Diplomarbeitspräsentation

FAKULTÄT FÜR !NFORMATIK
Faculty of Informatics

# Interpolation in First-Order Logic with Equality

Masterstudium:
Computational Intelligence

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## Craig Interpolation

**Theorem** (Craig). *Let $A$ and $B$ be first-order formulas such that* $\models A \supset B$ *where $A$ contains red and gray symbols and $B$ contains of blue and gray symbols. Then there is an interpolant $I$ containing of gray symbols for $A$ and $B$ such that:*

- $\models A \supset I$
- $\models I \supset B$

$$\begin{tikzcd} A \arrow[rr, Rightarrow] \arrow[dr, Rightarrow] & & B \\ & I \arrow[ur, Rightarrow] & \end{tikzcd}$$

$\Rightarrow$ Interpolants give a concise logical summary of the implication

## Applications of Craig Interpolation

Theoretical:

- Proof of Beth's Definability Theorem

Practical:

- Program analysis: Detect loop invariants
- Model checking: Overapproximate set of reachable states

## Aim and Scope of the Thesis

Give comprehensive account of existing techniques and extend them:

- Model-theoretic proof
- Reduction to first-order logic without equality
- Interpolant extraction from resolution proofs

## Model-theoretic proof

- Non-constructive proof:
  - Let $T_A$ and $T_{\neg B}$ be theories extending $A$ and $\neg B$
  - Build model from maximal consistent intersection of $T_A$ and $T_{\neg B}$ (assuming the non-existence of interpolants)
    $\Rightarrow A \land \neg B$ satisfiable
- Related to Robinson's Joint Consistency Theorem

## Reduction to first-order logic without equality [1]

Translate equality and function symbols:

$$\begin{aligned} \Big(P(c)\Big)^* &\equiv \exists x (C(x) \land P(x)) \\ \Big(P(f(c))\Big)^* &\equiv \exists x (\exists y (C(y) \land F(y,x)) \land P(x)) \\ \Big(s = t\Big)^* &\equiv E(s,t) \end{aligned}$$

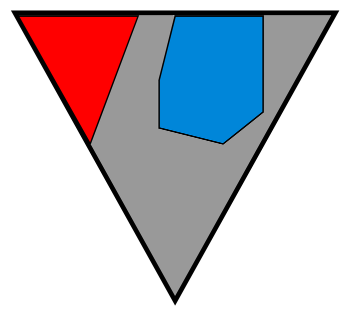
Add theory of equality:

$$\varphi \;\rightarrow\; T_E \supset \varphi^*$$

$\Rightarrow$ Then calculate interpolant in reduced logic

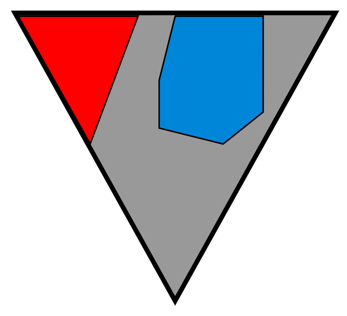
## Interpolant extraction from proofs in two phases [2]

Proof:



$\Downarrow$ *Extract propositional interpolant structure from proof*

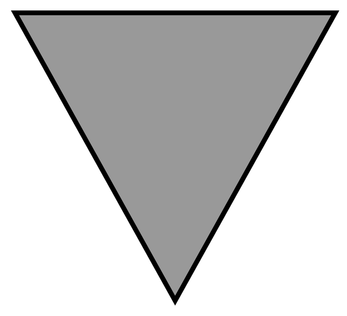
Propositional Interpolant:



$\ldots Q(f(c), c) \ldots$

$\Downarrow$ *Replace colored function and constant symbols*

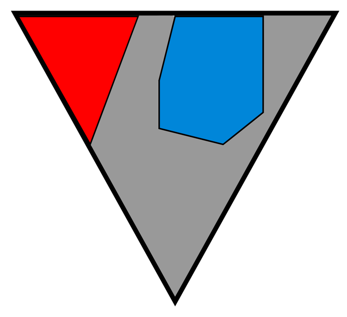
Prenex First-Order Interpolant:



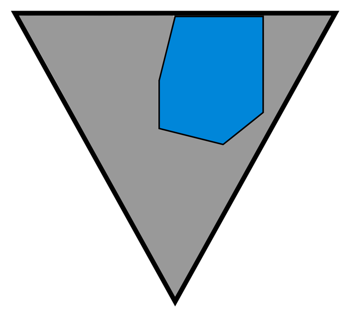
$\exists x_3 \forall x_5 \ldots Q(x_5, x_3) \ldots$

## Interpolant extraction from proofs in one phase

Proof:



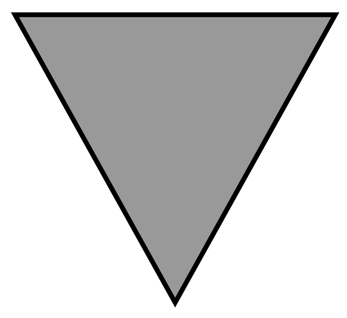
$\Downarrow \vdots \Downarrow$ *Combined extraction and replacing phases*



$\ldots \forall x_5 \ldots Q(x_5, c) \ldots$

$\Downarrow \vdots \Downarrow$ *Combined extraction and replacing phases*

Non-Prenex First-Order Interpolant:



$\exists x_3 \ldots \forall x_5 \ldots Q(x_5, x_3) \ldots$

## Contributions

- We introduced the one phase extraction approach.
- We showed that the number of quantifier alternations in the interpolant corresponds to the number of color alternations in terms.

## References

[1] William Craig. Linear Reasoning. A New Form of the Herbrand-Gentzen Theorem. *Journal of Symbolic Logic*, 22(3):250–268, 1957.

[2] Guoxiang Huang. Constructing Craig Interpolation

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