necessary and unnecessary arrows in literal

Ex 301a

$$\dfrac{\overset{\Pi}{\neg P(a,z)} \qquad \dfrac{\overset{\Sigma}{P(x,f(y)) \vee Q(y)} \qquad \overset{\Pi}{\neg Q(a)}}{Q(a) \mid P(x,f(a))}}{Q(a) \vee P(a,f(a)) \mid \square}$$

$$\dfrac{\overset{\Pi}{\forall x_1\ \neg P(x_1,z)} \qquad \dfrac{\overset{\Sigma}{P(x,f(y)) \vee Q(y)} \qquad \overset{\Pi}{\forall x_1\ \neg Q(x_1)}}{\forall x_1(Q(x_1) \mid P(x,f(x_1)))}}{\forall x_1(Q(x_1) \vee P(x_1,f(x_1))) \mid \square}$$

$$\dfrac{\overset{\Pi}{\forall x_1\ \neg P(x_1,z)} \qquad \overset{\forall x_1}{\exists y_2\ (Q(x_1) \mid P(x,y_2))}}{\overset{\forall x_1}{\exists y_3\ (Q(x_1) \vee P(x_1,y_3))} \mid \square} \qquad \dfrac{\overset{\Sigma}{\exists y_2\ P(x,y_2) \vee Q(y)} \qquad \overset{\Pi}{\forall x_1\ \neg Q(x_1)}}{\overset{\forall x_1}{\exists y_2\ (Q(x_1) \mid P(x,y_2))}}$$

all orderings work:

$\forall x_1 \exists y_2 (Q(x_1) \vee P(x_1,y_2))$ // need not be x_1 both times, that's just an accident of this example

$\exists y_2 \forall x_1 (Q(x_1) \vee P(x_1,y_2))$

Ex 302a

$$\dfrac{\overset{\Sigma}{P(u,f(u))} \qquad \overset{\Pi}{\neg P(a,z)}}{P(a,f(a)) \mid \square}$$

$$\dfrac{\overset{\Sigma}{P(u,f(u))} \qquad \overset{\Pi}{\forall x_1\ \neg P(x_1,z)}}{\forall x_1 P(x_1,f(x_1)) \mid \square} \qquad \dfrac{\overset{\Sigma}{\forall u \exists y_2\ P(u,y_1))} \qquad \overset{\Pi}{\forall x_1\ \neg P(x_1,z)}}{\forall x_1 \exists y_3 P(x_1,y_3) \mid \square}$$

(order matters)

$\forall x_1 \exists y_2 (P(x_1,y_2))$

~~$\exists y_2 \forall x_1 (P(x_1,y_2))$~~

Ex 302a' – inverse coloring

$$\dfrac{\overset{\Pi}{\forall y_2\ P(u,y_2)} \qquad \overset{\Sigma}{\neg P(a,z)}}{\forall y_3 P(a,y_3) \mid \square}$$

(can't really fix order, but matters)