

Interpolation in First-Order Logic with Equality

Masterstudium:
Computational Intelligence

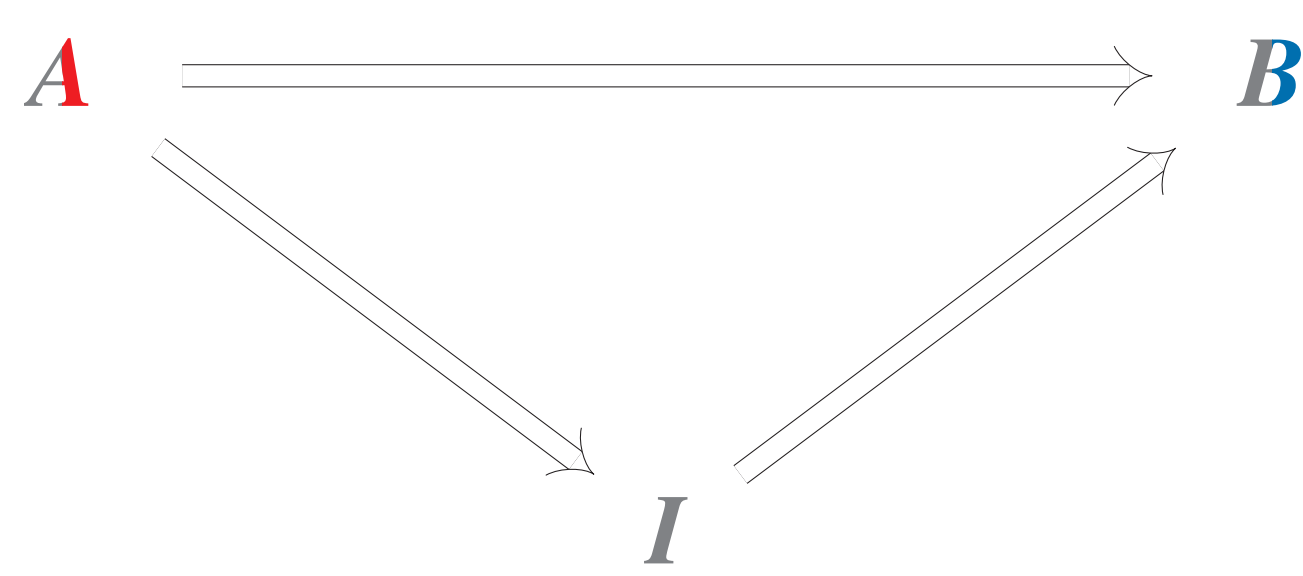
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Craig Interpolation

Theorem (Craig). Let A and B be first-order formulas such that $\models A \supset B$. Then there is an interpolant I for A and B such that:

- ▶ $\models A \supset I$
- ▶ $\models I \supset B$
- ▶ $\text{Lang}(I) \subseteq \text{Lang}(A) \cap \text{Lang}(B)$



⇒ Interpolants give a concise logical summary of the implication

Applications of Craig Interpolation

Theoretical:

- ▶ Proof of Beth's Definability Theorem

Practical:

- ▶ Program analysis: Detect loop invariants
- ▶ Model checking: Overapproximate set of reachable states

Aim and Scope of the Thesis

Give comprehensive account of existing techniques and extend them:

- ▶ Model-theoretic proof
- ▶ Reduction to first-order logic without equality
- ▶ Interpolant extraction from resolution proofs

Model-theoretic proof

- ▶ Non-constructive proof:
 - ▶ Let T_A and $T_{\neg B}$ be theories extending A and $\neg B$
 - ▶ Build model from maximal consistent intersection of T_A and $T_{\neg B}$ (feasible assuming the non-existence of interpolants)
⇒ $A \wedge \neg B$ satisfiable
- ▶ Related to Robinson's Joint Consistency Theorem

Reduction to first-order logic without equality [1]

Translate equality and function symbols:

$$\begin{aligned} P(c) &\rightarrow \exists x (C(x) \wedge P(x)) \\ P(f(c)) &\rightarrow \exists x (\exists y (C(y) \wedge F(y, x)) \wedge P(x)) \\ s = t &\rightarrow E(s, t) \end{aligned}$$

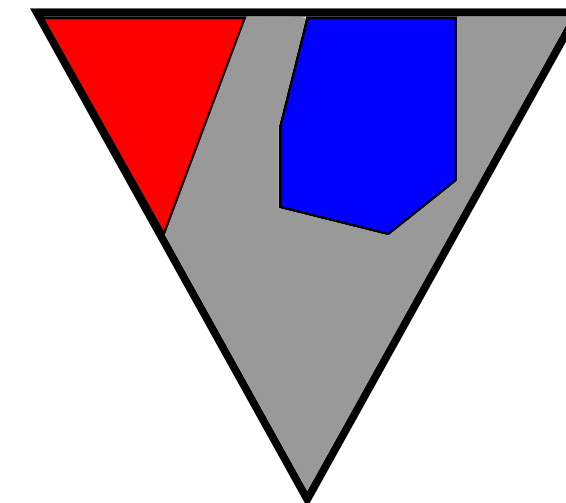
Add theory of equality:

$$\varphi \rightarrow T_E \supset \varphi^*$$

⇒ Then calculate interpolant in reduced logic

Interpolant extraction from proofs in two phases [2]

- ▶ Extract structure from proof
- ▶ Replace colored terms by quantified variables



$\dots Q(f(c), c) \dots$



$\exists x_3 \forall x_5 \dots Q(x_5, x_3) \dots$

Interpolant extraction from proofs in one phase

- ▶ Replace colored terms during extraction



$\dots Q(f(c), c) \dots$



$\dots \forall x_5 \dots Q(x_5, c) \dots$



$\exists x_3 \dots \forall x_5 \dots Q(x_5, x_3) \dots$

Contributions

- ▶ We introduced the one phase-approach.
- ▶ We showed that the number of quantifier alternations in the interpolant essentially corresponds to the number of color alternations in terms.

References

- [1] William Craig.
Linear Reasoning. A New Form of the Herbrand-Gentzen Theorem.
Journal of Symbolic Logic, 22(3):250–268, 1957.
- [2] Guoxiang Huang.
Constructing Craig Interpolation Formulas.
In *Proc COCOON '95*, p. 181–190, 1995.