# Contents



## 0.1 random notes

- As long as every pair of literal is variable disjoint, the quantifier ordering is arbitrary (proof idea: establish that some ordering works, then pull quantifier inwards and back outwards in arbitrary order).
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  - lifted terms which contain variables are disjoint for different clauses, but ground lifted terms can be the same (which does not appear to be necessarily so!)
  - the resolved/factorised literal should be the same (else this kind of proof doesn't go through)

⟨1⟩ **Lemma 1.** $\Gamma \models \mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$.

⟨2⟩ **Lemma 2.** $\Gamma \models \forall \bar{x}\, \exists \bar{y}\, (\mathrm{LI}(C) \vee \mathrm{LI}_{\mathrm{cl}}(C))$.

*Proof.* By 1, $\Gamma \models \mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)$.
Hence $\Gamma \models \forall \bar{x}\, (\mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C))$.
and also $\Gamma \models \forall \bar{x}\, \exists \bar{y}\, \ell_{\Gamma}[\mathrm{LI}^{\Delta}(C) \vee \mathrm{LI}^{\Delta}_{\mathrm{cl}}(C)]$.
by some lemma then $\Gamma \models \forall \bar{x}\, \exists \bar{y}\, (\mathrm{LI}(C) \vee \mathrm{LI}_{\mathrm{cl}}(C))$. □

but can't invert this idea:
Let $\hat{\Delta} = \Gamma$ and $\hat{\Gamma} = \Delta$.
Then with $\hat{\pi}$ and 2: $\hat{\Gamma} \models \forall \bar{\hat{x}}\, \exists \bar{\hat{y}}\, (\mathrm{LI}(\bar{\pi}))$
Hence (some lemma) $\Delta \models \forall \bar{y}\, \exists \bar{x}\, (\neg\, \mathrm{LI}(\pi))$.
Hence $\Delta \models \neg \exists \bar{y}\, \forall \bar{x}\, (\mathrm{LI}(\pi))$.
need some consistent ordering, so possibly just prove that all work, because we need to shuffle a lot anyway