border cases: arrows not within supposedly connected components

211a

$$\frac{Q(x)\lor P(f(x,a)) \qquad \neg Q(y)\lor R(f(y,b))}{Q(x)\mid P(f(x,a))\lor R(f(x,b))}$$

$\Rightarrow$ no arr between P and R

211a'

$$\dfrac{\dfrac{Q(x)\lor \overset{\Sigma}{P}(f(x,a)) \qquad \neg Q(y)\lor \overset{\Sigma}{R}(f(y,b))}{Q(x)\mid P(f(x,a))\lor R(f(x,b))} \qquad \dfrac{\neg P(f(u,z))\overset{\Sigma}{\lor} S(u)) \qquad \neg \overset{\Pi}{S}(c)}{S(c)\mid \neg P(f(c,z))}}{(P(f(c,a)\land S(c))\lor(\neg P(f(c,a))\land Q(c)))\mid R(f(c,b))}$$

$c\sim x_1$ $\qquad$ $f(c,a)\sim y_2$ $\qquad$ $f(c,b)\sim y_3$

$(P(y_2)\land S(x_1))\lor(\neg P(y_2)\land Q(x_1))\mid R(y_3)$ $\qquad$ NOTE: arrow merge on resolution is not drawn here (but is necessary)

$\forall x_1\exists y_2\exists y_3$

this is not valid per se as the left hand side only contains Σ-formulas, but it probably could be fixed by adding some Π-inferences

Lesson is: no extra arrows needed, if a term enters, it does so via x, but there is a variable from the grey x to both colored x.

211b

$$\frac{Q(x)\lor P(f(x)) \qquad R(y)\lor \neg P(f(y))}{P(f(x))\mid Q(x)\lor R(x)}$$

$\Rightarrow$ no arr between Q and R

NB: should be fixed by backwards merging special case

211b'

$$\dfrac{\dfrac{Q(x)\lor \overset{\Sigma}{P}(f(x)) \qquad R(y)\lor \overset{\Sigma}{\neg} P(f(y))}{P(f(x))\mid Q(x)\lor R(x)} \qquad \neg\overset{\Pi}{Q}(a)}{P(f(a))\lor Q(a)\mid R(a)}$$

WRONG: conjecture: Q and R do not need arrows as they are lifted by the same variable anyway, so constraints on Q do the work

211c

$$\frac{Q(f(x))\overset{\Sigma}{\lor} R(x) \qquad \neg\overset{\Pi}{R}(g(y))}{Q(f(g(y)))\lor R(g(y))}$$

Have same var but no merge arrow. The whole term $g(y)$ is somehow the "travelling term", there is no "renaming".

211d – problem cases with lemma grey->colored

currently not clear what the connetion between the arguments of R on the RHS is If we use factorisation, not sure how to handle yet, but could be like: $R(t[x],s[x])\lor Q(x)$

$$\frac{Q(y) \vee Q'(z) \vee P(f(y)) \vee R(g(y), g'(z)) \qquad \neg R(g(h(x)), g'(x))}{R(g(h(x)), g'(x)) \mid Q(h(x)) \vee Q'(x) \vee P(f(h(x)))}\; y \mapsto h(x), z \mapsto x$$

NB: this is different since x occurs grey as well (example not finished)

Problem case 1: x grey and colored, but not connection

$$\frac{Q'(z) \vee P(f(y)) \vee R(g(f(y)), g'(z)) \qquad \neg R(g(f(h(x))), g'(x))}{R(g(f(h(x))), g'(x)) \mid Q(x) \vee P(f(h(x)))}\; y \mapsto h(x), z \mapsto x$$

NB: no connection between Q and P

$\Rightarrow$ backwards merging

Problem case 2: x colored and colored, not sure what the connection is supposed to be

$$\frac{Q'(k(z)) \vee P(f(y)) \vee R(g(y), g'(z)) \qquad \neg R(g(h(x)), g'(x))}{R(g(h(x)), g'(x)) \mid Q'(k(x)) \vee P(f(h(x)))}\; y \mapsto h(x), z \mapsto x$$

lifting var doesn't correspond to actual term exactly in context of unifier arrows

212a

$$
\dfrac{Q(u) \lor \overset{\Sigma}{\neg} P(h(u)) \qquad \dfrac{\dfrac{R(g(x)\overset{\Pi}{)}) \lor S(x) \qquad \neg \overset{\Pi}{S}(a)}{S(a) \mid\mid R(g(a))}\ x \mapsto a \qquad P(h(f(y)\overset{\Sigma}{)})) \lor \neg R(y)}{S(a) \mid R(g(a)) \mid P(h(f(g(a))))}\ y \mapsto g(a)}{S(a) \mid R(g(a)), P(h(f(g(a)))) \mid Q(f(g(a)))}\ u \mapsto g(f(g(a)))
$$

lifted:

$$
\dfrac{Q(u) \lor \overset{\Sigma}{\neg} P(y_{h(u)}) \qquad \dfrac{\dfrac{R(x_{g(x)}\overset{\Pi}{)}) \lor S(x) \qquad \neg \overset{\Pi}{S}(x_a)}{S(x_a) \mid\mid R(x_{g(x)})}\ x \mapsto a \qquad P(y_{h(f(y))}\overset{\Sigma}{)}) \lor \neg R(y)}{S(x_a) \mid R(x_{g(a)}) \mid P(y_{h(f(y))})}\ y \mapsto g(a) \ast}{S(x_a) \mid R(x_{g(a)}), P(y_{h(f(g(a)))}) \mid Q(y_{f(g(a))})}\ u \mapsto g(f(g(a)))
$$

at $\ast$, $R(x_g(x))$ is not known to refer to $g(a)$. can we resort to check grey occurrences of $g(a)$?
need arrow from R to P (this situation should be more critical if it is a backwards arrow)
thought: concerns only stuff in literal, maybe can leverage something here (all lifting vars x_i point to same term or so)

212b – same but more info not present

$$
\dfrac{\dfrac{R(g'(g(x\overset{\Pi}{)})) \lor S(x) \qquad \neg \overset{\Pi}{S}(a)}{S(a) \mid\mid R(g'(g(a)))}\ x \mapsto a \qquad \dfrac{P(h(f(y\overset{\Sigma}{)})) \lor \neg T(y) \qquad T(z) \lor \overset{\Pi}{\neg} R(g'(z))}{T(y) \mid P(h(f(y))) \lor \neg R(g'(y))}\ z \mapsto y}{S(a) \mid \neg R(g'(g(a))) \lor T(g(a)) \mid P(h(f(g(a))))}\ y \mapsto g(a) \ast
$$

lifted w unifier arrows:

$$
\dfrac{\dfrac{R(x_{g'(g(x))}\overset{\Pi}{)}) \lor S(x) \qquad \neg \overset{\Pi}{S}(x_a)}{S(x_a) \mid\mid R(x_{g'(g(x))})}\ x \mapsto a \qquad \dfrac{P(y_{h(f(y))}\overset{\Sigma}{)}) \lor \neg T(y) \qquad T(z) \lor \overset{\Pi}{\neg} R(x_{g'(z)})}{T(y) \mid P(y_{h(f(y))}) \lor \neg R(x_{g'(y)})}\ z \mapsto y}{S(x_a) \mid \neg R(x_{g'(g(a))}) \lor T(x_{g(a)}) \mid P(y_{h(f(y))})}\ y \mapsto g(a) \ast
$$

$\ast$ is similar here, but the grey occurrence of y isn't even in the literal

212c – other approach

$$\dfrac{\dfrac{\dfrac{P(f(x),u) \overset{\Sigma}{\vee} Q(x) \vee R(u) \qquad \neg Q(g(z)\overset{\Pi}{)}) \vee R(z)}{Q(g(z)) \mid P(f(g(z)),u) \vee R(u) \vee R(z)}\; x \mapsto g(z)}{Q(g(z)) \mid P(f(g(z)),z) \vee R(z)}\; u \mapsto z \qquad S(h(u)) \overset{\Sigma}{\vee} \neg P(f(u),c)}{P(f(g(c)),c) \mid Q(g(c)) \mid S(h(g(c))) \vee R(c)}\; u \mapsto g(c), z \mapsto c$$

lifted:

$$\dfrac{\dfrac{\dfrac{P(y_{f(x)},u) \overset{\Sigma}{\vee} Q(x) \vee R(u) \qquad \neg Q(x_{g(z)}\overset{\Pi}{)} \vee R(z)}{Q(x_{g(z)}) \mid P(y_{f(g(z))},u) \vee R(u) \vee R(z)}}{Q(x_{g(z)}) \mid P(y_{f(g(z))}),z) \vee R(z)} \qquad S(y_{h(u)}) \overset{\Sigma}{\vee} \neg P(y_{f(u)},y_c)}{P(y_{f(g(c))},y_c) \mid Q(x_{g(z)}) \mid S(y_{h(u)})) \vee R(y_c)}$$

Need arrow to S. possibly work in C, not lifted variants.

Problem 2: lifting var in P is updated, but the one in Q is not, hence index of lifting vars don't match, which per se isn't a problem

Δ-lifted:

$$\dfrac{\dfrac{\dfrac{P(f(x),u) \overset{\Sigma}{\vee} Q(x) \vee R(u) \qquad \neg Q(x_{g(z)}\overset{\Pi}{)} \vee R(z)}{Q(x_{g(z)}) \mid P(f(x_{g(z)}),u) \vee R(u) \vee R(z)}}{Q(x_{g(z)}) \mid P(f(x_{g(z)}),z) \vee R(z)} \qquad S(h(u)) \overset{\Sigma}{\vee} \neg P(f(u),c)}{P(f(x_{g(c)}),y_c) \mid Q(x_{g(z)}) \mid S(h(x_{g(z)})) \vee R(c)}$$

can prove something here which doesn't work when both are lifted? don't think so

term unified which is just produced

$$\frac{P(g(x),x) \qquad \neg P(y,a)}{P(g(a),a)}\; y \mapsto g(a), x \mapsto a$$

$\Rightarrow$ can only add arrow from terms in C, as they do not exist before.

old examples with unifier arrows

214a (210f)

$$\frac{\dfrac{\dfrac{\bot \mid P(f(\overset{\Sigma}{x})) \vee Q(x) \qquad \bot \mid \neg Q(y\overset{\Sigma}{)} \vee R(g(y))}{Q(x) \mid \bot \mid P(f(x)) \vee R(g(x))}\; y \mapsto x \qquad \dfrac{\bot \mid \neg P(f\overset{\Sigma}{(}z)) \vee S(z) \qquad \top \mid \overset{\Pi}{\neg} S(a)}{S(a) \mid \neg P(f(a))}\; z \mapsto a}{Q(a), P(f(a) \mid S(a) \mid R(g(a))}\; x \mapsto a \qquad \top \mid \overset{\Pi}{\neg} R(u)}{Q(a), P(f(a)) \mid S(a) \vee R(g(a))}$$

what if different starting point:

$$\frac{\dfrac{\bot \mid P(f(x\overset{\Sigma}{)}) \vee R(g(x)) \qquad \dfrac{\bot \mid \neg P(f(\overset{\Sigma}{z})) \vee S(z) \qquad \top \mid \overset{\Pi}{\neg} S(a)}{S(a) \mid \neg P(f(a))}\; z \mapsto a}{P(f(a) \mid S(a) \mid R(g(a))}\; x \mapsto a \qquad \top \mid \overset{\Pi}{\neg} R(u)}{P(f(a) \mid S(a) \vee R(g(a))}$$