1 Proof of the correctness of Huang's algorithm without propositional refutations

Intuition of σ':
If we pull a substitution out of a lifting which replaces Δ-terms, we also have to replace the Δ-terms in the "codomain" of the substitution. This is the second case in the definition of σ' below.
There is just a problem in the following case: $\ell_{\Delta,x}[f(x)\sigma]$, where $x\sigma = a$ and f is a Δ-symbol. Then $\ell_{\Delta,x}[f(x)\sigma] = \ell_{\Delta,x}[f(a)] = x_i$, but $\ell_{\Delta,x}[f(x)]\sigma = x_j$ with $i \neq j$. The first case of the definition of x_j then fixes this by replacing x_j with x_i.

Lemma 1. *Let C be a clause and σ a substitution. Let $t_1, \ldots, t_n$ be all maximal Δ-terms in this context, i.e. those that occur in C or $C\sigma$, and $x_1, \ldots, x_n$ the corresponding fresh variables to replace the t_i. Define σ' such that for a variable z,*

$$z\sigma' = \begin{cases} x_l & \textit{if } z = x_k \textit{ and } t_k\sigma = t_l \\ \ell_{\Delta,x}[z\sigma] & \textit{otherwise} \end{cases}$$

Then $\ell_{\Delta,x}[C\sigma] = \ell_{\Delta,x}[C]\sigma'$.

Note that the definition of σ' only depends on the x_i and t_i.

Proof. We prove this for an atom $P(s_1, \ldots, s_m)$ in C, which works since lifting and substitution commute over binary connectives and into an atom.
We show that $\ell_{\Delta,x}[s_j\sigma] = \ell_{\Delta,x}[s_j]\sigma'$ for $1 \leq j \leq m$.
Note that anything in the term structure above a maximal Δ-term is unaffected by both substitution and the lifting.
Let t_i be a maximal Δ-term in $s_i\sigma$.
We show that $\ell_{\Delta,x}[t_i\sigma] = \ell_{\Delta,x}[t_i]\sigma'$, which proves the lemma.
Let $t_i\sigma = t_j$. Then $\ell_{\Delta,x}[t_i\sigma] = \ell_{\Delta,x}[t_j] = x_j$.
We show that $x_j = \ell_{\Delta,x}[t_i]\sigma'$.
Suppose that $t_i = t_j$, i.e. σ is trivial on t_i. Then $i = j$ as the Δ-terms have a unique number. Hence $\ell_{\Delta,x}[t_i]\sigma' = x_i\sigma' = x_i = x_j$.
Otherwise $t_i \neq t_j$. Then $i \neq j$ and $x_j \neq x_i$.
$\ell_{\Delta,x}[t_i]\sigma' = x_i\sigma'$. By the definition of σ', as $t_i\sigma = t_j$, $x_i\sigma' = x_j$. □

Lemma 2 (corresponds to Lemma 4.8 in thesis and Lemma 11 in Huang)**.** *Let A and B be first-order formulas and s and t be terms. Then it holds that:*

1. $\ell_{\Phi,x}[\neg A] \Leftrightarrow \neg\ell_{\Phi,x}[A]$
2. $\ell_{\Phi,x}[A \circ B] \Leftrightarrow (\ell_{\Phi,x}[A] \circ \ell_{\Phi,x}[B])$ *for* $\circ \in \{\wedge, \vee\}$
3. $\ell_{\Phi,x}[s = t] \Leftrightarrow (\ell_{\Phi,x}[s] = \ell_{\Phi,x}[t])$

Lemma 3. *Let s and t be terms such that no x_i occurs in them, Φ a set of formulas and M a model. Then $M \models \ell_{\Phi,x}[s] = \ell_{\Phi,x}[t]$ implies that $M \models s = t$.*

Proof. Suppose no Δ-term occurs in s or t. Then $\ell_{\Phi,x}[s] = s$ and $\ell_{\Phi,x}[t] = t$.
Otherwise let t_i be a maximal Δ-term in s. Suppose it occurs at position p. In $\ell_{\Phi,x}[s]$, it is replaced by x_i. But as $M \models \ell_{\Phi,x}[s] = \ell_{\Phi,x}[t]$, two situations can arise:

1. x_i occurs at p in $\ell_{\Phi,x}[t]$. As x_i does not occur in t, it is placed there by the lifting. But x_i is only employed in order to replace t_i, so at position p in t, we have t_i.
2. A term r occurs at p in $\ell_{\Phi,x}[t]$ which does not influence the evaluation of $\ell_{\Phi,x}[t]$ in M. This can be the case if r is contained in a subterm of u and in M, the function symbol of u is interpreted such that it does not depend on the argument that contains r.
   But as the maximal Δ-term t_i occurs in s at p and $M \models \ell_{\Phi,x}[s] = \ell_{\Phi,x}[t]$, there is a function symbol u' in $\ell_{\Phi,x}[s]$ corresponding to u which also does not depend on this argument.
   Hence even though s and t are not syntactically equal, $M \models s = t$ in this case. $\square$

We use basically the same definition of PI as Huang with minor adaptions for paramodulation (deviations are marked):

Definition 4 (Propositional interpolant extraction.)**.** Let π be a resolution refutation of $\Gamma \cup \Delta$. $\mathrm{PI}(\pi)$ is defined to be $\mathrm{PI}(\square)$, where $\square$ is the empty clause derived in π.
For a clause C in π, $\mathrm{PI}(C)$ is defined as follows:

Base case. If $C \in \Gamma$, $\mathrm{PI}(C) = \bot$. If otherwise $C \in \Delta$, $\mathrm{PI}(C) = \top$.

Resolution. If the clause C is the result of a resolution step of $C_1 : D \vee l$ and $C_2 : E \vee \neg l'$ using a unifier σ such that $l\sigma = l'\sigma$, then $\mathrm{PI}(C)$ is defined as follows:

1. If l is Γ-colored: $\mathrm{PI}(C) = [\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2)]\sigma$
2. If l is Δ-colored: $\mathrm{PI}(C) = [\mathrm{PI}(C_1) \wedge \mathrm{PI}(C_2)]\sigma$
3. If l is grey: $\mathrm{PI}(C) = [(l \wedge \mathrm{PI}(C_2)) \vee (\neg l' \wedge \mathrm{PI}(C_1))]\sigma$

Factorisation. If the clause C is the result of a factorisation of $C_1 : l \vee l' \vee D$ using a unifier σ such that $l\sigma = l'\sigma$, then $\mathrm{PI}(C) = \mathrm{PI}(C_1)\sigma$.

Paramodulation. If the clause C is the result of a paramodulation of $C_1 : s = t \vee C$ and $C_2 : D[r]$ using a unifier σ such that $r\sigma = s\sigma$, then $\mathrm{PI}(C)$ is defined according to the following case distinction:

1. If r occurs in a maximal Δ-term $h(r)$ in $D[r]$:
   $\mathrm{PI}(C) = [(s = t \wedge \mathrm{PI}(C_2)) \vee (s \neq t \wedge \mathrm{PI}(C_1))]\sigma \vee (s = t \wedge h[s] \neq h[t])\sigma$
2. If r occurs in a maximal Γ-term $h(r)$ in $D[r]$ and $h(r)$ occurs more than once in $D[r] \vee \mathrm{PI}(D[r])$:
   $\mathrm{PI}(C) = [(s = t \wedge \mathrm{PI}(C_2)) \vee (s \neq t \wedge \mathrm{PI}(C_1))]\sigma \wedge (s \neq t \vee h[s] = h[t])\sigma$
3. Otherwise:
   $\mathrm{PI}(C) = [(s = t \wedge (\mathrm{PI}(C_2) \vee h[s] \neq h[t]) \vee (s \neq t \wedge \mathrm{PI}(C_1))]\sigma$ $\triangle$

Now we show the "main" lemma of Huang's proof without using a propositional deduction P_P. The remaining part of his proof after this lemma does not use the restriction to propositional deductions and hence goes through.

Lemma 5 (corresponds to Lemma 12 in Huang and Lemma 4.9 in the thesis)**.** *Let* π *be a resolution refutation of* $\Gamma \cup \Delta$*. Then for* $C \in \pi$*,* $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C) \vee C]$.

Proof. By induction on the resolution refutation of the strengthening: $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C) \vee C_\Gamma]$, i.e. we only consider literals of C which are contained in $\mathrm{L}(\Gamma)$.
Base case: Either $C \in \Gamma$, then it does not contain Δ-terms. Otherwise $C \in \Delta$ and $\mathrm{PI}(C) = \top$.
Induction step:

Resolution.

$$\frac{C_1 : D \vee l \qquad C_2 : E \vee \neg l'}{C : (D \vee E)\sigma} \quad l\sigma = l'\sigma$$

By the induction hypothesis, we can assume that:

$\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1) \vee (D \vee l)_\Gamma]$ and $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_2) \vee (E \vee \neg l')_\Gamma]$

which by Lemma 2 implies that

$\Gamma \overset{(*)}{\models} \ell_{\Delta,x}[\mathrm{PI}(C_1)] \vee \ell_{\Delta,x}[D_\Gamma] \vee \ell_{\Delta,x}[l_\Gamma]$ and $\Gamma \overset{(\circ)}{\models} \ell_{\Delta,x}[\mathrm{PI}(C_2)] \vee \ell_{\Delta,x}[E_\Gamma] \vee \neg\ell_{\Delta,x}[l'_\Gamma]$

Let σ' be defined as in Lemma 1 with $t_1, \ldots, t_n$ all Δ-terms in this context (we need that every maximal Δ-term has a distinct index, so take all occurring in C_1, C_2, $\mathrm{PI}(C_1)$, $\mathrm{PI}(C_2)$, with and without σ applied to them).

Case distinction:

1. l is Γ-colored. Then $\mathrm{PI}(C) = [\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2)]\sigma$.

   We show that $\Gamma \models \ell_{\Delta,x}[(\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2))\sigma \vee (D \vee E)_\Gamma\sigma]$,
   i.e. $\Gamma \models \ell_{\Delta,x}[\Big(\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2) \vee D_\Gamma \vee E_\Gamma\Big)\sigma]$.

   Hence by Lemma 1, $\Gamma \models \ell_{\Delta,x}[(\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2) \vee D_\Gamma \vee E_\Gamma)]\sigma'$.

   Since $\sigma = \mathrm{mgu}(l, l')$, $l\sigma$ and $l'\sigma$ are syntactically equal and so $\ell_{\Delta,x}[l\sigma] = \ell_{\Delta,x}[l'\sigma]$.

   As by Lemma 1 $\ell_{\Delta,x}[l\sigma] = \ell_{\Delta,x}[l]\sigma'$ and $\ell_{\Delta,x}[l'\sigma] = \ell_{\Delta,x}[l']\sigma'$, we get $\ell_{\Delta,x}[l]\sigma' = \ell_{\Delta,x}[l']\sigma'$.

   So by applying σ' to $(*)$ and $(\circ)$ (note that $l_\Gamma = l$ and $l'_\Gamma = l'$ as they are Γ-colored), we can perform a resolution step on $\ell_{\Delta,x}[l]\sigma'$ and get

   $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1)]\sigma' \vee \ell_{\Delta,x}[D_\Gamma]\sigma' \vee \ell_{\Delta,x}[\mathrm{PI}(C_2)]\sigma' \vee \ell_{\Delta,x}[E_\Gamma]\sigma'$.

   and consequently $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2) \vee D_\Gamma \vee E_\Gamma]\sigma'$.

   So by Lemma 1,

   $\Gamma \models \ell_{\Delta,x}[\Big(\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2) \vee D_\Gamma \vee E_\Gamma\Big)\sigma]$.

2. l is Δ-colored. Then $\mathrm{PI}(C) = (\mathrm{PI}(C_1) \wedge \mathrm{PI}(C_2))\sigma$.

   We show that $\Gamma \models \ell_{\Delta,x}[(\mathrm{PI}(C_1) \wedge \mathrm{PI}(C_2))\sigma \vee (D_\Gamma \vee E_\Gamma)\sigma]$

   which by Lemma 2 is equivalent to
   $\Gamma \models \Big(\ell_{\Delta,x}[\mathrm{PI}(C_1)\sigma] \wedge \ell_{\Delta,x}[\mathrm{PI}(C_2)\sigma]\Big) \vee \ell_{\Delta,x}[D_\Gamma\sigma] \vee \ell_{\Delta,x}[E_\Gamma\sigma]$

and by Lemma 1 is equivalent to

$\Gamma \overset{(\circledast)}{\models} \Big(\ell_{\Delta,x}[\mathrm{PI}(C_1)]\sigma' \wedge \ell_{\Delta,x}[\mathrm{PI}(C_2)]\sigma'\Big) \vee \ell_{\Delta,x}[D_\Gamma]\sigma' \vee \ell_{\Delta,x}[E_\Gamma]\sigma'$

As l and l' are Δ-colored, we can simplify $(*)$ and $(\circ)$ as follows and apply σ':

$\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1)]\sigma' \vee \ell_{\Delta,x}[D_\Gamma]\sigma'$ and $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_2)]\sigma' \vee \ell_{\Delta,x}[E_\Gamma]\sigma'$

These clearly imply $(\circledast)$.

3. l is grey. Then $\mathrm{PI}(C) = [(l \wedge \mathrm{PI}(C_2)) \vee (\neg l' \wedge \mathrm{PI}(C_2))]\sigma$.

   We show that $\Gamma \models \ell_{\Delta,x}[\Big((l \wedge \mathrm{PI}(C_2)) \vee (\neg l' \wedge \mathrm{PI}(C_2)) \vee D_\Gamma \vee E_\Gamma\Big)\sigma]$, which by Lemma 2 and Lemma 1 is equivalent to

   $\Gamma \models \Big(\ell_{\Delta,x}[l]\sigma' \wedge \ell_{\Delta,x}[\mathrm{PI}(C_2)]\sigma'\Big) \vee \Big(\neg\ell_{\Delta,x}[l']\sigma' \wedge \ell_{\Delta,x}[\mathrm{PI}(C_2)]\sigma'\Big) \vee \ell_{\Delta,x}[D_\Gamma]\sigma' \vee \ell_{\Delta,x}[E_\Gamma]\sigma'$.

   Suppose for a model M of Γ that $M \not\models \ell_{\Delta,x}[D_\Gamma]\sigma'$ and $M \not\models \ell_{\Delta,x}[E_\Gamma]\sigma'$ as otherwise we would be done. But then by $(*)$ and $(\circ)$, $M \models \ell_{\Delta,x}[\mathrm{PI}(C_1)]\sigma' \vee \ell_{\Delta,x}[l]\sigma'$ and $M \models \ell_{\Delta,x}[\mathrm{PI}(C_2)]\sigma' \vee \neg\ell_{\Delta,x}[l']\sigma'$.

   As observed in case 1, $\ell_{\Delta,x}[l]\sigma' = \ell_{\Delta,x}[l']\sigma'$. By a case distinction on the truth value of $\ell_{\Delta,x}[l]\sigma'$, we obtain the result.

Factorisation.

$$\frac{C_1 : l \vee l' \vee D}{C : (l \vee D)\sigma} \qquad \sigma = \mathrm{mgu}(l, l')$$

Then $\mathrm{PI}(C) = \mathrm{PI}(C_1)\sigma$.

The induction hypothesis gives that $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1) \vee l \vee l' \vee D]$. Let σ' be as in Lemma 1.

Then $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1) \vee l \vee l' \vee D]\sigma'$ and by Lemma 1, $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1)\sigma \vee l\sigma \vee l'\sigma \vee D\sigma]$.

By Lemma 2, $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1)\sigma] \vee \ell_{\Delta,x}[l\sigma] \vee \ell_{\Delta,x}[l'\sigma] \vee \ell_{\Delta,x}[D\sigma]$.

As $\sigma = \mathrm{mgu}(l, l')$, $l\sigma$ and $l'\sigma$ are syntactically equal, hence $\ell_{\Delta,x}[l\sigma] = \ell_{\Delta,x}[l'\sigma]$.

But then we can apply a factorisation step and get $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1)\sigma] \vee \ell_{\Delta,x}[l\sigma] \vee \ell_{\Delta,x}[D\sigma]$ and by Lemma 1 and Lemma 2, $\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1)\sigma \vee l\sigma \vee D\sigma]$.

Paramodulation.

$$\frac{C_1 : D \vee s = t \qquad C_2 : E[r]_p}{C : (D \vee E[t]_p)\sigma} \qquad \sigma = \mathrm{mgu}(s, r)$$

By the induction hypothesis, we have:

$\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_1) \vee (D \vee s = t)_\Gamma]$

$\Gamma \models \ell_{\Delta,x}[\mathrm{PI}(C_2) \vee (E[r]_p)_\Gamma]$

By Lemma 1 and Lemma 2, we get that:

$\Gamma \overset{(\circ)}{\models} \ell_{\Delta,x}[\mathrm{PI}(C_1)] \vee \ell_{\Delta,x}[D_\Gamma] \vee \ell_{\Delta,x}[s] = \ell_{\Delta,x}[t]$

$\Gamma \overset{(*)}{\models} \ell_{\Delta,x}[\mathrm{PI}(C_2)] \vee \ell_{\Delta,x}[(E[r]_p)_\Gamma]$

We distinguish two cases:

1. Suppose s does not occur in a maximal Δ-term $h[s]$ in $E[s]_p$ which occurs more than once in $\mathrm{PI}(E(s)) \vee E[s]_p$.

   We show that $\Gamma \models \ell_{\Delta,x}[\Big((s = t \wedge \mathrm{PI}(C_2)) \vee (s \neq t \wedge \mathrm{PI}(C_1))\Big)\sigma \vee \Big((D \vee E[t]_p)_\Gamma\Big)\sigma]$, which subsumes the cases 2 and 3 of the definition of PI for paramodulation. By Lemma 2, we can pull the liftings inwards and by Lemma 1, we can commute substitution and lifting by employing σ' to arrive at

   $\Gamma \models \Big((\ell_{\Delta,x}[s]\sigma') = (\ell_{\Delta,x}[t]\sigma') \wedge \ell_{\Delta,x}[\mathrm{PI}(C_2)]\sigma'\Big) \vee \Big((\ell_{\Delta,x}[s]\sigma') \neq (\ell_{\Delta,x}[t]\sigma') \wedge \ell_{\Delta,x}[\mathrm{PI}(C_1)]\sigma'\Big) \vee \Big(\ell_{\Delta,x}[D_\Gamma]\sigma' \vee \ell_{\Delta,x}[(E[t]_p)_\Gamma]\sigma'\Big)$

   Let M be a model of Γ. Let $M \not\models \ell_{\Delta,x}[D_\Gamma]\sigma' \vee \ell_{\Delta,x}[(E[t]_p)_\Gamma]\sigma'$ as otherwise we would be done. We show that depending on the truth value of $(\ell_{\Delta,x}[s]) = (\ell_{\Delta,x}[t])$ in M, either the first or second conjunct of the above formula holds.

   Suppose that $M \models (\ell_{\Delta,x}[s]) \neq (\ell_{\Delta,x}[t])$. Then by ($\circ$), $M \models \ell_{\Delta,x}[\mathrm{PI}(C_1)]$ and hence $M \models \ell_{\Delta,x}[\mathrm{PI}(C_1)]\sigma'$.

   On the other hand, suppose that $M \models (\ell_{\Delta,x}[s]) = (\ell_{\Delta,x}[t])$. The following two lemmas show that $M \not\models \ell_{\Delta,x}[E[r]_p]\sigma'$, so by ($*$), we get that $M \models \ell_{\Delta,x}[\mathrm{PI}(C_2)]\sigma'$.

Lemma 6. *$M \models (\ell_{\Delta,x}[s]) = (\ell_{\Delta,x}[t])$ and $M \not\models \ell_{\Delta,x}[E[t]_p]$ imply that $M \not\models \ell_{\Delta,x}[E[s]_p]$ or, in case the term at position p in E is contained in a maximal Δ-colored term $g[t]$, $M \models s = t \;\wedge\; (\ell_{\Delta,x}[g[s]]) \neq (\ell_{\Delta,x}[g[t]])$.*

Proof. Suppose that the term at p in E is not contained in a Δ-colored term. Then $\ell_{\Delta,x}[E[t]_p]$ and $\ell_{\Delta,x}[E[s]_p]$ only differ at position p, where at the first, there is $\ell_{\Delta,x}[t]$, and at the latter, there is $\ell_{\Delta,x}[s]$. But in M, they are interpreted the same way, hence $M \models \ell_{\Delta,x}[E[t]_p] \Leftrightarrow \ell_{\Delta,x}[E[s]_p]$, which implies the result.

Otherwise as $g[t]$ and $g[s]$ in $E[t]_p$ and $E[s]_p$ respectively are distinct Δ-terms, they are replaced by distinct variables by the lifting. By Lemma 3, $M \models s = t$, so $M \models s = t \;\wedge\; (\ell_{\Delta,x}[g[s]]) \neq (\ell_{\Delta,x}[g[t]])$.

□

Lemma 7. *$\sigma = \mathrm{mgu}(s, r)$ and $M \not\models \ell_{\Delta,x}[E[s]_p]\sigma'$ imply that $M \not\models \ell_{\Delta,x}[E[r]_p]\sigma'$.*

Proof. By Lemma 1, $M \not\models \ell_{\Delta,x}[(E[s]_p)\sigma]$.

Due to $\sigma = \mathrm{mgu}(s, r)$, both $s\sigma$ and $r\sigma$ are syntactically equal. Suppose they are both not Δ-colored. Then the lifting does not affect them and $\ell_{\Delta,x}[(E[s]_p)\sigma] = \ell_{\Delta,x}[(E[r]_p)\sigma]$. Otherwise the lifting will replace them with the same variable and we as well get that $\ell_{\Delta,x}[(E[s]_p)\sigma] = \ell_{\Delta,x}[(E[r]_p)\sigma]$.

By Lemma 1, $\ell_{\Delta,x}[(E[s]_p)]\sigma' = \ell_{\Delta,x}[(E[r]_p)]\sigma'$, which implies the result. □

2. Otherwise s occurs in a maximal Δ-term $h[s]_q$ in $E[s]_p$ which occurs more than once in $\mathrm{PI}(E(s)) \vee E[s]_p$.

   Then we have to replace Lemma 6 by:

   Lemma 6′. *$M \models (\ell_{\Delta,x}[s]) = (\ell_{\Delta,x}[t])$ and $M \not\models \ell_{\Delta,x}[E[t]_p]\sigma'$ imply that $M \not\models \ell_{\Delta,x}[E[s]_p]\sigma'$ or that $\ell_{\Delta,x}[h[s]_q] \neq \ell_{\Delta,x}[h[t]_q]$.*

   Proof. If $\ell_{\Delta,x}[E[t]_p]$ and $\ell_{\Delta,x}[E[s]_p]$ differ only at position p, then the proof of Lemma 6 applies.

   Otherwise position p is in a maximal Δ-term $h[t]_q$, such that $h[t]_q$ and $h[s]_q$ are replaced with distinct variables. But then clearly $\ell_{\Delta,x}[h[s]_q] \neq \ell_{\Delta,x}[h[t]_q]$. □

   Hence the following holds:

   $$\Gamma \models \Big((\ell_{\Delta,x}[s]\sigma') = (\ell_{\Delta,x}[t]\sigma') \wedge \ell_{\Delta,x}[\mathrm{PI}(C_2)]\sigma'\Big) \vee \Big((\ell_{\Delta,x}[s]\sigma') \neq (\ell_{\Delta,x}[t]\sigma') \wedge \ell_{\Delta,x}[\mathrm{PI}(C_1)]\sigma'\Big) \vee \Big((\ell_{\Delta,x}[s]\sigma') = (\ell_{\Delta,x}[t]\sigma') \wedge (\ell_{\Delta,x}[h[s]_q]) \neq (\ell_{\Delta,x}[h[t]_q])\Big) \vee \Big(\ell_{\Delta,x}[D_\Gamma]\sigma' \vee \ell_{\Delta,x}[(E[t]_p)_\Gamma]\sigma'\Big)$$

   □

Then the following from the thesis (also same in Huang) seem to go through:
Lemma 4.10: swap Γ and Δ and obtain logical negation as interpolant
Corollary 4.11: $\Delta \models \ell_{\Gamma,y}[\neg\,\mathrm{PI}(C) \vee C]$
Lemma 4.12: not important if lifting delta or gamma terms first
Thm 4.13: ordering