1 Attempt without P_P

current state:

lemma 2 and 3 seem to not work together. they would need some kind of common σ

Definition 1. Overline as in paper, replace Δ-terms $t_1, \ldots, t_n$ by respective fresh variables in parenthesis $\triangle$

NOTE: variables are *not* replaced by overline.

Lemma 2. *Let $t_1, \ldots, t_n$ be the maximal Δ-terms in C.*
Let $\sigma'(x) = \begin{cases} x_i & \text{if } x = x_j \text{ for some } j \text{ and } t_j\sigma = t_i \\ \overline{x\sigma} & \text{otherwise} \end{cases}$
Then $(\overline{C\sigma}(x_1, \ldots, x_n)) = (\overline{C}(x_1, \ldots, x_n))\sigma'$.

Proof. TODO
// the first case handles the case where σ replaces x and $P(f(x)) \in C$ where f is Δ-colored. $\square$

Lemma 3. *If $l\sigma = l'\sigma$, then $\bar{l}\sigma' = \bar{l'}\sigma'$ for σ' defined as follows: TODO*

Proof. TODO $\square$

Lemma 4. *// currently unused*
$(\overline{C}(x_1, \ldots, x_n))\sigma = (\overline{C\sigma'}(x_1, \ldots, x_n))$ *if σ does not change any of $x_1, \ldots, x_n$ or any of $t_1, \ldots, t_n$.*

it would work to fix substitutions of x_i by substituting t_i for that instead, as long as the result isn't another t_i, but this isn't actually relevant here.

Proposition 5. $\Gamma = \overline{\Gamma}(x_1, \ldots, x_n)$.

Proof. By definition, Δ-terms only appear in Δ and not in Γ. $\square$

Lemma 6. $\Gamma \models \overline{(\mathrm{PI}(C) \vee C)}(x_1, \ldots, x_n)$.

Proof. By induction on the resolution refutation.
Base case: Either $C \in \Gamma$, then it does not contain Δ-terms. Otherwise $C \in \Delta$ and $\mathrm{PI}(C) = \top$.
Induction step:

Resolution.

$$\frac{C_1 : D \vee l \qquad C_2 : E \vee \neg l'}{C : (D \vee E)\sigma} \quad l\sigma = l'\sigma$$

By the induction hypothesis, we can assume that:

$\Gamma \models \overline{\mathrm{PI}(C_1) \vee (D \vee l)}(x_1, \dots, x_n)$

$\Gamma \models \overline{\mathrm{PI}(C_2) \vee (E \vee \neg l')}(x_1, \dots, x_n)$

1. $\mathrm{PS}(l) \in \mathrm{L}(\Gamma) \setminus \mathrm{L}(\Delta)$: Then $\mathrm{PI}(C) = [\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2)]\sigma$.

   We show that $\Gamma \models \overline{(\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2))\sigma \vee (D \vee E)\sigma}(x_1, \dots, x_n)$, i.e. $\Gamma \models \overline{(\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2) \vee D \vee E)\sigma}(x_1, \dots, x_n)$. This is by lemma 2 with σ' as in the lemma equivalent to $\Gamma \models \overline{(\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2) \vee D \vee E)}(x_1, \dots, x_n)\sigma'$.

   By Lemma 11 (Huang) and the induction hypothesis,

   $\Gamma \models \overline{\mathrm{PI}(C_1)} \vee \overline{D} \vee \overline{l}$

   $\Gamma \models \overline{\mathrm{PI}(C_2)} \vee \overline{E} \vee \neg \overline{l'}$

   By lemma 3 and since $l\sigma = l'\sigma$, $\overline{l}\sigma'' = \overline{l'}\sigma''$.

   Hence $\Gamma \models (\overline{\mathrm{PI}(C_1)} \vee \overline{D} \vee \overline{\mathrm{PI}(C_2)} \vee \overline{E})\sigma''$ and again by Lemma 11 (Huang), $\Gamma \models \overline{\mathrm{PI}(C_1) \vee D \vee \mathrm{PI}(C_2) \vee E}\sigma''$.

   TODO: show that from this, it follows that: $\Gamma \models \overline{(\mathrm{PI}(C_1) \vee \mathrm{PI}(C_2))\sigma \vee (D \vee E)}(x_1, \dots, x_n)\sigma'$,

2. $\mathrm{PS}(l) \in \mathrm{L}(\Delta) \setminus \mathrm{L}(\Gamma)$: Then $\mathrm{PI}(C) = [\mathrm{PI}(C_1) \wedge \mathrm{PI}(C_2)]\sigma$.

   We show that $\Gamma \models \overline{((\mathrm{PI}(C_1) \wedge \mathrm{PI}(C_2)) \vee D \vee E)\sigma}(x_1, \dots, x_n)$. By lemma 2 with σ' as in the lemma, $\Gamma \models \overline{((\mathrm{PI}(C_1) \wedge \mathrm{PI}(C_2)) \vee D \vee E)}(x_1, \dots, x_n)\sigma'$.

   TODO

Paramodulation.

$$\frac{C_1 : D \vee s = t \qquad C_2 : E[r]}{C : (D \vee E[t])\sigma} \quad \sigma = \mathrm{mgu}(s, r)$$

By the induction hypothesis, we have:

$\Gamma \models \overline{\mathrm{PI}(C_1) \vee (D \vee s = t)}$

$\Gamma \models \overline{\mathrm{PI}(C_2) \vee (E[r])}$

easy case: $\mathrm{PI}(C) = [(s = t \wedge \mathrm{PI}(C_2)) \vee (s \neq t \wedge \mathrm{PI}(C_1))]\sigma$

to show: $\Gamma \models \overline{[((s = t \wedge \mathrm{PI}(C_2)) \vee (s \neq t \wedge \mathrm{PI}(C_1))) \vee (D \vee E[t])]\sigma}$

proof idea: either $s = t$, then also $\mathrm{PI}(C_2)$, or else $s \neq t$, but then also $\mathrm{PI}(C_1)$

by lemma 2 for σ' as in lemma, $\Gamma \models \overline{((s = t \wedge \mathrm{PI}(C_2)) \vee (s \neq t \wedge \mathrm{PI}(C_1))) \vee (D \vee E[t])}\sigma'$

by lemma 11 (huang) $\Gamma \models [((\overline{s} = \overline{t} \wedge \overline{\mathrm{PI}(C_2)}) \vee (\overline{s \neq t} \wedge \overline{\mathrm{PI}(C_1)})) \vee (\overline{D} \vee \overline{E[t]})]\sigma'$

reformulate: $\Gamma \models ((\overline{s}\sigma' = \overline{t}\sigma' \wedge \overline{\mathrm{PI}(C_2)}\sigma') \vee (\overline{s}\sigma' \neq \overline{t}\sigma' \wedge \overline{\mathrm{PI}(C_1)}\sigma')) \vee (\overline{D}\sigma' \vee \overline{E[t]}\sigma')$

By the rule: $s\sigma = r\sigma$, hence also $\overline{s\sigma} = \overline{r\sigma}$ and $\overline{s}\sigma' = \overline{r}\sigma'$ REALLY TRUE? – think so…

Suppose $M \models \Gamma$ and $M \not\models (\overline{D}\sigma' \vee \overline{E[t]}\sigma')$.

Suppose $M \models \overline{s}\sigma' = \overline{t}\sigma'$.

By induction hypothesis (and lemma 11 (huang) and adding the substitution σ'), $\Gamma \models \overline{\mathrm{PI}(C_2)}\sigma' \vee \overline{(E[r])}\sigma'$.

However by assumption $\Gamma \not\models \overline{E[t]}\sigma'$.

Hence $\Gamma \not\models \overline{E[s]}\sigma'$, and $\Gamma \not\models \overline{E[r]}\sigma'$. Therefore $\Gamma \models \overline{\mathrm{PI}(C_2)}\sigma'$.

Suppose on the other hand $M \models \overline{s}\sigma' \neq \overline{t}\sigma'$.

By the induction hypothesis, $M \models \overline{\mathrm{PI}(C_1)}\sigma' \vee (\overline{D}\sigma' \vee (\overline{s} = \overline{t})\sigma')$, hence then $M \models \overline{\mathrm{PI}(C_1)}\sigma'$.

Consequently, $M \models (\overline{s}\sigma' \neq \overline{t}\sigma' \wedge \overline{\mathrm{PI}(C_1)}\sigma') \vee (\overline{s}\sigma' = \overline{t}\sigma' \wedge \overline{\mathrm{PI}(C_2)}\sigma')$.

By lemma 11 (huang), $M \models \overline{(s \neq t \wedge \mathrm{PI}(C_1) \vee (s = t \wedge \mathrm{PI}(C_2))}\sigma'$.

Hence $\Gamma \models \overline{(s \neq t \wedge \mathrm{PI}(C_1) \vee (s = t \wedge \mathrm{PI}(C_2))}\sigma' \vee (\overline{D} \vee \overline{E[t]})\sigma')$.

IS THIS REALLY WHAT I NEED TO SHOW?

$\square$