

## Example 101: canonical examples for overbinding w.r.t. order

### Ex 101a

$$\frac{\frac{P(u, f(u)) \vee Q(u)_{\Sigma} \quad \neg Q(a)_{\Pi} \quad u \mapsto a}{P(a, f(a))} \quad \neg P(x, y)_{\Pi} \quad x \mapsto a, y \mapsto f(a)}{\square}$$

$$\frac{\frac{\frac{\perp \quad \top}{Q(a)} \quad u \mapsto a \quad \top}{P(a, f(a)) \vee (\neg P(a, f(a)) \wedge Q(a))} \quad \top}{x \mapsto a, y \mapsto f(a)} \quad \frac{\frac{\frac{\perp \quad \top}{\forall x_1 Q(x_1)} \quad \top}{\forall x_1 \exists x_2 (P(x_1, x_2) \vee Q(x_1))}}{\square}$$

### Ex 101b – other resolution order

$$\frac{\frac{P(u, f(u)) \vee Q(u)_{\Sigma} \quad \neg P(x, y)_{\Pi} \quad y \mapsto f(u), x \mapsto u}{Q(u)} \quad \neg Q(a)_{\Pi} \quad u \mapsto a}{\square}$$

$$\frac{\frac{\frac{\perp \quad \top}{P(u, f(u))} \quad x \mapsto f(u), x \mapsto u \quad \top}{P(a, f(a)) \vee Q(a)} \quad \top}{u \mapsto a} \quad \frac{\frac{\frac{\perp \quad \top}{\exists x_1 P(u, x_1)} \quad \top}{\forall x_1 \exists x_2 (P(x_1, x_2) \vee Q(x_1))} \quad \top}{u \mapsto a}$$

### Ex 101c – $\Pi$ and $\Sigma$ swapped

$$\frac{\frac{P(u, f(u)) \vee Q(u)_{\Pi} \quad \neg P(x, y)_{\Sigma} \quad y \mapsto f(u), x \mapsto u}{Q(u)} \quad \neg Q(a)_{\Sigma} \quad u \mapsto a}{\square}$$

$$\frac{\frac{\frac{\top \quad \perp}{\neg P(u, f(u))} \quad x \mapsto f(u), x \mapsto u \quad \perp}{\neg P(a, f(a)) \wedge \neg Q(a)} \quad \perp}{u \mapsto a} \quad \frac{\frac{\frac{\top \quad \perp}{\forall x_2 \neg P(u, x_2)} \quad \perp}{\exists x_1 \forall x_2 (\neg P(x_1, x_2) \wedge \neg Q(x_1))} \quad \perp}{\square}$$

### Ex 101d – $\Pi$ and $\Sigma$ swapped, other resolution order

$$\frac{\frac{P(u, f(u)) \vee Q(u)_{\Pi} \quad \neg Q(a)_{\Sigma} \quad u \mapsto a}{P(a, f(a))} \quad \neg P(x, y)_{\Sigma} \quad x \mapsto a, y \mapsto f(a)}{\square}$$

$$\frac{\frac{\frac{\top \quad \perp}{\neg Q(a)} \quad y \mapsto a \quad \perp}{\neg Q(a) \wedge \neg P(a, f(a))} \quad \perp}{x \mapsto a, y \mapsto f(a)} \quad \frac{\frac{\frac{\top \quad \perp}{\exists x_1 \neg Q(x_1)} \quad \perp}{\exists x_1 \forall x_2 (\neg P(x_1, x_2) \wedge \neg Q(x_1))} \quad \perp}{\square}$$

# OLD EXAMPLES

Example 5b: no equality, but quantifier order still matters

$$\frac{P(u, g(u))_{\Sigma} \quad \neg P(a, x)_{\Pi}}{\square} u \mapsto a, x \mapsto g(a)$$

Prop Interpolant:  $P(a, g(a))$

Interpolant:  $\forall x_1 \exists x_2 P(x_1, x_2)$

Example 5b': order matters, construction in multiple steps:

$$\frac{\frac{P(u, v, f(u, v)) \vee Q(u)_{\Sigma} \quad \neg Q(a)_{\Pi}}{P(a, v, f(a, v))} u \mapsto a \quad \neg P(x, b, y)_{\Pi}}{\square} x \mapsto a, v \mapsto b, y \mapsto f(a, b)$$

$$\frac{\frac{\perp \quad \top}{Q(a)} u \mapsto a \quad \top}{P(a, b, f(a, b)) \vee (\neg P(a, b, f(a, b)) \wedge Q(a))} x \mapsto a, v \mapsto b, y \mapsto f(a, b)$$

Non-trivial interpolants:

$\forall x_1 Q(x_1)$

$\forall x_1 \forall x_2 \exists x_3 P(x_1, x_2, x_3) \vee Q(x_1)$

Example 5b'': 5b' with different resolution order

$$\frac{\frac{P(u, v, f(u, v)) \vee Q(u)_{\Sigma} \quad \neg P(x, b, y)_{\Pi}}{Q(u)} x \mapsto u, v \mapsto b, y \mapsto f(u, b) \quad \neg Q(a)_{\Pi}}{\square} u \mapsto a$$

$$\frac{\frac{\perp \quad \top}{P(u, b, f(u, b))} x \mapsto u, v \mapsto b, y \mapsto f(u, b) \quad \top}{P(a, b, f(a, b)) \vee Q(a)} u \mapsto a$$

Non-trivial interpolants:

$\forall x_2 \exists x_3 P(u, x_2, x_3)$

$\forall x_1 \forall x_2 \exists x_3 (P(x_1, x_2, x_3) \vee Q(x_1))$

Example 5c: example with  $\exists\forall$  necessarily in interpolant  
 $\Rightarrow$  as shown in Huang, swap  $\Sigma$  and  $\Pi$  from 5b'

$$\frac{\frac{P(u, v, f(u, v)) \vee Q(u)_{\Pi} \quad \neg Q(a)_{\Sigma}}{P(a, v, f(a, v))} \quad u \mapsto a \quad \neg P(x, b, y)_{\Sigma}}{x \mapsto a, v \mapsto b, y \mapsto f(a, b)} \quad \square$$

$$\frac{\frac{\top \quad \perp}{\neg Q(a)} \quad u \mapsto a \quad \perp}{\neg P(a, b, f(a, b)) \wedge \neg Q(a)} \quad x \mapsto a, v \mapsto b, y \mapsto f(a, b)$$

Non-trivial interpolants:

$\exists x_1 Q(x_1)$   
 $\exists x_1 \exists x_2 \forall x_3 (\neg P(x_1, x_2, x_3) \wedge \neg Q(x_1))$

$\Rightarrow$  similar for 5b''

$$\frac{\frac{P(u, v, f(u, v)) \vee Q(u)_{\Pi} \quad \neg P(x, b, y)_{\Sigma}}{Q(u)} \quad x \mapsto u, v \mapsto b, y \mapsto f(u, b) \quad \neg Q(a)_{\Sigma}}{u \mapsto a} \quad \square$$

$$\frac{\frac{\top \quad \perp}{\neg P(u, b, f(u, b))} \quad x \mapsto u, v \mapsto b, y \mapsto f(u, b) \quad \perp}{\neg Q(a) \wedge \neg P(a, b, f(a, b))} \quad u \mapsto a$$

Non-trivial interpolants:

$\exists x_2 \forall x_3 \neg (P(u, x_2, x_3))$   
 $\exists x_1 \exists x_2 \forall x_3 (\neg Q(x_1) \wedge \neg (P(x_2, x_3)))$