border cases: arrows not within supposedly connected components

211a

$$\frac{Q(x) \vee P(f(x,a)) \qquad \neg Q(y) \vee R(f(y,b))}{Q(x) \mid P(f(x,a)) \vee R(f(x,b))}$$

$\Rightarrow$ no arr between P and R

211a'

$$\cfrac{\cfrac{Q(x) \vee \overset{\Sigma}{P}(f(x,a)) \qquad \neg Q(y) \vee \overset{\Sigma}{R}(f(y,b))}{Q(x) \mid P(f(x,a)) \vee R(f(x,b))} \qquad \cfrac{\neg P(f(u,\overset{\Sigma}{z})) \vee S(u)) \qquad \neg\overset{\Pi}{S}(c)}{S(c) \mid \neg P(f(c,z))}}{(P(f(c,a) \wedge S(c)) \vee (\neg P(f(c,a)) \wedge Q(c))) \mid R(f(c,b))}$$

$c \sim x_1$ $\qquad f(c,a) \sim y_2$ $\qquad f(c,b) \sim y_3$

$(P(y_2) \wedge S(x_1)) \vee (\neg P(y_2) \wedge Q(x_1)) \mid R(y_3)$ NOTE: arrow merge on resolution is not drawn here (but is necessary)

$\forall x_1 \exists y_2 \exists y_3$

this is not valid per se as the left hand side only contains Σ-formulas, but it probably could be fixed by adding some Π-inferences

Lesson is: no extra arrows needed, if a term enters, it does so via x, but there is a variable from the grey x to both colored x.

211b

$$\frac{Q(x) \vee P(f(x)) \qquad R(y) \vee \neg P(f(y))}{P(f(x)) \mid Q(x) \vee R(x)}$$

$\Rightarrow$ no arr between Q and R

NB: should be fixed by backwards merging special case

211b'

$$\cfrac{\cfrac{Q(x) \vee \overset{\Sigma}{P}(f(x)) \qquad R(y) \vee \overset{\Sigma}{\neg} P(f(y))}{P(f(x)) \mid Q(x) \vee R(x)} \qquad \neg\overset{\Pi}{Q}(a)}{P(f(a)) \vee Q(a) \mid R(a)}$$

WRONG: conjecture: Q and R do not need arrows as they are lifted by the same variable anyway, so constraints on Q do the work

211c

$$\frac{Q(f(x\overset{\Sigma}{)}) \vee R(x) \qquad \neg \overset{\Pi}{R}(g(y))}{Q(f(g(y))) \vee R(g(y))}$$

Have same var but no merge arrow. The whole term $g(y)$ is somehow the "travelling term", there is no "renaming".

211d – problem cases with lemma grey->colored

currently not clear what the connetion between the arguments of R on the RHS is If we use factorisation, not sure how to handle yet, but could be like: $R(t[x], s[x]) \vee Q(x)$

$$\frac{Q(y) \vee Q'(z) \vee P(f(y)) \vee R(g(y), g'(z)) \qquad \neg R(g(h(x)), g'(x))}{R(g(h(x)), g'(x)) \mid Q(h(x)) \vee Q'(x) \vee P(f(h(x)))}\ y \mapsto h(x), z \mapsto x$$

NB: this is different since x occurs grey as well (example not finished)

Problem case 1: x grey and colored, but not connection

$$\frac{Q'(z) \vee P(f(y)) \vee R(g(f(y)), g'(z)) \qquad \neg R(g(f(h(x))), g'(x))}{R(g(f(h(x))), g'(x)) \mid Q(x) \vee P(f(h(x)))}\ y \mapsto h(x), z \mapsto x$$

NB: no connection between Q and P

$\Rightarrow$ backwards merging

Problem case 2: x colored and colored, not sure what the connection is supposed to be

$$\frac{Q'(k(z)) \vee P(f(y)) \vee R(g(y), g'(z)) \qquad \neg R(g(h(x)), g'(x))}{R(g(h(x)), g'(x)) \mid Q'(k(x)) \vee P(f(h(x)))}\ y \mapsto h(x), z \mapsto x$$

lifting var doesn't correspond to actual term exactly in context of unifier arrows

212a

$$\dfrac{Q(u) \lor \neg \overset{\Sigma}{P}(h(u)) \qquad \dfrac{\dfrac{\overset{\Pi}{R(g(x))} \lor S(x) \qquad \neg \overset{\Pi}{S}(a)}{S(a) \mid\mid R(g(a))}\; x \mapsto a \qquad \overset{\Sigma}{P(h(f(y)))} \lor \neg R(y)}{S(a) \mid R(g(a)) \mid P(h(f(g(a))))}\; y \mapsto g(a)}{S(a) \mid R(g(a)), P(h(f(g(a)))) \mid Q(f(g(a)))}\; u \mapsto g(f(g(a)))$$

lifted:

$$\dfrac{Q(u) \lor \neg \overset{\Sigma}{P}(y_{h(u)}) \qquad \dfrac{\dfrac{\overset{\Pi}{R(x_{g(x)})} \lor S(x) \qquad \neg \overset{\Pi}{S}(x_a)}{S(x_a) \mid\mid R(x_{g(x)})}\; x \mapsto a \qquad \overset{\Sigma}{P(y_{h(f(y))})} \lor \neg R(y)}{S(x_a) \mid R(x_{g(a)}) \mid P(y_{h(f(y))})}\; y \mapsto g(a) \ast}{S(x_a) \mid R(x_{g(a)}), P(y_{h(f(g(a)))}) \mid Q(y_{f(g(a))})}\; u \mapsto g(f(g(a)))$$

at $\ast$, $R(x_g(x))$ is not known to refer to $g(a)$. can we resort to check grey occurrences of $g(a)$?
need arrow from R to P (this situation should be more critical if it is a backwards arrow)
thought: concerns only stuff in literal, maybe can leverage something here (all lifting vars x_i point to same term or so)

212b – same but more info not present

$$\dfrac{\dfrac{R(g'(\overset{\Pi}{g(x)})) \lor S(x) \qquad \neg \overset{\Pi}{S}(a)}{S(a) \mid\mid R(g'(g(a)))}\; x \mapsto a \qquad \dfrac{P(h(f(\overset{\Sigma}{y}))) \lor \neg T(y) \qquad T(z) \lor \neg \overset{\Pi}{R}(g'(z))}{T(y) \mid P(h(f(y))) \lor \neg R(g'(y))}\; z \mapsto y}{S(a) \mid \neg R(g'(g(a))) \lor T(g(a)) \mid P(h(f(g(a))))}\; y \mapsto g(a) \ast$$

lifted w unifier arrows:

$$\dfrac{\dfrac{\overset{\Pi}{R(x_{g'(g(x))})} \lor S(x) \qquad \neg \overset{\Pi}{S}(x_a)}{S(x_a) \mid\mid R(x_{g'(g(x))})}\; x \mapsto a \qquad \dfrac{\overset{\Sigma}{P(y_{h(f(y))})} \lor \neg T(y) \qquad T(z) \lor \neg \overset{\Pi}{R}(x_{g'(z)})}{T(y) \mid P(y_{h(f(y))}) \lor \neg R(x_{g'(y)})}\; z \mapsto y}{S(x_a) \mid \neg R(x_{g'(g(a))}) \lor T(x_{g(a)}) \mid P(y_{h(f(y))})}\; y \mapsto g(a) \ast$$

$\ast$ is similar here, but the grey occurrence of y isn't even in the literal

212c – other approach

$$\dfrac{\dfrac{\dfrac{P(f(x),u) \overset{\Sigma}{\vee} Q(x) \vee R(u) \qquad \neg Q(g(z)\overset{\Pi}{)}) \vee R(z)}{Q(g(z)) \mid P(f(g(z)),u) \vee R(u) \vee R(z)}\; x \mapsto g(z)}{Q(g(z)) \mid P(f(g(z)),z) \vee R(z)}\; u \mapsto z \qquad \mathbf{S}(h(u)) \vee \overset{\Sigma}{\neg} P(f(u),c)}{P(f(g(c)),c) \mid Q(g(c)) \mid S(h(g(c))) \vee R(c)}\; u \mapsto g(c), z \mapsto c$$

(arrows for c not shown)

(the bold S receives the Δ-term here, the rest is technical details. it's about how S receives the arrow)

lifted:

$$\dfrac{\dfrac{\dfrac{P(y_{f(x)},u) \overset{\Sigma}{\vee} Q(x) \vee R(u) \qquad \neg Q(x_{g(z)}\overset{\Pi}{)}) \vee R(z)}{Q(x_{g(z)}) \mid P(y_{f(g(z))},u) \vee R(u) \vee R(z)}}{Q(x_{g(z)}) \mid P(y_{f(g(z))},z) \vee R(z)} \qquad S(y_{h(u)}) \vee \overset{\Sigma}{\neg} P(y_{f(u)},y_c)}{P(y_{f(g(c))},y_c) \mid Q(x_{g(z)}) \mid S(y_{h(u)})) \vee R(y_c)}$$

Need arrow to S. possibly work in C, not lifted variants.

Problem 2: lifting var in P is updated, but the one in Q is not, hence index of lifting vars don't match, which per se isn't a problem

Δ-lifted:

$$\dfrac{\dfrac{\dfrac{P(f(x),u) \overset{\Sigma}{\vee} Q(x) \vee R(u) \qquad \neg Q(x_{g(z)}\overset{\Pi}{)}) \vee R(z)}{Q(x_{g(z)}) \mid P(f(x_{g(z)}),u) \vee R(u) \vee R(z)}}{Q(x_{g(z)}) \mid P(f(x_{g(z)}),z) \vee R(z)} \qquad S(h(u)) \vee \overset{\Sigma}{\neg} P(f(u),c)}{P(f(x_{g(c)}),y_c) \mid Q(x_{g(c)}) \mid S(h(x_{g(c)})) \vee R(c)}$$

lifting var in Q is fixed here

term unified which is just produced

$$\frac{P(g(x),x) \qquad \neg P(y,a)}{P(g(a),a)}\; y \mapsto g(a), x \mapsto a$$

$\Rightarrow$ can only add arrow from terms in C, as they do not exist before.

old examples with unifier arrows

214a (210f)

$$\frac{\dfrac{\dfrac{\bot \mid P(f(\overset{\Sigma}{x})) \vee Q(x) \qquad \bot \mid \neg Q(\overset{\Sigma}{y}) \vee R(g(y))}{Q(x) \mid \bot \mid P(f(x)) \vee R(g(x))}\; y \mapsto x \qquad \dfrac{\bot \mid \neg P(f(\overset{\Sigma}{z})) \vee S(z) \qquad \top \mid \overset{\Pi}{\neg} S(a)}{S(a) \mid \neg P(f(a))}\; z \mapsto a}{Q(a), P(f(a) \mid S(a) \mid R(g(a))}\; x \mapsto a \qquad \top \mid \overset{\Pi}{\neg} R(u)}{Q(a), P(f(a)) \mid S(a) \vee R(g(a))}$$

what if different starting point:

$$\frac{\dfrac{\bot \mid P(f(\overset{\Sigma}{x})) \vee R(g(x)) \qquad \dfrac{\bot \mid \neg P(f(\overset{\Sigma}{z})) \vee S(z) \qquad \top \mid \overset{\Pi}{\neg} S(a)}{S(a) \mid \neg P(f(a))}\; z \mapsto a}{P(f(a) \mid S(a) \mid R(g(a))}\; x \mapsto a \qquad \top \mid \overset{\Pi}{\neg} R(u)}{P(f(a)) \mid S(a) \vee R(g(a))}$$

More special cases from proof:

215a

$$\frac{P(f(g(v)), f(v)) \qquad \neg P(f(g(a)), f(u)) \vee S(h(u))}{P(f(g(a)), f(a)) \mid S(h(a))}$$

need arrow to S

prequel to this situation:

RHS:

$$\frac{R(\overset{\Pi}{g}(a)) \qquad \neg R(x) \vee \neg P(f(\overset{\Sigma}{x}), f(u)) \vee S(h(u))}{\neg R(g(a)) \mid \neg P(f(g(a)), f(u)) \vee S(h(u))}$$

LHS1:

$$\cfrac{\cfrac{R(y) \vee P(f(\overset{\Sigma}{y}), f(v)) \vee Q(v) \qquad \neg R(g(\overset{\Pi}{z})) \vee Q(z)}{R(g(z)) \mid P(f(g(z)), f(v)) \vee Q(v) \vee Q(z)}}{R(g(v)) \mid P(f(g(v)), f(v)) \vee Q(v)}$$

$\Rightarrow$ have grey occ of v, it is only a problem if there isn't one