

# Interpolation in First-Order Logic with Equality

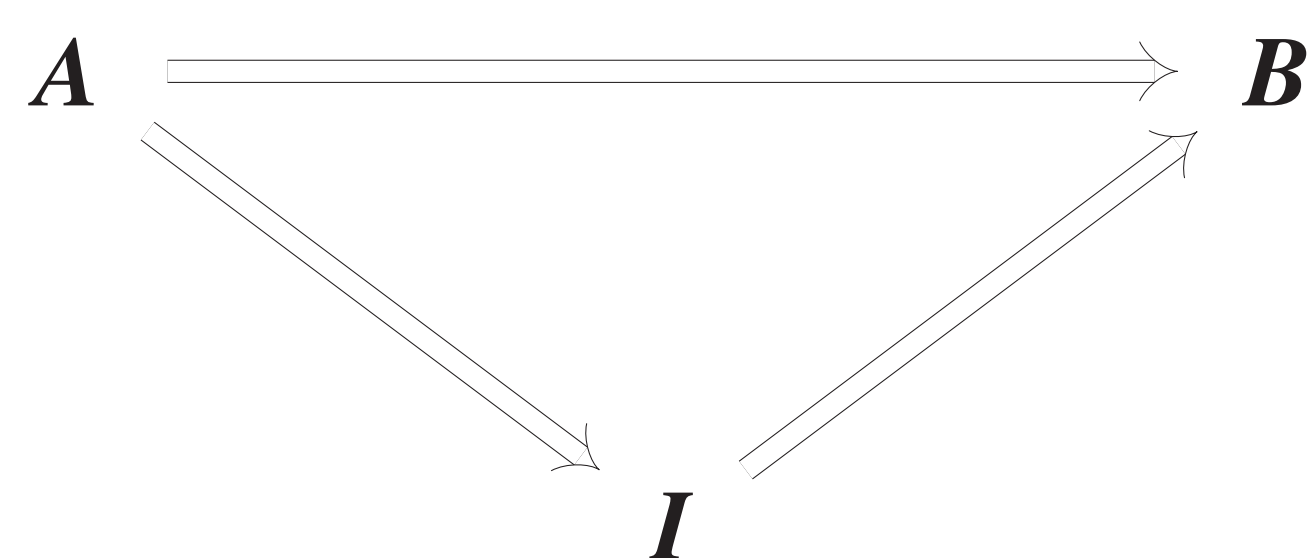
Masterstudium:  
Computational Intelligence

Bernhard Mallinger

Technische Universität Wien  
Institut für diskrete Mathematik und Geometrie  
Arbeitsbereich: Computational Logic  
Betreuer: Ass.Prof. Stefan Hetzl

## Interpolation

- ▶ Given two formulas  $A$  and  $B$  such that  $A$  implies  $B$ , an interpolant  $I$  is a formula which is implied by  $A$  and which itself implies  $B$ .



- ▶ Additionally, interpolants can only contain symbols which are common to both  $A$  and  $B$ .
- ▶ Hence interpolants succinctly capture the logical content which explains an implication.

**Theorem (Craig).** *Let  $A$  and  $B$  be first-order formulas such that  $A$  implies  $B$ . Then there is an interpolant for  $A$  and  $B$ .*

## Aim and Scope

Give comprehensive account of existing proofs and techniques and extend them:

- ▶ Reduction to first-order logic without equality
- ▶ Interpolant extraction from resolution proofs
- ▶ Model-theoretic proof

## Reduction to first-order logic without equality

This is the approach used by Craig for initial proof.

- ▶ Express equality and function symbols by means of fresh predicates with appropriate axioms
- ▶ Compute interpolants in first-order logic without equality and function symbols, for instance using Maehara's Lemma.

## Interpolant extraction from resolution proofs

This constructive proof by Huang consists of two phases:

- ▶ From a resolution proof inductively construct a propositional interpolant, which may still contain non-common terms.
- ▶ Replace non-common terms by variables and bind them in a quantifier prefix.

## Contributions:

- ▶ We showed that the number of quantifier alternations in the interpolant corresponds directly to the number of nested alternations of symbols which only occur in  $A$  or  $B$  respectively.
- ▶ We developed an improved version which combines these phases and produces non-prenex formulas.

## Model-theoretic proof

The interpolation theorem can also be proven semantically:

- ▶ Suppose that there is no interpolant.
- ▶ Then we can build a model in which  $A$  holds, but  $B$  does not.

⇒ If there is no formula which explains the logical relation between  $A$  and  $B$  (=interpolant), then this is possible.

TODO: applications of interpolation?

## References

- ▶ William Craig.  
Linear Reasoning. A New Form of the Herbrand-Gentzen Theorem.  
*Journal of Symbolic Logic*, 22(3):250–268, 1957.
- ▶ Guoxiang Huang.  
Constructing Craig Interpolation Formulas.  
In *Proc COCOON '95*, p. 181–190, 1995.