

border cases: arrows not within supposedly connected components

211a

$$\frac{Q(x) \vee P(f(x, a)) \quad \neg Q(y) \vee R(f(y, b))}{Q(x) \mid P(f(x, a)) \vee R(f(x, b))}$$

\Rightarrow no arr between P and R

211a'

$$\frac{\frac{Q(x) \vee \overset{\Sigma}{P}(f(x, a)) \quad \neg Q(y) \vee \overset{\Sigma}{R}(f(y, b))}{Q(x) \mid P(f(x, a)) \vee R(f(x, b))} \quad \frac{\overset{\Sigma}{\neg P(f(u, z)) \vee S(u)} \quad \overset{\Pi}{\neg S(c)}}{S(c) \mid \neg P(f(c, z))}}{(P(f(c, a) \wedge S(c)) \vee (\neg P(f(c, a) \wedge Q(c))) \mid R(f(c, b)))}$$

$c \sim x_1$

$f(c, a) \sim y_2$

$f(c, b) \sim y_3$

$$\frac{(P(y_2) \wedge S(x_1)) \vee (\neg P(y_2) \wedge Q(x_1)) \mid R(y_3)}{\forall x_1 \exists y_2 \exists y_3}$$

NOTE: arrow merge on resolution is not drawn here (but is necessary)

this is not valid per se as the left hand side only contains Σ -formulas, but it probably could be fixed by adding some Π -inferences
Lesson is: no extra arrows needed, if a term enters, it does so via x , but there is a variable from the grey x to both colored x .

211b

$$\frac{Q(x) \vee P(f(x)) \quad R(y) \vee \neg P(f(y))}{P(f(x)) \mid Q(x) \vee R(x)}$$

\Rightarrow no arr between Q and R

NB: should be fixed by backwards merging special case

211b'

$$\frac{\frac{Q(x) \vee \overset{\Sigma}{P}(f(x)) \quad R(y) \vee \overset{\Sigma}{\neg P}(f(y))}{P(f(x)) \mid Q(x) \vee R(x)} \quad \overset{\Pi}{\neg Q(a)}}{P(f(a)) \vee Q(a) \mid R(a)}$$

WRONG: conjecture: Q and R do not need arrows as they are lifted by the same variable anyway, so constraints on Q do the work

211c

$$\frac{Q(f(x)) \vee \overset{\Sigma}{R}(x) \quad \overset{\Pi}{\neg R(g(y))}}{Q(f(g(y))) \vee R(g(y))}$$

Have same var but no merge arrow. The whole term $g(y)$ is somehow the “travelling term”, there is no “renaming”.

211d – problem cases with lemma grey->colored

currently not clear what the connection between the arguments of R on the RHS is If we use factorisation, not sure how to handle yet, but could be like: $R(t[x], s[x]) \vee Q(x)$

$$\frac{Q(y) \vee Q'(z) \vee P(f(y)) \vee R(g(y), g'(z)) \quad \neg R(g(h(x)), g'(x))}{R(g(h(x)), g'(x)) \mid Q(h(x)) \vee Q'(x) \vee P(f(h(x)))} y \mapsto h(x), z \mapsto x$$

NB: this is different since x occurs grey as well (example not finished)

Problem case 1: x grey and colored, but not connection

$$\frac{Q'(z) \vee P(f(y)) \vee R(g(f(y)), g'(z)) \quad \neg R(g(f(h(x))), g'(x))}{R(g(f(h(x))), g'(x)) \mid Q(x) \vee P(f(h(x)))} y \mapsto h(x), z \mapsto x$$

NB: no connection between Q and P

\Rightarrow backwards merging

Problem case 2: x colored and colored, not sure what the connection is supposed to be

$$\frac{Q'(k(z)) \vee P(f(y)) \vee R(g(y), g'(z)) \quad \neg R(g(h(x)), g'(x))}{R(g(h(x)), g'(x)) \mid Q'(k(x)) \vee P(f(h(x)))} y \mapsto h(x), z \mapsto x$$

lifting var doesn't correspond to actual term exactly in context of unifier arrows

212a

$$\frac{Q(u) \vee \neg P(h(u)) \quad \frac{\frac{R(g(x)) \vee S(x) \quad \neg S(a)}{S(a) \parallel R(g(a))} x \mapsto a \quad \frac{P(h(f(y))) \vee \neg R(y)}{S(a) \mid R(g(a)) \mid P(h(f(g(a))))} y \mapsto g(a)}{S(a) \mid R(g(a)), P(h(f(g(a)))) \mid Q(f(g(a)))} u \mapsto g(f(g(a)))$$

lifted:

$$\frac{Q(u) \vee \neg P(y_{h(u)}) \quad \frac{\frac{R(x_{g(x)}) \vee S(x) \quad \neg S(x_a)}{S(x_a) \parallel R(x_{g(x)})} x \mapsto a \quad \frac{P(y_{h(f(y))}) \vee \neg R(y)}{S(x_a) \mid R(x_{g(a)}) \mid P(y_{h(f(y))))} y \mapsto g(a) *}{S(x_a) \mid R(x_{g(a)}), P(y_{h(f(g(a)))) \mid Q(y_{f(g(a))})} u \mapsto g(f(g(a)))$$

at *, $R(x_g(x))$ is not known to refer to $g(a)$. can we resort to check grey occurrences of $g(a)$?

need arrow from R to P (this situation should be more critical if it is a backwards arrow)

thought: concerns only stuff in literal, maybe can leverage something here (all lifting vars x_i point to same term or so)

212b – same but more info not present

$$\frac{\frac{R(g'(g(x))) \vee S(x) \quad \neg S(a)}{S(a) \parallel R(g'(g(a)))} x \mapsto a \quad \frac{\frac{P(h(f(y))) \vee \neg T(y) \quad T(z) \vee \neg R(g'(z))}{T(y) \mid P(h(f(y))) \vee \neg R(g'(y))} z \mapsto y}{S(a) \mid \neg R(g'(g(a))) \vee T(g(a)) \mid P(h(f(g(a))))} y \mapsto g(a) *$$

lifted w unifier arrows:

$$\frac{\frac{R(x_{g'(g(x))}) \vee S(x) \quad \neg S(x_a)}{S(x_a) \parallel R(x_{g'(g(x))})} x \mapsto a \quad \frac{\frac{P(y_{h(f(y))}) \vee \neg T(y) \quad T(z) \vee \neg R(x_{g'(z)})}{T(y) \mid P(y_{h(f(y))}) \vee \neg R(x_{g'(y)})} z \mapsto y}{S(x_a) \mid \neg R(x_{g'(g(a))}) \vee T(x_{g(a)}) \mid P(y_{h(f(y))})} y \mapsto g(a) *$$

* is similar here, but the grey occurrence of y isn't even in the literal

212c – other approach

$$\frac{\frac{\frac{P(f(x), u) \vee Q(x) \vee R(u)}{Q(g(z)) \mid P(f(g(z)), u) \vee R(u) \vee R(z)} \quad \frac{\neg Q(g(z)) \vee R(z)}{S(h(u)) \vee \neg P(f(u), c)} \quad x \mapsto g(z)}{\frac{Q(g(z)) \mid P(f(g(z)), z) \vee R(z)}{P(f(g(c)), c) \mid Q(g(c)) \mid S(h(g(c))) \vee R(c)} \quad u \mapsto z} \quad \text{S}(h(u)) \vee \neg P(f(u), c) \quad u \mapsto g(c), z \mapsto c$$

(arrows for c not shown)

(the bold S receives the Δ -term here, the rest is technical details. it's about how S receives the arrow)
lifted:

$$\frac{\frac{P(y_{f(x)}, u) \vee Q(x) \vee R(u) \quad \neg Q(x_{g(z)}) \vee R(z)}{Q(x_{g(z)}) \mid P(y_{f(g(z)}), u) \vee R(u) \vee R(z)}}{\frac{Q(x_{g(z)}) \mid P(y_{f(g(z)}), z) \vee R(z)}{P(y_{f(g(c))}, y_c) \mid Q(x_{g(z)}) \mid S(y_{h(u)}) \vee R(y_c)}} S(y_{h(u)}) \vee \neg P(y_{f(u)}, y_c) \quad \Sigma$$

Need arrow to S . possibly work in C , not lifted variants.

Problem 2: lifting var in P is updated, but the one in Q is not, hence index of lifting vars don't match, which per se isn't a problem

 Δ -lifted:

$$\frac{\frac{\frac{P(f(x), u) \vee^{\Sigma} Q(x) \vee R(u) \quad \neg Q(x_{g(z)}) \vee^{\Pi} R(z)}{Q(x_{g(z)}) \mid P(f(x_{g(z)}), u) \vee R(u) \vee R(z)}}{Q(x_{g(z)}) \mid P(f(x_{g(z)}), z) \vee R(z)} \quad S(h(u)) \vee^{\Sigma} \neg P(f(u), c)}{P(f(x_{g(c)}), y_c) \mid Q(x_{g(c)}) \mid S(h(x_{g(c)})) \vee R(c)} \quad \text{lifting var in } Q \text{ is fixed here}$$

212d – not in literal

$$\frac{P(g(x)) \overset{\Pi}{\vee} Q(k(x)) \quad \neg Q(k(a))}{Q(k(a)) \mid P(g(\textcolor{red}{a}))}$$

$$\frac{P(x_{g(x)}) \overset{\Pi}{\vee} Q(x_{k(x)}) \quad \neg Q(x_{k(a)})}{Q(x_{k(a)}) \mid P(x_{q(\textcolor{red}{x})})}$$

term unified which is just produced

$$\frac{P(g(x), x) \quad \neg P(y, a)}{P(g(a), a)} y \mapsto g(a), x \mapsto a$$

\Rightarrow can only add arrow from terms in C , as they do not exist before.

old examples with unifier arrows

214a (210f)

$$\frac{\frac{\frac{\perp \mid P(f(\textcolor{red}{x})) \vee Q(\textcolor{red}{x}) \quad \perp \mid \neg Q(\textcolor{blue}{y}) \vee R(g(\textcolor{blue}{y}))}{Q(x) \mid \perp \mid P(f(x)) \vee R(g(x))} y \mapsto x \quad \frac{\frac{\perp \mid \neg P(f(\textcolor{violet}{z})) \vee S(\textcolor{violet}{z}) \quad \top \mid \neg S(a)}{S(a) \mid \neg P(f(a))} z \mapsto a}{\frac{Q(a), P(f(a) \mid S(a) \mid R(g(a)) \quad S(a) \mid \neg P(f(a))}{Q(a), P(f(a)) \mid S(a) \vee R(g(a))} x \mapsto a} \top \mid \neg R(u)$$

what if different starting point:

$$\frac{\frac{\frac{\perp \mid P(f(x)) \vee R(g(x)) \quad \frac{\perp \mid \neg P(f(\textcolor{violet}{z})) \vee S(\textcolor{violet}{z}) \quad \top \mid \neg S(a)}{S(a) \mid \neg P(f(a))} z \mapsto a}{P(f(a) \mid S(a) \mid R(g(a))} x \mapsto a}{\frac{P(f(a) \mid S(a) \mid R(g(a)) \quad \top \mid \neg R(u)}{P(f(a)) \mid S(a) \vee R(g(a))}$$

More special cases from proof:

215a

$$\frac{P(f(g(v)), f(v)) \quad \neg P(f(g(a)), f(u)) \vee S(h(u))}{P(f(g(a)), f(a)) \mid S(h(a))}$$

need arrow to S

prequel to this situation:

RHS:

$$\frac{\frac{R(g(a))}{\neg R(g(a)) \mid \neg P(f(g(a)), f(u)) \vee S(h(u))} \quad \neg R(x) \vee \neg P(f(x), f(u)) \vee S(h(u))}{\neg R(x) \vee \neg P(f(x), f(u)) \vee S(h(u))}$$

LHS1:

$$\frac{\frac{R(y) \vee P(f(y), f(v)) \vee Q(v)}{R(g(z)) \mid P(f(g(z)), f(v)) \vee Q(v)} \quad \neg R(g(z)) \vee Q(z)}{R(g(v)) \mid P(f(g(v)), f(v)) \vee Q(v)}$$

\Rightarrow have grey occ of v , it is only a problem if there isn't one