necessary and unnecessary arrows in literal

Ex 301a' – simplified

$$\frac{\overset{\Pi}{\neg P(z)} \qquad \dfrac{\overset{\Sigma}{P(f(y)) \lor Q(y)} \qquad \overset{\Pi}{\neg Q(a)}}{Q(a) \mid P(f(a))}}{Q(a) \lor P(f(a)) \mid \Box}$$

$$\frac{\overset{\Pi}{\neg P(z)} \qquad \dfrac{\overset{\Sigma}{P(f(y)) \lor Q(y)} \qquad \overset{\Pi}{\forall x_1 \, \neg Q(x_1)}}{\forall x_1 (Q(x_1) \mid P(f(x_1)))}}{\forall x_1 (Q(x_1) \lor P(f(x_1))) \mid \Box}$$

$$\frac{\overset{\Pi}{\neg P(z)} \qquad \dfrac{\overset{\Sigma}{\exists y_2 \, P(y_2) \lor Q(y)} \qquad \overset{\Pi}{\forall x_1 \, \neg Q(x_1)}}{\overset{\forall x_1}{\exists y_2} \, (Q(x_1) \mid P(y_2))}}{\overset{\forall x_1}{\exists y_3} \, (Q(x_1) \lor P(y_3)) \mid \Box}$$

all orderings work:

$\forall x_1 \exists y_2 (Q(x_1) \lor P(y_2))$

$\exists y_2 \forall x_1 (Q(x_1) \lor P(y_2))$

Ex 302a

$$\frac{\overset{\Sigma}{P(u, f(u))} \qquad \overset{\Pi}{\neg P(a, z)}}{P(a, f(a)) \mid \Box}$$

$$\frac{\overset{\Sigma}{P(u, f(u))} \qquad \overset{\Pi}{\forall x_1 \, \neg P(x_1, z)}}{\forall x_1 P(x_1, f(x_1)) \mid \Box}$$

$$\frac{\overset{\Sigma}{\forall u \exists y_2 \, P(u, y_1))} \qquad \overset{\Pi}{\forall x_1 \, \neg P(x_1, z)}}{\forall x_1 \exists y_3 P(x_1, y_3) \mid \Box}$$

(order matters)

$\forall x_1 \exists y_2 (P(x_1, y_2))$

$\cancel{\exists y_2 \forall x_1 (P(x_1, y_2))}$

Ex 302a' – inverse coloring

$$\frac{\overset{\Pi}{\forall y_2 \, P(u, y_2)} \qquad \overset{\Sigma}{\neg P(a, z)}}{\forall y_3 P(a, y_3) \mid \Box}$$

(can't really fix order, but matters)

Lession of 30x:

LK-proof for $P(x, f(y)) \lor Q(y)$ works by multiple instantiations $\Rightarrow$ not possible for $P(x, f(x))$, as x is always instantiated with same thing. it really does mean something different) first version has in some sense more liberty (it really does mean something different).

situations arise where arrows should be present but aren't

Ex 303

$$\frac{P(f(x), y) \vee Q(g(x), h(y)) \qquad \neg Q(g(z), h(z))}{P(f(z), z)}$$

no arrow but $\exists x \forall z P(x, z)$ NOT valid.

countermodel:

$\mathcal{U} = \{0, 1\}$

$P^{\mathcal{I}} = \{(1, 0), (0, 1)\}$

$Q^{\mathcal{I}} = \{(1, 0), (0, 1)\}$

$f^{\mathcal{I}} = g^{\mathcal{I}} = h^{\mathcal{I}} = \mathrm{id}$