Example 101: canonical examples for overbinding w.r.t. order

Here, the second lower prooftree shows what huang would produce.

Ex 101a

$$\dfrac{\dfrac{P({\color{red}u}, f({\color{red}u})) \lor Q({\color{red}u})_{\Sigma} \qquad \neg Q(a)_{\Pi}}{P(a, f(a))}\, u \mapsto a \qquad \neg P(x, y)_{\Pi}}{\square}\, x \mapsto a, y \mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{Q(a)}\, u \mapsto a \qquad \top}{P(a, f(a)) \lor Q(a)}\, x \mapsto a, y \mapsto f(a) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1 Q(x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1, x_2) \lor Q(x_1))}$$

Direct overbinding would not work without merging same variables!: $\forall x_1 \forall x_2 \exists x_3 (P(x_2, x_3) \lor Q(x_1))$
counterexample: $Q \sim \{0\}, P \sim \{(1,0)\}$
Direct overbinding would work when considering original dependencies as highlighted above

Ex 101b – other resolution order

$$\dfrac{\dfrac{P(u, f(u)) \lor Q(u)_{\Sigma} \qquad \neg P(x, y)_{\Pi}}{Q(u))}\, y \mapsto f(u), x \mapsto u \qquad \neg Q(a)_{\Pi}}{\square}\, u \mapsto a$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(u, f(u))}\, x \mapsto f(u), x \mapsto u \qquad \top}{P(a, f(a)) \lor Q(a)}\, u \mapsto a \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(u, x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1, x_2) \lor Q(x_1))}\, u \mapsto a$$

Ex 101c – Π and Σ swapped

$$\dfrac{\dfrac{P(u, f(u)) \lor Q(u)_{\Pi} \qquad \neg P(x, y)_{\Sigma}}{Q(u))}\, y \mapsto f(u), x \mapsto u \qquad \neg Q(a)_{\Sigma}}{\square}\, u \mapsto a$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg P(u, f(u))}\, x \mapsto f(u), x \mapsto u \qquad \bot}{\neg P(a, f(a)) \land \neg Q(a)}\, u \mapsto a \qquad\qquad \dfrac{\dfrac{\top \qquad \bot}{\forall x_2 \neg P(u, x_2)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1, x_2) \land \neg Q(x_1))}$$

Ex 101d – Π and Σ swapped, other resolution order

$$\dfrac{\dfrac{P(u, f(u)) \lor Q(u)_{\Pi} \qquad \neg Q(a)_{\Sigma}}{P(a, f(a))}\, u \mapsto a \qquad \neg P(x, y)_{\Sigma}}{\square}\, x \mapsto a, y \mapsto f(a)$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg Q(a)}\, y \mapsto a \qquad\qquad \bot}{\neg Q(a) \land \neg P(a, f(a))}\, x \mapsto a, y \mapsto f(a) \qquad\qquad \dfrac{\dfrac{\top \qquad \bot}{\exists x_1 \neg Q(x_1)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1, x_2) \land \neg Q(x_1))}$$

102 – similar to 101 but with intra-clause-set inferences

Ex 102a

$$\dfrac{\dfrac{P(f(x)) \lor Q(f(x), z)_\Sigma \qquad \neg P(y)_\Pi}{Q(f(x), z)} \qquad \dfrac{\neg Q(x_1, y) \lor R(y)_\Sigma \qquad \neg R(g(z_1))_\Pi}{\neg Q(x_1, g(z_1))}\, y \mapsto g(z_1), x_1 \mapsto f(x)}{\square}\, x_1 \mapsto f(x), z \mapsto g(z_1)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(f(x))} \qquad \dfrac{\bot \qquad \top}{R(g(z_1))}\, y \mapsto g(z_1), x_1 \mapsto f(x)}{P(f(x)) \lor R(g(z_1))}\, x_1 \mapsto f(x), z \mapsto g(z_1) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(x_1)} \qquad \dfrac{\bot \qquad \top}{\forall x_2 R(x_2)}}{\exists x_1 \forall x_2 (P(x_1) \lor R(x_2)) \quad \text{(order irrelevant!)}}$$

Ex 102b

$$\dfrac{\dfrac{P(f(x)) \lor Q(f(x), z)_\Sigma \qquad \neg P(y)_\Pi}{Q(f(x), z)} \qquad \dfrac{\neg Q(f(y), z_1) \lor R(y)_\Sigma \qquad \neg R(a)_\Pi}{\neg Q(f(a), z_1)}\, y \mapsto a}{\square}\, x \mapsto a, z \mapsto z_1$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(f(x))} \qquad \dfrac{\bot \qquad \top}{R(a)}\, y \mapsto a}{P(f(a)) \lor R(a)}\, x \mapsto a, z \mapsto z_1 \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(x_1)} \qquad \dfrac{\bot \qquad \top}{\forall x_2 R(x_2)}\, y \mapsto a}{\forall x_2 \exists x_1 (P(x_1) \lor R(x_2))}\, x \mapsto a, z \mapsto z_1$$

Example 103: variables in interpolant get unified and order might have to be changed

103a: length changes but old version is still valid, despite huang's algorithm doing a sort

$$\dfrac{\dfrac{\dfrac{Q(f(x)) \vee P(y) \vee R(x)_\Sigma \qquad \neg Q(y_1)_\Pi}{P(y) \vee R(x)}\, y_1 \mapsto f(x) \qquad \neg P(g(g(a)))_\Pi}{R(x)}\, y \mapsto g(g(a)) \qquad \neg R(g(g(a)))_\Pi}{\Box}\, x \mapsto g(g(a))$$

$$\dfrac{\dfrac{\dfrac{\bot \qquad \top}{Q(f(x))}\, y_1 \mapsto f(x) \qquad \top}{Q(f(x)) \vee P(g(g(a)))}\, y \mapsto g(g(a)) \qquad \top}{Q(f(g(g(a)))) \vee P(g(g(a))) \vee R(g(g(a)))}\, x \mapsto g(g(a)) \qquad\qquad \dfrac{\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 Q(x_1)} \qquad \top}{\exists x_1 \forall x_2 Q(x_1) \vee P(x_2)} \qquad \top}{X}$$

X:
Huang's algo gives:
$\forall x_2 \exists x_1 Q(x_1) \vee P(x_2) \vee R(x_2)$
Direct overbinding gives ($x_3 < x_1$, rest arbitrary):
$\forall x_3 \exists x_1 \forall x_2 Q(x_1) \vee P(x_2) \vee R(x_3)$

103b: length changes "uniformly"

$$\dfrac{\dfrac{\dfrac{Q(f(f(x))) \vee P(f(x)) \vee R(x)_\Sigma \qquad \neg Q(y_1)_\Pi}{P(f(x)) \vee R(x)}\, y_1 \mapsto f(f(x)) \qquad \neg P(y_2)_\Pi}{R(x)}\, y_2 \mapsto f(x) \qquad \neg R(g(a))_\Pi}{\Box}\, x \mapsto g(a)$$

$$\dfrac{\dfrac{\dfrac{\bot \qquad \top}{Q(f(f(x)))}\, y_1 \mapsto f(f(x)) \qquad \top}{Q(f(f(x))) \vee P(f(x))}\, y_2 \mapsto f(x) \qquad \top}{Q(f(f(g(a)))) \vee P(f(g(a))) \vee R(g(a))}\, x \mapsto g(a) \qquad\qquad \dfrac{\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 Q(x_1)} \qquad \top}{\exists x_2 \exists x_1 (Q(x_1) \vee P(x_2))} \qquad \top}{\forall x_3 \exists x_2 \exists (x_1 Q(x_1) \vee P(x_2) \vee R(x_3))}$$

Huang and direct overbinding somewhat coincide as $x_2 < x_1$ in both cases, and $x_3 < x_1$ and $x_3 < x_2$

103c: different variables, accidentally the same terms appear but no logical connection

$$\dfrac{P(a, x)_\Sigma \qquad \dfrac{\neg Q(a)_\Sigma \qquad \neg P(y, f(z)) \vee Q(z)_\Pi}{\neg P(y, f(a))}\, z \mapsto a}{\Box}\, y \mapsto a, x \mapsto f(a)$$

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg Q(a)}\, z \mapsto a}{P(a, f(a)) \wedge \neg Q(a)}\, y \mapsto a, x \mapsto f(a) \qquad\qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg Q(x_1)}}{\exists x_1 \forall x_2 (P(x_1, x_2) \wedge \neg Q(x_1))}$$

Error: no sorting requirement is just for Σ Again, Huang sorts, but no order is required.

SECOND ATTEMPT:

$$\dfrac{P(a)_\Sigma \qquad \dfrac{Q(z)_\Sigma \qquad \dfrac{\neg S(a)_\Sigma \qquad \neg P(y) \vee \neg Q(f({\color{red}x})) \vee S({\color{red}x})_\Pi}{\neg P(y) \vee \neg Q(f(a))}\, x \mapsto a}{\neg P(y)}\, z \mapsto f(a)}{\square}\, y \mapsto a$$

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg S(a)}\, x \mapsto a}{\neg S(a) \wedge Q(f(a))}\, z \mapsto f(a)}{P(a) \wedge \neg S(a) \wedge Q(f(a))}\, y \mapsto a$$

Huang:

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg S(x_1)}}{\exists x_1 \forall x_2 (\neg S(x_1) \wedge Q(x_2))}}{\exists x_1 \forall x_2 (P(x_1) \wedge \neg S(x_1) \wedge Q(x_2))}$$

$\forall x_1 \exists x_2 (\neg P(x_1) \vee S(x_1) \vee \neg Q(x_2))$

similar fail

$\Rightarrow$ anytime there is $P(a, f(a))$, either they have a dependency or they are not both differently colored (grey is uncolored)

for the record, direct method anyway:

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg S(x_1)}\, x \sim a}{\exists x_1 \forall x_2 \neg S(x_1) \wedge Q(x_2)}\, z \sim f(a); x_1 < x_2}{\exists x_1 \forall x_2 \exists x_3 P(x_3) \wedge \neg S(x_1) \wedge Q(x_2)}\, x_3 \sim a; x_3 \text{ need not be merged w } x_1$$

Example with ordering on both ancestors which where the merge forces a new ordering

202a – canonical

$$\dfrac{\dfrac{P(a,x_1) \lor R(y)_\Sigma \qquad \neg P(x, f(x))_\Pi}{R(y)} \; \begin{array}{c} x_1 \mapsto f(a) \\ x \mapsto a \end{array} \qquad \dfrac{Q(x_2, g(x_2)) \lor \neg R(u)_\Sigma \qquad \dfrac{\neg S(a)_\Sigma \qquad \neg Q(f(z), x_3) \lor S(z)_\Pi}{\neg Q(f(a), x_3)} \; z \mapsto a}{\neg R(u)} \; \begin{array}{c} x_2 \mapsto f(a), \\ x_3 \mapsto g(f(a)) \end{array}}{\square}$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a, f(a))} \; \begin{array}{c} x_1 \mapsto f(a) \\ x \mapsto a \end{array} \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg S(a)} \; z \mapsto a}{Q(f(a), g(f(a))) \land \neg S(a)} \; \begin{array}{c} x_2 \mapsto f(a), \\ x_3 \mapsto g(f(a)) \end{array}}{P(a, f(a)) \lor (Q(f(a), g(f(a))) \land \neg S(a))} \qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \forall x_2 P(x_1, x_2))} \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg S(x_1)}}{\exists x_1 \forall x_2 \exists x_3 Q(x_2, x_3) \land \neg S(x_1)}}{\exists x_1 \forall x_2 \exists x_3 P(x_1, x_2) \lor (Q(x_2, x_3)) \land \neg S(x_1))}$$

(Huang)

direct:

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \forall x_2 P(x_1, x_2))} \; \begin{array}{c} x_1 \sim a, x_2 \sim f(a) \\ x_1 < x_2 \end{array} \qquad \begin{array}{c} x_3 \sim a, x_4 \sim f(a), x_5 \sim g(f)a) \\ x_3 < x_4, x_4 < x_5 \end{array} \; \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_3 \neg S(x_3)} \; x_3 \sim a}{\exists x_3 \forall x_4 \exists x_5 Q(x_4, x_3) \land \neg S(x_3)}}{\exists x_1 \forall x_2 \exists x_3 P(x_1, x_2) \lor (Q(x_2, x_5) \land \neg S(x_3))} \; \begin{array}{c} x_3 \mapsto x_1, x_4 \mapsto x_2 \\ x_1 < x_2, x_2 < x_5 \end{array}$$

without merge in end:

$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5 P(x_1, x_2) \lor (Q(x_4, x_5) \land \neg S(x_3))$

202b – just a a lot of terms for random mass test

TODO

Example with transitive order constraint

203a

$$
\dfrac{\neg S(x_1)_\Pi \qquad \dfrac{\neg R(a)_\Pi \qquad \dfrac{\dfrac{R(x) \vee \neg P(f(x))_\Sigma \qquad P(z) \vee Q(g(z))_\Pi}{R(x) \vee Q(g(f(x)))}\, z \mapsto f(x) \qquad \neg Q(y) \vee S(h(y))_\Sigma}{R(x) \vee S(h(g(f(x))))}\, y \mapsto g(f(x))}{S(h(g(f(a))))}\, x \mapsto a}{\Box}\, x_1 \mapsto h(g(f(a)))
$$

$$
\dfrac{\top \qquad \dfrac{\top \qquad \dfrac{\dfrac{\bot \qquad \top}{\neg P(f(x))}\, z \mapsto f(x) \qquad \bot}{\neg Q(g(f(x))) \wedge \neg P(f(x))}\, y \mapsto g(f(x))}{\neg Q(g(f(a))) \wedge \neg P(f(a)) \vee R(a)}\, x \mapsto a}{\neg Q(g(f(a))) \wedge \neg P(f(a)) \vee R(a) \vee S(h(g(f(a))))}\, x_1 \mapsto h(g(f(a)))
$$

Huang:

$$
\dfrac{\top \qquad \dfrac{\top \qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \neg P(x_1)} \qquad \bot}{\exists x_1 \forall x_2 \neg(Q(x_2) \wedge \neg P(x_1))}}{\forall x_0 \exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0))}}{\forall x_0 \exists x_1 \forall x_2 \exists x_3 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0) \vee S(x_3))}
$$

Direct:

$$
\dfrac{\top \qquad \dfrac{\top \qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \neg P(x_1)}\, x_1 \sim f(x) \qquad \bot}{\exists x_1 \forall x_2 \neg Q(x_2)) \wedge \neg P(f(x))}\, x_2 \sim g(f(x)); x_1 < x_2}{\forall x_0 \exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0))}\, x_0 \sim a; x_0 < x_1, x_0 < x_2}{\forall x_0 \exists x_1 \forall x_2 \exists x_3 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0) \vee S(x_3))}\, x_3 \sim h(g(f(a))); x_0 < x_3, x_1 < x_3, x_2 < x_3
$$

misc examples

201a

$$\dfrac{\dfrac{P(x,y)\vee\neg Q(y)_\Sigma \qquad \neg P(a,y_2)_\Pi}{\neg Q(y)}\,x\mapsto a \qquad \dfrac{Q(f(z))\vee R(z)_\Sigma \qquad \neg R(a)_\Pi}{Q(f(a)}\,z\mapsto a}{\square}\,y\mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a,y)}\,x\mapsto a \qquad \dfrac{\bot \qquad \top}{R(a)}\,z\mapsto a}{P(a,f(a))\vee R(a)}\,y\mapsto f(a) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1 P(x_1,y)}\,x\mapsto a \qquad \dfrac{\bot \qquad \top}{\forall x_3 R(x_3)}\,z\mapsto a}{\forall x_3\forall x_1\exists x_2(P(x_1,x_2)\vee R(x_3))}\,y\mapsto f(a)$$

Huang would produce: $\forall x_1\exists x_2 P(x_1,x_2)\vee R(x_1)$

201b

$$\dfrac{\dfrac{P(x,f(y))\vee\neg Q(f(y))_\Sigma \qquad \neg P(a,y_2)_\Pi}{\neg Q(f(y))}\,x\mapsto a \qquad \dfrac{Q(f(z))\vee R(z)_\Sigma \qquad \neg R(a)_\Pi}{Q(f(a)}\,z\mapsto a}{\square}\,y\mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a,f(y))}\,x\mapsto a \qquad \dfrac{\bot \qquad \top}{R(a)}\,z\mapsto a}{P(a,f(a))\vee R(a)}\,y\mapsto a \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1\exists x_2 P(x_1,x_2)}\,x\mapsto a \qquad \dfrac{\bot \qquad \top}{\forall x_3 R(x_3)}\,z\mapsto a}{\forall x_3\forall x_1\exists x_2 P(x_1,x_2)\vee R(x_3)}\,y\mapsto f(a)$$

Huang would produce: $\forall x_1\exists x_2 P(x_1,x_2)\vee R(x_1)$