

Interpolation in First-Order Logic with Equality

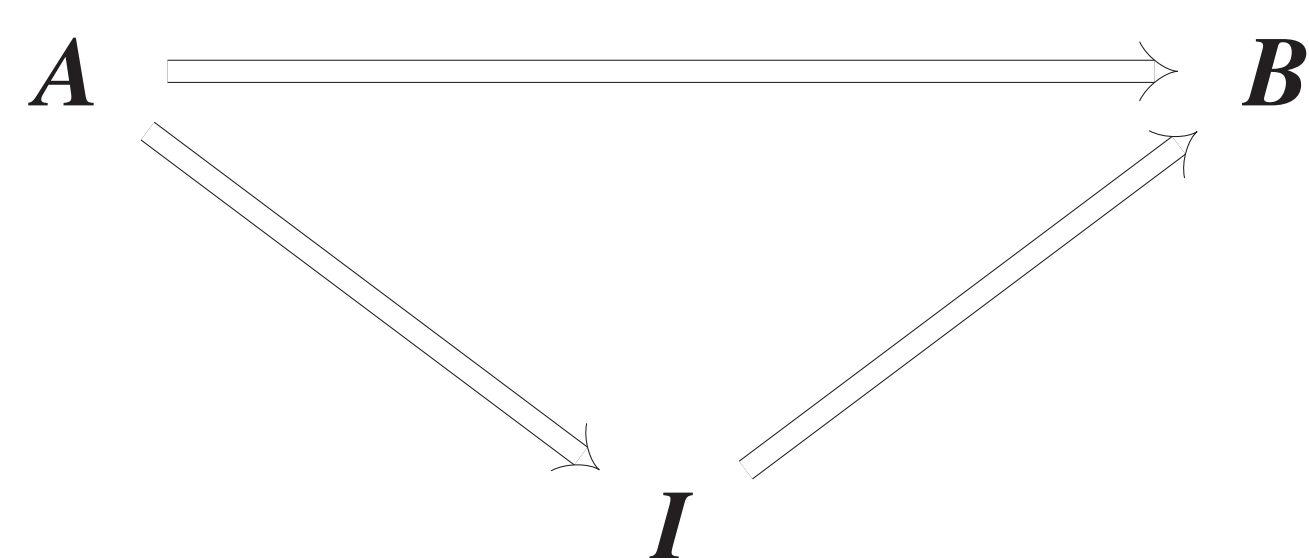
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Interpolation

- ▶ Given two formulas A and B such that A implies B , an interpolant I is a formula which is implied by A and which itself implies B .



- ▶ Additionally, interpolants can only contain symbols which are common to both A and B .
- ▶ Hence interpolants succinctly capture the logical content which explains an implication.

Theorem (Craig). *Let A and B be first-order formulas such that A implies B . Then there is an interpolant for A and B .*

Aim and Scope

Give comprehensive account of existing proofs and techniques and extend them:

- ▶ Reduction to first-order logic without equality
- ▶ Interpolant extraction from resolution proofs
- ▶ Model-theoretic proof

Reduction to first-order logic without equality

This is the approach used by Craig for initial proof.

- ▶ Express equality and function symbols by means of fresh predicates with appropriate axioms
- ▶ Compute interpolants in first-order logic without equality and function symbols, for instance using Maehara's Lemma.

Interpolant extraction from resolution proofs

This constructive proof by Huang consists of two phases:

- ▶ From a resolution proof inductively construct a propositional interpolant, which may still contain non-common terms.
- ▶ Replace non-common terms by variables and bind them in a quantifier prefix.

Contributions:

- ▶ We showed that the number of quantifier alternations in the interpolant corresponds directly to the number of nested alternations of symbols which only occur in A or B respectively.
- ▶ We developed an improved version which combines these phases and produces non-prenex formulas.

Model-theoretic proof

The interpolation theorem can also be proven semantically:

- ▶ Suppose that there is no interpolant.
- ▶ Then we can build a model in which A holds, but B does not.

⇒ If there is no formula which explains the logical relation between A and B (=interpolant), then this is possible.

TODO: applications of interpolation? (wie in introduction / abstract vom märz)

References

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