Example 101: canonical examples for overbinding w.r.t. order

Here, the second lower prooftree shows what huang would produce.

Ex 101a

$$\dfrac{\dfrac{P(u,f(u)) \vee Q(u)_\Sigma \qquad \neg Q(a)_\Pi}{P(a,f(a))}\,u \mapsto a \qquad \neg P(x,y)_\Pi}{\square}\,x \mapsto a, y \mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{Q(a)}\,u \mapsto a \qquad \top}{P(a,f(a)) \vee Q(a)}\,x \mapsto a, y \mapsto f(a) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1 Q(x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1,x_2) \vee Q(x_1))}$$

Direct overbinding would not work without merging same variables!: $\forall x_1 \forall x_2 \exists x_3 (P(x_2,x_3) \vee Q(x_1))$

Ex 101b – other resolution order

$$\dfrac{\dfrac{P(u,f(u)) \vee Q(u)_\Sigma \qquad \neg P(x,y)_\Pi}{Q(u))}\,y \mapsto f(u), x \mapsto u \qquad \neg Q(a)_\Pi}{\square}\,u \mapsto a$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(u,f(u))}\,x \mapsto f(u), x \mapsto u \qquad \top}{P(a,f(a)) \vee Q(a)}\,u \mapsto a \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(u,x_1)} \qquad \top}{\forall x_1 \exists x_2 (P(x_1,x_2) \vee Q(x_1))}\,u \mapsto a$$

Ex 101c – Π and Σ swapped

$$\dfrac{\dfrac{P(u,f(u)) \vee Q(u)_\Pi \qquad \neg P(x,y)_\Sigma}{Q(u))}\,y \mapsto f(u), x \mapsto u \qquad \neg Q(a)_\Sigma}{\square}\,u \mapsto a$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg P(u,f(u))}\,x \mapsto f(u), x \mapsto u \qquad \bot}{\neg P(a,f(a)) \wedge \neg Q(a)}\,u \mapsto a \qquad\qquad \dfrac{\dfrac{\top \qquad \bot}{\forall x_2 \neg P(u,x_2)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1,x_2) \wedge \neg Q(x_1))}$$

Ex 101d – Π and Σ swapped, other resolution order

$$\dfrac{\dfrac{P(u,f(u)) \vee Q(u)_\Pi \qquad \neg Q(a)_\Sigma}{P(a,f(a))}\,u \mapsto a \qquad \neg P(x,y)_\Sigma}{\square}\,x \mapsto a, y \mapsto f(a)$$

$$\dfrac{\dfrac{\top \qquad \bot}{\neg Q(a)}\,y \mapsto a \qquad \bot}{\neg Q(a) \wedge \neg P(a,f(a))}\,x \mapsto a, y \mapsto f(a) \qquad\qquad \dfrac{\dfrac{\top \qquad \bot}{\exists x_1 \neg Q(x_1)} \qquad \bot}{\exists x_1 \forall x_2 (\neg P(x_1,x_2) \wedge \neg Q(x_1))}$$

102 – similar to 101 but with intra-clause-set inferences

Ex 102a

$$\dfrac{\dfrac{P(f(x)) \lor Q(f(x),z)_\Sigma \qquad \neg P(y)_\Pi}{Q(f(x),z)} \qquad \dfrac{\neg Q(x_1,y) \lor R(y)_\Sigma \qquad \neg R(g(z_1))_\Pi}{\neg Q(x_1,g(z_1))}\; y \mapsto g(z_1), x_1 \mapsto f(x)}{\Box}\; x_1 \mapsto f(x), z \mapsto g(z_1)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(f(x))} \qquad \dfrac{\bot \qquad \top}{R(g(z_1))}\; y \mapsto g(z_1), x_1 \mapsto f(x)}{P(f(x)) \lor R(g(z_1))}\; x_1 \mapsto f(x), z \mapsto g(z_1)$$

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(x_1)} \qquad \dfrac{\bot \qquad \top}{\forall x_2 R(x_2)}}{\exists x_1 \forall x_2 (P(x_1) \lor R(x_2)) \quad \text{(order irrelevant!)}}$$

Ex 102b

$$\dfrac{\dfrac{P(f(x)) \lor Q(f(x),z)_\Sigma \qquad \neg P(y)_\Pi}{Q(f(x),z)} \qquad \dfrac{\neg Q(f(y),z_1) \lor R(y)_\Sigma \qquad \neg R(a)_\Pi}{\neg Q(f(a),z_1)}\; y \mapsto a}{\Box}\; x \mapsto a, z \mapsto z_1$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(f(x))} \qquad \dfrac{\bot \qquad \top}{R(a)}\; y \mapsto a}{P(f(a)) \lor R(a)}\; x \mapsto a, z \mapsto z_1$$

$$\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 P(x_1)} \qquad \dfrac{\bot \qquad \top}{\forall x_2 R(x_2)}\; y \mapsto a}{\forall x_2 \exists x_1 (P(x_1) \lor R(x_2))}\; x \mapsto a, z \mapsto z_1$$

Example 103: variables in interpolant get unified and order might have to be changed

103a: length changes but old version is still valid, despite huang's algorithm doing a sort

$$\dfrac{\dfrac{\dfrac{Q(f(x)) \lor P(y) \lor R(x)_\Sigma \qquad \neg Q(y_1)_\Pi}{P(y) \lor R(x)}\; y_1 \mapsto f(x) \qquad \neg P(g(g(a)))_\Pi}{R(x)}\; y \mapsto g(g(a)) \qquad \neg R(g(g(a)))_\Pi}{\Box}\; x \mapsto g(g(a))$$

$$\dfrac{\dfrac{\dfrac{\bot \qquad \top}{Q(f(x))}\; y_1 \mapsto f(x) \qquad \top}{Q(f(x)) \lor P(g(g(a)))}\; y \mapsto g(g(a)) \qquad \top}{Q(f(g(g(a)))) \lor P(g(g(a))) \lor R(g(g(a)))}\; x \mapsto g(g(a)) \qquad\qquad \dfrac{\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 Q(x_1)} \qquad \top}{\exists x_1 \forall x_2 Q(x_1) \lor P(x_2)} \qquad \top}{X}$$

X:
Huang's algo gives:
$\forall x_2 \exists x_1 Q(x_1) \lor P(x_2) \lor R(x_2)$
Direct overbinding gives ($x_3 < x_1$, rest arbitrary):
$\forall x_3 \exists x_1 \forall x_2 Q(x_1) \lor P(x_2) \lor R(x_3)$

103b: length changes "uniformly"

$$\dfrac{\dfrac{\dfrac{Q(f(f(x))) \lor P(f(x)) \lor R(x)_\Sigma \qquad \neg Q(y_1)_\Pi}{P(f(x)) \lor R(x)}\; y_1 \mapsto f(f(x)) \qquad \neg P(y_2)_\Pi}{R(x)}\; y_2 \mapsto f(x) \qquad \neg R(g(a))_\Pi}{\Box}\; x \mapsto g(a)$$

$$\dfrac{\dfrac{\dfrac{\bot \qquad \top}{Q(f(f(x)))}\; y_1 \mapsto f(f(x)) \qquad \top}{Q(f(f(x))) \lor P(f(x))}\; y_2 \mapsto f(x) \qquad \top}{Q(f(f(g(a)))) \lor P(f(g(a))) \lor R(g(a))}\; x \mapsto g(a) \qquad \dfrac{\dfrac{\dfrac{\bot \qquad \top}{\exists x_1 Q(x_1)} \qquad \top}{\exists x_2 \exists x_1 (Q(x_1) \lor P(x_2))} \qquad \top}{\forall x_3 \exists x_2 \exists (x_1 Q(x_1) \lor P(x_2) \lor R(x_3))}$$

Huang and direct overbinding somewhat coincide as $x_2 < x_1$ in both cases, and $x_3 < x_1$ and $x_3 < x_2$

103c: different variables, accidentally the same terms appear but no logical connection

$$\dfrac{P(a,x)_\Sigma \qquad \dfrac{\neg Q(a)_\Sigma \qquad \neg P(y, f(z)) \lor Q(z)_\Pi}{\neg P(y, f(a))}\; z \mapsto a}{\Box}\; y \mapsto a, x \mapsto f(a)$$

$$\dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\neg Q(a)}\; z \mapsto a}{P(a, f(a)) \land \neg Q(a)}\; y \mapsto a, x \mapsto f(a) \qquad\qquad \dfrac{\bot \qquad \dfrac{\bot \qquad \top}{\exists x_1 \neg Q(x_1)}}{\exists x_1 \forall x_2 (P(x_1, x_2) \land \neg Q(x_1))}$$

Again, Huang sorts, but no order is required.

misc examples

201a

$$\dfrac{\dfrac{P(x,y)\lor\neg Q(y)_\Sigma \qquad \neg P(a,y_2)_\Pi}{\neg Q(y)}\;x\mapsto a \qquad \dfrac{Q(f(z))\lor R(z)_\Sigma \qquad \neg R(a)_\Pi}{Q(f(a)}\;z\mapsto a}{\Box}\;y\mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a,y)}\;x\mapsto a \qquad \dfrac{\bot \qquad \top}{R(a)}\;z\mapsto a}{P(a,f(a))\lor R(a)}\;y\mapsto f(a) \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1 P(x_1,y)}\;x\mapsto a \qquad \dfrac{\bot \qquad \top}{\forall x_3 R(x_3)}\;z\mapsto a}{\forall x_3\forall x_1\exists x_2(P(x_1,x_2)\lor R(x_3))}\;y\mapsto f(a)$$

Huang would produce: $\forall x_1\exists x_2 P(x_1,x_2)\lor R(x_1)$

201b

$$\dfrac{\dfrac{P(x,f(y))\lor\neg Q(f(y))_\Sigma \qquad \neg P(a,y_2)_\Pi}{\neg Q(f(y))}\;x\mapsto a \qquad \dfrac{Q(f(z))\lor R(z)_\Sigma \qquad \neg R(a)_\Pi}{Q(f(a)}\;z\mapsto a}{\Box}\;y\mapsto f(a)$$

$$\dfrac{\dfrac{\bot \qquad \top}{P(a,f(y))}\;x\mapsto a \qquad \dfrac{\bot \qquad \top}{R(a)}\;z\mapsto a}{P(a,f(a))\lor R(a)}\;y\mapsto a \qquad\qquad \dfrac{\dfrac{\bot \qquad \top}{\forall x_1\exists x_2 P(x_1,x_2)}\;x\mapsto a \qquad \dfrac{\bot \qquad \top}{\forall x_3 R(x_3)}\;z\mapsto a}{\forall x_3\forall x_1\exists x_2 P(x_1,x_2)\lor R(x_3)}\;y\mapsto f(a)$$

Huang would produce: $\forall x_1\exists x_2 P(x_1,x_2)\lor R(x_1)$