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Definition 1 ($\tau(\iota)$, deprecated version)**.** For an inference ι with $\sigma = \mathrm{mgu}(\iota)$, we define the infinite substitution $\tau(\iota)$ with $\mathrm{dom}(\tau(\iota)) = \{z_s \mid s\sigma \neq s\}$ as follows for a variable x:

$$x\tau(\iota) = \begin{cases} x & x \text{ is a non-lifting variable} \\ z_t & x \text{ is a lifting variable } z_s \text{ and } s\sigma = t \end{cases}$$

$\triangle$

Definition 2 ($\tau(\iota)$, current version)**.** For an inference ι with $\sigma = \mathrm{mgu}(\iota)$, we define the infinite substitution $\tau(\iota)$ with $\mathrm{dom}(\tau(\iota)) = \mathrm{dom}(\sigma) \cup \{z_s \mid s\sigma \neq s\}$ as follows for a variable x:

$$x\tau(\iota) = \begin{cases} x\sigma & x \text{ is a non-lifting variable} \\ z_t & x \text{ is a lifting variable } z_s \text{ and } s\sigma = t \end{cases}$$

$\triangle$

Definition 3 (Incremental lifting)**.** Let π be a resolution refutation of $\Gamma \cup \Delta$.

For a clause C in π, we define $\mathrm{LI}(C)$ and $\mathrm{LI}_{\mathrm{cl}}(C)$ as follows:

Base case. If $C \in \Gamma$, $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} \bot$. If otherwise $C \in \Delta$, $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} \top$.

In any case, $\mathrm{LI}_{\mathrm{cl}}(C) \stackrel{\mathrm{def}}{=} \ell[C]$.

Resolution. If the clause C is the result of a resolution step ι of $C_1 : D \vee l$ and $C_2 : E \vee \neg l'$ using a unifier σ such that $l\sigma = l'\sigma$, then let $\tau = \tau(\iota)$ and define $\mathrm{LI}(C)$ and $\mathrm{LI}_{\mathrm{cl}}(C)$ as follows:

$\mathrm{LI}_{\mathrm{cl}}(C) \stackrel{\mathrm{def}}{=} \ell[(\mathrm{LI}_{\mathrm{cl}}(C_1)\backslash\{l_{\mathrm{LI}_{\mathrm{cl}}}\})\tau] \ \vee \ \ell[(\mathrm{LI}_{\mathrm{cl}}(C_2)\backslash\{l'_{\mathrm{LI}_{\mathrm{cl}}}\})\tau]$

say something about $l_{\mathrm{LI}_{\mathrm{cl}}}$ and/or rewrite that (also below)

1. If l is Γ-colored: $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} \ell[\mathrm{LI}(C_1)\tau] \vee \ell[\mathrm{LI}(C_2)\tau]$
2. If l is Δ-colored: $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} \ell[\mathrm{LI}(C_1)\tau] \wedge \ell[\mathrm{LI}(C_2)\tau]$
3. If l is grey: $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} (\neg\ell[l'_{\mathrm{LI}_{\mathrm{cl}}}\tau] \wedge \ell[\mathrm{LI}(C_1)\tau]) \ \vee \ (\ell[l_{\mathrm{LI}_{\mathrm{cl}}}\tau] \wedge \ell[\mathrm{LI}(C_2)\tau])$

Factorisation. If the clause C is the result of a factorisation step ι of $C_1 : l \vee l' \vee D$ using a unifier σ such that $l\sigma = l'\sigma$, then $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} \ell[\mathrm{LI}(C_1)\tau(\iota)]$ and $\mathrm{LI}_{\mathrm{cl}}(C) \stackrel{\mathrm{def}}{=} \ell[(\mathrm{LI}_{\mathrm{cl}}(C_1)\backslash\{l'_{\mathrm{LI}_{\mathrm{cl}}}\})\tau(\iota)]$. $\triangle$

Definition 4. $\mathrm{LI}^{\Delta}(C)$ ($\mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$) for a clause C is defined as $\mathrm{LI}(C)$ ($\mathrm{LI}_{\mathrm{cl}}(C)$) with the difference that in its inductive definition, every lifting $\ell[\varphi]$ for a formula or term φ is replaced by a lifting of only the Δ-terms $\ell_{\Delta}[\varphi]$. $\triangle$

?⟨def:arrow_quantifier_block⟩? **Definition 5** (Quantifier block)**.** Let C be a clause in a resolution refutation π of $\Gamma \cup \Delta$ and $\bar{x}$ be the Δ-lifting variables and $\bar{y}$ the Γ-lifting variables occurring in $\mathrm{LI}(C)$ and $\mathrm{LI}_{\mathrm{cl}}(C)$. $Q(C)$ denotes an arrangement of the elements of $\{\forall x_t \mid x_t \in \bar{x}\} \cup \{\exists y_t \mid y_t \in \bar{y}\}$ such that for two lifting variable z_s and z_r, if s is a subterm of r, then z_s is listed before z_r. We denote $Q(\Box)$ by $Q(\pi)$. $\triangle$

Conjectured Lemma 6. *For a clause C of a resolution refutation of $\Gamma \cup \Delta$, $\Gamma \models \mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$.*

Proof. Induction of the strengthening $\Gamma \models \mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C_{\Gamma})$

Base case. ✓

Resolution.

Ind hyp gives $\Gamma \models \mathrm{LI}^{\Delta}(C_1) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(D) \vee l_{\mathrm{LI}^{\Delta}_{\mathrm{cl}}}$ and similar for C_2.

Don't really say $\mathrm{LI}^{\Delta}_{\mathrm{cl}}(D)$ here, we only have $\mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$

with τ:

$\Gamma \models \mathrm{LI}^{\Delta}(C_1) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(D) \vee l_{\mathrm{LI}^{\Delta}_{\mathrm{cl}}}$

have that $l\sigma = l'\sigma$, get also that $\ell_{\Delta}[l_{\mathrm{LI}^{\Delta}_{\mathrm{cl}}}\tau] = \ell_{\Delta}[l'_{\mathrm{LI}^{\Delta}_{\mathrm{cl}}}\tau]$. Proof: Suppose not lifted, then same. Otw. lifting variables, but then for p pos of lft var z_t in $l_{\mathrm{LI}^{\Delta}_{\mathrm{cl}}}$, $l|_p$ is t after applying τ. Hence have z_t for both.

- supp Γ resolved literals not removed due to coloring. literals are equal,

□

Conjectured Lemma 7. *For a clause C of a resolution refutation of* $\Gamma \cup \Delta$*,* $\Gamma \models Q(C)(\mathrm{LI}(C) \vee \mathrm{LI}_{\mathrm{cl}}(C))$*.*