FAKULTÄT FÜR !NFORMATIK

Faculty of Informatics

Diplomarbeitspräsentation

Interpolation in First-Order Logic with Equality

Masterstudium:
Computational Intelligence

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Interpolation

- Given two formulas A and B such tht A implies B, an interpolant I is a formula which is implied by A and which itself implies B.

$$\begin{tikzcd} A \arrow[rr, Rightarrow] \arrow[dr, Rightarrow] & & B \\ & I \arrow[ur, Rightarrow] & \end{tikzcd}$$

- Additionally, interpolants can only contains symbols which are common to both A and B.
- Hence interpolants succinctly capture the logical content which explains an implication.

Theorem (Craig). *Let A and B be first-order formulas such that A implies B. Then there is an interpolant for A and B.*

Aim and Scope

Give comprehensive account of existing proofs and techniques and extend them:

- Reduction to first-order logic without equality
- Interpolant extraction from resolution proofs
- Model-theoretic proof

Reduction to first-order logic without equality

This is the approach used by Craig for inital proof.

- Express equality and function symbols by means of fresh predicates with appropriate axioms
- Compute interpolants in first-order logic without equality and function symbols, for instance using Maehara's Lemma.

Interpolant extraction from resolution proofs

This constructive proof by Huang consists of two phases:

- From a resolution proof inductively construct a propositional interpolant, which may still contain non-common terms.
- Replace non-common terms by variables and bind them in a quantifier prefix.

Contributions:

- We showed that the number of quantifier alternations in the interpolant corresponds directly to the number of nested alternations of symbols which only occur in A or B respectively.
- We developed an improved version which combines these phases and produces non-prenex formulas.

Model-theoretic proof

The interpolation theorem can also be proven semantically:

- Suppose that there is no interpolant.
- Then we can build a model in which A holds, but B does not.

$\Rightarrow$ If there is no formula which explains the logical relation between A and B (=interpolant), then this is possible.

TODO: applications of interpolation?

References

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