Example 101: canonical examples for overbinding w.r.t. order

Ex 101a

$$\dfrac{\dfrac{P(u,f(u)) \vee Q(u)_\Sigma \qquad \neg Q(a)_\Pi}{P(a,f(a))}\; u \mapsto a \qquad \neg P(x,y)_\Pi}{\Box}\; x \mapsto a, y \mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{Q(a)}\; u \mapsto a \qquad \top}{P(a,f(a)) \vee Q(a)}\; x \mapsto a, y \mapsto f(a) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1 Q(x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1,x_2) \vee Q(x_1))}$$

Ex 101b – other resolution order

$$\dfrac{\dfrac{P(u,f(u)) \vee Q(u)_\Sigma \qquad \neg P(x,y)_\Pi}{Q(u))}\; y \mapsto f(u), x \mapsto u \qquad \neg Q(a)_\Pi}{\Box}\; u \mapsto a$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(u,f(u))}\; x \mapsto f(u), x \mapsto u \qquad \top}{P(a,f(a)) \vee Q(a)}\; u \mapsto a \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(u,x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1,x_2) \vee Q(x_1))}\; u \mapsto a$$

Ex 101c – Π and Σ swapped

$$\dfrac{\dfrac{P(u,f(u)) \vee Q(u)_\Pi \qquad \neg P(x,y)_\Sigma}{Q(u))}\; y \mapsto f(u), x \mapsto u \qquad \neg Q(a)_\Sigma}{\Box}\; u \mapsto a$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg P(u,f(u))}\; x \mapsto f(u), x \mapsto u \qquad \bot}{\neg P(a,f(a)) \wedge \neg Q(a)}\; u \mapsto a \qquad\qquad \dfrac{\dfrac{\top \qquad \bot}{\forall x_2 \neg P(u,x_2)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1,x_2) \wedge \neg Q(x_1))}$$

Ex 101d – Π and Σ swapped, other resolution order

$$\dfrac{\dfrac{P(u,f(u)) \vee Q(u)_\Pi \qquad \neg Q(a)_\Sigma}{P(a,f(a))}\; u \mapsto a \qquad \neg P(x,y)_\Sigma}{\Box}\; x \mapsto a, y \mapsto f(a)$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg Q(a)}\; y \mapsto a \qquad \bot}{\neg Q(a) \wedge \neg P(a,f(a)}\; x \mapsto a, y \mapsto f(a) \qquad\qquad \dfrac{\dfrac{\top \qquad \bot}{\exists x_1 \neg Q(x_1)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1,x_2) \wedge \neg Q(x_1))}$$

OLD EXAMPLES

Example 5b: no equality, but quantifier order still matters

$$\frac{P(u,g(u))_{\Sigma} \qquad \neg P(a,x)_{\Pi}}{\Box}\; u \mapsto a, x \mapsto g(a)$$

Prop Interpolant: $P(a, g(a)$
Interpolant: $\forall x_1 \exists x_2 P(x_1, x_2)$

Example 5b': order matters, construction in multiple steps:

$$\cfrac{\cfrac{P(u,v,f(u,v)) \lor Q(u)_{\Sigma} \qquad \neg Q(a)_{\Pi}}{P(a,v,f(a,v))}\; u \mapsto a \qquad \neg P(x,b,y)_{\Pi}}{\Box}\; x \mapsto a, v \mapsto b, y \mapsto f(a,b)$$

$$\cfrac{\cfrac{\bot \qquad \top}{Q(a)}\; u \mapsto a \qquad \top}{P(a,b,f(a,b)) \lor (\neg P(a,b,f(a,b)) \land Q(a))}\; x \mapsto a, v \mapsto b, y \mapsto f(a,b)$$

Non-trivial interpolants:
$\forall x_1 Q(x_1)$
$\forall x_1 \forall x_2 \exists x_3 P(x_1, x_2, x_3) \lor Q(x_1)$

Example 5b": 5b' with different resolution order

$$\cfrac{\cfrac{P(u,v,f(u,v)) \lor Q(u)_{\Sigma} \qquad \neg P(x,b,y)_{\Pi}}{Q(u)}\; x \mapsto u, v \mapsto b, y \mapsto f(u,b) \qquad \neg Q(a)_{\Pi}}{\Box}\; u \mapsto a$$

$$\cfrac{\cfrac{\bot \qquad \top}{P(u,b,f(u,b))}\; x \mapsto u, v \mapsto b, y \mapsto f(u,b) \qquad \top}{P(a,b,f(a,b)) \lor Q(a)}\; u \mapsto a$$

Non-trivial interpolants:
$\forall x_2 \exists x_3 P(u, x_2, x_3)$
$\forall x_1 \forall x_2 \exists x_3 (P(x_1, x_2, x_3) \lor Q(x_1))$

Example 5c: example with $\exists\forall$ necessarily in interpolant
$\Rightarrow$ as shown in Huang, swap Σ and Π from 5b'

$$\dfrac{\dfrac{P(u,v,f(u,v)) \lor Q(u)_{\Pi} \qquad \neg Q(a)_{\Sigma}}{P(a,v,f(a,v))}\; u \mapsto a \qquad \neg P(x,b,y)_{\Sigma}}{\square}\; x \mapsto a, v \mapsto b, y \mapsto f(a,b)$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg Q(a)}\; u \mapsto a \qquad \bot}{\neg P(a,b,f(a,b)) \land \neg Q(a)}\; x \mapsto a, v \mapsto b, y \mapsto f(a,b)$$

Non-trivial interpolants:
$\exists x_1 Q(x_1)$
$\exists x_1 \exists x_2 \forall x_3 (\neg P(x_1, x_2, x_3) \land \neg Q(x_1))$

$\Rightarrow$ similar for 5b"

$$\dfrac{\dfrac{P(u,v,f(u,v)) \lor Q(u)_{\Pi} \qquad \neg P(x,b,y)_{\Sigma}}{Q(u)}\; x \mapsto u, v \mapsto b, y \mapsto f(u,b) \qquad \neg Q(a)_{\Sigma}}{\square}\; u \mapsto a$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg P(u,b,f(u,b))}\; x \mapsto u, v \mapsto b, y \mapsto f(u,b) \qquad \bot}{\neg Q(a) \land \neg P(a,b,f(a,b))}\; u \mapsto a$$

Non-trival interpolants:
$\exists x_2 \forall x_3 \neg(P(u, x_2, x_3))$
$\exists x_1 \exists x_2 \forall x_3 (\neg Q(x_1) \land \neg(P(x_2, x_3)))$