border cases: arrows not within supposedly connected components

211a

$$\dfrac{Q(x) \lor P(f(x,a)) \qquad \neg Q(y) \lor R(f(y,b))}{Q(x) \mid P(f(x,a)) \lor R(f(x,b))}$$

$\Rightarrow$ no arr between P and R

211a'

$$\dfrac{\dfrac{Q(x) \lor \overset{\Sigma}{P}(f(x,a)) \qquad \neg Q(y) \lor \overset{\Sigma}{R}(f(y,b))}{Q(x) \mid P(f(x,a)) \lor R(f(x,b))} \qquad \dfrac{\neg P(f(u,\overset{\Sigma}{z})) \lor S(u)) \qquad \neg \overset{\Pi}{S}(c)}{S(c) \mid \neg P(f(c,z))}}{(P(f(c,a)) \land S(c)) \lor (\neg P(f(c,a)) \land Q(c))) \mid R(f(c,b))}$$

$c \sim x_1$ $\qquad$ $f(c,a) \sim y_2$ $\qquad$ $f(c,b) \sim y_3$

$(P(y_2) \land S(x_1)) \lor (\neg P(y_2) \land Q(x_1)) \mid R(y_3)$ $\qquad$ NOTE: arrow merge on resolution is not drawn here (but is necessary)

$\forall x_1 \exists y_2 \exists y_3$

this is not valid per se as the left hand side only contains Σ-formulas, but it probably could be fixed by adding some Π-inferences

Lesson is: no extra arrows needed, if a term enters, it does so via x, but there is a variable from the grey x to both colored x.

211b

$$\dfrac{Q(x) \lor P(f(x)) \qquad R(y) \lor \neg P(f(y))}{P(f(x)) \mid Q(x) \lor R(x)}$$

$\Rightarrow$ no arr between Q and R

NB: should be fixed by backwards merging special case

211b'

$$\dfrac{\dfrac{Q(x) \lor \overset{\Sigma}{P}(f(x)) \qquad R(y) \lor \overset{\Sigma}{\neg} P(f(y))}{P(f(x)) \mid Q(x) \lor R(x)} \qquad \neg \overset{\Pi}{Q}(a)}{P(f(a)) \lor Q(a) \mid R(a)}$$

WRONG: conjecture: Q and R do not need arrows as they are lifted by the same variable anyway, so constraints on Q do the work

211c

$$\dfrac{Q(f(x)\overset{\Sigma}{)} \lor R(x) \qquad \neg R(\overset{\Pi}{g}(y))}{Q(f(g(y))) \lor R(g(y))}$$

Have same var but no merge arrow. The whole term $g(y)$ is somehow the "travelling term", there is no "renaming".

211d – problem cases with lemma grey->colored

currently not clear what the connetion between the arguments of R on the RHS is If we use factorisation, not sure how to handle yet, but could be like: $R(t[x], s[x]) \vee Q(x)$

$$\frac{Q(y) \vee Q'(z) \vee P(f(y)) \vee R(g(y), g'(z)) \qquad \neg R(g(h(x)), g'(x))}{R(g(h(x)), g'(x)) \mid Q(h(x)) \vee Q'(x) \vee P(f(h(x)))}\; y \mapsto h(x), z \mapsto x$$

NB: this is different since x occurs grey as well (example not finished)

Problem case 1: x grey and colored, but not connection

$$\frac{Q'(z) \vee P(f(y)) \vee R(g(f(y)), g'(z)) \qquad \neg R(g(f(h(x))), g'(x))}{R(g(f(h(x))), g'(x)) \mid Q(x) \vee P(f(h(x)))}\; y \mapsto h(x), z \mapsto x$$

NB: no connection between Q and P

$\Rightarrow$ backwards merging

Problem case 2: x colored and colored, not sure what the connection is supposed to be

$$\frac{Q'(k(z)) \vee P(f(y)) \vee R(g(y), g'(z)) \qquad \neg R(g(h(x)), g'(x))}{R(g(h(x)), g'(x)) \mid Q'(k(x)) \vee P(f(h(x)))}\; y \mapsto h(x), z \mapsto x$$

lifting var doesn't correspond to actual term exactly in context of unifier arrows

$$\dfrac{Q(u) \vee \overset{\Sigma}{\neg} P(h(u)) \qquad \dfrac{\dfrac{R(g(x)\overset{\Pi}{)}) \vee S(x) \qquad \neg \overset{\Pi}{S}(a)}{S(a) \mid\mid R(g(a))}\; x \mapsto a \qquad P(h(f(y))\overset{\Sigma}{)}) \vee \neg R(y)}{S(a) \mid R(g(a)) \mid P(h(f(g(a))))}\; y \mapsto g(a)}{S(a) \mid R(g(a)), P(h(f(g(a)))) \mid Q(f(g(a)))}\; u \mapsto g(f(g(a)))$$

lifted:

$$\dfrac{Q(u) \vee \overset{\Sigma}{\neg} P(y_{h(u)}) \qquad \dfrac{\dfrac{R(x_{g(x)}\overset{\Pi}{)}) \vee S(x) \qquad \neg \overset{\Pi}{S}(x_a)}{S(x_a) \mid\mid R(x_{g(x)})}\; x \mapsto a \qquad P(y_{h(f(y))}\overset{\Sigma}{)}) \vee \neg R(y)}{S(x_a) \mid R(x_{g(a)}) \mid P(y_{h(f(y))})}\; y \mapsto g(a) \ast}{S(x_a) \mid R(x_{g(a)}), P(y_{h(f(g(a)))}) \mid Q(y_{f(g(a))})}\; u \mapsto g(f(g(a)))$$

at $\ast$, $R(x_g(x))$ is not known to refer to $g(a)$. can we resort to check grey occurrences of $g(a)$?
need arrow from R to P (this situation should be more critical if it is a backwards arrow)
thought: concerns only stuff in literal, maybe can leverage something here (all lifting vars x_i point to same term or so)