Conjectured Lemma 1. *Let a variable x occur twice in C such that in one occ, the smallest colored term containing x is a Γ-term and for the other, the smallest colored term containing x is a Δ-term. Then x occurs grey in $\mathrm{AI}_*(C)$.*

Proof.

- Suppose that in C_i, $\gamma[x]$ occurs and in C_j, we have $\delta[y]$ such that x occurs grey in $y\sigma$.

  Then y occurs in l at $l|_{\hat{y}}$ such that $l'|_{\hat{y}}$ is an abstraction of a term containing a grey occurrence of x.

  Suppose that $l|_{\hat{y}}$ (and therefore also $l|_{\hat{y}}$) is not a grey occurrence as otherwise we are done.

  So either both are Γ or Δ; suppose Δ because symmetric.

  Then induction hypothesis.

  TODO: what if var occurs in multiple colors (as below) ?

- Suppose that in C_i, $\gamma[z]$ occurs and in C_j, $\delta[y]$ occurs such that x occurs grey in $y\sigma$ and in $z\sigma$.

  Then $\lambda|_{\hat{z}}$ such that $\lambda'|_{\hat{z}}$ is abstraction of term with grey occ of x

  and $\kappa|_{\hat{y}}$ such that $\kappa'|_{\hat{y}}$ is abstraction of term with grey occ of x.

  Suppose that x does not occur grey as otherwise we are done.

  By the variable convention, it only occurs in one clause.

  Suppose x does not occur such that induction hypothesis would kick in.

  As x must occur grey in both $y\sigma$ and $z\sigma$,

  INTUITION: either induction hypothesis, or we end up with terms which do not unify.

□