fund.**

Risk-Adjusted Performance Measures

- **Information Ratio** measures the alpha per unit of unsystematic risk.

$$IR = \frac{\alpha}{\sigma_\alpha}$$

  - Alpha is also called the abnormal return
  - unsystematic risk is also called residual risk, or tracking error

- When the fund manager optimally combined with the **benchmark portfolio** with original portfolio, the maximum improvement in the Sharpe measure will be determined by its information ratio

$$SR_p^2 = SR_B^2 + IR^2$$

$\alpha = IC \cdot \sigma \cdot Score$

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$$SR_p^2 = SR_B^2 + IR^2$$

把不识的基金经理可以实现的最大的SR

$$IR = \begin{cases} \dfrac{\alpha}{\sigma(\alpha)} \\ IC\sqrt{BR} \\ \sqrt{SR_p^2 - SR_B^2} \end{cases}$$

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Exercise

- Portfolio manager returns 10% with a volatility of 20%. The benchmark returns 8% with risk of 14%. The correlation between the two is 0.98. The risk-free rate is 3%. Which of the following statements is correct?

A. The portfolio has higher SR than the benchmark

B. The portfolio has negative IR

C. The IR is 0.35

D. The IR is 0.29

Correct Answer: D

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D. The IR is 0.29

$$IR = \frac{\alpha}{\sigma(\alpha)} = \frac{R_p - R_B}{10\% - 8\%}$$

$$SR_B = \frac{8\% - 3\%}{14\%}$$

$$SR_p = \frac{10\% - 3\%}{20\%}$$

Correct Answer: D

$$\sigma(R_p - R_B) = \sqrt{\sigma_p^2 + \sigma_B^2 - 2\rho\sigma_p\sigma_B} \approx |\sigma_p - \sigma_B|$$

Exercise

- **Analysis of portfolio P and Q**

Performance Statistics			
	Portfolio P	Portfolio Q	Portfolio M
Sharpe Ratio	0.43	0.49	0.19
M^2	2.16	2.66	0.00
SCL Regression Statistics			
Alpha	1.63	5.26	0.00
Beta	0.70	1.40	1.00
Treynor	3.97	5.38	1.64
T^2	2.34	3.74	0.00
$\sigma(e)$	2.02	9.81	0.00
Information Ratio	0.81	0.54	0.00

Exercise

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Information Ratio	0.81	0.54	0.00

Statistical Significance of Alpha

- **Alpha plays a critical role in determining portfolio performance.**
  - A positive alpha produces an indication of superior performance;
  - A negative alpha produces an indication of inferior performance;
  - zero alpha produces an indication of normal performance matching the benchmark.
  - In order to assess a manger's ability of generate alpha, we conduct a t-test under the following hypotheses:
    - ✓ H_0: True alpha is zero
    - ✓ H_A: True alpha is not zero

$$t = \frac{\alpha - 0}{se(\alpha)} = \frac{\alpha}{\sigma/\sqrt{N}}$$

Where

- ✓ α = alpha estimate;
- ✓ σ = alpha estimate volatility
- ✓ N = sample number of observations

$\alpha = 0.1$

$H_0: \alpha = 0$ $\qquad$ $H_a: \alpha \neq 0$

$$t = \frac{\alpha(\text{样}) - 0}{\sigma_{(\alpha)} / \sqrt{n}}$$

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Where

- ✓ α = alpha estimate;
- ✓ σ = alpha estimate volatility
- ✓ N = sample number of observations

Exercise

- Based on 60 monthly returns, you estimate an actively managed portfolio alpha=1.24% and standard error of alpha=0.1278%. The portfolio manager wants to get due credit for producing positive alpha and believes that the probability of observing such a large alpha by chance is only 1%. Calculate the t-statistic, and based on the estimated t-value would you accept (or reject) the **claim** made by the portfolio manager.

A. t=9.70, accept

~~B. t=2.66, accept~~

~~C. t=2.66, reject~~

D. t=9.70, reject

$$t = \frac{1.24\%}{0.1278\%} = 9.7$$

Correct Answer: A

Exercise

- Based on 60 monthly returns, you estimate an actively managed portfolio alpha=1.24% and standard error of alpha=0.1278%. The portfolio manager wants to get due credit for producing positive alpha and believes that the probability of observing such a large alpha by chance is only 1%. Calculate the t-statistic, and based on the estimated t-value would you accept (or reject) the **claim** made by the portfolio manager.

A. t=9.70, accept

B. t=2.66, accept

C. t=2.66, reject

D. t=9.70, reject

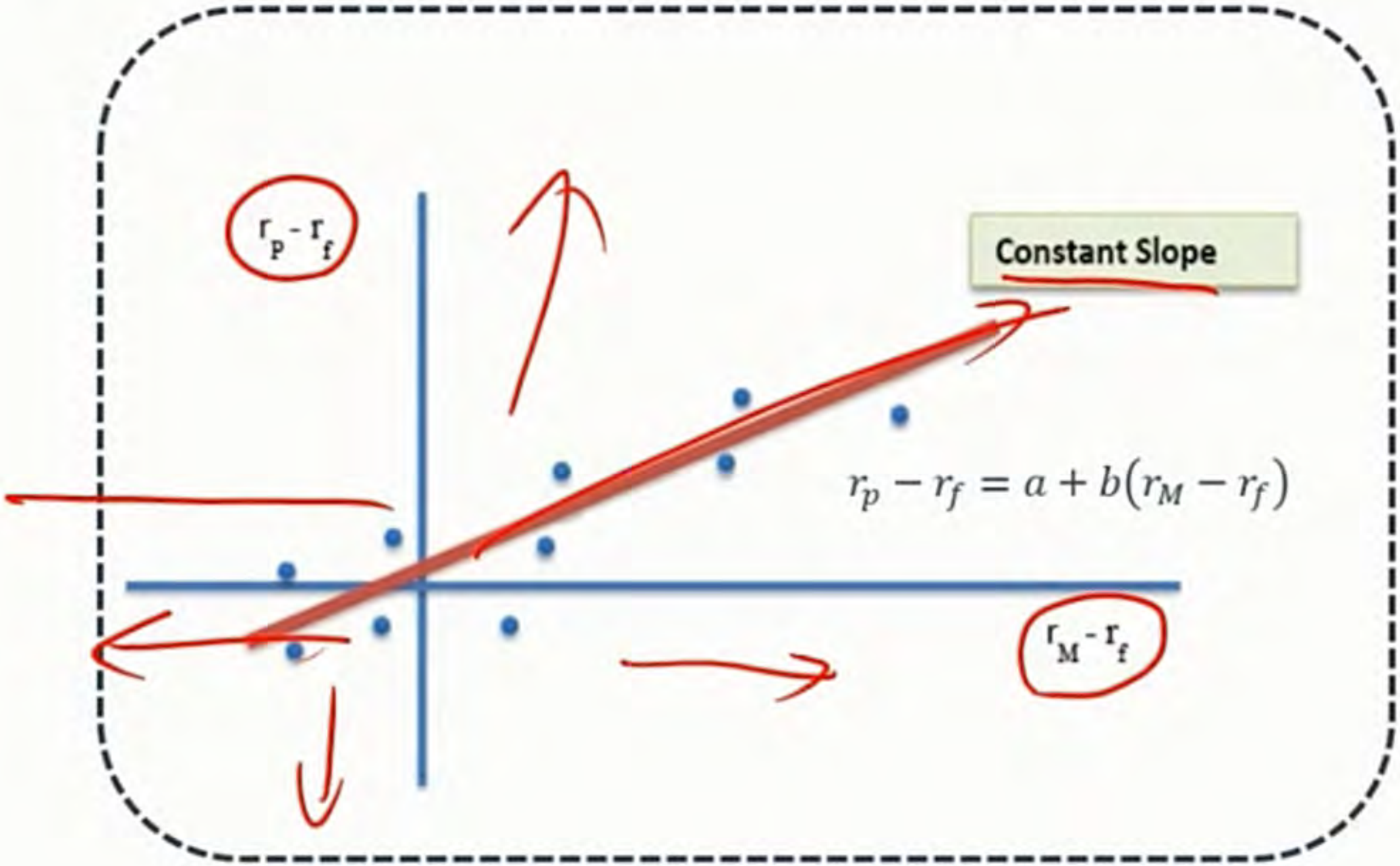
impossible

Correct Answer: A

Market Timing Ability 择时能力

- **Market Timing**
  - The ability to predict the future direction of market and shifting funds between the market index portfolio and risk-free assets depending of whether the market will outperform the risk-free assets.
- **No Market Timing**
  - Assuming no market timing, and a constant beta, the security characteristic line will be straight line with a constant slop.

Market Timing Ability



Market Timing Ability

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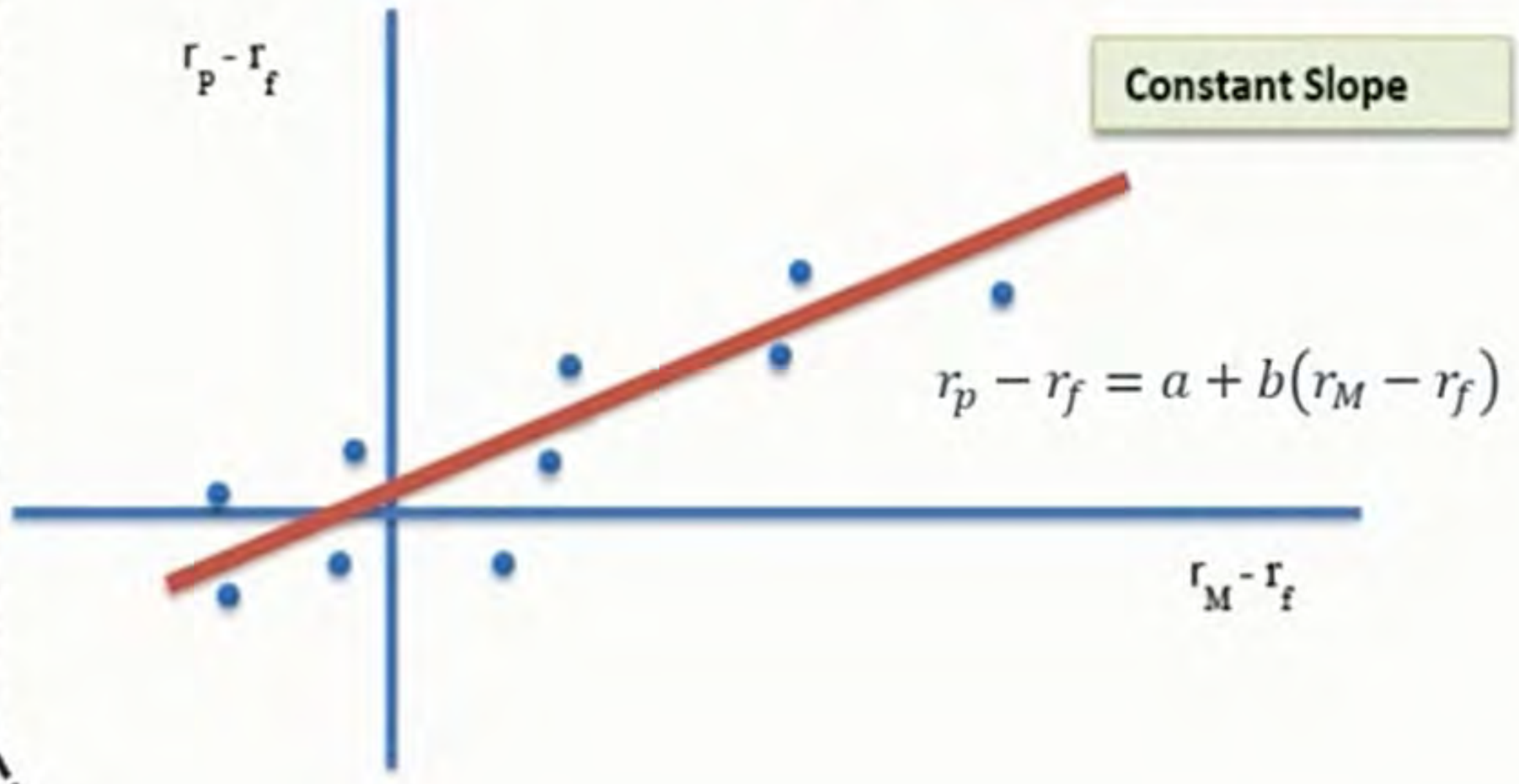


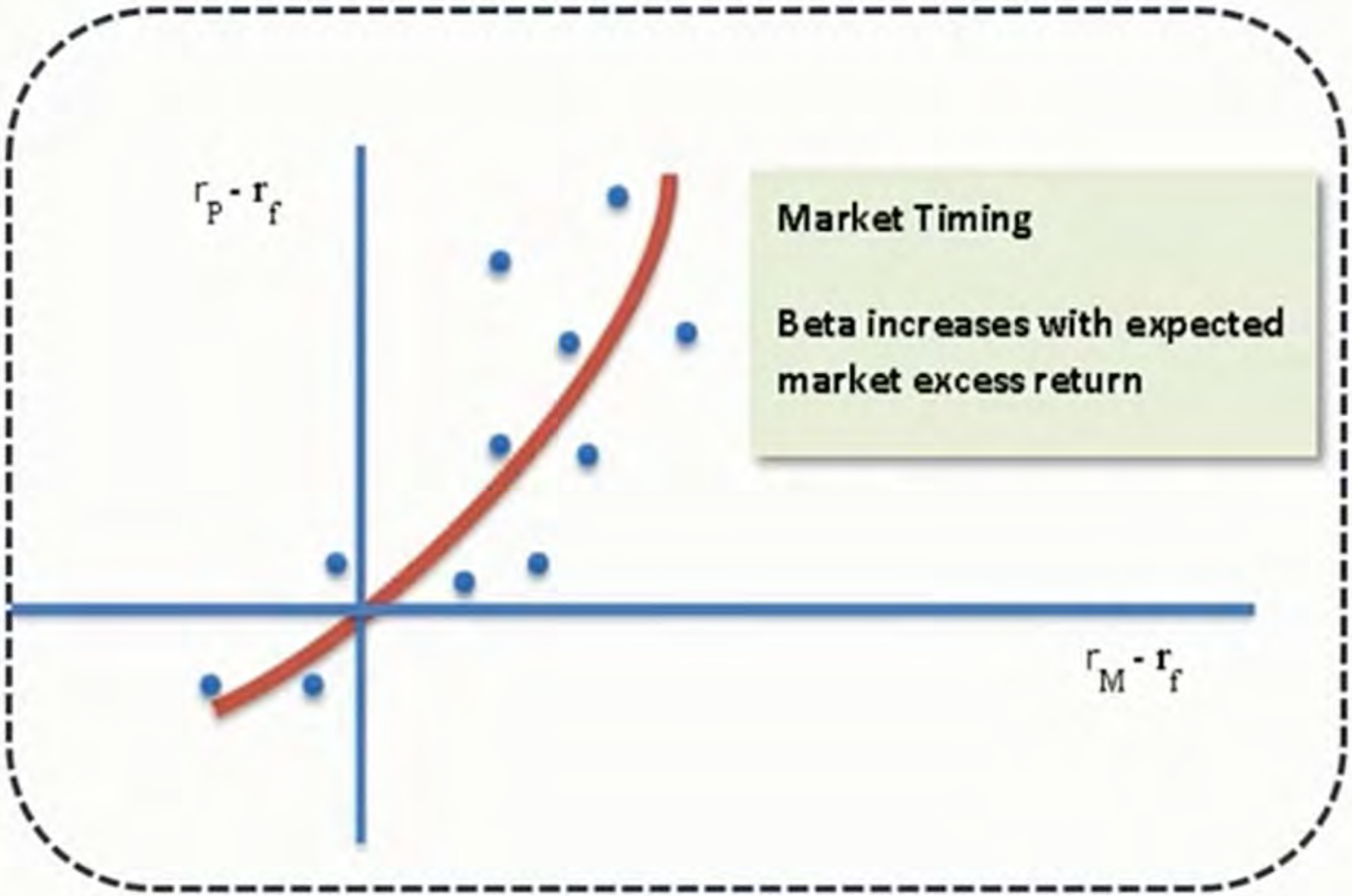
Market Timing Ability

- **Market Timing (Treynor and Mazuy)**
  - The investor is able to accurately time the market and is shifting the funds in to the market when it does well and withdraws and puts funds in safe assets when the market in going downwards.
  - As the market returns increase, the portfolio beta will also increase, resulting in a curved line.

$$r_P - r_f = a + b(r_M - r_f) + c(r_m - r_f)^2 + e_P$$

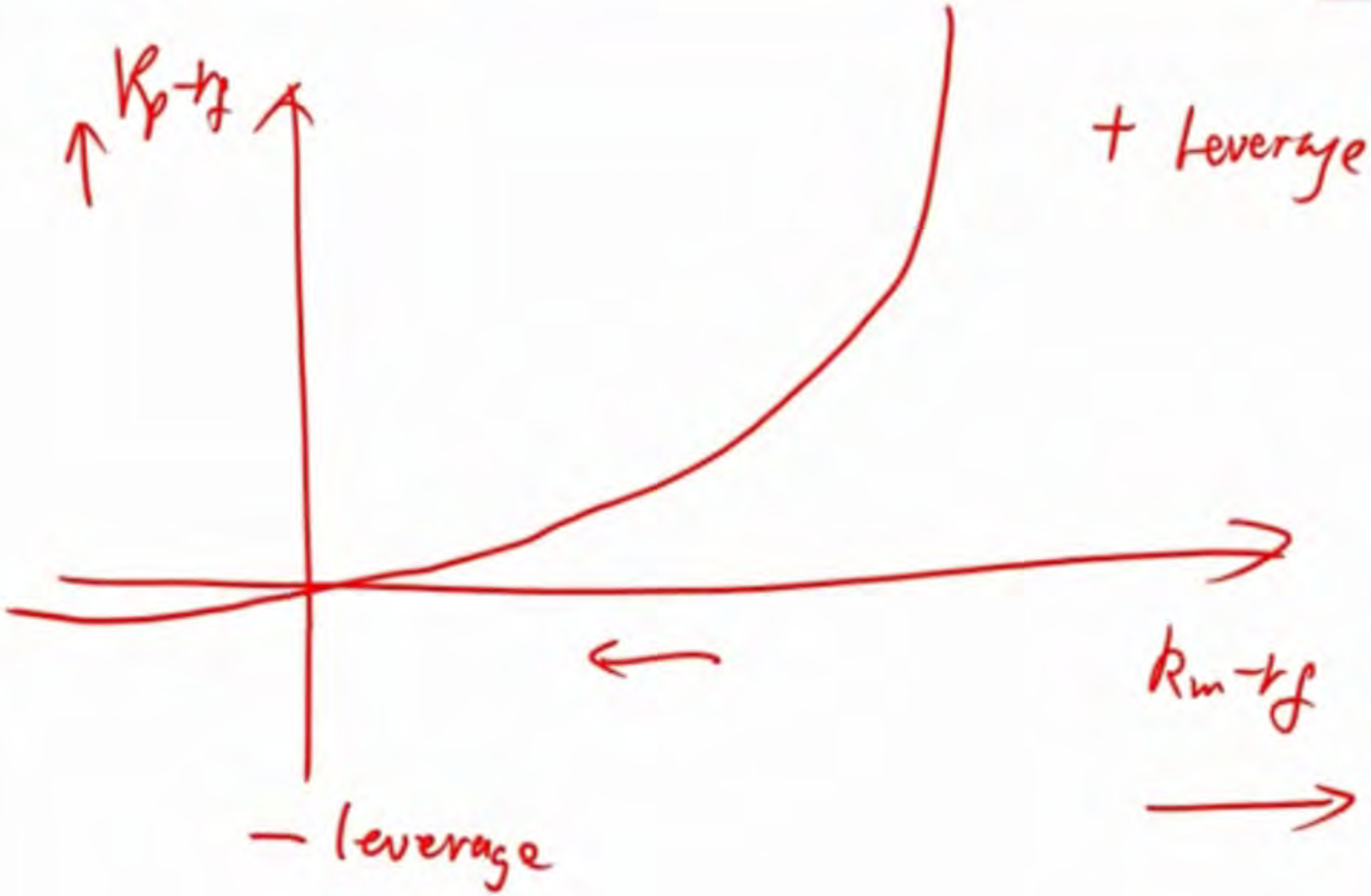
  - The squared term represents the market-timing factor. A positive c is an indication of the fund manager's timing ability.

Market Timing Ability



Market Timing

Beta increases with expected market excess return


$R_p - r_f$
+ Leverage
$R_m - r_f$
− leverage

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Market Timing Ability

- **Market Timing (Henriksson and Merton)**
  - This approach assumes that the beta of the portfolio can take only two values: a large value if the market is expected to do well, otherwise a smaller value.

$$r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)D + e_P$$

  - D is a dummy variable. D = 1 when $r_M > r_f$, otherwise D = 0.
  - The portfolio's beta is b in bear market and b + c in bull market. A positive c is an indication of the fund manager's timing ability.

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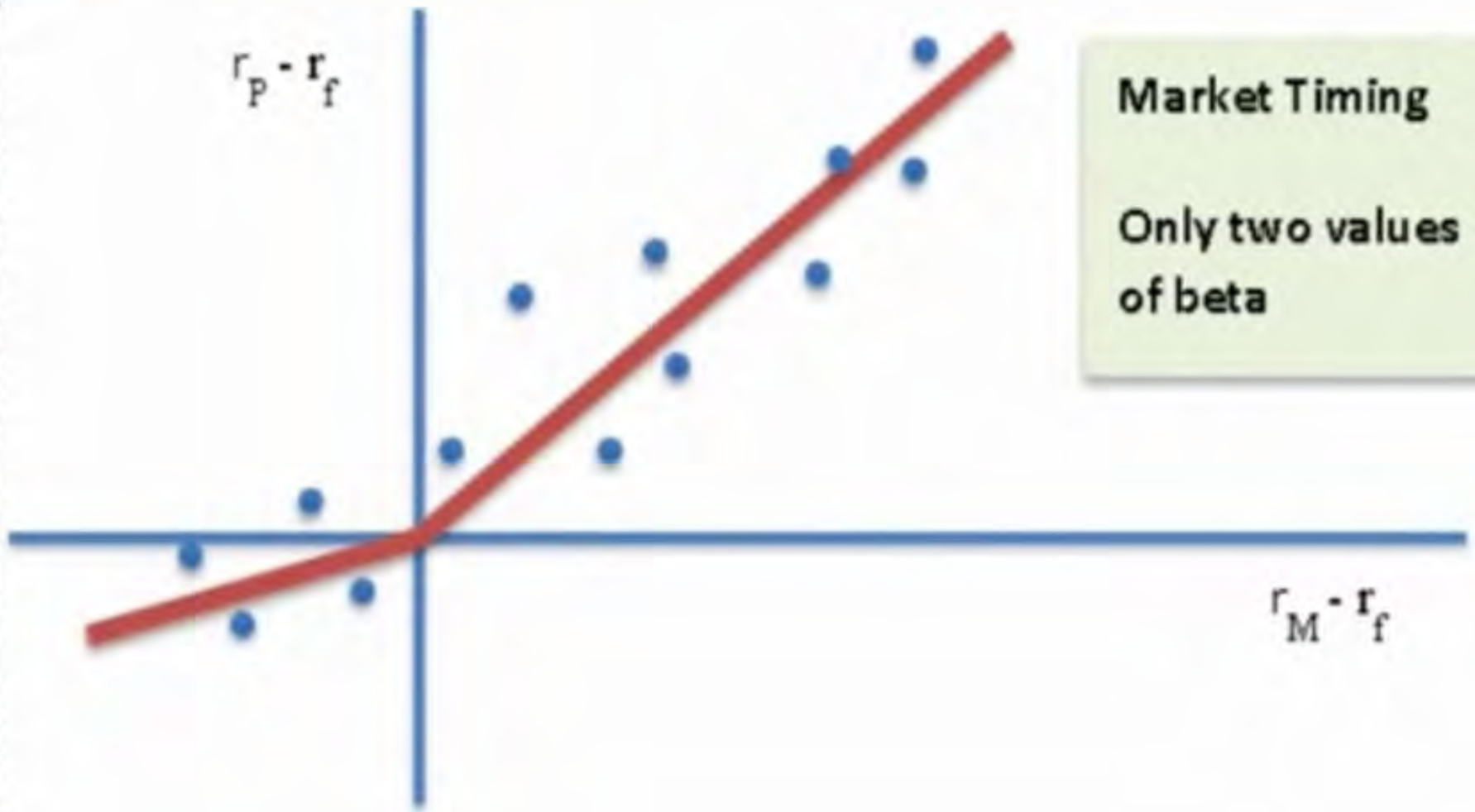

Market Timing

Only two values of beta

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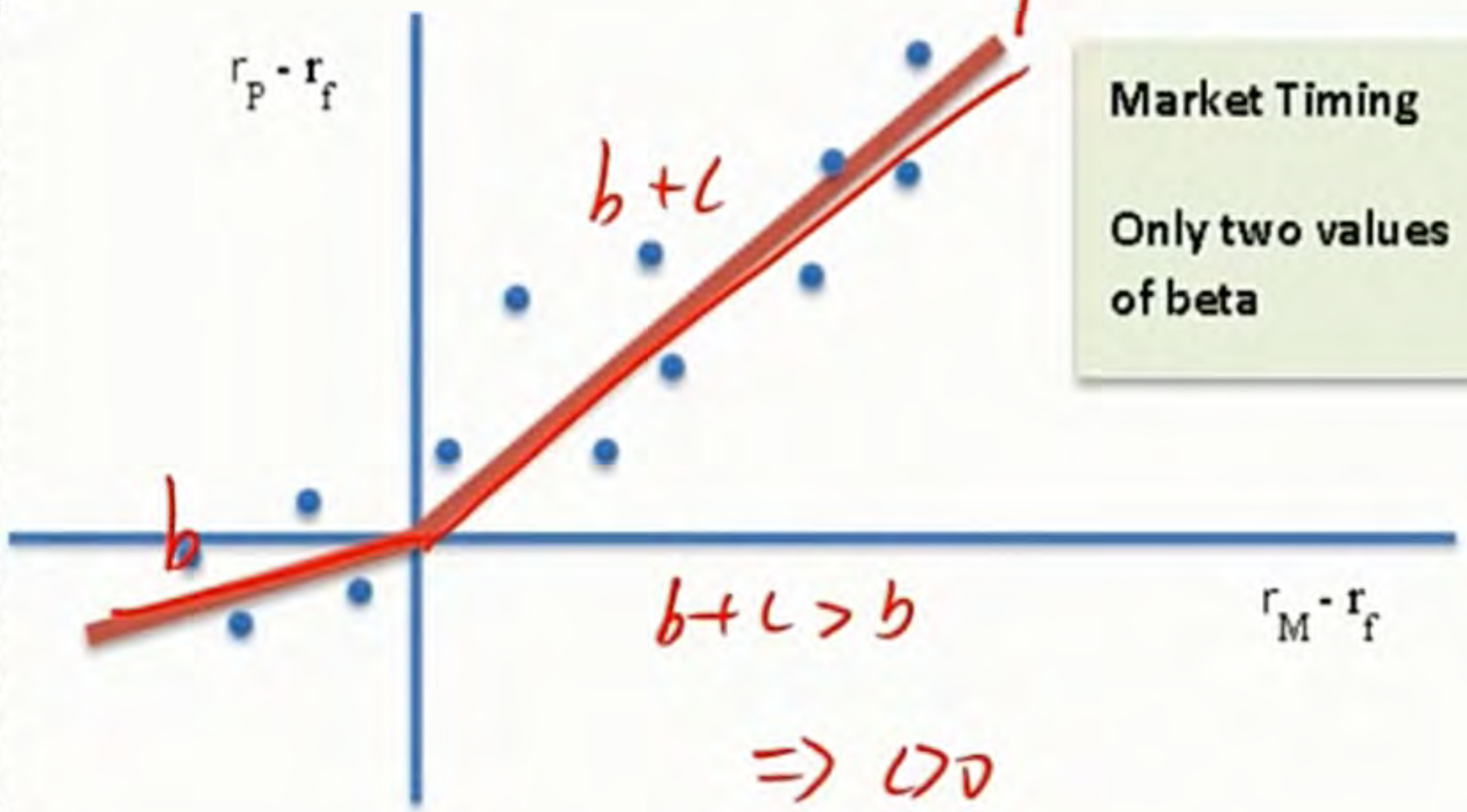
$$\begin{cases} r_p - r_f = a + (b+c)(r_m - r_f) & r_m > r_f \rightarrow D=1 \\ r_p - r_f = a + b(r_m - r_f) & r_m < r_f \rightarrow D=0 \end{cases}$$

Market Timing Ability



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100M

A（有）
+1% +3% 合理

B（无）
+1%

Value Market Timing Ability

Merton

- **Call Option Model**
  - The key to valuing market timing ability is to recognize that perfect foresight is equivalent to holding a call option on the equity portfolio.
    - ✓ The perfect timer invests 100% in either the safe asset or the equity portfolio, whichever will provide the higher return.
  - To see the value of information as an option, suppose that the market index currently is at S_0 and that a call option on the index has an exercise price of $X = S(1+r_f)$.

	$S_T < X$	$S_T > X$
Bills	$S_0(1+r_f)$	$S_0(1+r_f)$
Option	0	$S_T - X$
Total	$S_0(1+r_f)$	S_T

  - ✓ The value or appropriate fee for perfect foresight should equal to the price of the call option on the market index.

$\underbrace{100m}$ → A < $\underline{\max(r_{股}, r_f)}$

① call option → $\underline{100mx(1+r_f)}$ → $\underline{x}$ (premium支出)

② 100m → risk free bond

---

① 股 > r_f，回报 $100mx(1+r_f)$，行权，亏损 $100mx(1+r)$
$-100mx(1+r_f)$

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$r_f = 3\%$ $\quad r \begin{cases} 10\% \\ 1\% \end{cases}$

初始 $t=0$, $\begin{cases} S = 100元 \\ Np = 100元 \end{cases}$

$t=0$ $\begin{cases} 买\ option\ (X = 103) \\ 买\ bond\ (P_0 = 100) \rightarrow t=1\quad P = 103 \end{cases}$

$t=1$ 若 $r=10\%$, $S = 100\times(1+10\%) = 110$ $\longrightarrow$ 10%

若 $r=1\%$ $P_{bond} = 103$, $S = 101$ $\longrightarrow$ 3%

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$X = S \times (1+r_f)$

	$S_T < X$	$S_T > X$
Bills	$S_0(1+r_f)$	$S_0(1+r_f)$
Option	0	$S_T - X$
Total	$S_0(1+r_f)$	S_T

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call option (X=1 x 1Hrs) P=3元

整 P=3元

P=2

付

Performance Attribution

- **Performance Attribution**
  - Identify the sources of value addition to the portfolio. 板+1
  - How much of the performance (excess returns above **bogey portfolio**) is attributable to the selection of the risk asset classes.
  - How much is attributable to selection of right sector or security within an asset class.

birdie 多标 板-1

eagle -2

Performance Attribution

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R_B

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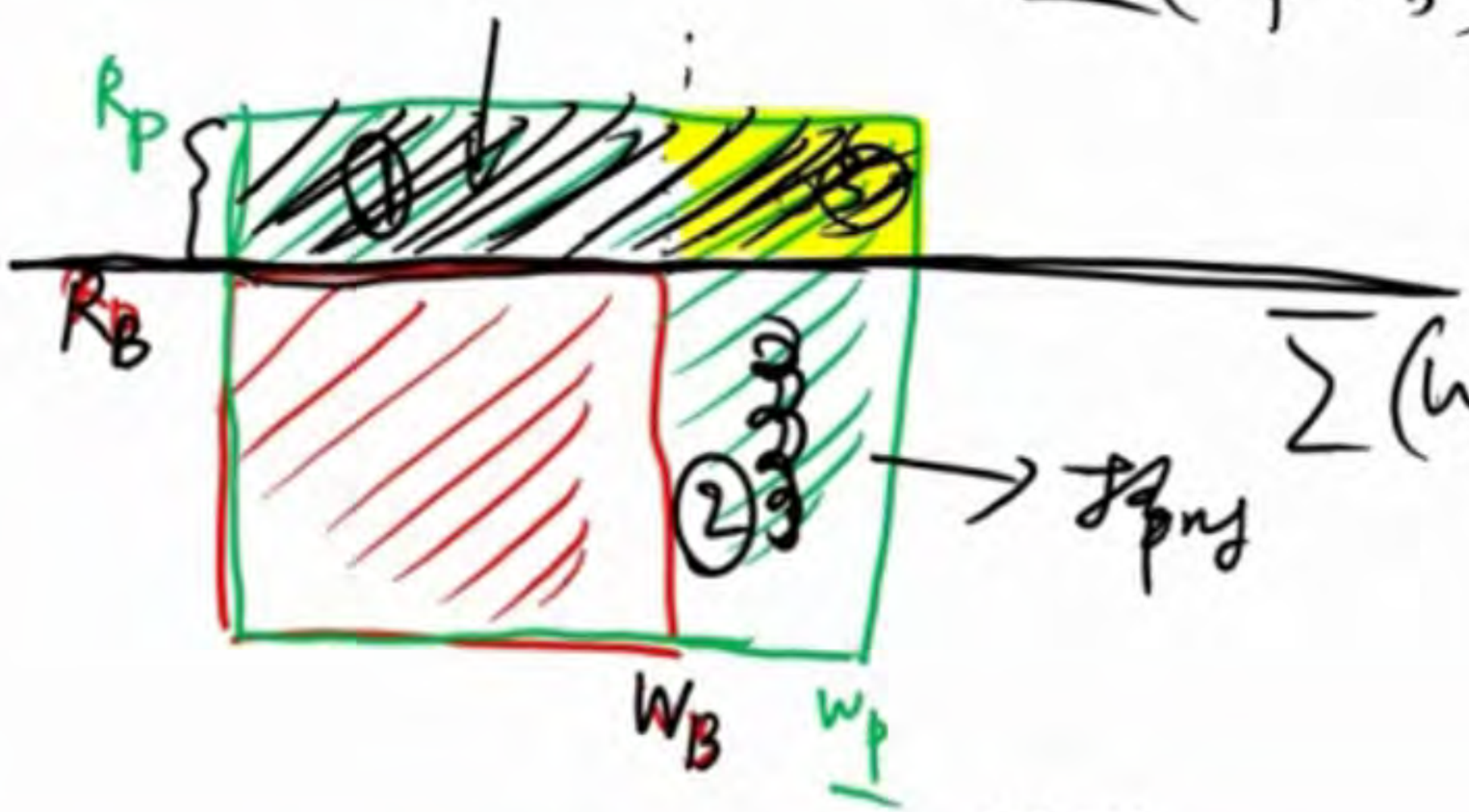
$R_P - R_B$

- 选股 ✓ >0
- 择时 >0

	R_P	W_P	R_B	W_B
Stock	9%	50%	7%	40%
bond	3%	50%	2.5%	60%

$$\sum (R_P^i - R_B^i) \times W_P^i$$



$$\sum (W_P^i - W_B^i) \times R_B^i$$

Performance Attribution

➢ **Example**

- Let's assume that we have a benchmark portfolio with funds allocated in three asset classes namely, equity, bonds, and money markets. We also have a managed portfolio with assets allocated in the same asset classes with but a different composition.

	Managed Portfolio		Benchmark Portfolio	
	Weights	Performance	Weights	Performance
Equity	0.75	7.50%	0.50	6.00%
Bonds	0.10	2.00%	0.40	2.00%
Cash	0.15	0.50%	0.10	0.50%

Managed Portfolio Returns	0.75×7.5%+0.1×2%+0.15×0.5%	5.90%
Benchmark Portfolio Returns	0.50×6%+0.4×2%+0.1×0.5%	3.85%
Excess Returns		2.05%

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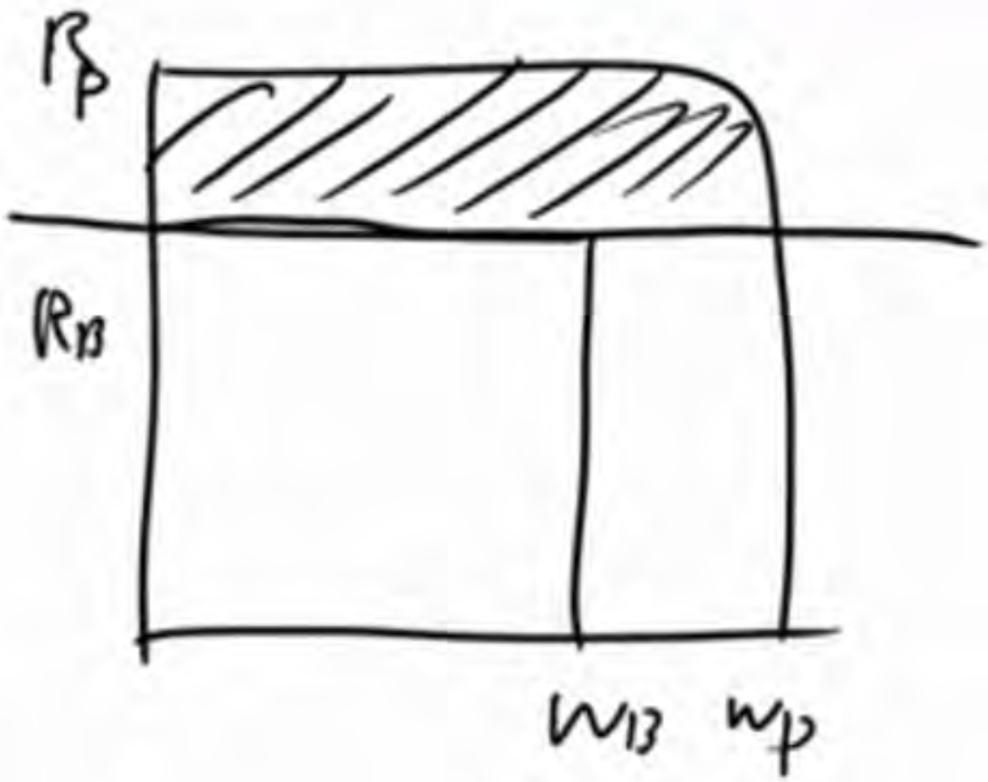

$(R_p - R_B) \times W_p$

E: (75%-60%) × 75%

B: _____

C: _____

Total

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Performance Attribution

➢ **Answers:**

timing ability

Contribution of Asset Allocation to Performance					
Market	**Actual Weight**	**Benchmark Weight**	**Active/Excess Weight**	**Market Return**	**Contrib. to Perform.**
Equity	0.75	0.50	0.25	6.00%	1.5000%
Bonds	0.10	0.40	-0.30	2.00%	-0.6000%
Cash	0.15	0.10	0.05	0.50%	0.0250%
Total					0.9250%

Contribution of Selection to Performance					
Market	**Portfolio Perform.**	**Benchmark Perfom.**	**Excess Perform.**	**Port. Weight**	**Contrib.**
Equity	7.50%	6.00%	1.50%	0.75	1.1250%
Bonds	2.00%	2.00%	0.00%	0.10	0.0000%
Cash	0.50%	0.50%	0.00%	0.15	0.0000%
Total					1.1250%

Tool 1 red/yellow/green

Tool 2 SR / IR / TR

① TWR vs. DWR

② SR $\longrightarrow$ $M^2 = \sigma_m \times (SR_p - SR_m)$

③ TR $\Rightarrow$ $T^2 = TR_p - (R_m - r_f)$

④ IR $= \alpha / \sigma(\alpha) = \sqrt{SR_p^2 - SR_B^2}$

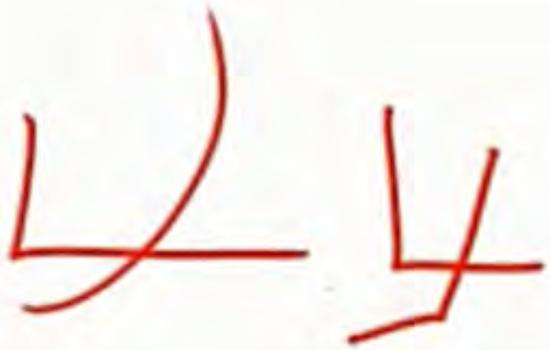
Tool 3/4 alpha (benchmark / peer group)

1. alpha / excess return / Jensen's α

2. 回归 $\longrightarrow$ slope (β) $\longrightarrow$ Leverage

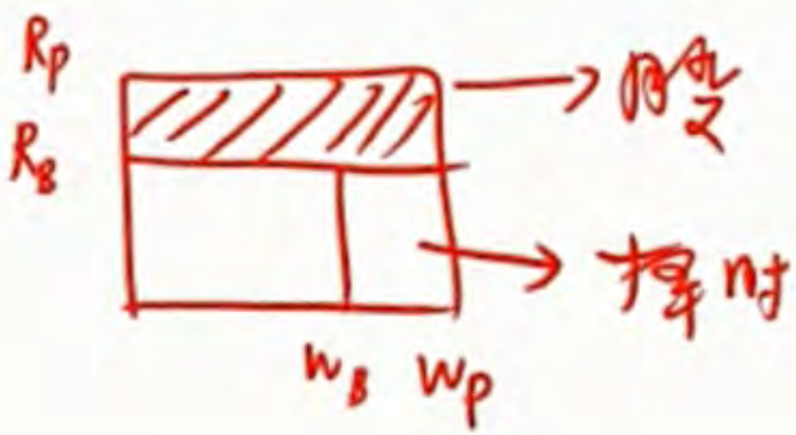
3、alpla (sig. test)

$$t = \frac{\alpha}{\sigma/\sqrt{n}} = IR \times \sqrt{N}$$



4、performance attribution

择时
- 定性 判断
- merton



Reading 6

Hedge Fund

158-201

Introduction of Hedge Funds

- **Hedge Funds**
  - Private partnership funds that have very few limitations on investment strategy and can take long and short positions in various markets.

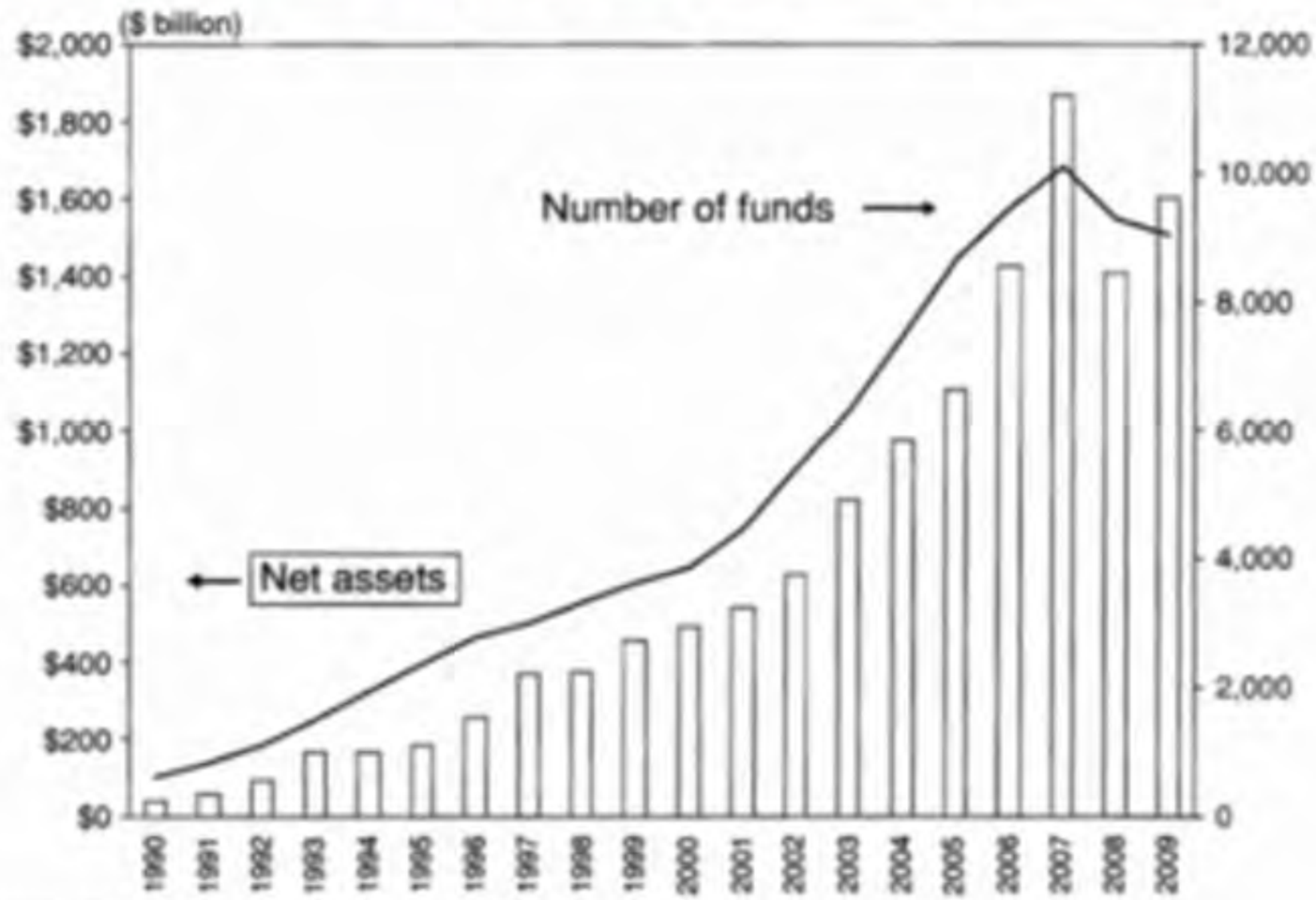


FIGURE 30.1 Growth of Hedge Fund Industry
Source: Hedge Fund Research. Data as of December of each year.

Evolution of the Hedge Fund Industry

- **Early Days, 1987-1996**
  - Hedge funds easily outperformed the S&P index by a wide margin
  - Survivorship and selection biases

- **Collapse of Long Term Capital Management in Fall of 1998**
  - The collapse of a large, well-known hedge fund like LTCM had a dramatic impact on the private world of hedge fund investors.
  - LTCM was a turning point in the capital formation process of the hedge fund industry. **There is a significant reduction in investors' appetite for hedge funds.** By the end of 1998, the hedge fund investment community had generally accepted that the first two moments of an expected return distribution may be woefully inadequate for capturing the risk of these dynamic, nonlinear, leveraged strategies.

Evolution of the Hedge Fund Industry

- **Rapid Expansion, 2000-**
  - The ensuing two year period, 2000-2001**, witnessed the burst of the dot-com bubble.**
  - For the first time since 1996, the cumulative performance of hedge funds exceeded that of the S&P index. Not only had hedge funds outperformed the S&P index, their return standard deviations were just over half of the S&P index's return standard deviation.
  - **Institutional investors' appetite for hedge funds improved**. Arrival of Institutional Investors (foundations, endowments, pension funds, and insurance companies, etc.) (policy放松，HF比利)
  - By the end of 2007, the total Asset-Under-Management of the hedge fund industry had grown to $1,390 billion from $197 billion at the end of 1999. Taken together these observations are consistent with a shift in the investor clientele from an industry dominated by private wealthy investors to institutional investors.

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Introduction of Hedge Funds

- **Hedge Funds versus Mutual Funds**
  - Private versus public
    - ✓ Historically, hedge funds are private investment vehicles not open to the general investment public.
    - ✓ Consequently, hedge funds face less regulation than publicly traded mutual funds.
  - Ability to take short positions
    - ✓ Typically hedge fund managers generate profit from both long as well as short positions.
  - Freedom to use high leverage
  - Ability to employ derivatives

Introduction of Hedge Funds

- **Basic Features**
  - To keep other traders from mimicking or "front running" their trades, they offer very little transparency, even to their investors.
  - A typical hedge fund charges a fixed management fee based on the value of assets they manage. The lower end of this range is comparable to the **management fees** charged by actively managed mutual funds. However, unlike mutual funds, hedge funds generally charge an **incentive fee** which are only payable when new profits are made. Sometimes, the incentive fee is paid after the performance exceeds a hurdle rate, such as LIBOR.

2/20

Example

- As an example, assume that the Net Asset Value (NAV) goes from 100 to 120 over the year before fees (gross) and that LIBOR is at 5%. Fees are at the usual 2 and 20.
- The total fees is 100 * 2% + [(120 – 102) – 0.05 * 100] * 20%. In this case, the gross return is 20%, the total fees to the manager are 4.6%, and the net return is 15.4%.
- Without a hurdle, the net return would be 14.4%

Example (hurdle rate) L+300bp

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100 × 2% = 2 → mgt. fee (2)

120 − 2 = 18

18 − 5 = 13 —×20%→ incentive fee (2.6)

(4.6)

Introduction of Hedge Funds

- **Hedge Fund Manager versus Institutional Investors**
  - Hedge Fund Manager
    - ✓ Is the best judge of the appropriate risk/reward trade-off of the portfolio; should be given broad discretion
    - ✓ Trading strategies are highly proprietary and must be jealously guarded lest they be reverse-engineered/copied
    - ✓ Return is the ultimate, often the only, objective
    - ✓ Risk management is not central to success of a hedge fund
    - ✓ Regulatory constraints and compliance issues are generally a drag on performance; the whole point of a hedge fund is to avoid these issues.
    - ✓ There is little intellectual property involved in the fund; the general partner is the fund.

Introduction of Hedge Funds

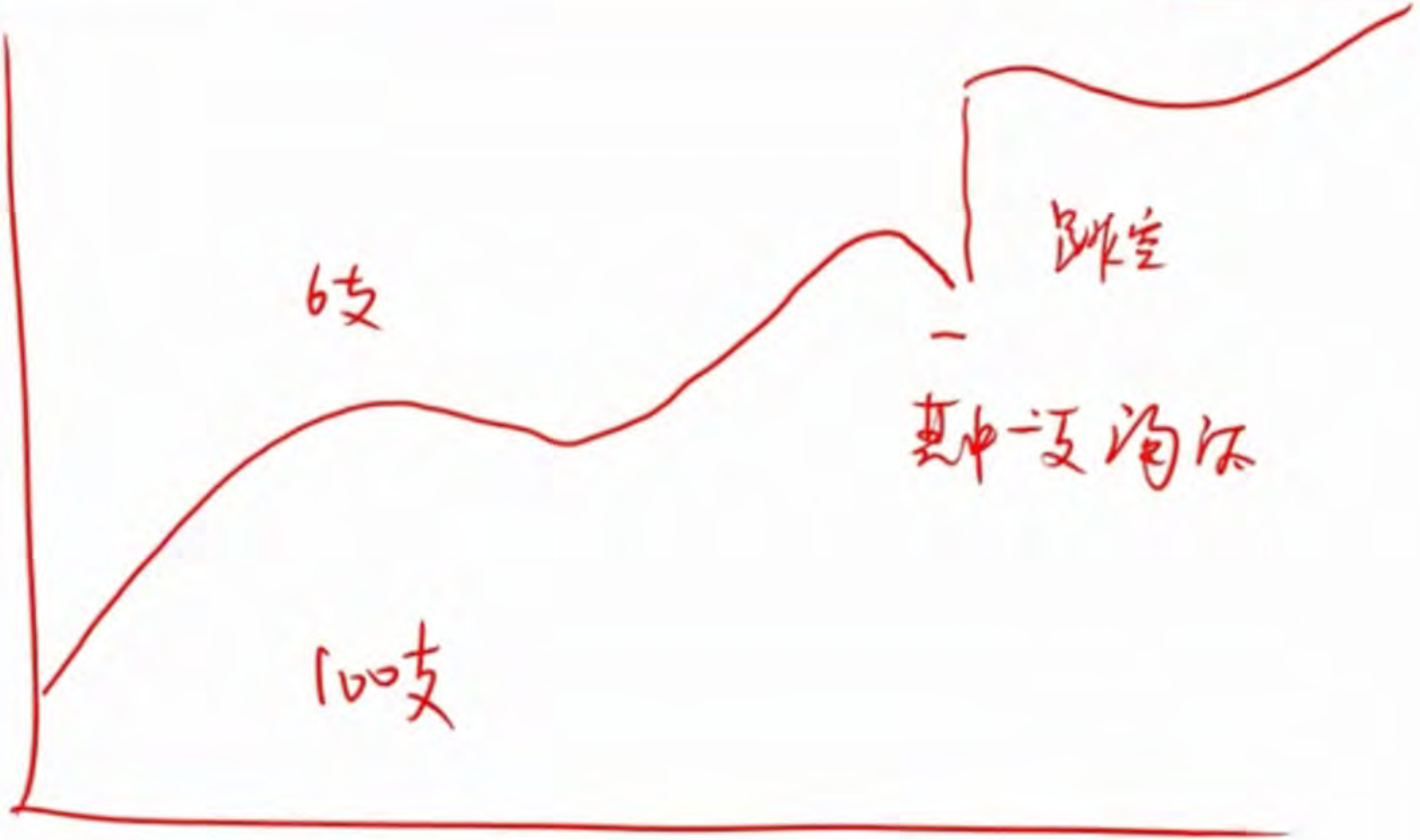
- Institutional Investors
  - ✓ As fiduciaries, institutions need to understand the investment process before committing to it.
  - ✓ Must fully understand the risk exposures of each manager; may have to circumscribe a manager's strategies to be consistent with the institution's investment objectives.
  - ✓ Performance is not measured solely by return. Other factors; e.g., risk, tracking error, peer comparisons.
  - ✓ Risk management and risk transparency are essential.
  - ✓ Operate in highly regulated environment
  - ✓ Desire structure, stability, and consistency in a well-defined investment process that is institutionalized and not dependent on any single individual.

Evolution of the Hedge Fund Industry

- **Bias in Hedge Fund Databases**
  - **Survivorship Bias**
    - ✓ Few hedge-fund databases maintain histories of funds that have shut down.
  - **Self-Selection Bias**
    - ✓ If a manager operates several hedge funds, it is questionable whether the poor performing ones will find their way into databases.
  - **Backfill Bias**
    - ✓ A related and important form is sometime referred to as the **"instant history"** bias.
    - ✓ When a new fund enters the database some of its performance history during its incubation period is incorporated without clear distinction from the live performance data going forward.

2018.12.31 ✓

2018.1.1 — 2018.12.31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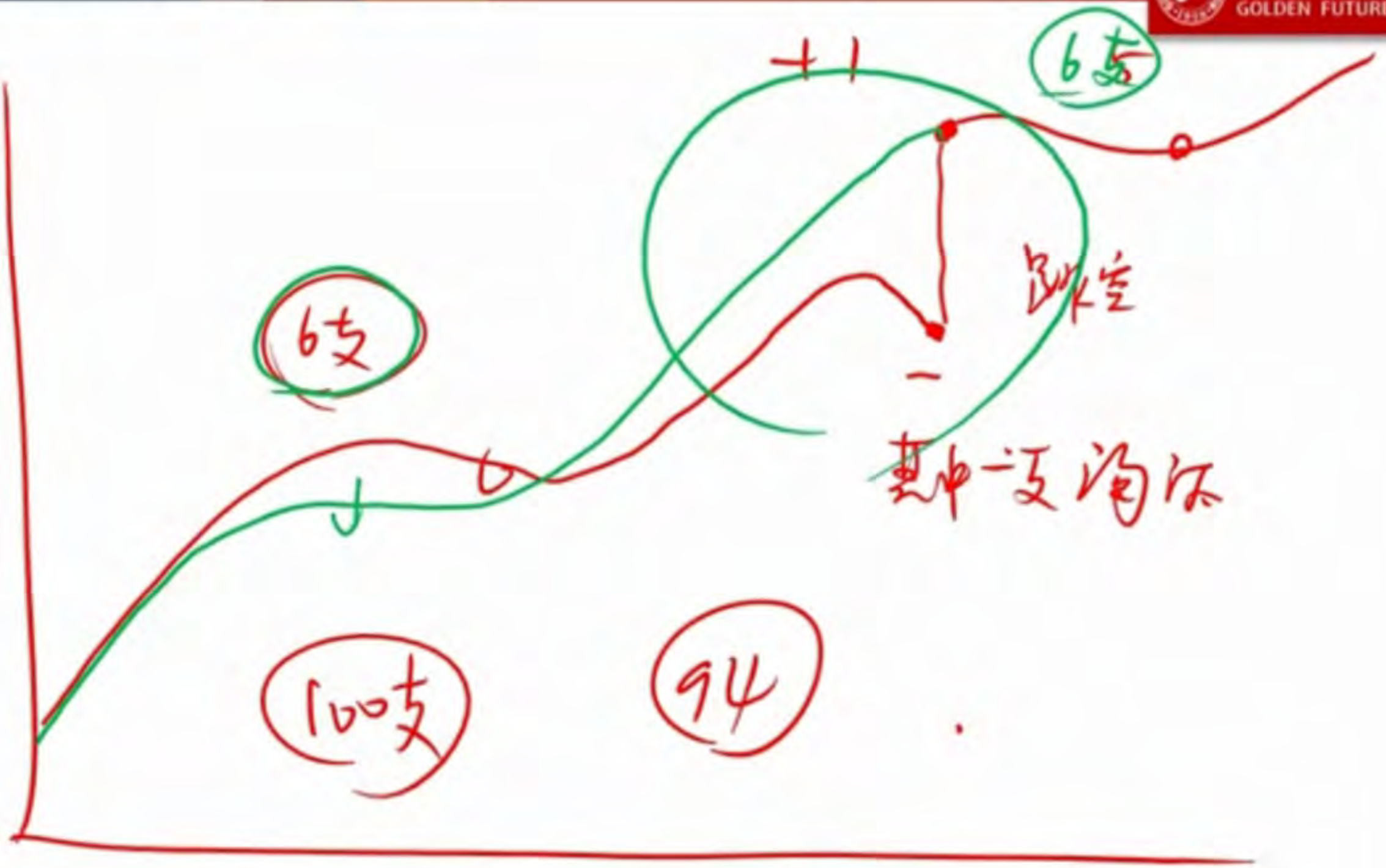
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100支
94

Evolution of the Hedge Fund Industry

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Exercise

- A significant percentage of hedge funds stop trading each year and drop out of hedge fund databases. Which of the following best describes the impact this has historically had on hedge fund analyses performed using these databases?
  A. The average performance of hedge funds is overstated.
  B. The average volatility of hedge funds is overstated.
  C. The average correlation of hedge fund returns is understated.
  D. The average Sharpe ratio of hedge fund returns is understated.

  Correct Answer: A

- Every year, BusinessWeek reports the performance of a group of existing equity mutual funds, selected for their popularity. Taking the average performance of this group of fund will create
  A. Survivorship bias only
  B. Selection bias only
  C. Both survivorship and selection bias
  D. Instant-history bias only

  Correct Answer: C

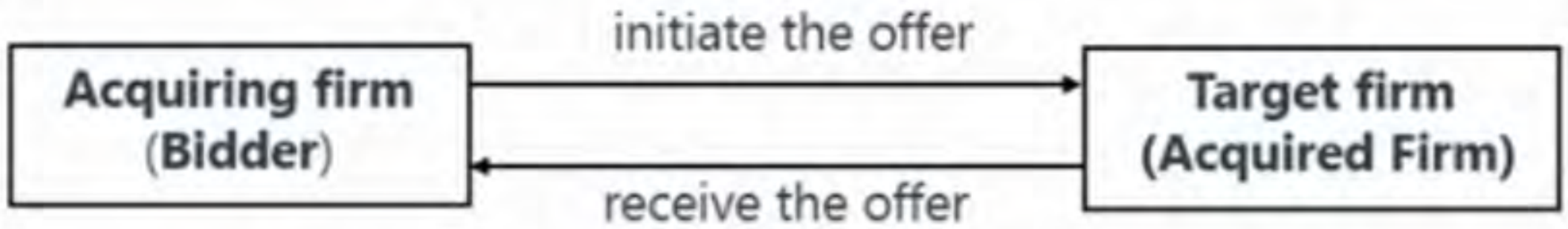
Hedge Fund Strategies

- **Directional Strategies: Trend Followers (Managed Futures) and Global Macro**
  - **Managed Futures**
    - ✓ Managed futures funds typically focus on investing in listed bond, equity, commodity futures and currency markets, globally.
    - ✓ Managed futures fund managers tend to employ systematic trading programs that largely rely upon historical price data and market trends.
  - **Global Macro**
    - ✓ Global macro funds typically focus on identifying mis-pricings in equity, currency, interest rate and commodity markets.
    - ✓ Managers typically employ a top-down global approach to analyze how political trends and global macroeconomic events may affect the valuation of financial instruments.

Hedge Fund Strategies

② Event-Driven Strategies: Risk Arbitrage & Distressed

- **Risk Arbitrage**
  - ✓ As known as Merger Arbitrage.



  - ✓ Attempt to capture the spreads in merger or acquisition transactions involving public companies after the terms of the transaction have been announced.
  - ✓ For a stock deal, the bidder offers to exchange each target share for X shares of the bidder. In a fixed exchange ratio stock merger, one would go long the target stock and short the acquirer's stock according to the merger ratio, in order to isolate the spread and hedge out market risk.
  - ✓ The principal risk is usually deal risk, should the deal fail to close.

Hedge Fund Strategies

3 **Relative Value and Arbitrage-like Strategies:** Fixed Income Arbitrage, Convertible Arbitrage and Long/Short Equity

- **Fixed Income Arbitrage**
  - ✓ Generate profits by exploiting inefficiencies and price anomalies between related fixed income securities.
  - ✓ Strategies may include leveraging long and short positions in similar fixed income securities that are related either mathematically or economically.
  - ✓ Seek to limit volatility by hedging out exposure to the market and interest-rate risk.

Hedge Fund Strategies

4 **Niche Strategies: Dedicated Short Bias, Emerging Market and Equity Market Neutral**

- **Dedicated Short Bias**
  - ✓ Take more short positions than long positions and earn returns by maintaining net short exposure in long and short equities.
  - ✓ Detailed individual company research typically forms the core alpha generation driver of dedicated short bias managers, and a focus on companies with weak cash flow generation is common.
- **Emerging Market**
  - ✓ Invest in currencies, debt instruments, equities and other instruments of developing countries' markets.
- **Equity Market Neutral**
  - ✓ Return behavior suggests that different funds apply different trading strategies with a similar goal of achieving almost zero beta(s) against a broad set of equity indices.

Hedge Fund Strategies

- **Directional Strategies: Trend Followers (Managed Futures) and Global Macro** ①
  - **Managed Futures** (futures only)
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    - ✓ Managed futures fund managers tend to employ systematic trading programs that largely rely upon historical price data and market trends.
  - **Global Macro** 一国24h
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Hedge Fund Strategies

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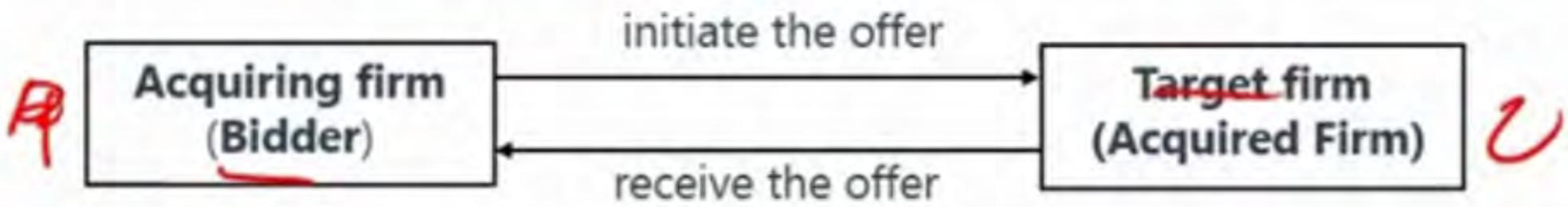

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A　　买　　B

P↓　　P↑

Hedge Fund Strategies

- **Example - Merger Arbitrage**
  - On December 1, 1998, Exxon (Acquiring firm) confirmed that it had agreed to buy Mobil (Target firm), another major oil company. Under the terms of the agreement, each shareholder of Mobil would receive X = 1.32015 shares of Exxon in exchange. Before the announcement, the initial prices of Mobil and Exxon were $78.4 and $72.7, respectively.
  - Over the three days around the announcement, Mobil's stock price went up to $84.2 and Exxon's price went down to $71.6.
  - The exchange was consummated on November 30, 1999, after regulatory and shareholder approval. On that day, the respective stock prices for Mobil and Exxon were $104.4 and $79.3. Multiplying the latter by 1.32015, we get $104.7, which is close to the final stock price for Mobil. So, **the two prices converged to the same converted value.**
  - Profit from the risk arbitrage trade was ($104.4 - $84.2) – 1.32015*($79.3 - $71.6) = $10

E　　1998.12.1　　M

P↓　　P↑

78.4　×　↔　72.7

T=0 →　71.6　　84.2

1999.11.30

79.3　　104.4

Hedge Fund Strategies

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P↓　　P↑

78.4 ×　　↔　　72.7

T=0 →　　71.6　　84.2

sell 1.32015份　　buy 1份

1999.11.30

79.3 × 1.32015 =　　104.4

(104.4-84.2)×1 − (79.3-71.6)×1.32015

Hedge Fund Strategies

Event-Driven Strategies: Risk Arbitrage & Distressed

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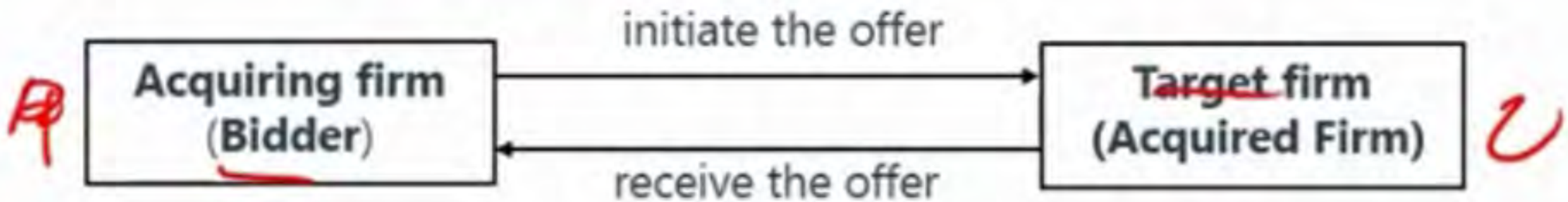

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  - ✓ The principal risk is usually deal risk, should the deal fail to close.

Hedge Fund Strategies

- **Distressed Securities** 13
  - ✓ Invest across the capital structure of companies subject to financial or operational distress or bankruptcy proceedings.
  - ✓ Such securities often trade **at discounts** to intrinsic value due to difficulties in assessing their proper value, lack of research coverage, or an inability of traditional investors to continue holding them.
  - ✓ The strategy may **focus on mispricing** caused by expected restructuring, reorganization, legal or regulatory issues, or corporate transactions.

fallen angle

Hedge Fund Strategies

elephant strategy

- **Relative Value and Arbitrage-like Strategies:** Fixed Income Arbitrage, Convertible Arbitrage and Long/Short Equity

3

  - **Fixed Income Arbitrage**

    Convexity strategy

    - ✓ Generate profits by exploiting inefficiencies and price anomalies between related fixed income securities.
    - ✓ Strategies may include leveraging long and short positions in similar fixed income securities that are related either mathematically or economically.
    - ✓ Seek to limit volatility by hedging out exposure to the market and interest-rate risk.

Dur = 9

A D = 8

B D = 8

$$D_{swap} = D(\text{fixed coupon bond}) - D(\text{float} \quad \text{——}) \approx 0$$

$$D_{swap} = D_{fix}$$

Hedge Fund Strategies

- **Example**
  - Long-Term Capital Management (LTCM) started as a fixed-income arbitrage fund, taking positions in relative value trades, such as duration-matched positions in long swap, short Treasuries.
  - On August 21, 1998, the 10-year Treasury yield dropped from 5.38% to 5.32%. The swap rate, in contrast, increased from 6.01% to 6.05%.
  - This divergence was highly unusual. Assuming a notional position of $50 billion and modified duration of eight years, this leads to a value change of:

$$\textit{Long Treasury Position}: -8 \times (5.32 - 5.38)/100 \times \$50{,}000 = +\$240 million$$
$$\textit{Long Swap Position}: -8 \times (6.05 - 6.01)/100 \times \$50{,}000 = -\$160\ million$$

  - As the spread position is long the swap and short Treasury, this leads to a total loss of $400 million, close to 10% of capital.

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Hedge Fund Strategies

- **Convertible Arbitrage**
  - ✓ Seek to profit from investments in convertible securities.
  - ✓ Managers typically build long positions of convertible and other equity hybrid securities and then hedge the equity component of the long securities positions by shorting the underlying stock or options of that company. Interest rate, volatility and credit hedges may also be employed.
  - ✓ Positions are typically designed with the objective of creating profit opportunities irrespective of market moves.**(net long gamma and vega)**

    Convertible bond = pure bond + call option on stock
- **Long/Short Equity**
  - ✓ Invest in both long and short sides of equity markets, often with a specific focus on certain sectors, regions or market capitalizations.

Bond (Convertible)

= Bond (pure)

+ Call on Stock

+ Convertible bond

− pure bond Call on stock

− Δ · stock

Gamma / Vega / ρ / θ

r_f

Arr

Hedge Fund Strategies

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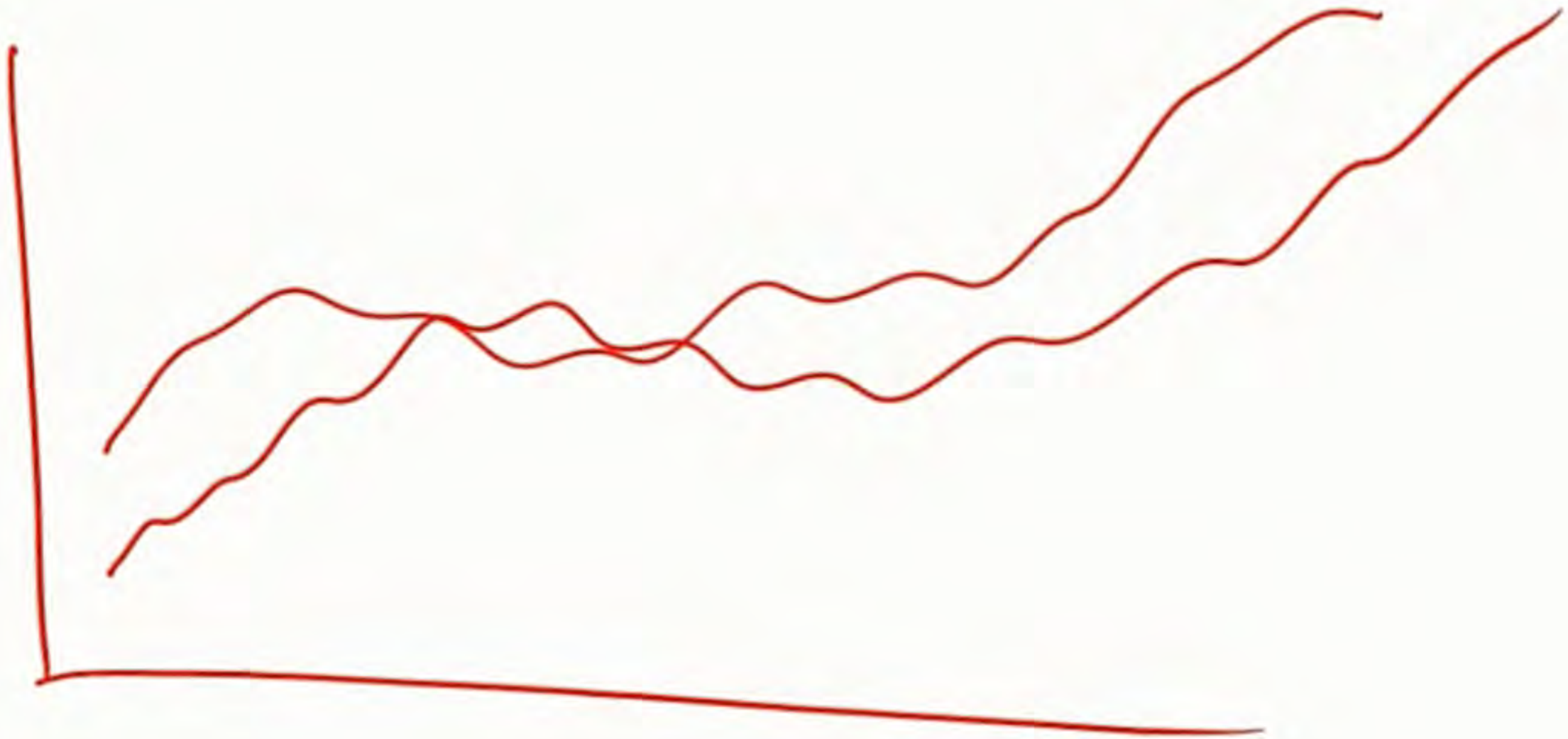
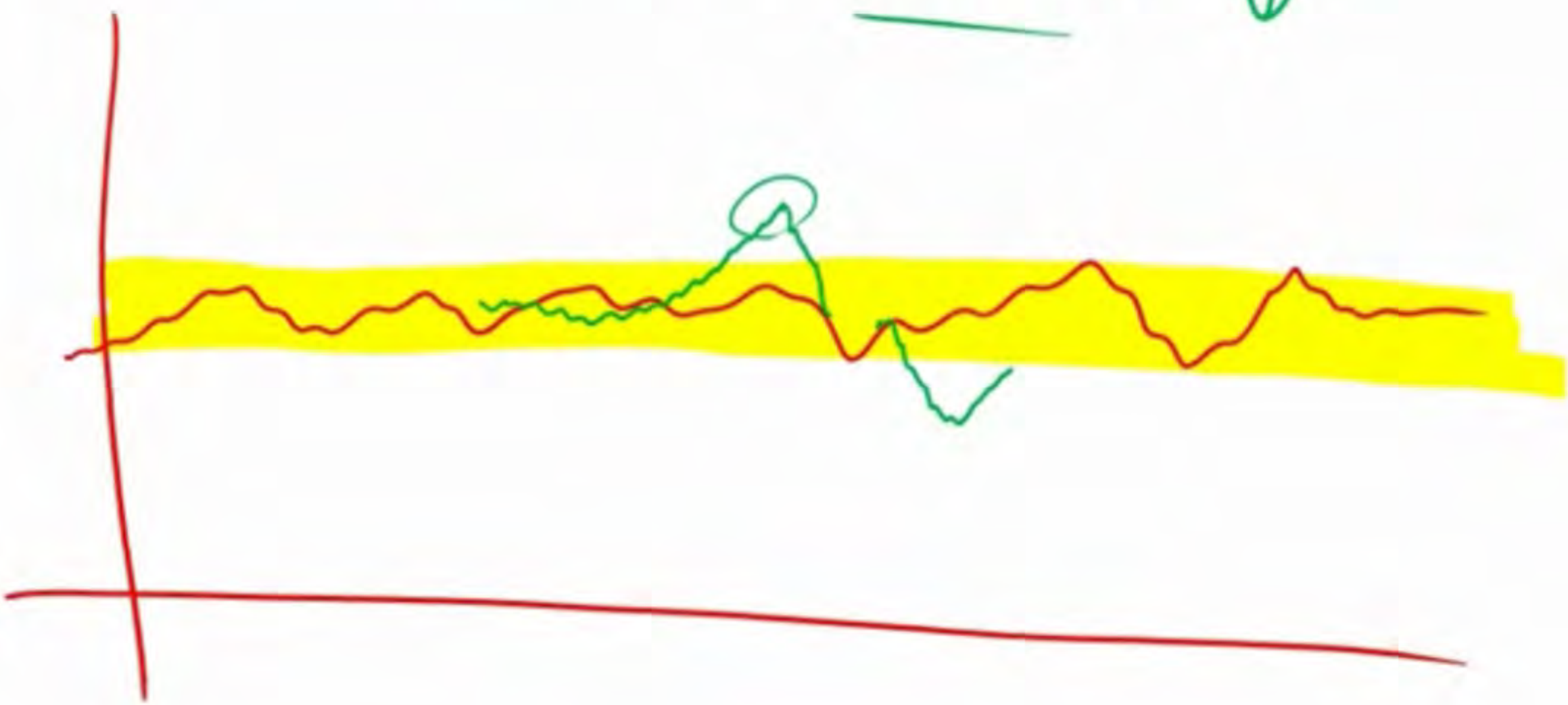


long 证 short 证

$P_{证} - P_{证}$ ↓



Hedge Fund Strategies

- **Niche Strategies: Dedicated Short Bias, Emerging Market and Equity Market Neutral** ④
  - **Dedicated Short Bias** (Short only)
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    - ✓ Detailed individual company research typically forms the core alpha generation driver of dedicated short bias managers, and a focus on companies with weak cash flow generation is common.
  - **Emerging Market**
    - ✓ Invest in currencies, debt instruments, equities and other instruments of developing countries' markets.
  - **Equity Market Neutral** ($\beta=0$)
    - ✓ Return behavior suggests that different funds apply different trading strategies with a similar goal of achieving almost zero beta(s) against a broad set of equity indices.

Hedge Fund Strategies

- **Funds of Hedge Funds** FoF
  - A fund of hedge funds are portfolios of hedge funds, which add value by providing automatic diversification and careful selection of styles and investment managers.
  - A major objective of the fund of hedge funds is optimal diversification.
  - Funds of funds charge additional management fees on top of those levied by the underlying funds, typically around 1%.

双重征费

Exercise

- A fund of hedge funds combines a mix of strategy sectors, managers, and styles, and therefore fund of funds risk managers need to understand the common attributes of hedge fund strategies. Which of the following statements is incorrect?

  A. Equity market neutral funds aim to generate returns that have low correlation with the overall equity market and to insulate their portfolios from broad market risk factors.

  B. Convertible arbitrage funds typically purchase securities that are convertible into the issuer's stock and simultaneously short the underlying stock. These funds earn returns in part form gamma trading on the stock's volatility.

  C. Merger arbitrage funds buy the stock of an acquisition target company and simultaneously short the bidding company's stock. These funds have large exposure to deal risk.

  D. Equity short-selling funds sell stocks not currently owned by the seller in order to take a directional bet that the stock price will decline. These funds tend to be uncorrelated with traditional long-only equity portfolios.

卖空

Correct Answer: D

Exercise

- Identify the risks in a fixed-income arbitrage strategy that takes long positions in interest rate swaps hedged with short positions in Treasuries.
  A. The strategy could lose from decreases in the swap-Treasury spread.
  B. The strategy could lose from increases in the Treasury rate, all else fixed.
  C. The payoff in the strategy has negative skewness.
  D. The payoff in the strategy has positive skewness.
  Correct Answer: C

- Identify the risks in a convertible arbitrage strategy that takes long positions in convertible bonds hedged with short positions in Treasuries and the underlying stock.
  A. Short implied volatility
  B. Long duration
  C. Long stock delta
  D. Positive gamma
  Correct Answer: D

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  $r_s - r_t$

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  C. Long stock delta
  D. Positive gamma

  Correct Answer: D

Exercise

- A major acquisition has just been announced, targeting Company B. The bid from Company A is an exchange offer with a ratio of 2. Just after the announcement, the prices of A and B are $50 and $90, respectively. A hedge fund takes a long position in Company B hedged with A's stock. After the acquisition goes through, the prices move to $60 and $120. For each share of B, the gain is

A. $30

B. $20

C. $10

D. $0 since the acquisition is successful

Correct Answer: C

Risk Management of Hedge Fund

- **Risk Management Challenges**
  - Are heterogeneous
  - Hedge funds invest in illiquid assets
  - Most employ leverage
  - Some have high turnover
  - Exhibit a lack of transparency
  - Survivorship bias is a major consideration when hedge funds are evaluated as a group. This could significantly exaggerate their returns.

Risk Management of Hedge Fund

- **Hedge fund risks**
  - Liquidity risk
  - Leverage Risk
  - Agency Risk
  - Style Drift Risk
  - Fraud risk

Risk Management of Hedge Fund

- **Liquidity Risk**
  - On the asset side, liquidity risk is a function of the size of the positions, as well as of the price impact of a trade.
  - On the liabilities side, funding risk arises when the hedge fund cannot renew funding from its broker, or when losses in marked-to-market positions or increases in haircuts lead to cash outflows. Finally, funding risk also arises when the fund faces investor redemptions.

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asset liq risk

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rollover risk

Risk Management of Hedge Fund

- Some funds invest in illiquid assets such as convertible bonds, which trade infrequently. In this case, risk measures based on monthly returns are misleading (the closing net asset value, NAV, does not reflect transaction prices)
- This create two biases:
  - ✓ Low correlations: correlations with other asset classes will be artificially lowered, giving the appearance of low systemic risk.
  - ✓ Low volatility: volatility will be artificially lowered, giving the appearance of low total risk. Such illiquidity, however, will show up in positive serial autocorrelation in returns.

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risk (低)

return (高)

autocorrelation (高)

infrequent sampling

Risk Management of Hedge Fund

- Using **Autocorrelation of Return** as a Measure of liquidity of the Asset.
  - ✓ The degree of serial autocorrelation in an asset's return is a proxy for the amount of illiquidity friction that is present.
  - ✓ More liquid assets should exhibit less serial autocorrelation than illiquid assets.

    $= f(\rho^2_{i,i+1})$
  - ✓ A **Q-statistic** is used as a summary measure of the overall statistical significance of autocorrelations.
    - ◆This statistical measure is approximately chi-squared distributed in large samples under the null hypothesis of no autocorrelations.
    - ◆Funds with large positive or negative autocorrelation coefficients will result in large Q-statistic.

autocorrelation 高 → liq 差

Q-statistic 高 → liq 差

Risk Management of Hedge Fund

Leverage Risk

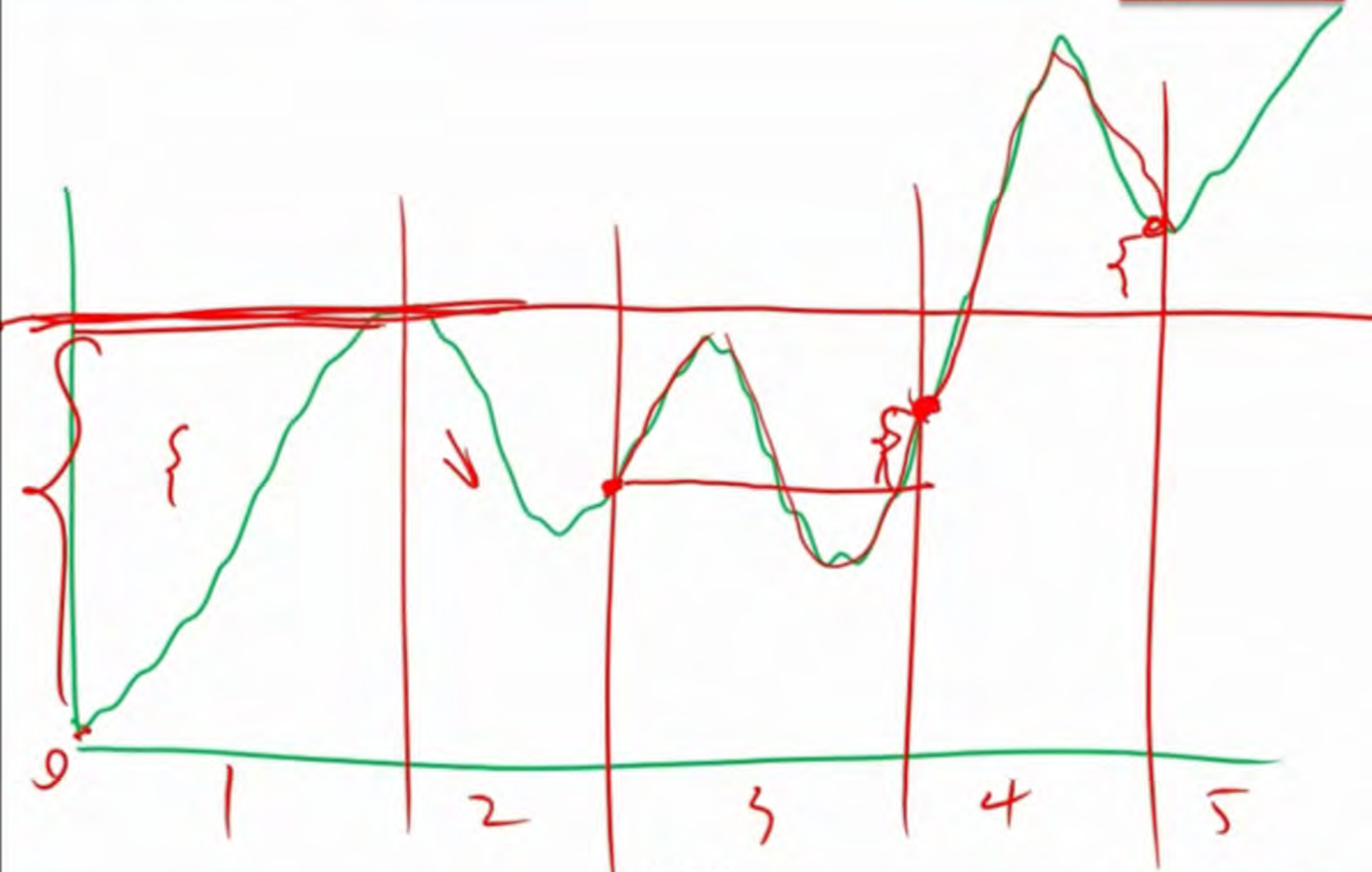
- Leverage can create the problems, which can be classified as crowded trade risk.
- **Crowded trade risk** arises when many leveraged investors are on the same side of a trade. A loss in their portfolio may require them to post additional margin, which may be satisfied by several funds selling similar assets at the same time, which can create disruptions in markets.
- While wholesale liquidation of risky assets can occur globally for a variety of reasons inflicting stress on conventional long-biased strategies as well as hedge funds, it is the effect of leverage that makes hedge fund investment risk different from conventional long-bias strategies.
- Take LTCM and Bear Stearns for an example.

Risk Management of Hedge Fund

- **Agency Risk**
  - The principle-agent problem arises when the **incentive fee** to which hedge fund managers are entitled, typically at 15%-20% of new profits (or profits above a high water mark), entices a fund manager to take unreasonable bets.
  - A manager may be tempted to take an unreasonable risk **so long as the option of closing shop and restarting in a new business if a large bet goes wrong is a feasible proposition**.

Risk Management of Hedge Fund

- Ways to reduce agency risk:
  - ✓ The incentive to take risk can be lessened with **high water marks** clause, which states that following a year in which the fund declined in value, the hedge fund would first have to recover those losses before any incentive fee would be paid.
  - ✓ The **cost** of fund closure being higher for large funds does partially mitigate this principal (investor) and agent (manager) conflict.
  - ✓ In addition, investors have a preference for funds in which the agents (fund managers) **invest a sizeable amount of their own wealth**.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Risk Management of Hedge Fund

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Risk Management of Hedge Fund

- **Style Drift** ($ 变多3 / 变小3 ; Leverage变化)
  - Another potential problem is that of **style drift**, which refers to the fund manager' departure from his specialized skill area. Hedge fund style drift can occur in two ways:
    - ✓ Changes in the risk factor exposures of the fund, including changes in exposures to pre-existing risk factors or the emergence of exposures to new risk factors outside the scope of the manager's investment strategy.
    - ✓ Changes in the overall risk of the fund, primarily through changes in leverage.

Reasons for Style Drift:
- Poor style market performance
- Excessive cash inflows
- Poor manager performance
- Recent losses
- Personnel change
- Regulatory change

Risk Management of Hedge Fund

- **Fraud risk**
  - **Unscrupulous hedge fund managers** have succumbed to the temptation to misreport the value of the fund's assets in order to hide their trading losses and even steal investors' assets. Improper valuation of assets arises when assets do not have market-clearing prices at the end of the reporting period and when fund managers calculate the NAV themselves.
  - **Ponzi scheme**: the scheme using new investor funds to repay earlier investors. The most famous case of a Ponzi scheme is the case of Bernard Madoff Investment Secuties (BMIS). The possibility of fraud can be lessened when a fund has an independent administrator, an outside auditor and an external custodian.
  - **Due diligence** is the process of systematically investigating the fund before investing. This involves an analysis of the fund documents, of the key personnel, of the fund service providers, of the regulatory registration, and of the operations and valuation procedures.

Risk Management of Hedge Fund

- **Limitations of the VaR Measure**
  - VaR cannot capture the heterogeneous spectrum of risks among various hedge funds.
  - VaR is a purely statistical measure of risk "with little or no economic structure underlying its computation"
  - VaR is notoriously difficult to estimate. Tail events happen rarely: "generally too small a sample to yield reliable estimates of tail probabilities" ES
  - VaR is an unconditional measure of risk: calculations are almost always based on the unconditional distribution of a portfolio's profit-and-loss. But for purposes of active risk management, conditional measures are more relevant. ES

Performance Evaluations for Hedge Fund

- **The difficulties in measuring the performance of hedge funds**
  - In practice, evaluating hedge funds poses considerable practical challenges. We can briefly mention a few of the difficulties:
    - ✓ The risk profile of hedge funds (both total volatility and exposure to relevant systematic factors) may change rapidly. SR
    - ✓ Hedge funds tend to invest in illiquid assets.
    - ✓ Many hedge funds pursue strategies that may provide apparent profits over long periods of time, but expose the fund to infrequent but severe losses.
    - ✓ Hedge funds have ample latitude to change their risk profiles and therefore considerable ability to manipulate conventional performance measures.
    - ✓ When hedge funds are evaluated as a group, survivorship bias can be a major consideration.

金程教育 GOLDEN FUTURE

$$SR = \frac{R_p - r_f}{\sigma_p}$$

$$TR = \frac{R_p - r_f}{\beta_p}$$

专业·创新·增值

Exercise

- Risk management of hedge funds has challenges not generally faced in traditional investment management companies. Which of the following statements are correct about hedge fund risk management?

  I. Because hedge funds can hold long and short positions, and can use derivatives and leverage, their exposure to market risks can experience large and rapid changes that make it difficult to assess these exposures using only monthly returns.

  II. Many hedge funds use over-the counter derivatives, which are valued by models or quoted prices and often hold illiquid assets, as a result, the returns of these strategies generally exhibit much lower serial correlation than mutual fund returns.

  III. For hedge fund strategies that use leverage to amplify returns and rely on their ability to move out of trades quickly when they turn against them, liquidity risk must be closely monitored and managed.

  IV. Hedge fund returns are often similar to the return of a basket of exotic derivatives with nonlinear payoffs, and therefore assessing risk based on past performance can be misleading.

  A. I, II, III, and IV

  B. I, III, and IV

  C. I and III

  D. II and IV

  Correct Answer: B

194-201

Exercise

- You are asked to estimate the exposure of a hedge fund to the S&P 500. Though the fund claims to mark to market weekly, it does not do so and marks to market once a month. The fund also does not tell investors that it simply holds an Exchange Traded Fund (ETF) indexed to the S&P 500. Because of the claims of the hedge fund, you decide to estimate the market exposure by regressing weekly returns of the fund on the weekly return of the S&P 500. Which of the following correctly describes a property of your regression estimates?

A. The intercept of your regression will be positive, showing that the fund has a positive alpha when estimated using an OLS regression.

B. The beta will be misestimated because hedge fund exposures are nonlinear.

C. The beta of your regression will be one because the fund holds the S&P 500.

D. The beta of your regression will be zero because the fund returns are not synchronous with the S&P 500 returns.

Correct Answer: D

195-201

$$\underset{\sim}{y} = \alpha + \beta \underset{\sim}{x} + \varepsilon$$

$y_1\ y_2\ y_3\ y_4$

weelly return of the fund

$x_1\ x_2\ x_3\ x_4$

weely data of S&P 500

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Correct Answer: D

Due Diligence

- **Definition**
  - Is the term used to describe the process of evaluation and analysis that an investor follows to **get comfortable** with a strategy, a manager, and a fund prior to making an investment.
  - Investors would be wise to assess all three skill sets needed to run a modern-day hedge fund – investment, operational, and business skills – before committing capital to any fund.
  - It includes all the steps that are needed to get to know:
    - ✓ Why and how a fund came into being.
    - ✓ The skills its founders or current partners claim to have mastered.
    - ✓ The evaluation of the timeliness, accuracy, and consistency of manager and fund information.
    - ✓ The reliability and independence of service providers;
    - ✓ And far more.

Due Diligence

- **Reasons for the Failures of Funds in the Past**
  - Funds can fail as a result of **bad investment decisions.**
  - Funds can also fail due to all sorts of **frauds,** including accounting frauds, valuation frauds, or misappropriation of funds.
  - Funds can fail due **to excessive leverage, improbable probabilities, unexpected events, and tail risk.**
  - Fund can get **caught in squeezes by other hedge funds.**
  - Funds can fail as a result of **a lack of supervision or compliance controls related to insider trading.**
  - Fund can fail due to **a flood of unanticipated withdrawals of capital at the least opportune time.** Funds can fail when liquidity dries up, and they can't meet redemptions.
  - Funds can fail because of their own actions or the acts of other.

Due Diligence

- **Conclusion:**
  - Investors need to evaluate each strategy and each manager with open eyes and take great care not to be blinded by the past success of any one person or strategy.
  - The more an investor assumes things can and will go wrong, the better the due diligence process and the more likely it is that an investor will uncover problems and/or ensure that attractive opportunities have a higher probability of success.

Due Diligence Process

- **The due diligence process**
  - Investment process
    - ✓ What is your strategy, and how does it work?
    - ✓ How is equity ownership allocated among the portfolio management, trading, and research Teams?
    - ✓ Is the track record reliable?
    - ✓ Who are the principals, and are they trustworthy?
  - Related risk controls
    - ✓ How is risk measured and managed?
    - ✓ How are securities valued?
    - ✓ What is the portfolio leverage and liquidity?
    - ✓ Does the strategy expose the investor to tail risk?
    - ✓ How often do investors get risk reports, and what do they include?
    - ✓ Do the fund terms make sense for the strategy?

Due Diligence Process

- **The due diligence process**
  - Operational Environment
    - ✓ Internal Control Assessment
    - ✓ Documents and Disclosures
    - ✓ Service Provider Evaluation
  - Model Risk and Fraud Risk Ponzi
    - ✓ Due diligence process **of business model risk** deals with the risks encompassed in **simply running the business of being a hedge fund.**
    - ✓ Investors in hedge funds always need to be on the lookout for fraud. Despite the due diligence done on the investment, risk management, and operational practices of a manager or fund, and even where there is a complete understanding of the business model risk, investors can still find themselves defrauded.