

necessary and unnecessary arrows in literal

Ex 301a

$$\begin{array}{c}
 \frac{\neg P(a, z) \quad \frac{P(x, f(y)) \vee Q(y) \quad \neg Q(a)}{Q(a) \mid P(x, f(a))}}{Q(a) \vee P(a, f(a)) \mid \square} \\
 \\
 \frac{\forall x_1 \neg P(x_1, z) \quad \frac{P(x, f(y)) \vee Q(y) \quad \forall x_1 \neg Q(x_1)}{\forall x_1 (Q(x_1) \mid P(x, f(x_1)))}}{\forall x_1 (Q(x_1) \vee P(x_1, f(x_1))) \mid \square} \qquad \frac{\exists y_2 P(x, y_2) \vee Q(y) \quad \forall x_1 \neg Q(x_1)}{\forall x_1 \neg P(x_1, z) \quad \frac{\forall x_1 \exists y_2 (Q(x_1) \mid P(x, y_2))}}{\forall x_1 (Q(x_1) \vee P(x_1, y_3)) \mid \square}
 \end{array}$$

all orderings work:

$\forall x_1 \exists y_2 (Q(x_1) \vee P(x_1, y_2))$ // need not be x_1 both times, that's just an accident of this example

$\exists y_2 \forall x_1 (Q(x_1) \vee P(x_1, y_2))$

Ex 301a' – simplified

$$\begin{array}{c}
 \frac{\neg P(z) \quad \frac{P(f(y)) \vee Q(y) \quad \neg Q(a)}{Q(a) \mid P(f(a))}}{Q(a) \vee P(f(a)) \mid \square} \\
 \\
 \frac{\neg P(z) \quad \frac{P(f(y)) \vee Q(y) \quad \forall x_1 \neg Q(x_1)}{\forall x_1 (Q(x_1) \mid P(f(x_1)))}}{\forall x_1 (Q(x_1) \vee P(f(x_1))) \mid \square} \qquad \frac{\exists y_2 P(y_2) \vee Q(y) \quad \forall x_1 \neg Q(x_1)}{\neg P(z) \quad \frac{\forall x_1 \exists y_2 (Q(x_1) \mid P(y_2))}}{\forall x_1 (Q(x_1) \vee P(y_3)) \mid \square}
 \end{array}$$

all orderings work:

$\forall x_1 \exists y_2 (Q(x_1) \vee P(y_2))$

$\exists y_2 \forall x_1 (Q(x_1) \vee P(y_2))$

Ex 302a

$$\begin{array}{c}
 \frac{P(u, f(u)) \quad \neg P(a, z)}{P(a, f(a)) \mid \square} \\
 \\
 \frac{P(u, f(u)) \quad \forall x_1 \neg P(x_1, z)}{\forall x_1 P(x_1, f(x_1)) \mid \square} \qquad \frac{\forall u \exists y_2 P(u, y_1) \quad \forall x_1 \neg P(x_1, z)}{\forall x_1 \exists y_3 P(x_1, y_3) \mid \square} \\
 \text{(order matters)}
 \end{array}$$

$\forall x_1 \exists y_2 (P(x_1, y_2))$

~~$\exists y_2 \forall x_1 (P(x_1, y_2))$~~

Ex 302a' – inverse coloring

$$\frac{\forall y_2 P(u, y_2) \quad \neg P(a, z)}{\forall y_3 P(a, y_3) \mid \square}$$

(can't really fix order, but matters)

Lesson of 30x:

LK-proof for $P(x, f(y)) \vee Q(y)$ works by multiple instantiations \Rightarrow not possible for $P(x, f(x))$, as x is always instantiated with same thing. it really does mean something different) first version has in some sense more liberty (it really does mean something different).