FAKULTÄT FÜR !NFORMATIK
Faculty of Informatics

Diplomarbeitspräsentation

Interpolation in First-Order Logic with Equality

Masterstudium:
Computational Intelligence

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Craig Interpolation

Theorem (Craig). *Let* A *and* B *be first-order formulas where*

- A *contains red and gray symbols and*
- B *contains blue and gray symbols*

such that:

- $\models A \supset B$

Then there is a interpolant I *containing only gray symbols such that:*

- $\models A \supset I$
- $\models I \supset B$

$$\begin{tikzcd} A \arrow[rr, Rightarrow] \arrow[dr, Rightarrow] & & B \\ & I \arrow[ur, Rightarrow] & \end{tikzcd}$$

$\Rightarrow$ Interpolants give a concise logical summary of the implication

Applications of Craig Interpolation

Theoretical:

- Proof of Beth's Definability Theorem

Practical:

- Program analysis: Detect loop invariants
- Model checking: Overapproximate set of reachable states

Aim and Scope of the Thesis

Provide overview of existing techniques and extend them:

- Model-theoretic proof
- Reduction to first-order logic without equality
- Interpolant extraction from resolution proofs

Model-theoretic proof

- Non-constructive proof by contradiction:
  - Let T_A and $T_{\neg B}$ be theories extending A and $\neg B$ respectively
  - Build a model from maximal consistent intersection of T_A and $T_{\neg B}$ (assuming the non-existence of interpolants) $\Rightarrow A \wedge \neg B$ satisfiable
- Related to Robinson's Joint Consistency Theorem

Reduction to first-order logic without equality [1]

Translate equality and function symbols:

$$\begin{aligned} \big(P(c)\big)^* &\equiv \exists x(C(x) \wedge P(x)) \\ \big(P(f(c))\big)^* &\equiv \exists x(\exists y(C(y) \wedge F(y,x)) \wedge P(x)) \\ \big(s = t\big)^* &\equiv E(s,t) \end{aligned}$$

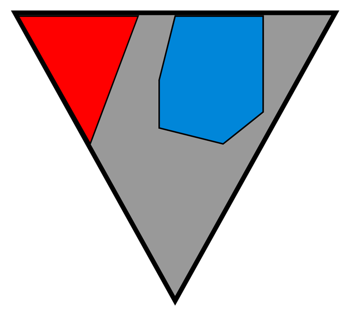
Add theory of equality:

$$\varphi \;\rightarrow\; T_E \supset \varphi^*$$

$\Rightarrow$ Then calculate interpolant in reduced logic

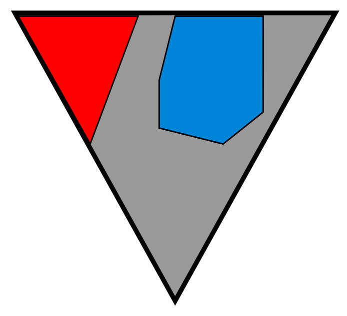
Interpolant extraction from proofs in two phases [2]

Proof:



$\Downarrow$ *Extract propositional interpolant structure from proof*

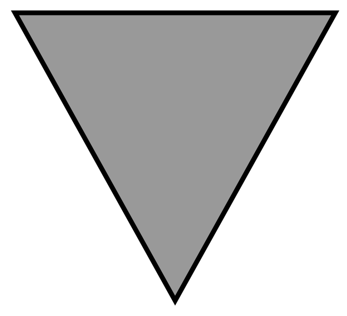
Propositional Interpolant:



$\ldots Q(f(c), c) \ldots$

$\Downarrow$ *Replace colored function and constant symbols*

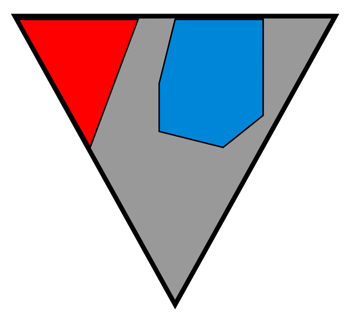
Prenex First-Order Interpolant:



$\exists x_3 \forall x_5 \ldots Q(x_5, x_3) \ldots$

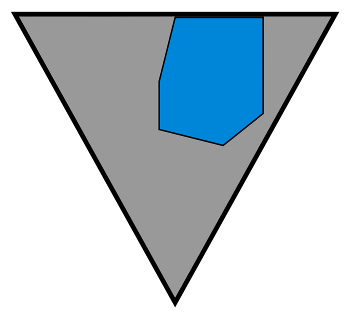
Interpolant extraction from proofs in one phase

Proof:



$\Downarrow$ *Combined structure extraction and replacing of colored symbols*

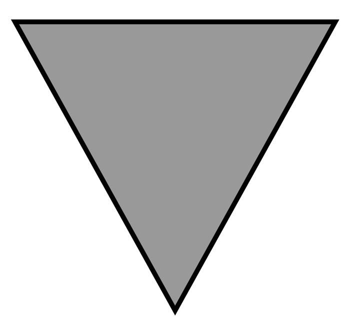
Interpolant modulo current clause:



$\forall x_5 \ldots Q(x_5, c) \ldots$

Recursively applied to all inferences of the proof results in:

Non-Prenex First-Order Interpolant:



$\exists x_3 \ldots \forall x_5 \ldots Q(x_5, x_3) \ldots$

Contributions

- We introduced the one phase extraction approach.
- We showed that the number of *quantifier alternations* in the extracted interpolant essentially corresponds to the number of *color alternations* in the terms of the proof.

References

[1] William Craig. Linear Reasoning. A New Form of the Herbrand-Gentzen Theorem. *Journal of Symbolic Logic*, 22(3):250–268, 1957.

[2] Guoxiang Huang. Constructing Craig Interpolation Formulas. In *Proc COCOON '95*, p. 181–190, 1995.

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