Interpolation in First-Order Logic with Equality

Master Thesis Presentation

Bernhard Mallinger

Advisor: Stefan Hetzl

Institute of Discrete Mathematics and Geometry
TU Wien

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Agenda

1. Introduction
2. Craig Interpolation (10 min)
   - Interpolation and Equality
   - Applications (include Beth proof?)
3. Craig's Proof (6 min)
   - Translation (brief, by example)
   - Maehara/BL (mention lifting?)
4. Huang's Proof (10 min)
   - Phase one (propositional, inductive)
   - Phase two (lifting, ordering)
   - Optional: Number of quantifier alternations
   - Proof with one phase: Can lift earlier.
5. Semantic Proof (6 min)
6. Conclusion

Introduction

- Want concrete algorithms for FOL/EQ
  $\Rightarrow$ Little attention so far
- Present different constructive proofs

Craig Interpolation

Theorem (Craig). *Let A and B be first-order formulas where*

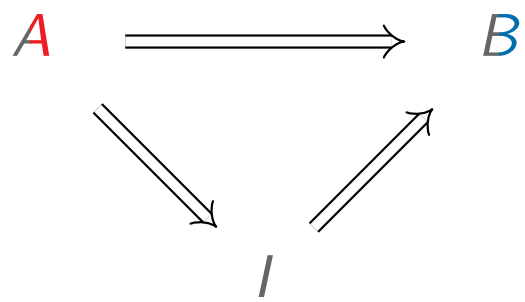
- *A contains red and gray symbols and*
- *B contains blue and gray symbols*

such that:

- $\models A \supset B$

Then there is a interpolant I containing only gray symbols such that:

- $\models A \supset I$
- $\models I \supset B$



Applications (include Beth proof?)

Translation (brief, by example)

Maehara/BL (mention lifting?)

Phase one (propositional, inductive)

Phase two (lifting, ordering)

Optional: Number of quantifier alternations

Proof with one phase: Can lift earlier.

Conclusion

- Craig's and Huang's proof based interpolant extraction from proofs
  $\Rightarrow$ differ in applicability
- Craig shows that the interpolation theorem holds also in FOL/EQ
- Huang shows that interpolants can efficiently be extracted in FOL/EQ
  - Does not require different methods
  - Little attention so far in research
- Interpolation also allows for a model theoretic approach