Contents

0.1 random notes

- As long as every pair of literal is variable disjoint, the quantifier ordering is arbitrary (proof idea: establish that some ordering works, then pull quantifier inwards and back outwards in arbitrary order).
- - lifted terms which contain variables are disjoint for different clauses, but ground lifted terms can be the same (which does not appear to be necessarily so!)
  - the resolved/factorised literal should be the same (else this kind of proof doesn't go through)
- $\forall x \exists y \varphi \Leftrightarrow \exists y \forall x$ does not hold for formula coding $f(0) = 1$, $f(1) = 0$: $(Z(y) \supset O(x)) \wedge (O(y) \supset Z(x)$, $\mathcal{U} = \{0, 1\}$, $Z/1$ and $O/1$ encode being 0 or 1 respectively.

TODO NOW: check if we can encode that counterexample in a resolution refutation, because that would be a counterexample to the conjecture that the quantifier order is arbitrary and try to learn something from success or failure respectively

⟨1⟩ **Lemma 1.** $\Gamma \models \mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$.

⟨2⟩ **Lemma 2.** $\Gamma \models \forall \bar{x}\, \exists \bar{y}\, (\mathrm{LI}(C) \vee \mathrm{LI}_{\mathrm{cl}}(C))$.

Proof. By 1, $\Gamma \models \mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$.
Hence $\Gamma \models \forall \bar{x}\, (\mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C))$.
and also $\Gamma \models \forall \bar{x}\, \exists \bar{y}\, \ell_{\Gamma}[\mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)]$.
by some lemma then $\Gamma \models \forall \bar{x}\, \exists \bar{y}\, (\mathrm{LI}(C) \vee \mathrm{LI}_{\mathrm{cl}}(C))$. ∎

but can't invert this idea:
Let $\hat{\Delta} = \Gamma$ and $\hat{\Gamma} = \Delta$.

Then with $\hat{\pi}$ and 2: $\hat{\Gamma} \models \forall \bar{\hat{x}} \exists \bar{\hat{y}} \,(\mathrm{LI}(\bar{\pi}))$

Hence (some lemma) $\Delta \models \forall \bar{y} \exists \bar{x} \,(\neg \mathrm{LI}(\pi))$.

Hence $\Delta \models \neg \exists \bar{y} \forall \bar{x} \,(\mathrm{LI}(\pi))$.

need some consistent ordering, so possibly just prove that all work, because we need to shuffle a lot anyway

example with same lifting var in two children of a connective:

601 – lifting vars interleaved so quantifier pull in/out trick doesn't work

$$\dfrac{\dfrac{P(f(x)) \overset{\Sigma}{\vee} S(f(x)) \qquad \neg P(z) \vee \overset{\Pi}{Q}(g(y)) \vee R(g(y))}{P(f(x)) \mid S(f(x)) \vee Q(g(y)) \vee R(g(y))} \qquad \overset{\Sigma}{\neg Q(z)}}{\neg Q(g(y)) \wedge P(f(x)) \mid S(f(x)) \vee R(g(y))}$$

$\Sigma \models \forall u \exists v \big((\neg Q(u_{g(y)}) \wedge P(v_{f(x)})) \;\vee\; S(v_{f(x)}) \vee R(u_{g(y)})\big)$
$\Rightarrow$ not interesting as R is not mentioned, so it collapses.

$\Pi \models \exists u \forall v \big((Q(u_{g(y)}) \vee \neg P(v_{f(x)})) \;\vee\; S(v_{f(x)}) \vee R(u_{g(y)})\big)$

$$\dfrac{\neg Q(g(y)) \wedge P(f(x)) \overset{(\text{cont})}{\mid} S(f(x)) \vee R(g(y)) \qquad \neg \overset{\Pi}{S}(x_7)}{S(f(x)) \vee (\neg Q(g(y)) \wedge P(f(x))) \mid R(g(y))}$$

$\Sigma \models \forall u \exists v \Big(S(v) \vee (\neg Q(u) \vee P(v)) \vee R(u)\Big)$

$\Pi \models \exists u \forall v \Big((\neg S(v_{f(x)}) \wedge (Q(u_{g(y)}) \vee \neg P(v_{f(x)}))) \;\vee\; R(u_{g(y)})\Big)$

Can't see much of interest, but can not apply quantifier pulling in and out trick

same again with direct overbinding:

$$\dfrac{\exists v(P(v) \overset{\Sigma}{\vee} S(v)) \qquad \forall u(\neg P(z) \vee \overset{\Pi}{Q}(u) \vee R(u))}{\overset{\exists v}{\forall u}\;(P(v) \mid S(v) \vee Q(u) \vee R(u))}$$

only Δ: $\forall u (P(f(x)) \mid S(f(x)) \vee Q(u) \vee R(u))$

602 – lifting vars interleaved so quantifier pull in/out trick doesn't work