Example 101: canonical examples for overbinding w.r.t. order

Here, the second lower prooftree shows what huang would produce.

Ex 101a

$$\dfrac{\dfrac{\overset{\Sigma}{P({\color{red}u},f({\color{red}u}))\lor Q({\color{red}u})} \qquad \overset{\Pi}{\neg Q(a)}}{P(a,f(a))}\; u\mapsto a \qquad \overset{\Pi}{\neg P(x,y)}}{\square}\; x\mapsto a, y\mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{Q(a)}\; u\mapsto a \qquad \top}{P(a,f(a))\lor Q(a)}\; x\mapsto a, y\mapsto f(a) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1 Q(x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1,x_2)\lor Q(x_1))}$$

Direct overbinding would not work without merging same variables!: $\forall x_1 \forall x_2 \exists x_3 (P(x_2,x_3)\lor Q(x_1))$
counterexample: $Q\sim\{0\}, P\sim\{(1,0)\}$
Direct overbinding would work when considering original dependencies as highlighted above

Ex 101b – other resolution order

$$\dfrac{\dfrac{\overset{\Sigma}{P(u,f(u))\lor Q(u)} \qquad \overset{\Pi}{\neg P(x,y)}}{Q(u))}\; y\mapsto f(u), x\mapsto u \qquad \overset{\Pi}{\neg Q(a)}}{\square}\; u\mapsto a$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(u,f(u))}\; x\mapsto f(u), x\mapsto u \qquad \top}{P(a,f(a))\lor Q(a)}\; u\mapsto a \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(u,x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1,x_2)\lor Q(x_1))}\; u\mapsto a$$

Ex 101c – Π and Σ swapped

$$\dfrac{\dfrac{\overset{\Pi}{P(u,f(u))\lor Q(u)} \qquad \overset{\Sigma}{\neg P(x,y)}}{Q(u))}\; y\mapsto f(u), x\mapsto u \qquad \overset{\Sigma}{\neg Q(a)}}{\square}\; u\mapsto a$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg P(u,f(u))}\; x\mapsto f(u), x\mapsto u \qquad \bot}{\neg P(a,f(a))\land \neg Q(a)}\; u\mapsto a \qquad\qquad \dfrac{\dfrac{\top \qquad \bot}{\forall x_2 \neg P(u,x_2)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1,x_2)\land \neg Q(x_1))}$$

Ex 101d – Π and Σ swapped, other resolution order

$$\dfrac{\dfrac{\overset{\Pi}{P(u,f(u))\lor Q(u)} \qquad \overset{\Sigma}{\neg Q(a)}}{P(a,f(a))}\; u\mapsto a \qquad \overset{\Sigma}{\neg P(x,y)}}{\square}\; x\mapsto a, y\mapsto f(a)$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg Q(a)}\; y \mapsto a \qquad \bot}{\neg Q(a) \land \neg P(a, f(a))}\; x \mapsto a, y \mapsto f(a) \qquad\qquad \dfrac{\dfrac{\top \qquad \bot}{\exists x_1 \neg Q(x_1)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1, x_2) \land \neg Q(x_1))}$$

102 – similar to 101 but with intra-clause-set inferences

Ex 102a

$$\dfrac{\dfrac{P(f(x)) \overset{\Sigma}{\lor} Q(f(x), z) \qquad \overset{\Pi}{\neg P(y)}}{Q(f(x), z)} \qquad \dfrac{\neg Q(x_1, \overset{\Sigma}{y}) \lor R(y) \qquad \overset{\Pi}{\neg R(g(z_1))}}{\neg Q(x_1, g(z_1))}\; y \mapsto g(z_1), x_1 \mapsto f(x)}{\square}\; x_1 \mapsto f(x), z \mapsto g(z_1)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(f(x))} \qquad \dfrac{\bot \qquad \top}{R(g(z_1))}\; y \mapsto g(z_1), x_1 \mapsto f(x)}{P(f(x)) \lor R(g(z_1))}\; x_1 \mapsto f(x), z \mapsto g(z_1) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(x_1)} \qquad \dfrac{\bot \qquad \top}{\forall x_2 R(x_2)}}{\exists x_1 \forall x_2 (P(x_1) \lor R(x_2)) \quad \text{(order irrelevant!)}}$$

Ex 102b

$$\dfrac{\dfrac{P(f(x)) \overset{\Sigma}{\lor} Q(f(x), z) \qquad \overset{\Pi}{\neg P(y)}}{Q(f(x), z)} \qquad \dfrac{\neg Q(f(\overset{\Sigma}{y}), z_1) \lor R(y) \qquad \overset{\Pi}{\neg R(a)}}{\neg Q(f(a), z_1)}\; y \mapsto a}{\square}\; x \mapsto a, z \mapsto z_1$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(f(x))} \qquad \dfrac{\bot \qquad \top}{R(a)}\; y \mapsto a}{P(f(a)) \lor R(a)}\; x \mapsto a, z \mapsto z_1 \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(x_1)} \qquad \dfrac{\bot \qquad \top}{\forall x_2 R(x_2)}\; y \mapsto a}{\forall x_2 \exists x_1 (P(x_1) \lor R(x_2))}\; x \mapsto a, z \mapsto z_1$$

direct:

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(x_1)}\; x_1 \sim f(x) \qquad \dfrac{\bot \qquad \top}{\forall x_2 R(x_2)}\; x_2 \sim a}{\exists x_1 \forall x_2 (P(x_1) \lor R(x_2))}\; \text{order irrelevant!}$$

Ex 102b' with Q grey

$$\dfrac{\dfrac{\overset{\Sigma}{P(f(x)) \vee Q(f(x), z)} \qquad \overset{\Pi}{\neg P(y)}}{Q(f(x), z)} \qquad \dfrac{\overset{\Sigma}{\neg Q(f(y), z_1) \vee R(y)} \qquad \overset{\Pi}{\neg R(a)}}{\neg Q(f(a), z_1)}\; y \mapsto a}{\square}\; x \mapsto a, z_1 \mapsto z$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(f(x))} \qquad \dfrac{\bot \qquad \top}{R(a)}\; y \mapsto a}{(\neg Q(f(a), z) \wedge P(f(a))) \vee (Q(f(a), z) \wedge R(a))}\; x \mapsto a, z_1 \mapsto z$$

Huang:

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(x_1)} \qquad \dfrac{\bot \qquad \top}{\forall x_2 R(x_2)}\; y \mapsto a}{\forall x_1 \exists x_2 (\neg Q(x_2, z) \wedge P(x_2)) \vee (Q(x_2, z) \wedge R(x_1))}\; x \mapsto a, z_1 \mapsto z$$

direct:

$$\cfrac{\cfrac{\cfrac{\bot \qquad \top}{\exists x_1 P(x_1)}\; x_1 \sim f(x) \qquad \cfrac{\bot \qquad \top}{\forall x_2 R(x_2)}\; x_2 \sim a}{\forall x_1 \exists x_2 \exists x_3 (\neg Q(x_3, z) \wedge P(x_2)) \vee (Q(x_3, z) \wedge R(x_1))}\; x_3 \sim f(a); x_1 \parallel x_3, x_2 < x_3}{\text{OR: } \exists x_2 \forall x_1 \exists x_3 (\neg Q(x_3, z) \wedge P(x_2)) \vee (Q(x_3, z) \wedge R(x_1))}}{\text{OR: } \exists x_2 \exists x_3 \forall x_1 (\neg Q(x_3, z) \wedge P(x_2)) \vee (Q(x_3, z) \wedge R(x_1))}$$

Huang-Version passt weil man für existenzielle quantoren eh gleichen term einsetzt

TODO: algo-formulierung hier überprüfen

Example 103: variables in interpolant get unified and order might have to be changed

103a: length changes but old version is still valid, despite huang's algorithm doing a sort

$$\dfrac{\dfrac{\dfrac{\overset{\Sigma}{Q(f({\color{red}x})) \lor P(y) \lor R({\color{red}x})} \qquad \overset{\Pi}{\neg Q(y_1)}}{P(y) \lor R(x)}\; y_1 \mapsto f(x) \qquad \overset{\Pi}{\neg P(h(g(a)))}}{R(x)}\; y \mapsto h(g(a)) \qquad \overset{\Pi}{\neg R(g(g(a)))}}{\square}\; x \mapsto g(g(a))$$

$$\dfrac{\dfrac{\dfrac{\bot \qquad \top}{Q(f(x))}\; y_1 \mapsto f(x) \qquad \top}{Q(f(x)) \lor P(h(g(a)))}\; y \mapsto h(g(a)) \qquad \top}{Q(f(g(g(a)))) \lor P(h(g(a))) \lor R(g(g(a)))}\; x \mapsto g(g(a)) \qquad\qquad \dfrac{\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 Q(x_1)} \qquad \top}{\exists x_1 \forall x_2 (Q(x_1) \lor P(x_2))} \qquad \top}{X}$$

X:
Huang's algo gives:
$\forall x_2 \forall x_3 \exists x_1 (Q(x_1) \lor P(x_2) \lor R(x_3))$
Direct overbinding gives: $x_3 < x_1$, rest arbitrary, hence:
$\forall x_3 \exists x_1 \forall x_2 (Q(x_1) \lor P(x_2) \lor R(x_3))$ <- this you do not get with huang
$\forall x_2 \forall x_3 \exists x_1 (Q(x_1) \lor P(x_2) \lor R(x_3))$
$\forall x_3 \forall x_2 \exists x_1 (Q(x_1) \lor P(x_2) \lor R(x_3))$

103b: length changes "uniformly"

$$\dfrac{\dfrac{\dfrac{\overset{\Sigma}{Q(f(f({\color{red}x}))) \lor P(f({\color{red}x})) \lor R({\color{red}x})} \qquad \overset{\Pi}{\neg Q(y_1)}}{P(f(x)) \lor R(x)}\; y_1 \mapsto f(f(x)) \qquad \overset{\Pi}{\neg P(y_2)}}{R(x)}\; y_2 \mapsto f(x) \qquad \overset{\Pi}{\neg R(g(a))}}{\square}\; x \mapsto g(a)$$

$$\dfrac{\dfrac{\dfrac{\bot \qquad \top}{Q(f(f(x)))}\; y_1 \mapsto f(f(x)) \qquad \top}{Q(f(f(x))) \lor P(f(x))}\; y_2 \mapsto f(x) \qquad \top}{Q(f(f(g(a)))) \lor P(f(g(a))) \lor R(g(a))}\; x \mapsto g(a) \qquad \dfrac{\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 Q(x_1)} \qquad \top}{\exists x_2 \exists x_1 (Q(x_1) \lor P(x_2))} \qquad \top}{\forall x_3 \exists x_2 \exists x_1 (Q(x_1) \lor P(x_2) \lor R(x_3))}$$

Huang and direct overbinding somewhat coincide as $x_2 < x_1$ in both cases, and $x_3 < x_1$ and $x_3 < x_2$

103c: different variables, accidentally the same terms appear but no logical connection

$$\dfrac{\overset{\Sigma}{P(a,x)} \qquad \dfrac{\overset{\Sigma}{\neg Q(a)} \qquad \overset{\Pi}{\neg P(y,f(z)) \lor Q(z)}}{\neg P(y,f(a))}\; z \mapsto a}{\square}\; y \mapsto a, x \mapsto f(a)$$

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg Q(a)}\; z \mapsto a}{P(a,f(a)) \land \neg Q(a)}\; y \mapsto a, x \mapsto f(a) \qquad\qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg Q(x_1)}}{\exists x_1 \forall x_2 (P(x_1,x_2) \land \neg Q(x_1))}$$

Error:

no sorting requirement is just for Σ Again, Huang sorts, but no order is required.

SECOND ATTEMPT:

$$\dfrac{\overset{\Sigma}{P(a)} \qquad \dfrac{\overset{\Sigma}{Q(z)} \qquad \dfrac{\overset{\Sigma}{\neg S(a)} \qquad \overset{\Pi}{\neg P(y) \lor \neg Q(f(x)) \lor S(x)}}{\neg P(y) \lor \neg Q(f(a))}\; x \mapsto a}{\neg P(y)}\; z \mapsto f(a)}{\square}\; y \mapsto a$$

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg S(a)}\; x \mapsto a}{\neg S(a) \land Q(f(a))}\; z \mapsto f(a)}{P(a) \land \neg S(a) \land Q(f(a))}\; y \mapsto a$$

Huang:

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg S(x_1)}}{\exists x_1 \forall x_2 (\neg S(x_1) \land Q(x_2))}}{\exists x_1 \forall x_2 (P(x_1) \land \neg S(x_1) \land Q(x_2))}$$

$\forall x_1 \exists x_2 (\neg P(x_1) \lor S(x_1) \lor \neg Q(x_2))$

similar fail

$\Rightarrow$ anytime there is $P(a,f(a))$, either they have a dependency or they are not both differently colored (grey is uncolored)

for the record, direct method anyway:

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg S(x_1)}\; x \sim a}{\exists x_1 \forall x_2 \neg S(x_1) \land Q(x_2)}\; z \sim f(a); x_1 < x_2}{\exists x_1 \forall x_2 \exists x_3 P(x_3) \land \neg S(x_1) \land Q(x_2)}\; x_3 \sim a; x_3 \text{ need not be merged w } x_1$$

Example: ordering on both ancestors where the merge forces a new ordering

202a – canonical

$$\dfrac{\dfrac{P(a,\overset{\Sigma}{x_1)}) \lor R(y) \qquad \neg P(\overset{\Pi}{x}, fx)}{R(y)}\begin{array}{c}x_1\mapsto fa\\ x\mapsto a\end{array} \qquad \dfrac{Q(x_2, g\overset{\Sigma}{x_2})) \lor \neg R(u) \qquad \dfrac{\overset{\Sigma}{\neg S(a)} \qquad \overset{\Pi}{\neg Q(fz,x_3)\lor S(z)}}{\neg Q(fa,x_3)}\, z\mapsto a}{\neg R(u)}\begin{array}{l}x_2\mapsto fa,\\ x_3\mapsto gfa\end{array}}{\square}$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a,f(a))}\begin{array}{c}x_1\mapsto f(a)\\ x\mapsto a\end{array} \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg S(a)}\, z\mapsto a}{Q(f(a),g(f(a)))\land \neg S(a)}\begin{array}{l}x_2\mapsto f(a),\\ x_3\mapsto g(f(a))\end{array}}{P(a,f(a))\lor(Q(f(a),g(f(a)))\land\neg S(a))}$$

Huang

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_1\forall x_2 P(x_1,x_2))} \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1\neg S(x_1)}}{\exists x_1\forall x_2\exists x_3 Q(x_2,x_3)\land\neg S(x_1)}}{\exists x_1\forall x_2\exists x_3 P(x_1,x_2)\lor(Q(x_2,x_3))\land\neg S(x_1))}$$

direct:

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_1\forall x_2 P(x_1,x_2))}\begin{array}{c}x_1\sim a, x_2\sim fa\\ x_1<x_2\end{array} \qquad \begin{array}{c}x_3\sim a, x_4\sim fa, x_5\sim gfa)\\ x_3<x_4, x_4<x_5\end{array} \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_3\neg S(x_3)}\, x_3\sim a}{\exists x_3\forall x_4\exists x_5 Q(x_4,x_3)\land\neg S(x_3)}}{\exists x_1\forall x_2\exists x_5 P(x_1,x_2)\lor(Q(x_2,x_5)\land\neg S(x_5))}\begin{array}{l}x_3\mapsto x_1, x_4\mapsto x_2\\ x_1<x_2, x_2<x_5\end{array}$$

without merge in end: $(x_1<x_2, x_3<x_4, x_4<x_5)$
$\exists x_1\forall x_2\exists x_3\forall x_4\exists x_5 P(x_1,x_2)\lor(Q(x_4,x_5)\land\neg S(x_3))$
$\exists x_3\forall x_4\exists x_5\exists x_1\forall x_2 P(x_1,x_2)\lor(Q(x_4,x_5)\land\neg S(x_3))$
(also interwoven ones appear to work)

202b – just a a lot of terms for random mass test

TODO

Example with transitive order constraint

203a

$$\dfrac{\overset{\Pi}{\neg S(x_1)} \qquad \dfrac{\overset{\Pi}{\neg R(a)} \qquad \dfrac{\dfrac{R(x) \lor \overset{\Sigma}{\neg} P(f(x)) \qquad P(z) \lor \overset{\Pi}{Q}(g(z))}{R(x) \lor Q(g(f(x)))}\; z \mapsto f(x) \qquad \neg Q(y) \overset{\Sigma}{\lor} S(h(y))}{R(x) \lor S(h(g(f(x))))}\; y \mapsto g(f(x))}{S(h(g(f(a))))}\; x \mapsto a}{\square}\; x_1 \mapsto h(g(f(a)))$$

$$\dfrac{\top \qquad \dfrac{\top \qquad \dfrac{\dfrac{\bot \qquad \top}{\neg P(f(x))}\; z \mapsto f(x) \qquad \bot}{\neg Q(g(f(x))) \land \neg P(f(x))}\; y \mapsto g(f(x))}{\neg Q(g(f(a))) \land \neg P(f(a)) \lor R(a)}\; x \mapsto a}{\neg Q(g(f(a))) \land \neg P(f(a)) \lor R(a) \lor S(h(g(f(a))))}\; x_1 \mapsto h(g(f(a)))$$

Huang:

$$\dfrac{\top \qquad \dfrac{\top \qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \neg P(x_1)} \qquad \bot}{\exists x_1 \forall x_2 \neg(Q(x_2) \land \neg P(x_1))}}{\forall x_0 \exists x_1 \forall x_2 (\neg Q(x_2) \land \neg P(x_1) \lor R(x_0))}}{\forall x_0 \exists x_1 \forall x_2 \exists x_3 (\neg Q(x_2) \land \neg P(x_1) \lor R(x_0) \lor S(x_3))}$$

Direct:

$$\dfrac{\top \qquad \dfrac{\top \qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \neg P(x_1)}\; x_1 \sim f(x) \qquad \bot}{\exists x_1 \forall x_2 \neg Q(x_2)) \land \neg P(f(x))}\; x_2 \sim g(f(x)); x_1 < x_2}{\forall x_0 \exists x_1 \forall x_2 (\neg Q(x_2) \land \neg P(x_1) \lor R(x_0))}\; x_0 \sim a; x_0 < x_1, x_0 < x_2}{\forall x_0 \exists x_1 \forall x_2 \exists x_3 (\neg Q(x_2) \land \neg P(x_1) \lor R(x_0) \lor S(x_3))}\; x_3 \sim h(g(f(a))); x_0 < x_3, x_1 < x_3, x_2 < x_3$$

misc examples

201a

$$\dfrac{\dfrac{P(x,y)\overset{\Sigma}{\vee}\neg Q(y) \qquad \neg\overset{\Pi}{P}(a,y_2)}{\neg Q(y)}\, x\mapsto a \qquad \dfrac{Q(f(z))\overset{\Sigma}{\vee} R(z) \qquad \neg\overset{\Pi}{R}(a)}{Q(f(a))}\, z\mapsto a}{\Box}\, y\mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a,y)}\, x\mapsto a \qquad \dfrac{\bot \qquad \top}{R(a)}\, z\mapsto a}{P(a,f(a))\vee R(a)}\, y\mapsto f(a) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1 P(x_1,y)}\, x\mapsto a \qquad \dfrac{\bot \qquad \top}{\forall x_3 R(x_3)}\, z\mapsto a}{\forall x_3\forall x_1\exists x_2(P(x_1,x_2)\vee R(x_3))}\, y\mapsto f(a)$$

Huang would produce: $\forall x_1\exists x_2 P(x_1,x_2)\vee R(x_1)$

201b

$$\dfrac{\dfrac{P(x,f(y))\overset{\Sigma}{\vee}\neg Q(f(y)) \qquad \neg\overset{\Pi}{P}(a,y_2)}{\neg Q(f(y))}\, x\mapsto a \qquad \dfrac{Q(f(z))\overset{\Sigma}{\vee} R(z) \qquad \neg\overset{\Pi}{R}(a)}{Q(f(a))}\, z\mapsto a}{\Box}\, y\mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a,f(y))}\, x\mapsto a \qquad \dfrac{\bot \qquad \top}{R(a)}\, z\mapsto a}{P(a,f(a))\vee R(a)}\, y\mapsto a \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1\exists x_2 P(x_1,x_2)}\, x\mapsto a \qquad \dfrac{\bot \qquad \top}{\forall x_3 R(x_3)}\, z\mapsto a}{\forall x_3\forall x_1\exists x_2 P(x_1,x_2)\vee R(x_3)}\, y\mapsto f(a)$$

Huang would produce: $\forall x_1\exists x_2 P(x_1,x_2)\vee R(x_1)$