501 – like 502 but easier

$$\frac{\forall x_a \exists y_{f(a)} Q(x_a, y_{f(a)}) \mid A \qquad \forall x_a (S(x_a)) \mid \neg A}{\forall x_a (S(x_a)) \,\wedge\, \forall x_a \exists y_{f(a)} Q(x_a, y_{f(a)}) \mid \Box}$$

no first order operation in this last inference $\Rightarrow$ nothing to prove

502 – example with multiple, independent a's

derivation:

$$\dfrac{\dfrac{\dfrac{P(f(x),x) \overset{\Sigma}{\vee} Q(z) \vee R(z) \qquad \overset{\Pi}{\neg R(a)}}{R(a) \mid P(f(x),x) \vee Q(a)} \qquad \overset{\Pi}{\neg Q(u)}}{Q(a) \vee R(a) \mid P(f(x),x)} \qquad \overset{\Pi}{\neg P(z,a)}}{P(f(a),a) \vee Q(a) \vee R(a) \mid \Box}$$

invariant: $\ell_\Delta[\mathrm{LI}(C)] \mid \ell_\Delta[C]$

$$\dfrac{\dfrac{\dfrac{P(f(x),x) \overset{\Sigma}{\vee} Q(z) \vee R(z) \qquad \overset{\Pi}{\neg R(x_a)}}{R(x_a) \mid P(f(x),x) \vee Q(x_a)} \qquad \overset{\Pi}{\neg Q(u)}}{\forall x_a (Q(x_a) \vee R(x_a)) \mid P(f(x),x)} \qquad \overset{\Pi}{\neg P(z,x_a)}}{\forall x_a \exists y_{f(a)} \Big(P(y_{f(a)}, x_a) \vee \forall x_a (Q(x_a) \vee R(x_a)) \Big) \mid \Box}$$

possibly important intermediate step before last formula:

$P(f(x_a), x_a) \vee \forall x_a (Q(x_a) \vee R(x_a))$

lifting: $\mathrm{LI}(C) \mid C$

$$\begin{array}{c} \dfrac{\dfrac{\dfrac{P(f(x),x) \overset{\Sigma}{\vee} Q(z) \vee R(z) \qquad \overset{\Pi}{\neg R(a)}}{R(a) \mid P(f(x),x) \vee Q(a)} \qquad \overset{\Pi}{\neg Q(u)}}{\begin{array}{c} Q(a) \vee R(a) \mid P(f(x),x) \\ \forall x_a (Q(x_a) \vee R(x_a)) \mid P(f(x),x) \end{array}} \qquad \overset{\Pi}{\neg P(z,a)}}{P(f(a),a) \vee \forall x_a (Q(x_a) \vee R(x_a)) \mid \Box} \\ \forall x_a \exists y_{f(a)} \Big(P(y_{f(a)}, x_a) \vee \forall x_a (Q(x_a) \vee R(x_a)) \Big) \mid \Box \end{array}$$

TODO figure out why this always works

possibly write down logical justification of steps down very precisely as for paramod special case