undirected edges (from $\mathcal{M}$) are to be interpreted as two directed edges.
$E(C) = \mathcal{A}(C) \cup \mathcal{M}(C)$
$V(C) = V(E(C))$
$G(C) = (V(C), E(C))$
color of component is color of some term in it (all the same)
per resolution step: oppositely colored components are not unifiable

Components

nodes: max col term occurrences and variables in grey occurrences.

1. components initially: for every variable, all grey occurrences and all colored occurrences

2. resolution: components of C_1 and C_2 are carried over, some are merged.

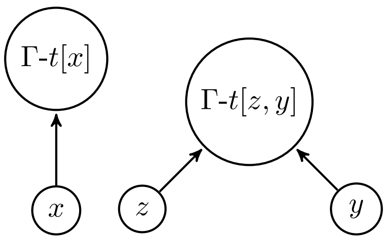
   $l\sigma = l'\sigma$

   For each max col term t in $l\sigma$: merge component of t and t'.

quantifier ordering: Build $\mathcal{A}(C)$, which is the condensation of $G(C)$. If in the condensation there is a path from a node containing a term containing u_i to a node containing term containing u_j, then $u_i <_{\mathcal{A}(C)} u_j$.

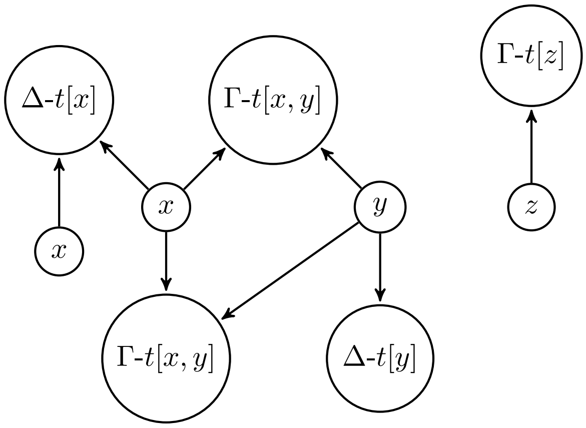
graph components visualised

initially



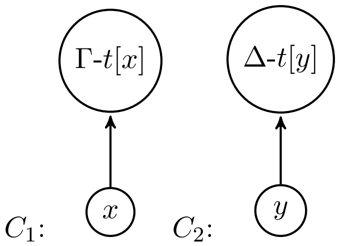
Note: initially, all colored terms are in one component

in the derivation, single color

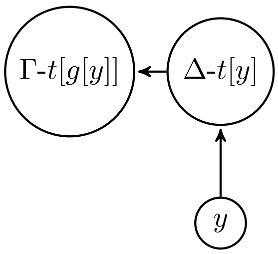


Note: Γ- and Δ-terms can not be merged (unified). All other combinations are possible.

in the derivation



C with x unified with a Δ-term, say $g[y]$:



proofs

Conjecture 1 (Lemma 1)**.** ***continue with other proof before fixing this***
If a variable has a grey and a colored occurrence, there is an arrow between the component containing the grey occurrence and the component containing the colored occurrence (in the condensation graph).

Proof. Induction start is easy.
Induction step: Suppose the statement holds C_1 and C_2.
Note that no new grey components can be added, just merged. Hence it suffices to show that the component of an arbitrary in C newly added colored occurrence of a variable, say x, has an arrow starting from a grey component.
No component is ever added. x is not unified as otherwise it would not exist anymore (the lemma statment requires the variable to occur). A new colored occurrence of x can be created by either putting x into a colored term or by a colored occurrence of x in the codomain of the unifier.

1. Putting x into a colored term. Then there is some $\gamma[y]$ with $y\sigma = t[x]$. In the easy case, y is just unified with $t[x]$. Let $\hat{y}$ be the occurrence of y in the resolved literal which causes a change of y in the unification algorithm and $\hat{t}[x]$ the corresponding term at the same position in the other resolved literal.

   "induction" until we hit a term containing x

   Then the component of $\hat{y}$ is merged with the component of $\hat{t}[x]$.

   Afterwards, we have some other component of x as well. This could be:

   a) in the same clause as $\hat{t}[x]$.

      Then distinguish on the the shape of $\hat{t}[x]$:

      - Either it is grey, then $\hat{y}$ is grey as well and we have an arrow by the induction hypothesis from $\hat{y}$ to $\gamma[y]$.
      - Otherwise it is colored. Then by the induction hypothesis, as there exists a grey component of x in this clause, there is an arrow to $\hat{t}[x]$. By some Lemma yet to define, there either is a merge arrow between $\gamma[y]$ and $\hat{y}$, which is also a colored term, or there is a grey occurrence of y with arrows to the two colored occurrences. in the first case, we are done, and in the second ???

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?? as some var which is unified to x ???

b) in the same clause as $\hat{y}$ in the form of a component which is called y in C_i for some i. By the subst, it's now x. Then we have an arrow by the induction hypothesis from the component of y to the component of $\gamma[y]$.

TODO: what if y is substituted by a colored term containing x?

□

Conjecture 2 (Conjecture 4)**.** *Suppose in* $\mathrm{AI}^{\Delta}(C)$ *a maximal* Γ*-term* $\gamma_j[z_i]$ *contains a lifting variable* z_i*. Then* $z_i <_{\mathcal{A}(C)} z_j$*.*

random notes

1. if two variable-nodes in the condensation are connected when disregarding the arrow direction, they occur in the same clause.