Example 101: canonical examples for overbinding w.r.t. order

Here, the second lower prooftree shows what huang would produce.

Ex 101a

$$\frac{\dfrac{\overset{\Sigma}{P({\color{red}u}, f({\color{red}u})) \vee Q({\color{red}u})} \qquad \overset{\Pi}{\neg Q(a)}}{P(a, f(a))}\; u \mapsto a \qquad \overset{\Pi}{\neg P(x, y)}}{\square}\; x \mapsto a, y \mapsto f(a)$$

$$\frac{\dfrac{\bot \qquad \top}{Q(a)}\; u \mapsto a \qquad \top}{P(a, f(a)) \vee Q(a)}\; x \mapsto a, y \mapsto f(a) \qquad\qquad \frac{\dfrac{\bot \qquad \top}{\forall x_1 Q(x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1, x_2) \vee Q(x_1))}$$

Direct overbinding would not work without merging same variables!: $\forall x_1 \forall x_2 \exists x_3 (P(x_2, x_3) \vee Q(x_1))$
counterexample: $Q \sim \{0\}, P \sim \{(1, 0)\}$
Direct overbinding would work when considering original dependencies as highlighted above

Ex 101b – other resolution order

$$\frac{\dfrac{\overset{\Sigma}{P(u, f(u)) \vee Q(u)} \qquad \overset{\Pi}{\neg P(x, y)}}{Q(u))}\; y \mapsto f(u), x \mapsto u \qquad \overset{\Pi}{\neg Q(a)}}{\square}\; u \mapsto a$$

$$\frac{\dfrac{\bot \qquad \top}{P(u, f(u))}\; x \mapsto f(u), x \mapsto u \qquad \top}{P(a, f(a)) \vee Q(a)}\; u \mapsto a \qquad\qquad \frac{\dfrac{\bot \qquad \top}{\exists x_1 P(u, x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1, x_2) \vee Q(x_1))}\; u \mapsto a$$

Ex 101c – Π and Σ swapped

$$\frac{\dfrac{\overset{\Pi}{P(u, f(u)) \vee Q(u)} \qquad \overset{\Sigma}{\neg P(x, y)}}{Q(u))}\; y \mapsto f(u), x \mapsto u \qquad \overset{\Sigma}{\neg Q(a)}}{\square}\; u \mapsto a$$

$$\frac{\dfrac{\top \qquad \bot}{\neg P(u, f(u))}\; x \mapsto f(u), x \mapsto u \qquad \bot}{\neg P(a, f(a)) \wedge \neg Q(a)}\; u \mapsto a \qquad\qquad \frac{\dfrac{\top \qquad \bot}{\forall x_2 \neg P(u, x_2)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1, x_2) \wedge \neg Q(x_1))}$$

Ex 101d – Π and Σ swapped, other resolution order

$$\frac{\dfrac{\overset{\Pi}{P(u, f(u)) \vee Q(u)} \qquad \overset{\Sigma}{\neg Q(a)}}{P(a, f(a))}\; u \mapsto a \qquad \overset{\Sigma}{\neg P(x, y)}}{\square}\; x \mapsto a, y \mapsto f(a)$$

$$\cfrac{\cfrac{\top \qquad \bot}{\neg Q(a)}\, y \mapsto a \qquad\qquad \bot}{\neg Q(a) \wedge \neg P(a, f(a))}\, x \mapsto a, y \mapsto f(a) \qquad\qquad \cfrac{\cfrac{\top \qquad \bot}{\exists x_1 \neg Q(x_1)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1, x_2) \wedge \neg Q(x_1))}$$

102 – similar to 101 but with intra-clause-set inferences

Ex 102a

$$\cfrac{\cfrac{P(f(x)) \overset{\Sigma}{\vee} Q(f(x), z) \qquad \overset{\Pi}{\neg P(y)}}{Q(f(x), z)} \qquad \cfrac{\neg Q(x_1, \overset{\Sigma}{y}) \vee R(y) \qquad \overset{\Pi}{\neg R(g(z_1))}}{\neg Q(x_1, g(z_1))}\, y \mapsto g(z_1), x_1 \mapsto f(x)}{\square}\, x_1 \mapsto f(x), z \mapsto g(z_1)$$

$$\cfrac{\cfrac{\bot \qquad \top}{P(f(x))} \qquad \cfrac{\bot \qquad \top}{R(g(z_1))}\, y \mapsto g(z_1), x_1 \mapsto f(x)}{P(f(x)) \vee R(g(z_1))}\, x_1 \mapsto f(x), z \mapsto g(z_1) \qquad\qquad \cfrac{\cfrac{\bot \qquad \top}{\exists x_1 P(x_1)} \qquad \cfrac{\bot \qquad \top}{\forall x_2 R(x_2)}}{\exists x_1 \forall x_2 (P(x_1) \vee R(x_2)) \quad \text{(order irrelevant!)}}$$

Ex 102b

$$\cfrac{\cfrac{P(f(x)) \overset{\Sigma}{\vee} Q(f(x), z) \qquad \overset{\Pi}{\neg P(y)}}{Q(f(x), z)} \qquad \cfrac{\neg Q(f(y), \overset{\Sigma}{z_1}) \vee R(y) \qquad \overset{\Pi}{\neg R(a)}}{\neg Q(f(a), z_1)}\, y \mapsto a}{\square}\, x \mapsto a, z \mapsto z_1$$

$$\cfrac{\cfrac{\bot \qquad \top}{P(f(x))} \qquad \cfrac{\bot \qquad \top}{R(a)}\, y \mapsto a}{P(f(a)) \vee R(a)}\, x \mapsto a, z \mapsto z_1 \qquad\qquad \cfrac{\cfrac{\bot \qquad \top}{\exists x_1 P(x_1)} \qquad \cfrac{\bot \qquad \top}{\forall x_2 R(x_2)}\, y \mapsto a}{\forall x_2 \exists x_1 (P(x_1) \vee R(x_2))}\, x \mapsto a, z \mapsto z_1$$

direct:

$$\cfrac{\cfrac{\bot \qquad \top}{\exists x_1 P(x_1)}\, x_1 \sim f(x) \qquad \cfrac{\bot \qquad \top}{\forall x_2 R(x_2)}\, x_2 \sim a}{\exists x_1 \forall x_2 (P(x_1) \vee R(x_2))}\, \text{order irrelevant!}$$

Ex 102b' with Q grey

$$\dfrac{\dfrac{\overset{\Sigma}{P(f({\color{red}x})) \lor Q(f({\color{red}x}),z)} \qquad \overset{\Pi}{\neg P(y)}}{Q(f(x),z)} \qquad \dfrac{\overset{\Sigma}{\neg Q(f({\color{blue}y}),z_1) \lor R({\color{blue}y})} \qquad \overset{\Pi}{\neg R(a)}}{\neg Q(f(a),z_1)}\ y \mapsto a}{\Box}\ x \mapsto a, z_1 \mapsto z$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(f(x))} \qquad \dfrac{\bot \qquad \top}{R(a)}\ y \mapsto a}{(\neg Q(f(a),z) \land P(f(a))) \lor (Q(f(a),z) \land R(a))}\ x \mapsto a, z_1 \mapsto z$$

Huang:

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_2 P(x_2)} \qquad \dfrac{\bot \qquad \top}{\forall x_1 R(x_1)}\ y \mapsto a}{\forall x_1 \exists x_2 (\neg Q(x_2,z) \land P(x_2)) \lor (Q(x_2,z) \land R(x_1))}\ x \mapsto a, z_1 \mapsto z$$

direct:

$$\dfrac{\dfrac{\dfrac{\bot \qquad \top}{\exists x_2 P(x_2)}\ x_2 \sim f(x) \qquad \dfrac{\bot \qquad \top}{\forall x_1 R(x_1)}\ x_1 \sim a}{\forall x_1 \exists x_2 \exists x_3 (\neg Q(x_3,z) \land P(x_2)) \lor (Q(x_3,z) \land R(x_1))}\ x_3 \sim f(a); x_2 \parallel x_3, x_1 < x_3}{\dfrac{\text{OR: } \exists x_2 \forall x_1 \exists x_3 (\neg Q(x_3,z) \land P(x_2)) \lor (Q(x_3,z) \land R(x_1))}{\text{OR: } \exists x_1 \exists x_3 \forall x_2 (\neg Q(x_3,z) \land P(x_2)) \lor (Q(x_3,z) \land R(x_1))}}$$

Huang-Version passt weil man für existenzielle quantoren eh gleichen term einsetzt

Example 103: variables in interpolant get unified and order might have to be changed

103a: length changes but old version is still valid, despite huang's algorithm doing a sort

$$\dfrac{\dfrac{\dfrac{\overset{\Sigma}{Q(f({\color{red}x})) \lor P(y) \lor R({\color{red}x})} \qquad \overset{\Pi}{\neg Q(y_1)}}{P(y) \lor R(x)}\ y_1 \mapsto f(x) \qquad \overset{\Pi}{\neg P(h(g(a)))}}{R(x)}\ y \mapsto h(g(a)) \qquad \overset{\Pi}{\neg R(g(g(a)))}}{\Box}\ x \mapsto g(g(a))$$

$$\dfrac{\dfrac{\dfrac{\bot \qquad \top}{Q(f(x))}\ y_1 \mapsto f(x) \qquad \top}{Q(f(x)) \lor P(h(g(a)))}\ y \mapsto h(g(a)) \qquad \top}{Q(f(g(g(a)))) \lor P(h(g(a))) \lor R(g(g(a)))}\ x \mapsto g(g(a)) \qquad \dfrac{\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 Q(x_1)} \qquad \top}{\exists x_1 \forall x_2 (Q(x_1) \lor P(x_2))} \qquad \top}{X}$$

X:
Huang's algo gives:
$\forall x_2 \forall x_3 \exists x_1 (Q(x_1) \lor P(x_2) \lor R(x_3))$
Direct overbinding gives: $x_3 < x_1$, rest arbitrary, hence:
$\forall x_3 \exists x_1 \forall x_2 (Q(x_1) \lor P(x_2) \lor R(x_3))$ <- this you do not get with huang
$\forall x_2 \forall x_3 \exists x_1 (Q(x_1) \lor P(x_2) \lor R(x_3))$
$\forall x_3 \forall x_2 \exists x_1 (Q(x_1) \lor P(x_2) \lor R(x_3))$

103b: length changes "uniformly"

$$\dfrac{\dfrac{\dfrac{\overset{\Sigma}{Q(f(f({\color{red}x}))) \lor P(f({\color{red}x})) \lor R({\color{red}x})} \qquad \overset{\Pi}{\neg Q(y_1)}}{P(f(x)) \lor R(x)}\ y_1 \mapsto f(f(x)) \qquad \overset{\Pi}{\neg P(y_2)}}{R(x)}\ y_2 \mapsto f(x) \qquad \overset{\Pi}{\neg R(g(a))}}{\Box}\ x \mapsto g(a)$$

$$\dfrac{\dfrac{\dfrac{\bot \qquad \top}{Q(f(f(x)))}\ y_1 \mapsto f(f(x)) \qquad \top}{Q(f(f(x))) \lor P(f(x))}\ y_2 \mapsto f(x) \qquad \top}{Q(f(f(g(a)))) \lor P(f(g(a))) \lor R(g(a))}\ x \mapsto g(a) \qquad \dfrac{\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 Q(x_1)} \qquad \top}{\exists x_2 \exists x_1 (Q(x_1) \lor P(x_2))} \qquad \top}{\forall x_3 \exists x_2 \exists x_1 (Q(x_1) \lor P(x_2) \lor R(x_3))}$$

Huang and direct overbinding somewhat coincide as $x_2 < x_1$ in both cases, and $x_3 < x_1$ and $x_3 < x_2$

103c: Failed attempt: different variables, accidentally the same terms appear but no logical connection

$$\dfrac{\overset{\Sigma}{P(a, x)} \qquad \dfrac{\overset{\Sigma}{\neg Q(a)} \qquad \overset{\Pi}{\neg P(y, f({\color{red}z})) \lor Q({\color{red}z})}}{\neg P(y, f(a))}\ z \mapsto a}{\Box}\ y \mapsto a, x \mapsto f(a)$$

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg Q(a)}\, z \mapsto a}{P(a, f(a)) \wedge \neg Q(a)}\, y \mapsto a, x \mapsto f(a) \qquad\qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg Q(x_1)}}{\exists x_1 \forall x_2 (P(x_1, x_2) \wedge \neg Q(x_1))}$$

order required for Π

direct:

$$\dfrac{\dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg Q(x_1)}\, x_1 \sim a}{\exists x_1 \exists x_2 \forall x_3 (P(x_2, x_3) \wedge \neg Q(x_1))}\, x_2 \sim a, x_3 \sim f(a); x_1 < x_3}{\text{OR: } \exists x_1 \forall x_3 \exists x_2 (P(x_2, x_3) \wedge \neg Q(x_1))}$$

invariant:

$$\dfrac{\dfrac{\exists x_2 (P(x_2, x) \vee \bot) \qquad \dfrac{\exists x_1 (Q(x_1) \vee \bot) \qquad \forall x_3 ((\neg P(y, x_3) \vee Q(z)) \vee \top)}{\exists x_1 \forall x_3 \neg P(y, x_3) \vee \neg Q(x_1)}\, x_1 \sim a}{\exists x_1 \exists x_2 \forall x_3 (P(x_2, x_3) \wedge \neg Q(x_1))}\, x_2 \sim a, x_3 \sim f(a); x_1 < x_3}{\text{OR: } \exists x_1 \forall x_3 \exists x_2 (P(x_2, x_3) \wedge \neg Q(x_1))}$$

invariant in other resolution order

$$\dfrac{\dfrac{\bot \qquad \dfrac{\bot \qquad \top}{Q(z) \vee \exists x_2 \forall x_3 P(x_2, x_3)}\, x_2 \sim a, x_3 \sim f(z)}{\exists x_1 \exists x_2 \forall x_3 (P(x_2, x_3) \wedge \neg Q(x_1))}\, x_1 \sim a; x_1 < x_3}{\text{OR: } \exists x_1 \forall x_3 \exists x_2 (P(x_2, x_3) \wedge \neg Q(x_1))}$$

invariant if Σ and Π swapped:

$$\dfrac{\dfrac{\bot \qquad \dfrac{\top \qquad \bot}{\neg P(y, f(x_1)) \vee \forall x_1 Q(x_1)}\, x_1 \sim a}{\forall x_1 \forall x_2 \exists x_3 (\neg P(x_2, x_3) \vee Q(x_1))}\, x_2 \sim a, x_3 \sim f(a); x_1 < x_3}{\text{OR: } \forall x_1 \exists x_3 \forall x_2 (\neg P(x_2, x_3) \vee Q(x_1))}$$

SECOND ATTEMPT:

$$\dfrac{\overset{\Sigma}{P(a)} \qquad \dfrac{\overset{\Sigma}{Q(z)} \qquad \dfrac{\overset{\Sigma}{\neg S(a)} \qquad \overset{\Pi}{\neg P(y) \vee \neg Q(f(x)) \vee S(x)}}{\neg P(y) \vee \neg Q(f(a))}\, x \mapsto a}{\neg P(y)}\, z \mapsto f(a)}{\square}\, y \mapsto a$$

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg S(a)}\, x \mapsto a}{\neg S(a) \wedge Q(f(a))}\, z \mapsto f(a)}{P(a) \wedge \neg S(a) \wedge Q(f(a))}\, y \mapsto a$$

Huang:

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg S(x_1)}}{\exists x_1 \forall x_2 (\neg S(x_1) \wedge Q(x_2))}}{\exists x_1 \forall x_2 (P(x_1) \wedge \neg S(x_1) \wedge Q(x_2))}$$

$\forall x_1 \exists x_2 (\neg P(x_1) \vee S(x_1) \vee \neg Q(x_2))$

similar fail

$\Rightarrow$ anytime there is $P(a, f(a))$, either they have a dependency or they are not both differently colored (grey is uncolored)

for the record, direct method anyway:

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg S(x_1)}\, x \sim a}{\exists x_1 \forall x_2 \neg S(x_1) \wedge Q(x_2)}\, z \sim f(a); x_1 < x_2}{\exists x_1 \forall x_2 \exists x_3 P(x_3) \wedge \neg S(x_1) \wedge Q(x_2)}\, x_3 \sim a; x_3 \text{ need not be merged w } x_1$$

Example: ordering on both ancestors where the merge forces a new ordering

202a – canonical

$$\dfrac{\dfrac{P(a, \overset{\Sigma}{x_1)} \vee R(y) \qquad \neg P(\overset{\Pi}{x}, fx)}{R(y)} \begin{smallmatrix} x_1 \mapsto fa \\ x \mapsto a \end{smallmatrix} \qquad \dfrac{Q(x_2, \overset{\Sigma}{g x_2)} \vee \neg R(u) \qquad \dfrac{\overset{\Sigma}{\neg S(a)} \qquad \overset{\Pi}{\neg Q(fz, x_3) \vee S(z)}}{\neg Q(fa, x_3)} z \mapsto a}{\neg R(u)} \begin{smallmatrix} x_2 \mapsto fa, \\ x_3 \mapsto gfa \end{smallmatrix}}{\square}$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a, f(a))} \begin{smallmatrix} x_1 \mapsto f(a) \\ x \mapsto a \end{smallmatrix} \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg S(a)} z \mapsto a}{Q(f(a), g(f(a))) \wedge \neg S(a)} \begin{smallmatrix} x_2 \mapsto f(a), \\ x_3 \mapsto g(f(a)) \end{smallmatrix}}{P(a, f(a)) \vee (Q(f(a), g(f(a))) \wedge \neg S(a))}$$

Huang

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \forall x_2 P(x_1, x_2))} \qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg S(x_1)}}{\exists x_1 \forall x_2 \exists x_3 Q(x_2, x_3) \wedge \neg S(x_1)}}{\exists x_1 \forall x_2 \exists x_3 P(x_1, x_2) \vee (Q(x_2, x_3)) \wedge \neg S(x_1))}$$

direct:

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \forall x_2 P(x_1, x_2))} \begin{smallmatrix} x_1 \sim a, x_2 \sim fa \\ x_1 < x_2 \end{smallmatrix} \qquad \begin{smallmatrix} x_3 \sim a, x_4 \sim fa, x_5 \sim gfa) \\ x_3 < x_4, x_4 < x_5 \end{smallmatrix} \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_3 \neg S(x_3)} x_3 \sim a}{\exists x_3 \forall x_4 \exists x_5 Q(x_4, x_3) \wedge \neg S(x_3)}}{\exists x_1 \forall x_2 \exists x_5 P(x_1, x_2) \vee (Q(x_2, x_5) \wedge \neg S(x_5))} \begin{smallmatrix} x_3 \mapsto x_1, x_4 \mapsto x_2 \\ x_1 < x_2, x_2 < x_5 \end{smallmatrix}$$

without merge in end: $(x_1 < x_2, x_3 < x_4, x_4 < x_5)$
$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5 P(x_1, x_2) \vee (Q(x_4, x_5) \wedge \neg S(x_3))$
$\exists x_3 \forall x_4 \exists x_5 \exists x_1 \forall x_2 P(x_1, x_2) \vee (Q(x_4, x_5) \wedge \neg S(x_3))$
(also interwoven ones appear to work)

202b – just a a lot of terms for random mass test

TODO

Example with transitive order constraint

203a

$$
\dfrac{\overset{\Pi}{\neg S(x_1)} \qquad \dfrac{\overset{\Pi}{\neg R(a)} \qquad \dfrac{\dfrac{R(x) \vee \overset{\Sigma}{\neg} P(f(x)) \qquad P(z) \overset{\Pi}{\vee} Q(g(z))}{R(x) \vee Q(g(f(x)))}\, z \mapsto f(x) \qquad \neg Q(y) \overset{\Sigma}{\vee} S(h(y))}{R(x) \vee S(h(g(f(x))))}\, y \mapsto g(f(x))}{S(h(g(f(a))))}\, x \mapsto a}{\square}\, x_1 \mapsto h(g(f(a)))
$$

$$
\dfrac{\top \qquad \dfrac{\top \qquad \dfrac{\dfrac{\bot \qquad \top}{\neg P(f(x))}\, z \mapsto f(x) \qquad \bot}{\neg Q(g(f(x))) \wedge \neg P(f(x))}\, y \mapsto g(f(x))}{\neg Q(g(f(a))) \wedge \neg P(f(a)) \vee R(a)}\, x \mapsto a}{\neg Q(g(f(a))) \wedge \neg P(f(a)) \vee R(a) \vee S(h(g(f(a))))}\, x_1 \mapsto h(g(f(a)))
$$

Huang:

$$
\dfrac{\top \qquad \dfrac{\top \qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \neg P(x_1)} \qquad \bot}{\exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(x_1))}}{\forall x_0 \exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0))}}{\forall x_0 \exists x_1 \forall x_2 \exists x_3 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0) \vee S(x_3))}
$$

Direct:

$$
\dfrac{\top \qquad \dfrac{\top \qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 \neg P(x_1)}\, x_1 \sim f(x) \qquad \bot}{\exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(f(x)))}\, x_2 \sim g(f(x)); x_1 < x_2}{\forall x_0 \exists x_1 \forall x_2 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0))}\, x_0 \sim a; x_0 < x_1, x_0 < x_2}{\forall x_0 \exists x_1 \forall x_2 \exists x_3 (\neg Q(x_2) \wedge \neg P(x_1) \vee R(x_0) \vee S(x_3))}\, x_3 \sim h(g(f(a))); x_0 < x_3, x_1 < x_3, x_2 < x_3
$$

Example where variables are not the outermost symbol but order is still relevant

204a

$\Sigma = \{P(f(x), g(f(x))\}$
$\Pi = \{P(f(a), y\}$
$\Rightarrow \forall x_1 \exists x_2 P(f(x_1), x_2)$

misc examples

201a

$$\dfrac{\dfrac{P(x,y) \overset{\Sigma}{\vee} \neg Q(y) \qquad \overset{\Pi}{\neg P(a,y_2)}}{\neg Q(y)}\;x \mapsto a \qquad \dfrac{\overset{\Sigma}{Q(f(z))} \vee R(z) \qquad \overset{\Pi}{\neg R(a)}}{Q(f(a))}\;z \mapsto a}{\Box}\;y \mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a,y)}\;x \mapsto a \qquad \dfrac{\bot \qquad \top}{R(a)}\;z \mapsto a}{P(a,f(a)) \vee R(a)}\;y \mapsto f(a) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1 P(x_1,y)}\;x \mapsto a \qquad \dfrac{\bot \qquad \top}{\forall x_3 R(x_3)}\;z \mapsto a}{\forall x_3 \forall x_1 \exists x_2 (P(x_1,x_2) \vee R(x_3))}\;y \mapsto f(a)$$

Huang would produce: $\forall x_1 \exists x_2 P(x_1,x_2) \vee R(x_1)$

201b

$$\dfrac{\dfrac{P(x,f(y)) \overset{\Sigma}{\vee} \neg Q(f(y)) \qquad \overset{\Pi}{\neg P(a,y_2)}}{\neg Q(f(y))}\;x \mapsto a \qquad \dfrac{\overset{\Sigma}{Q(f(z))} \vee R(z) \qquad \overset{\Pi}{\neg R(a)}}{Q(f(a))}\;z \mapsto a}{\Box}\;y \mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a,f(y))}\;x \mapsto a \qquad \dfrac{\bot \qquad \top}{R(a)}\;z \mapsto a}{P(a,f(a)) \vee R(a)}\;y \mapsto a \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1 \exists x_2 P(x_1,x_2)}\;x \mapsto a \qquad \dfrac{\bot \qquad \top}{\forall x_3 R(x_3)}\;z \mapsto a}{\forall x_3 \forall x_1 \exists x_2 P(x_1,x_2) \vee R(x_3)}\;y \mapsto f(a)$$

Huang would produce: $\forall x_1 \exists x_2 P(x_1,x_2) \vee R(x_1)$