Conjecture 1. *Let C be a clause in a resolution refutation. Suppose that $\mathrm{AI}^{\Delta}(C)$ contains a maximal Γ-term $\gamma_j[x_i]$ which contains a lifting variable x_i (lifting a Δ-term δ_i). Then there is an arrow in $\mathcal{A}^*(C)$ from an occurrence of x_k in $\mathrm{AI}^{\Delta}(C)$ such that δ_k is an abstraction of δ_i to $\gamma_j[x_i]$.*

Proof. We proceed by induction. For the base case, note that no multicolored terms occur in inital clauses, so no lifting term can occur inside of a Γ-term.
Suppose a clause C is the result of a resolution of $C_1 : D \vee l$ and $C_2 : E \vee \neg l$ with $l\sigma = l'\sigma$. Furthermore suppose that for every lifting term inside a Γ-term in the clauses C_1 and C_2 of the refutation the desired arrows exist. We show that the same holds true for every new term of the form $\gamma_j[x_i]$ for some j, i in $\mathrm{AI}^{\Delta}(C)$. By "new", we mean terms which are not present in $\mathrm{AI}^{\Delta}(C_1)$ or $\mathrm{AI}^{\Delta}(C_2)$. Note that new terms in $\mathrm{AI}^{\Delta}(C)$ are of the form $\ell_{\Delta,x}[t\sigma]\tau$ for some $t \in \mathrm{AI}^{\Delta}(C_1) \cup \mathrm{AI}^{\Delta}(C_2)$. Hence a new term of the form $\gamma_j[x_i]$ is created either by introducing a Δ-term into a Γ-term or by introducing $\gamma_j[\delta_i]$ via σ, both followed by lifting. Note that τ only substitutes lifting variables by other lifting variables and hence does not change the form of the term.
Also note that a substitution, in particular σ, only changes a finite number of variables. Furthermore a result of a run of the unification algorithm is acyclic in the sense that if a substitution $u \mapsto s$ is added to the resulting substitution, it is never the case that at a later stage $s \mapsto u$ is added. This can easily be seen by considering that at the point when $u \mapsto s$ is added to the resulting substitution, every occurrence of u is replaced by s, so u is not encountered by the algorithm at a later stage. Therefore considering the different cases which arise by substitution and reducing some cases to others is a valid proof strategy.

Suppose for some Γ-term $\tilde{\gamma}_j[u]$ occurring in $\mathrm{AI}^{\Delta}(C_1)$ or $\mathrm{AI}^{\Delta}(C_2)$, $(\tilde{\gamma}_j[u])\sigma = \gamma_j[\delta_i]$ for some i.

□

Suppose σ introduces a term of the form $\gamma_j[\delta_i]$. T