undirected edges (from $\mathcal{M}$) are to be interpreted as two directed edges.
$E(C) = \mathcal{A}(C) \cup \mathcal{M}(C)$
$V(C) = V(E(C))$
$G(C) = (V(C), E(C))$
color of component is color of some term in it (all the same)
per resolution step: oppositely colored components are not unifiable

Components

nodes: max col term occurrences and variables in grey occurrences.

1. components initially: for every variable, all grey occurrences and all colored occurrences

2. resolution: components of C_1 and C_2 are carried over, some are merged.

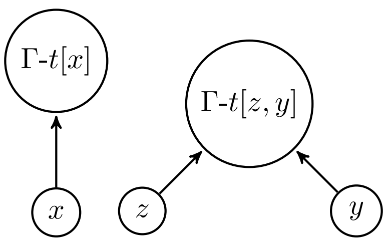
   $l\sigma = l'\sigma$

   For each max col term t in $l\sigma$: merge component of t and t'.

quantifier ordering: Build $\mathcal{A}(C)$, which is the condensation of $G(C)$. If in the condensation there is a path from a node containing a term containing u_i to a node containing term containing u_j, then $u_i <_{\mathcal{A}(C)} u_j$.

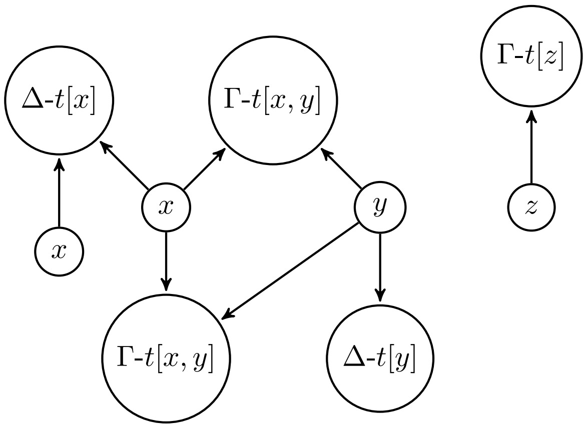
graph components visualised

initially



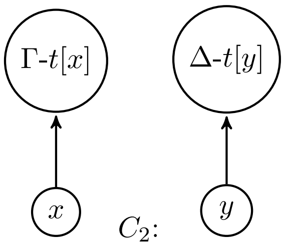
Note: initially, all colored terms are in one component

in the derivation, single color



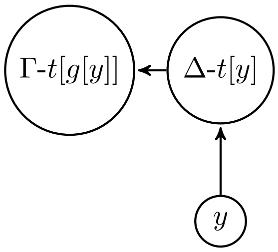
Note: Γ- and Δ-terms can not be merged (unified). All other combinations are possible.

in the derivation



C_1: C_2:

C with x unified with a Δ-term, say $g[y]$:



proofs

Conjecture 1 (Lemma 1)**.** *If a variable has a grey and a colored occurrence, there is an arrow between the component containing the grey occurrence and the component containing the colored occurrence (in the condensation graph).*

Proof. Induction start is easy.
Induction step: Suppose the statement holds C_1 and C_2.

Note that no new grey components can be added, just merged. Hence it suffices to show that the component of an arbitrary in C newly added colored occurrence of a variable, say x, has an arrow starting from a grey component.
No component is ever added. x is not unified as otherwise it would not exist anymore (the lemma statment requires the variable to occur). A new colored occurrence of x can be created by either putting x into a colored term or by a colored occurrence of x in the codomain of the unifier.

1. Putting x into a colored term. Then there is some $\gamma[y]$ with $y\sigma = t[x]$. In the easy case, y is just unified with $t[x]$. Let $\hat{y}$ be the occurrence of y in the resolved literal which causes a change of y in the unification algorithm and $\hat{t}[x]$ the corresponding term at the same position in the other resolved literal.

   “induction” until we hit a term containing x

   Then the component of $\hat{y}$ is merged with the component of $\hat{t}[x]$.

   Afterwards, we have some other component of x as well. This could be:

   a) in the same clause as $\hat{t}[x]$.

      Then distinguish on the the shape of $\hat{t}[x]$:

      - Either it is grey, then $\hat{y}$ is grey as well and we have an arrow by the induction hypothesis from $\hat{y}$ to $\gamma[y]$.
      - Otherwise it is colored. Then by the induction hypothesis, as there exists a grey component of x in this clause, there is an arrow to $\hat{t}[x]$. By some Lemma yet to define, there either is a merge arrow between $\gamma[y]$ and $\hat{y}$, which is also a colored term, or there is a grey occurrence of y with arrows to the two colored occurrences. in the first case, we are done, and in the second ???

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?? as some var which is unified to x ???

b) in the same clause as $\hat{y}$ in the form of a component which is called y in C_i for some i. By the subst, it's now x. Then we have an arrow by the induction hypothesis from the component of y to the component of $\gamma[y]$.

TODO: what if y is substituted by a colored term containing x?

$\square$

Conjecture 2 (Conjecture 4)**.** *Suppose in* $\mathrm{AI}^{\Delta}(C)$ *a maximal* Γ*-term* $\gamma_j[z_i]$ *contains a lifting variable* z_i*. Then* $z_i <_{\mathcal{A}(C)} z_j$*.*

random notes

1. if two variable-nodes in the condensation are connected when disregarding the arrow direction, they occur in the same clause.