CHAPTER 1

trying to overbind mostly right away does not look promising

Definition 1 (Q)**.** For a literal/clause φ, $Q(\varphi)$ denotes the quantifier block consisting of every lifting variable in φ with appropriate quantifier type. The order is yet to be defined △

For $l \in C$ for $C \in \Gamma$: $Q(l) = \exists\bar{y}$
For $l \in C$ for $C \in \Delta$: $Q(l) = \forall\bar{x}$

basic axioms which should be fulfilled for a reasonable procedure

start

- $\Gamma \vDash \mathrm{LI}_{\mathrm{cl}}(C)$

  $\Gamma = \{P(f(x))\} \Rightarrow \mathrm{LI}_{\mathrm{cl}}(C) \overset{\text{must be}}{=} \exists x P(x)$

  $\Gamma = \{\neg P(f(x))\} \Rightarrow \mathrm{LI}_{\mathrm{cl}}(C) \overset{\text{must be}}{=} \exists x \neg P(x)$

- $\Delta \vDash ?$

inferences LI is always basically just Γ-part
always want: $\Gamma \vDash \mathrm{LI}$, $\Delta \vDash \neg\,\mathrm{LI}$

- $\Gamma : P(f(x)) \Rightarrow \exists x P(x)$

  $\Delta : \neg P(y) \Rightarrow \forall y \neg P(x)$

- $\Gamma : \neg P(f(x)) \Rightarrow \exists x \neg P(x)$

  $\Delta : P(y) \Rightarrow \forall y P(x)$

- $\Gamma : \neg P(x) \Rightarrow \forall x \neg P(x)$

  $\Delta : P(g(y)) \Rightarrow \exists y P(y)$

- $\Gamma : P(x) \Rightarrow \forall x P(x)$

  $\Delta : \neg P(g(y)) \Rightarrow \neg\exists y P(y)$

but must not tear apart $P(x) \vee \neg P(x)$ to $\forall x P(x) \vee \forall x \neg P(x)$

example for "var does not occur in clause any more-condition":

$$\frac{R(f(z)) \qquad \neg R(x) \vee P(x)}{\neg R(x) \mid P(x)}$$

Note that $(\forall y_{f(x)} \neg R(y_{f(x)})) \vee P(x)$ is not valid!

attempt for a definition

Definition 2 (LI)**.**

Base case.

For $l \in C$ for $C \in \Gamma \cup \Delta$: $Q(l)\ell[C] \in \mathrm{LI}_{\mathrm{cl}}(C)$

LI as usual

Resolution.

Definition 3 (χ: lifting with quantification on literal level)**.**

$\chi(F \circ G) \stackrel{\text{def}}{=} \chi(F) \circ \chi(G)$

$\chi(\neg G) \stackrel{\text{def}}{=} \neg\chi(F)$

$\chi(Q(\lambda)\lambda) \stackrel{\text{def}}{=} Q(\lambda\sigma)\lambda\sigma$

where $Q(\lambda\sigma)$ is $Q(\lambda)$ with quantifiers and lifting variables for additional maximal colored terms introduced by σ into λ △

$\mathrm{LI}_{\mathrm{cl}}\, C \stackrel{\text{def}}{=} \chi(\mathrm{LI}_{\mathrm{cl}}(C_1) \backslash \{l_{\mathrm{LIcl}}\}) \vee \chi(\mathrm{LI}_{\mathrm{cl}}(C_2) \backslash \{l'_{\mathrm{LIcl}}\})$

1. If l is Γ-colored: $\mathrm{LI}(C) \stackrel{\text{def}}{=} \chi(\mathrm{LI}(C_1)) \vee \chi(\mathrm{LI}(C_2))$
2. If l is Δ-colored: $\mathrm{LI}(C) \stackrel{\text{def}}{=} \chi(\mathrm{LI}(C_1)) \wedge \chi(\mathrm{LI}(C_2))$
3. If l is grey: $\mathrm{LI}(C) \stackrel{\text{def}}{=} (l_{\mathrm{LIcl}}\tau \wedge \mathrm{LI}(C_2)\tau) \vee (\neg\ell[l'_{\mathrm{LIcl}}\tau] \wedge \ell[\mathrm{LI}(C_1)\tau])$

△

Conjectured Lemma 4. $\Gamma \models \mathrm{LI}(C) \vee \mathrm{LI}_{\mathrm{cl}}(C)$

Proof. Start works.
Step:
resolved literals: have same coloring

IH:
$\Gamma \models \mathrm{LI}(C_1) \vee \mathrm{LI}_{\mathrm{cl}}(C_1^*) \vee l_{\mathrm{LIcl}}$
$\Gamma \models \mathrm{LI}(C_2) \vee \mathrm{LI}_{\mathrm{cl}}(C_2^*) \vee l'_{\mathrm{LIcl}}$ □

Chapter 2

overbind just within thight constraints

⟨A⟩ **Lemma 5.** *If a variable does occurs in $\bar{C}$ but does not in C, then it is not modified by any* mgu *of a subsequent inference.*

2.1 naive interpolant extraction based on 5

Definition 6 (LI with stepwise prenex interplants but globally non-prenex ones)**.**

Base case.

For $l \in C$ for $C \in \Gamma \cup \Delta$: $C \in \mathrm{LI}_{\mathrm{cl}}(C)$

LI as usual

Resolution.

$\mathrm{LI}_{\mathrm{cl}}(C) \stackrel{\mathrm{def}}{=} \mathrm{LI}_{\mathrm{cl}}(C_1) \backslash \{l_{\mathrm{LIcl}}\}\sigma \vee \mathrm{LI}_{\mathrm{cl}}(C_2) \backslash \{l'_{\mathrm{LIcl}}\}\sigma$

$\Rightarrow \mathrm{LI}_{\mathrm{cl}}(C) = C$

$\chi(F)$: lift all terms which do contain a variable which do not contain variables which occur in $\mathrm{LI}_{\mathrm{cl}}(C)$ and quantify prenex

TODO: not sure where we can quantify ground terms as they can be added arbitrarily (possibly lift every occurrence of a ground term t distinctly)

TODO: need not be prenex here

1. If l is Γ-colored: $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} \chi(\mathrm{LI}(C_1) \vee \mathrm{LI}(C_2))\sigma$
2. If l is Δ-colored: $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} \chi(\mathrm{LI}(C_1) \wedge \mathrm{LI}(C_2))\sigma$
3. If l is grey: $\mathrm{LI}(C) \stackrel{\mathrm{def}}{=} (l_{\mathrm{LIcl}}\sigma \mathrm{LI}(C_2))\tau \vee (\neg l'_{\mathrm{LIcl}} \wedge \mathrm{LI}(C_1))\sigma$

△

$\Gamma \models \mathrm{LI}(C) \vee C$
$(\Delta \models \neg\, \mathrm{LI}(C) \vee C)$

2.2 lifting only Δ-terms in this way for now

Conjectured Lemma 7. $\Gamma \models \mathrm{LI}^{\Delta}(C) \vee C$

Proof. induction on strenghtening, as always.
$C_\Gamma = C^*_{1\,\Gamma} \vee C^*_{2\,\Gamma}$
IH:
$\Gamma \models \mathrm{LI}^{\Delta}(C_1) \vee C^*_{1\,\Gamma} \vee l_\Gamma$
$\Gamma \models \mathrm{LI}^{\Delta}(C_2) \vee C^*_{2\,\Gamma} \vee \neg l'_\Gamma$
Hence:
$\Gamma \models (\mathrm{LI}^{\Delta}(C_1) \vee C^*_{1\,\Gamma} \vee l_\Gamma)\sigma$
$\Gamma \models (\mathrm{LI}^{\Delta}(C_2) \vee C^*_{2\,\Gamma} \vee \neg l'_\Gamma)\sigma$
Supp grey:
$\Gamma \models (l \wedge \mathrm{LI}^{\Delta}(C_2))\sigma \vee (l' \wedge \mathrm{LI}^{\Delta}(C_1))\sigma \ \vee \ C_\Gamma$
$\Gamma \models \mathrm{LI}^{\Delta}(C) \ \vee \ C_\Gamma$

???

□

$X = \mathrm{LV}(\mathrm{LI}^{\Delta}(C)) \setminus \mathrm{LV}(\mathrm{LI}^{\Delta}_{\mathrm{cl}}(C_\Gamma))$
X': take from X those lifting variables, which contain variables which do not occur in C (this is safer than only $\mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$)
$Y = \mathrm{LV}(\ell_\Gamma[\mathrm{LI}^{\Delta}(C)])$
$Y' = \{z_t \in Y \mid t \text{ contains a variable which does not occur in } C\}$
From other pdf: $\Gamma \models \mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$
Hence $\Gamma \models (Q(Y')\,\mathrm{LI}^{\Delta}(C)) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$

2.3 random ideas

– we can pull apart existentially quantified variables: $\exists x(P(x) \vee Q(x))$ implies $\exists x P(x) \vee \exists y P(y)$. this does not work with universally quantified variables $(P(f(x)) \vee \neg P(f(x))$