lored_container⟩?

Conjectured Lemma 1. *Let a variable x occur twice in C such that in one occ, the smallest colored term containing x is a Γ-term and for the other, the smallest colored term containing x is a Δ-term. Then x occurs grey in $\mathrm{AI}_*(C)$.*

Proof. **missing: variables don't have to occur grey in $y\sigma$, e.g. in $\gamma[y]$, $y\sigma$ might be $f(x)$ with f Γ-colored.**

- Suppose that in C_i, $\gamma[x]$ occurs and in C_j, we have $\delta[y]$ such that x occurs grey in $y\sigma$.

  Then y occurs in l at $l|_{\hat{y}}$ such that $l'|_{\hat{y}}$ is an abstraction of a term containing a grey occurrence of x.

  Suppose that $l|_{\hat{y}}$ (and therefore also $l|_{\hat{y}}$) is not a grey occurrence as otherwise we are done.

  As $l\sigma l'\sigma$, $l|_{\hat{y}}$ and $l|_{\hat{y}}$ share their prefix, so their color is the same.

  Then induction hypothesis.

- Suppose that in C_i, $\gamma[z]$ occurs and in C_j, $\delta[y]$ occurs such that x occurs grey in $y\sigma$ and in $z\sigma$.

  By Lemma **??**, exists $y_1, \dots, y_n$ and $z_1, \dots z_m$ such that x occurs grey in $y_i\sigma$ and in $z_i\sigma$ and term opposite of y_n and z_m actually contains x.

  If any y_i, z_j occurs grey, done, so assume all occur colored.

  z_m and y_n opposite of actual x, as x only in one clause, z_m and y_n in same clause. they do share prefix with the occurrences of x in the clause where x is.

  if they there are contained in smallest col terms of opposite color $\Rightarrow$ ind hyp

  otw of same smallest term color there.

  Note that every y_i, z_j occurs at least twice: once as opposite var of the last one, once to unify with the next one.

  as originally different colors and at meeting point at x same color, there has to be one alternation, where we use the ind hyp.

- Suppose that $\gamma[x]$ in C_i and $\delta[x]$ in $z\sigma$ such that z occurs grey in C_j.

  If $\delta[x]$ occurs in C_i (cannot occur in other clause), ind hyp.

  Suppose it does not occur. Then however exists $\delta[y]$ s.t. x occurs grey in $y\sigma \Rightarrow$ other case.

- Suppose that $\gamma[x]$ in $y\sigma$ such that y occurs grey in C_i and $\delta[x]$ in $z\sigma$ such that z occurs grey in C_j.

  If $\gamma[x]$ and $\delta[x]$ occur, ind hyp.

  If just one occurs, $\Rightarrow$ other case.

  If none of them occur, then occur $\delta[\alpha]$ s.t. x grey in $\alpha\sigma$ and similar for $\gamma[\beta]$ $\Rightarrow$ other case.

$\square$

Conjectured Lemma 2. *Let σ unifier. exists unification order $\sigma = \sigma_1 \dots \sigma_n$ with $\sigma_i = \{x_i \mapsto r_i\}$ s.t. x_i does not occur in $\{r_i, r_{i+1}, \dots, r_n\}$.*

Proof. Suppose ordering does not exist, i.e. $l\sigma = l'\sigma$, but every x_i occurs in some r_j for j‘i. But then last variable does not occur later.. □

Lemma 3. *Let σ unifier.*
At any stage in the run of the unification algo, exists var x s.t. x does not occur in a function symbol in a difference pair.

Proof. Suppose no such var exists.
resolve all differences $x_i \sim r_i$ such that r_i does not contain a variable in a function symbol.
all variables, in particular the remaining x_i, occur in a function symbol in r_j for some j.
Iteratively resolve in some order: $x_i \mapsto r_i$, where every r_i contains at least one variable.
Hence as every x_i occurs in some r_j, the variable in r_i then occurs in r_j.
so after a step, for the remaining difference pairs, it is still the case that every variable occurs in some r_j.
We do not get an occurs check error as by assumptions, the term are unifiable.
when we get to the point where there is only one subst left, it has to be of the form $x_i \mapsto r_i[x_i]$, so we do get an occurs check error, which contradicts the assumptions that the terms are unifiable. □

TODO ICI: does this mean that there is a variable which does not have a variable at its RHS?

(all difference pairs have a variable at some side, let's call it LHR and the other one RHS)

possibly: do induction along this order: take subst which has no var to the right, then this one occurs in the term. next term then does not actually exists necessarily, so need to show some induction property.

evil examples:
$P(z, z, \delta), \neg P(f(x), f(y), y)$
$P(z, f(z), f(f(\delta))), \neg P(f(x), y, y)$