CHAPTER 1

Number of quantifier alternations in Huang and nested

1.1 Outline

Goal: try proof Huang and nested equal.

Method: proof for both:

Conjectured Proposition 1. *Let I be an interpolant created by $algorithm. If I contains a term t such that t has n color changes, then I has at least n quantifier alternations.*

1.2 Preliminaries

Quantifier alternations in I usually assumes the quantifier-alternation-minimising arrangement of quantifiers in I

Definition 2 (Color alternation col-alt)**.** Colors Γ and Δ, term t:

$\text{col-alt}(t) \stackrel{\text{def}}{=} \text{col-alt}_{\perp}(t)$

Let $t = f(t_1, \dots, t_n)$ for constant, function and variable symbols (syntax abuse)

$$\text{col-alt}_{\Phi}(t) \stackrel{\text{def}}{=} \begin{cases} \max(\text{col-alt}_{\Phi}(t_1), \dots, \text{col-alt}_{\Phi}(t_n)) & f \text{ is grey} \\ \max(\text{col-alt}_{\Phi}(t_1), \dots, \text{col-alt}_{\Phi}(t_n)) & f \text{ is of color } \Phi \\ 1 + \max(\text{col-alt}_{\Psi}(t_1), \dots, \text{col-alt}_{\Psi}(t_n)) & f \text{ is of color } \Psi,\ \Phi \neq \Psi \end{cases}$$

△

1.3 Random thoughts

- Quantifiers are introduced for lifting variables which actually occur in the interpolant
- If term t with $\text{col-alt}(t) = n$ enters I, we need subterm s of t with $\text{col-alt}(s) = n - 1$ to be in I (of course colors of t and s are exactly opposite)

1.3.1 Proof

- Induction over $\ell^x_\Delta[\mathrm{PI}(C) \vee C]$ and also about Γ-terms with Δ-lifting vars in that formula. Cf. `-final`

1.4 Proof port attempt from `-final`

Conjectured Lemma 3. *Resolution or factorisation step ι from $\bar{C}$.*
If x col-change var (where?), then x also occurs grey (where?).