Definition 1 (Q)**.** For a literal/clause φ, $Q(\varphi)$ denotes the quantifier block consisting of every lifting variable in φ with appropriate quantifier type. The order is yet to be defined △

For $l \in C$ for $C \in \Gamma$: $Q(l) = \exists \bar{y}$
For $l \in C$ for $C \in \Delta$: $Q(l) = \forall \bar{x}$

basic axioms which should be fulfilled for a reasonable procedure

start

- $\Gamma \models \mathrm{LI}_{\mathrm{cl}}(C)$

  $\Gamma = \{P(f(x))\} \Rightarrow \mathrm{LI}_{\mathrm{cl}}(C) \overset{\text{must be}}{=} \exists x P(x)$

  $\Gamma = \{\neg P(f(x))\} \Rightarrow \mathrm{LI}_{\mathrm{cl}}(C) \overset{\text{must be}}{=} \exists x \neg P(x)$

- $\Delta \models ?$

inferences LI is always basically just Γ-part
always want: $\Gamma \models \mathrm{LI}$, $\Delta \models \neg\, \mathrm{LI}$

- $\Gamma : P(f(x)) \Rightarrow \exists x P(x)$

  $\Delta : \neg P(y) \Rightarrow \forall y \neg P(x)$

- $\Gamma : \neg P(f(x)) \Rightarrow \exists x \neg P(x)$

  $\Delta : P(y) \Rightarrow \forall y P(x)$

- $\Gamma : \neg P(x) \Rightarrow \forall x \neg P(x)$

  $\Delta : P(g(y)) \Rightarrow \exists y P(y)$

- $\Gamma : P(x) \Rightarrow \forall x P(x)$

  $\Delta : \neg P(g(y)) \Rightarrow \neg \exists y P(y)$

but must not tear apart $P(x) \vee \neg P(x)$ to $\forall x P(x) \vee \forall x \neg P(x)$

example for "var does not occur in clause any more-condition":

$$\frac{R(f(z)) \qquad \neg R(x) \vee P(x)}{\neg R(x) \mid P(x)}$$

Note that $(\forall y_{f(x)} \neg R(y_{f(x)})) \vee P(x)$ is not valid!

attempt for a definition

Definition 2 (LI).

Base case.

For $l \in C$ for $C \in \Gamma \cup \Delta$: $Q(l)\ell[C] \in \mathrm{LI}_{\mathrm{cl}}(C)$

LI as usual

Resolution.

Definition 3 (χ: lifting with quantification on literal level).

$\chi(F \circ G) \stackrel{\mathrm{def}}{=} \chi(F) \circ \chi(G)$

$\chi(\neg G) \stackrel{\mathrm{def}}{=} \neg\chi(F)$

$\chi(Q(\lambda)\lambda) \stackrel{\mathrm{def}}{=} Q(\lambda\sigma)\lambda\sigma$

where $Q(\lambda\sigma)$ is $Q(\lambda)$ with quantifiers and lifting variables for additional maximal colored terms introduced by σ into λ $\triangle$

$\mathrm{LI}_{\mathrm{cl}}\, C \stackrel{\mathrm{def}}{=} \chi(\mathrm{LI}_{\mathrm{cl}}(C_1)\backslash\{l_{\mathrm{LIcl}}\}) \vee \chi(\mathrm{LI}_{\mathrm{cl}}(C_2)\backslash\{l'_{\mathrm{LIcl}}\})$

1. If l is Γ-colored: $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} \chi(\mathrm{LI}(C_1)) \vee \chi(\mathrm{LI}(C_2))$
2. If l is Δ-colored: $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} \chi(\mathrm{LI}(C_1)) \wedge \chi(\mathrm{LI}(C_2))$
3. If l is grey: $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} (\ell[l_{\mathrm{LIcl}}\tau] \wedge \ell[\mathrm{LI}(C_2)\tau]) \vee (\neg\ell[l'_{\mathrm{LIcl}}\tau] \wedge \ell[\mathrm{LI}(C_1)\tau])$

$\triangle$

Conjectured Lemma 4. $\Gamma \models \mathrm{LI}(C) \vee \mathrm{LI}_{\mathrm{cl}}(C)$

Proof. Start works.
Step:
resolved literals: have same coloring

IH:
$\Gamma \models \mathrm{LI}(C_1) \vee \mathrm{LI}_{\mathrm{cl}}(C_1^*) \vee l_{\mathrm{LIcl}}$
$\Gamma \models \mathrm{LI}(C_2) \vee \mathrm{LI}_{\mathrm{cl}}(C_2^*) \vee l'_{\mathrm{LIcl}}$ $\square$