necessary and unnecessary arrows in literal

Ex 301a

$$\dfrac{\neg P(\overset{\Pi}{a},z) \qquad \dfrac{P(x,\overset{\Sigma}{f}(y)) \vee Q(y) \qquad \neg \overset{\Pi}{Q}(a)}{Q(a) \mid P(x,f(a))}}{Q(a) \vee P(a,f(a)) \mid \square}$$

$$\dfrac{\forall x_1\, \neg P(\overset{\Pi}{x_1},z) \qquad \dfrac{P(x,\overset{\Sigma}{f}(y)) \vee Q(y) \qquad \forall x_1\, \neg \overset{\Pi}{Q}(x_1)}{\forall x_1(Q(x_1) \mid P(x,f(x_1)))}}{\forall x_1(Q(x_1) \vee P(x_1,f(x_1))) \mid \square}$$

$$\dfrac{\forall x_1\, \neg P(\overset{\Pi}{x_1},z) \qquad \dfrac{\exists y_2\, P(x,\overset{\Sigma}{y_2}) \vee Q(y) \qquad \forall x_1\, \neg \overset{\Pi}{Q}(x_1)}{\overset{\forall x_1}{\exists y_2}\, (Q(x_1) \mid P(x,y_2))}}{\overset{\forall x_1}{\exists y_3}\, (Q(x_1) \vee P(x_1,y_3)) \mid \square}$$

all orderings work:

$\forall x_1 \exists y_2 (Q(x_1) \vee P(x_1,y_2))$ // need not be x_1 both times, that's just an accident of this example

$\exists y_2 \forall x_1 (Q(x_1) \vee P(x_1,y_2))$

Ex 302a

$$\dfrac{P(u,\overset{\Sigma}{f}(u)) \qquad \neg P(\overset{\Pi}{a},z)}{P(a,f(a)) \mid \square}$$

$$\dfrac{P(u,\overset{\Sigma}{f}(u)) \qquad \forall x_1\, \neg P(\overset{\Pi}{x_1},z)}{\forall x_1 P(x_1,f(x_1)) \mid \square}$$

$\forall x_1 \exists y_2 (P(x_1,y_2))$

$\cancel{\exists y_2 \forall x_1 (P(x_1,y_2))}$

$$\dfrac{\forall u \exists y_2\, P(\overset{\Sigma}{u},y_1)) \qquad \forall x_1\, \neg P(\overset{\Pi}{x_1},z)}{\forall x_1 \exists y_3 P(x_1,y_3) \mid \square}$$

(order matters)

Ex 302a' – inverse coloring

$$\dfrac{\forall y_2\, P(\overset{\Pi}{u},y_2) \qquad \neg P(\overset{\Sigma}{a},z)}{\forall y_3 P(a,y_3) \mid \square}$$

(can't really fix order, but matters)

Lession of 30x:

LK-proof for $P(x,f(y)) \vee Q(y)$ works by multiple instantiations $\Rightarrow$ not possible for $P(x,f(x))$, as x is always instantiated with same thing. Hence first version has in some sense more liberty (it really does mean something different).