

Conjectured Lemma 1. *Let a variable x occur twice in C such that in one occ, the smallest colored term containing x is a Γ -term and for the other, the smallest colored term containing x is a Δ -term. Then x occurs grey in $\text{AI}_*(C)$.*

Proof.

- Suppose that in C_i , $\gamma[x]$ occurs and in C_j , we have $\delta[y]$ such that x occurs grey in $y\sigma$.
Then y occurs in l at $l|_{\hat{y}}$ such that $l'|_{\hat{y}}$ is an abstraction of a term containing a grey occurrence of x .

Suppose that $l|_{\hat{y}}$ (and therefore also $l'|_{\hat{y}}$) is not a grey occurrence as otherwise we are done.

As $l\sigma l'\sigma$, $l|_{\hat{y}}$ and $l'|_{\hat{y}}$ share their prefix, so their color is the same.

Then induction hypothesis.

- Suppose that in C_i , $\gamma[z]$ occurs and in C_j , $\delta[y]$ occurs such that x occurs grey in $y\sigma$ and in $z\sigma$.

By Lemma ??, exists y_1, \dots, y_n and z_1, \dots, z_m such that x occurs grey in $y_i\sigma$ and in $z_i\sigma$ and term opposite of y_n and z_m actually contains x .

If any y_i, z_j occurs grey, done, so assume all occur colored.

z_m and y_n opposite of actual x , as x only in one clause, z_m and y_n in same clause. they do share prefix with the occurrences of x in the clause where x is.

if they there are contained in smallest col terms of opposite color \Rightarrow ind hyp

otw of same smallest term color there.

Note that every y_i, z_j occurs at least twice: once as opposite var of the last one, once to unify with the next one.

as originally different colors and at meeting point at x same color, there has to be one alternation, where we use the ind hyp.

- Suppose that $\gamma[x]$ in C_i and $\delta[x]$ in $z\sigma$ such that z occurs grey in C_j .

If $\delta[x]$ occurs in C_i (cannot occur in other clause), ind hyp.

Suppose it does not occur. Then however exists $\delta[y]$ s.t. x occurs grey in $y\sigma \Rightarrow$ other case.

- Suppose that $\gamma[x]$ in $y\sigma$ such that y occurs grey in C_i and $\delta[x]$ in $z\sigma$ such that z occurs grey in C_j .

If $\gamma[x]$ and $\delta[x]$ occur, ind hyp.

If just one occurs, \Rightarrow other case.

If none of them occur, then occur $\delta[\alpha]$ s.t. x grey in $\alpha\sigma$ and similar for $\gamma[\beta] \Rightarrow$ other case.

□