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Definition 1 ($\tau(\iota)$, deprecated version)**.** For an inference ι with $\sigma = \mathrm{mgu}(\iota)$, we define the infinite substitution $\tau(\iota)$ with $\mathrm{dom}(\tau(\iota)) = \{z_s \mid s\sigma \neq s\}$ as follows for a variable x:

$$x\tau(\iota) = \begin{cases} x & x \text{ is a non-lifting variable} \\ z_t & x \text{ is a lifting variable } z_s \text{ and } s\sigma = t \end{cases}$$

△

Definition 2 ($\tau(\iota)$, current version)**.** For an inference ι with $\sigma = \mathrm{mgu}(\iota)$, we define the infinite substitution $\tau(\iota)$ with $\mathrm{dom}(\tau(\iota)) = \mathrm{dom}(\sigma) \cup \{z_s \mid s\sigma \neq s\}$ as follows for a variable x:

$$x\tau(\iota) = \begin{cases} x\sigma & x \text{ is a non-lifting variable} \\ z_t & x \text{ is a lifting variable } z_s \text{ and } s\sigma = t \end{cases}$$

△

Definition 3 (Incremental lifting)**.** Let π be a resolution refutation of $\Gamma \cup \Delta$.
For a clause C in π, we define $\mathrm{PI}_{\mathrm{inc}}(C)$ and $\ell i[C]$ as follows:

Base case. If $C \in \Gamma$, $\mathrm{PI}_{\mathrm{inc}}(C) \overset{\mathrm{def}}{=} \bot$. If otherwise $C \in \Delta$, $\mathrm{PI}_{\mathrm{inc}}(C) \overset{\mathrm{def}}{=} \top$.

In any case, $\ell i[C] \overset{\mathrm{def}}{=} \ell[C]$.

Resolution. If the clause C is the result of a resolution step ι of $C_1 : D \vee l$ and $C_2 : E \vee \neg l'$ using a unifier σ such that $l\sigma = l'\sigma$, then let $\tau = \tau(\iota)$ and define $\mathrm{PI}_{\mathrm{inc}}(C)$ and $\ell i[C]$ as follows:

$\ell i[C] \overset{\mathrm{def}}{=} \ell[(\ell i[C_1] \backslash \{l_{\mathrm{AIcl}}\})\tau] \ \vee\ \ell[(\ell i[C_2] \backslash \{l'_{\mathrm{AIcl}}\})\tau]$

say something about l_{AIcl} and/or rewrite that (also below)

1. If l is Γ-colored: $\mathrm{PI}_{\mathrm{inc}}(C) \overset{\mathrm{def}}{=} \ell[\mathrm{PI}_{\mathrm{inc}}(C_1)\tau] \vee \ell[\mathrm{PI}_{\mathrm{inc}}(C_2)\tau]$
2. If l is Δ-colored: $\mathrm{PI}_{\mathrm{inc}}(C) \overset{\mathrm{def}}{=} \ell[\mathrm{PI}_{\mathrm{inc}}(C_1)\tau] \wedge \ell[\mathrm{PI}_{\mathrm{inc}}(C_2)\tau]$
3. If l is grey: $\mathrm{PI}_{\mathrm{inc}}(C) \overset{\mathrm{def}}{=} (\neg\ell[l'_{\mathrm{AIcl}}\tau] \wedge \ell[\mathrm{PI}_{\mathrm{inc}}(C_1)\tau]) \ \vee\ (\ell[l_{\mathrm{AIcl}}\tau] \wedge \ell[\mathrm{PI}_{\mathrm{inc}}(C_2)\tau])$

Factorisation. If the clause C is the result of a factorisation step ι of $C_1 : l \vee l' \vee D$ using a unifier σ such that $l\sigma = l'\sigma$, then $\mathrm{PI}_{\mathrm{inc}}(C) \overset{\mathrm{def}}{=} \ell[\mathrm{PI}_{\mathrm{inc}}(C_1)\tau(\iota)]$ and $\ell i[C] \overset{\mathrm{def}}{=} \ell[(\ell i[C_1] \backslash \{l'_{\mathrm{AIcl}}\})\tau(\iota)]$. △

?⟨def:arrow_quantifier_block⟩? **Definition 4** (Quantifier block)**.** Let C be a clause in a resolution refutation π of $\Gamma \cup \Delta$ and $\bar{x}$ be the Δ-lifting variables and $\bar{y}$ the Γ-lifting variables occurring in $\mathrm{PI}_{\mathrm{inc}}(C)$ and $\ell i[(]C)$. $Q(C)$ denotes an arrangement of the elements of $\{\forall x_t \mid x_t \in \bar{x}\} \cup \{\exists y_t \mid y_t \in \bar{y}\}$ such that for two lifting variable z_s and z_r, if s is a subterm of r, then z_s is listed before z_r. We denote $Q(\Box)$ by $Q(\pi)$. △

Conjectured Lemma 5. *For a clause C of a resolution refutation of $\Gamma \cup \Delta$, $\Gamma \models Q(C)(\mathrm{PI}_{\mathrm{inc}}(C) \vee \ell i[C])$.*