

Example 101: canonical examples for overbinding w.r.t. order

Here, the second lower prooftree shows what huang would produce.

Ex 101a

$$\begin{array}{c}
 \frac{\frac{P(u, f(u)) \vee Q(u)}{P(a, f(a))} \quad \frac{\neg Q(a)}{u \mapsto a}}{\square} \quad \frac{\neg P(x, y)}{x \mapsto a, y \mapsto f(a)} \\
 \\
 \frac{\frac{\perp}{Q(a)} \quad \top}{P(a, f(a)) \vee Q(a)} \quad \top \quad x \mapsto a, y \mapsto f(a) \quad \frac{\frac{\perp}{\forall x_1 Q(x_1)} \quad \top}{\forall x_1 \exists x_2 (P(x_1, x_2) \vee Q(x_1))}
 \end{array}$$

Direct overbinding would not work without merging same variables!: $\forall x_1 \forall x_2 \exists x_3 (P(x_2, x_3) \vee Q(x_1))$

counterexample: $Q \sim \{0\}, P \sim \{(1, 0)\}$

Direct overbinding would work when considering original dependencies as highlighted above

Ex 101b – other resolution order

$$\begin{array}{c}
 \frac{\frac{P(u, f(u)) \vee Q(u)}{Q(u)} \quad \frac{\neg P(x, y)}{y \mapsto f(u), x \mapsto u}}{\square} \quad \frac{\neg Q(a)}{u \mapsto a} \\
 \\
 \frac{\frac{\perp}{P(u, f(u))} \quad \top}{P(a, f(a)) \vee Q(a)} \quad \top \quad u \mapsto a \quad \frac{\frac{\perp}{\exists x_1 P(u, x_1)} \quad \top}{\forall x_1 \exists x_2 (P(x_1, x_2) \vee Q(x_1))} \quad u \mapsto a
 \end{array}$$

Ex 101c – Π and Σ swapped

$$\begin{array}{c}
 \frac{\frac{P(u, f(u)) \vee Q(u)}{Q(u)} \quad \frac{\neg P(x, y)}{y \mapsto f(u), x \mapsto u}}{\square} \quad \frac{\neg Q(a)}{u \mapsto a} \\
 \\
 \frac{\frac{\top}{\neg P(u, f(u))} \quad \perp}{\neg P(a, f(a)) \wedge \neg Q(a)} \quad \perp \quad u \mapsto a \quad \frac{\frac{\top}{\forall x_2 \neg P(u, x_2)} \quad \perp}{\exists x_1 \forall x_2 (\neg P(x_1, x_2) \wedge \neg Q(x_1))} \quad \perp
 \end{array}$$

Ex 101d – Π and Σ swapped, other resolution order

$$\frac{\frac{P(u, f(u)) \vee Q(u)}{P(a, f(a))} \quad \frac{\neg Q(a)}{u \mapsto a}}{\square} \quad \frac{\neg P(x, y)}{x \mapsto a, y \mapsto f(a)}$$

$$\frac{\frac{\top}{\neg Q(a)} \quad \perp \quad y \mapsto a}{\neg Q(a) \wedge \neg P(a, f(a))} \quad \perp \quad x \mapsto a, y \mapsto f(a)$$

$$\frac{\frac{\top}{\exists x_1 \neg Q(x_1)} \quad \perp}{\exists x_1 \forall x_2 (\neg P(x_1, x_2) \wedge \neg Q(x_1))} \quad \perp$$

102 – similar to 101 but with intra-clause-set inferences

Ex 102a

$$\frac{\frac{P(f(\textcolor{red}{x})) \vee Q(f(\textcolor{red}{x}), z) \quad \neg P(y)}{Q(f(x), z)} \quad \frac{\neg Q(x_1, \textcolor{blue}{y}) \vee R(\textcolor{blue}{y}) \quad \neg R(g(z_1))}{\neg Q(x_1, g(z_1))} \quad y \mapsto g(z_1), x_1 \mapsto f(x)}{x_1 \mapsto f(x), z \mapsto g(z_1)} \quad \square$$

$$\frac{\frac{\perp}{P(f(x))} \quad \top \quad \frac{\perp}{R(g(z_1))} \quad \top \quad y \mapsto g(z_1), x_1 \mapsto f(x)}{P(f(x)) \vee R(g(z_1))} \quad x_1 \mapsto f(x), z \mapsto g(z_1) \quad \frac{\frac{\perp}{\exists x_1 P(x_1)} \quad \top \quad \frac{\perp}{\forall x_2 R(x_2)} \quad \top}{\exists x_1 \forall x_2 (P(x_1) \vee R(x_2))} \quad (\text{order irrelevant!})$$

Ex 102b

$$\frac{\frac{P(f(\textcolor{red}{x})) \vee Q(f(\textcolor{red}{x}), z) \quad \neg P(y)}{Q(f(x), z)} \quad \frac{\neg Q(f(\textcolor{blue}{y}), z_1) \vee R(\textcolor{blue}{y}) \quad \neg R(a)}{\neg Q(f(a), z_1)} \quad y \mapsto a}{x \mapsto a, z \mapsto z_1} \quad \square$$

$$\frac{\frac{\perp}{P(f(x))} \quad \top \quad \frac{\perp}{R(a)} \quad \top \quad y \mapsto a}{P(f(a)) \vee R(a)} \quad x \mapsto a, z \mapsto z_1 \quad \frac{\frac{\perp}{\exists x_1 P(x_1)} \quad \top \quad \frac{\perp}{\forall x_2 R(x_2)} \quad \top \quad y \mapsto a}{\forall x_2 \exists x_1 (P(x_1) \vee R(x_2))} \quad x \mapsto a, z \mapsto z_1$$

direct:

$$\frac{\frac{\perp}{\exists x_1 P(x_1)} \quad \top \quad x_1 \sim f(x) \quad \frac{\perp}{\forall x_2 R(x_2)} \quad \top \quad x_2 \sim a}{\exists x_1 \forall x_2 (P(x_1) \vee R(x_2))} \quad \text{order irrelevant!}$$

Ex 102b' with Q grey

$$\begin{array}{c}
 \frac{P(f(\textcolor{red}{x})) \vee Q(f(\textcolor{red}{x}), z) \quad \neg P(y)}{Q(f(x), z)} \quad \frac{\neg Q(f(\textcolor{blue}{y}), z_1) \vee R(\textcolor{blue}{y}) \quad \neg R(a)}{\neg Q(f(a), z_1)} \quad \frac{\neg R(a)}{y \mapsto a} \\
 \hline
 \frac{\quad}{\square} \quad \frac{\quad}{x \mapsto a, z_1 \mapsto z} \\
 \frac{\frac{\perp \quad \top}{P(f(x))} \quad \frac{\perp \quad \top}{R(a)} y \mapsto a}{(\neg Q(f(a), z) \wedge P(f(a))) \vee (Q(f(a), z) \wedge R(a))} x \mapsto a, z_1 \mapsto z
 \end{array}$$

Huang:

$$\frac{\frac{\perp \quad \top}{\exists x_2 P(x_2)} \quad \frac{\perp \quad \top}{\forall x_1 R(x_1)} y \mapsto a}{\forall x_1 \exists x_2 (\neg Q(x_2, z) \wedge P(x_2)) \vee (Q(x_2, z) \wedge R(x_1))} x \mapsto a, z_1 \mapsto z$$

direct:

$$\begin{array}{c}
 \frac{\frac{\perp \quad \top}{\exists x_2 P(x_2)} x_2 \sim f(x) \quad \frac{\perp \quad \top}{\forall x_1 R(x_1)} x_1 \sim a}{\forall x_1 \exists x_2 \exists x_3 (\neg Q(x_3, z) \wedge P(x_2)) \vee (Q(x_3, z) \wedge R(x_1))} x_3 \sim f(a); x_2 \parallel x_3, x_1 < x_3 \\
 \hline
 \text{OR: } \exists x_2 \forall x_1 \exists x_3 (\neg Q(x_3, z) \wedge P(x_2)) \vee (Q(x_3, z) \wedge R(x_1)) \\
 \hline
 \text{OR: } \exists x_1 \exists x_3 \forall x_2 (\neg Q(x_3, z) \wedge P(x_2)) \vee (Q(x_3, z) \wedge R(x_1))
 \end{array}$$

Huang-Version passt weil man für existenzielle quantoren eh gleichen term einsetzt

Example 103: variables in interpolant get unified and order might have to be changed

103a: length changes but old version is still valid, despite huang's algorithm doing a sort

$$\frac{\frac{\frac{Q(f(\mathbf{x})) \vee \overset{\Sigma}{P}(y) \vee R(\mathbf{x})}{P(y) \vee R(x)} \quad \overset{\Pi}{\neg Q(y_1)}}{y_1 \mapsto f(x)} \quad \overset{\Pi}{\neg P(h(g(a)))}}{y \mapsto h(g(a))} \quad \overset{\Pi}{\neg R(g(g(a)))}}{x \mapsto g(g(a))} \quad \square$$

$$\frac{\frac{\frac{\perp}{Q(f(x))} \quad \top}{y_1 \mapsto f(x)} \quad \top}{Q(f(x)) \vee P(h(g(a)))} \quad \top}{Q(f(g(g(a)))) \vee P(h(g(a))) \vee R(g(g(a)))} \quad \top}{x \mapsto g(g(a))} \quad \top \quad \frac{\frac{\frac{\perp}{\exists x_1 Q(x_1)} \quad \top}{\exists x_1 \forall x_2 (Q(x_1) \vee P(x_2))} \quad \top}{X}$$

X:

Huang's algo gives:

$$\forall x_2 \forall x_3 \exists x_1 (Q(x_1) \vee P(x_2) \vee R(x_3))$$

Direct overbinding gives: $x_3 < x_1$, rest arbitrary, hence:

$$\forall x_3 \exists x_1 \forall x_2 (Q(x_1) \vee P(x_2) \vee R(x_3)) \leftarrow \text{this you do not get with huang}$$

$$\forall x_2 \forall x_3 \exists x_1 (Q(x_1) \vee P(x_2) \vee R(x_3))$$

$$\forall x_3 \forall x_2 \exists x_1 (Q(x_1) \vee P(x_2) \vee R(x_3))$$

103b: length changes “uniformly”

$$\frac{\frac{\frac{Q(f(f(\mathbf{x}))) \vee \overset{\Sigma}{P}(f(\mathbf{x})) \vee R(\mathbf{x})}{P(f(x)) \vee R(x)} \quad \overset{\Pi}{\neg Q(y_1)}}{y_1 \mapsto f(f(x))} \quad \overset{\Pi}{\neg P(y_2)}}{y_2 \mapsto f(x)} \quad \overset{\Pi}{\neg R(g(a))}}{x \mapsto g(a)} \quad \square$$

$$\frac{\frac{\frac{\perp}{Q(f(f(x)))} \quad \top}{y_1 \mapsto f(f(x))} \quad \top}{Q(f(f(x))) \vee P(f(x))} \quad \top}{Q(f(f(g(a)))) \vee P(f(g(a))) \vee R(g(a))} \quad \top}{x \mapsto g(a)} \quad \top \quad \frac{\frac{\frac{\perp}{\exists x_1 Q(x_1)} \quad \top}{\exists x_2 \exists x_1 (Q(x_1) \vee P(x_2))} \quad \top}{\forall x_3 \exists x_2 \exists x_1 (Q(x_1) \vee P(x_2) \vee R(x_3))} \quad \top$$

Huang and direct overbinding somewhat coincide as $x_2 < x_1$ in both cases, and $x_3 < x_1$ and $x_3 < x_2$

103c: Failed attempt: different variables, accidentally the same terms appear but no logical connection

$$\frac{\overset{\Sigma}{P(a, x)} \quad \frac{\overset{\Sigma}{\neg Q(a)} \quad \overset{\Pi}{\neg P(y, f(\mathbf{z})) \vee Q(\mathbf{z})}}{\neg P(y, f(a))} \quad z \mapsto a}{y \mapsto a, x \mapsto f(a)} \quad \square$$

$$\frac{\perp \quad \frac{\perp \quad \top}{\neg Q(a)} z \mapsto a}{P(a, f(a)) \wedge \neg Q(a)} y \mapsto a, x \mapsto f(a) \qquad \frac{\perp \quad \frac{\perp \quad \top}{\exists x_1 \neg Q(x_1)}}{\exists x_1 \forall x_2 (P(x_1, x_2) \wedge \neg Q(x_1))}$$

order required for Π

direct:

$$\frac{\perp \quad \frac{\perp \quad \top}{\exists x_1 \neg Q(x_1)} x_1 \sim a}{\frac{\exists x_1 \exists x_2 \forall x_3 (P(x_2, x_3) \wedge \neg Q(x_1))}{\text{OR: } \exists x_1 \forall x_3 \exists x_2 (P(x_2, x_3) \wedge \neg Q(x_1))} x_2 \sim a, x_3 \sim f(a); x_1 < x_3}$$

invariant:

$$\frac{\exists x_2 (P(x_2, x) \vee \perp) \quad \frac{\exists x_1 (Q(x_1) \vee \perp) \quad \forall x_3 ((\neg P(y, \mathbf{x}_3) \vee Q(\mathbf{z})) \vee \top)}{\exists x_1 \forall x_3 \neg P(y, \mathbf{x}_3) \vee \neg Q(\mathbf{x}_1)} x_1 \sim a}{\frac{\exists x_1 \exists x_2 \forall x_3 (P(x_2, x_3) \wedge \neg Q(x_1))}{\text{OR: } \exists x_1 \forall x_3 \exists x_2 (P(x_2, x_3) \wedge \neg Q(x_1))} x_2 \sim a, x_3 \sim f(a); x_1 < x_3}$$

invariant in other resolution order

$$\frac{\perp \quad \frac{\perp \quad \top}{Q(\mathbf{z}) \vee \exists x_2 \forall x_3 P(x_2, \mathbf{x}_3)} x_2 \sim a, x_3 \sim f(\mathbf{z})}{\frac{\exists x_1 \exists x_2 \forall x_3 (P(x_2, x_3) \wedge \neg Q(x_1))}{\text{OR: } \exists x_1 \forall x_3 \exists x_2 (P(x_2, x_3) \wedge \neg Q(x_1))} x_1 \sim a; x_1 < x_3}$$

invariant if Σ and Π swapped:

$$\frac{\perp \quad \frac{\top \quad \perp}{\neg P(y, f(x_1)) \vee \forall x_1 Q(x_1)} x_1 \sim a}{\frac{\forall x_1 \forall x_2 \exists x_3 (\neg P(x_2, x_3) \vee Q(x_1))}{\text{OR: } \forall x_1 \exists x_3 \forall x_2 (\neg P(x_2, x_3) \vee Q(x_1))} x_2 \sim a, x_3 \sim f(a); x_1 < x_3}$$

SECOND ATTEMPT:

$$\frac{\frac{\Sigma}{P(a)} \quad \frac{\frac{\Sigma}{Q(z)} \quad \frac{\frac{\Sigma}{\neg S(a)} \quad \frac{\Pi}{\neg P(y) \vee \neg Q(f(\mathbf{x})) \vee S(\mathbf{x})} x \mapsto a}{\neg P(y) \vee \neg Q(f(a))} z \mapsto f(a)}{\neg P(y)} y \mapsto a \quad \square$$

$$\frac{\perp \quad \frac{\perp \quad \top}{\neg S(a)} x \mapsto a}{\frac{\perp \quad \frac{\perp \quad \top}{\neg S(a) \wedge Q(f(a))} z \mapsto f(a)}{P(a) \wedge \neg S(a) \wedge Q(f(a))} y \mapsto a$$

Huang:

$$\frac{\frac{\perp}{\exists x_1 \neg S(x_1)} \quad \top}{\exists x_1 \forall x_2 (\neg S(x_1) \wedge Q(x_2))} \quad \perp \quad \frac{\perp}{\exists x_1 \forall x_2 (P(x_1) \wedge \neg S(x_1) \wedge Q(x_2))}$$

$$\forall x_1 \exists x_2 (\neg P(x_1) \vee S(x_1) \vee \neg Q(x_2))$$

similar fail

\Rightarrow anytime there is $P(a, f(a))$, either they have a dependency or they are not both differently colored (grey is uncolored)

for the record, direct method anyway:

$$\frac{\frac{\perp}{\exists x_1 \neg S(x_1)} \quad \top \quad x \sim a}{\exists x_1 \forall x_2 \neg S(x_1) \wedge Q(x_2)} \quad z \sim f(a); x_1 < x_2 \quad \frac{\perp}{\exists x_1 \forall x_2 \exists x_3 P(x_3) \wedge \neg S(x_1) \wedge Q(x_2)} \quad x_3 \sim a; x_3 \text{ need not be merged w } x_1$$

Example: ordering on both ancestors where the merge forces a new ordering

202a – canonical

$$\begin{array}{c}
 \frac{\frac{P(a, x_1) \vee R(y)}{R(y)} \quad \frac{\neg P(\textcolor{violet}{x}, f\textcolor{violet}{x})}{x_1 \mapsto fa} \quad \frac{Q(\textcolor{red}{x}_2, g\textcolor{red}{x}_2) \vee \neg R(u)}{\neg R(u)} \quad \frac{\frac{\neg S(a)}{\neg Q(f\textcolor{blue}{z}, x_3) \vee S(\textcolor{blue}{z})} \quad \frac{\neg Q(fa, x_3)}{x_2 \mapsto fa, x_3 \mapsto gfa}}{z \mapsto a}}{\square}
 \\
 \\
 \frac{\frac{\frac{\perp}{P(a, f(a))} \quad \frac{\top}{x_1 \mapsto f(a)}}{x \mapsto a} \quad \frac{\frac{\perp}{Q(f(a), g(f(a))) \wedge \neg S(a)} \quad \frac{\frac{\perp}{\neg S(a)} \quad \top}{z \mapsto a}}{x_2 \mapsto f(a), x_3 \mapsto g(f(a))}}{P(a, f(a)) \vee (Q(f(a), g(f(a))) \wedge \neg S(a))}
 \end{array}$$

Huang

$$\frac{\frac{\perp}{\exists x_1 \forall x_2 P(x_1, x_2)} \quad \frac{\top}{\exists x_1 \forall x_2 \exists x_3 Q(x_2, x_3) \wedge \neg S(x_1)}}{\exists x_1 \forall x_2 \exists x_3 P(x_1, x_2) \vee (Q(x_2, x_3) \wedge \neg S(x_1))}$$

direct:

$$\frac{\frac{\perp}{\exists x_1 \forall x_2 P(x_1, x_2)} \quad \frac{\top}{x_1 \sim a, x_2 \sim fa} \quad \frac{\top}{x_3 \sim a, x_4 \sim fa, x_5 \sim gfa)}{\frac{\frac{\frac{\perp}{\exists x_1 \forall x_2 \exists x_5 P(x_1, x_2) \vee (Q(x_2, x_5) \wedge \neg S(x_5))} \quad \frac{\frac{\perp}{\exists x_3 \forall x_4 \exists x_5 Q(x_4, x_3) \wedge \neg S(x_3)} \quad \top}{x_3 \sim a}}{x_3 \mapsto x_1, x_4 \mapsto x_2, x_1 < x_2, x_2 < x_5}}$$

without merge in end: $(x_1 < x_2, x_3 < x_4, x_4 < x_5)$

$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5 P(x_1, x_2) \vee (Q(x_4, x_5) \wedge \neg S(x_3))$

$\exists x_3 \forall x_4 \exists x_5 \exists x_1 \forall x_2 P(x_1, x_2) \vee (Q(x_4, x_5) \wedge \neg S(x_3))$

(also interwoven ones appear to work)

202b – just a a lot of terms for random mass test

TODO

203a

$$\frac{\frac{\frac{\perp}{\neg P(f(x))} z \mapsto f(x)}{\top \frac{\neg Q(g(f(x))) \wedge \neg P(f(x))}{x \mapsto a}} \frac{\perp}{y \mapsto g(f(x))}}{\top \frac{\neg Q(g(f(a))) \wedge \neg P(f(a)) \vee R(a)}{x_1 \mapsto h(g(f(a)))}} \frac{\neg Q(g(f(a))) \wedge \neg P(f(a)) \vee R(a)}{\neg Q(g(f(a))) \wedge \neg P(f(a)) \vee R(a) \vee S(h(g(f(a))))}$$

$$\frac{\frac{\frac{\perp}{\exists x_1 \neg P(x_1)}}{\exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(x_1))} \quad \perp}{\frac{\top}{\forall x_0 \exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0))}} \quad \top$$
$$\frac{\frac{\frac{\perp}{\exists x_1 \neg P(x_1)} \quad \top}{x_1 \sim f(x)} \quad \perp}{\frac{\top}{\exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(f(x)))} \quad x_2 \sim g(f(x)); x_1 < x_2} \quad \frac{\top}{\forall x_0 \exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0))} \quad x_0 \sim a; x_0 < x_1, x_0 < x_2 \\ \frac{\top}{\forall x_0 \exists x_1 \forall x_2 \forall x_3 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0) \vee S(x_3))} \quad x_3 \sim h(g(f(a))); x_0 < x_3, x_1 < x_3, x_2 < x_3$$

Example where variables are not the outermost symbol but order is still relevant

204a

$$\Sigma = \{P(f(x), g(f(x)))\}$$

$$\Pi = \{P(f(a), y)\}$$

$$\Rightarrow \forall x_1 \exists x_2 P(f(x_1), x_2)$$

misc examples

201a

$$\begin{array}{c}
 \frac{\frac{P(x,y) \vee \neg Q(y)}{\neg Q(y)} \quad \frac{\neg P(a,y_2)}{x \mapsto a} \quad \frac{\frac{Q(f(z)) \vee R(z)}{Q(f(a))} \quad \frac{\neg R(a)}{z \mapsto a}}{y \mapsto f(a)} \\
 \hline
 \square
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\perp}{P(a,y)} \quad \frac{\top}{x \mapsto a} \quad \frac{\perp}{R(a)} \quad \frac{\top}{z \mapsto a}}{P(a, f(a)) \vee R(a)} \quad y \mapsto f(a)
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\perp}{\forall x_1 P(x_1, y)} \quad \frac{\top}{x \mapsto a} \quad \frac{\perp}{\forall x_3 R(x_3)} \quad \frac{\top}{z \mapsto a}}{\forall x_3 \forall x_1 \exists x_2 (P(x_1, x_2) \vee R(x_3))} \quad y \mapsto f(a)
 \end{array}$$

Huang would produce: $\forall x_1 \exists x_2 P(x_1, x_2) \vee R(x_1)$

201b

$$\begin{array}{c}
 \frac{\frac{P(x, f(y)) \vee \neg Q(f(y))}{\neg Q(f(y))} \quad \frac{\neg P(a, y_2)}{x \mapsto a} \quad \frac{\frac{Q(f(z)) \vee R(z)}{Q(f(a))} \quad \frac{\neg R(a)}{z \mapsto a}}{y \mapsto f(a)} \\
 \hline
 \square
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\perp}{P(a, f(y))} \quad \frac{\top}{x \mapsto a} \quad \frac{\perp}{R(a)} \quad \frac{\top}{z \mapsto a}}{P(a, f(a)) \vee R(a)} \quad y \mapsto a
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\perp}{\forall x_1 \exists x_2 P(x_1, x_2)} \quad \frac{\top}{x \mapsto a} \quad \frac{\perp}{\forall x_3 R(x_3)} \quad \frac{\top}{z \mapsto a}}{\forall x_3 \forall x_1 \exists x_2 P(x_1, x_2) \vee R(x_3)} \quad y \mapsto f(a)
 \end{array}$$

Huang would produce: $\forall x_1 \exists x_2 P(x_1, x_2) \vee R(x_1)$