Project 1 Report

Group member

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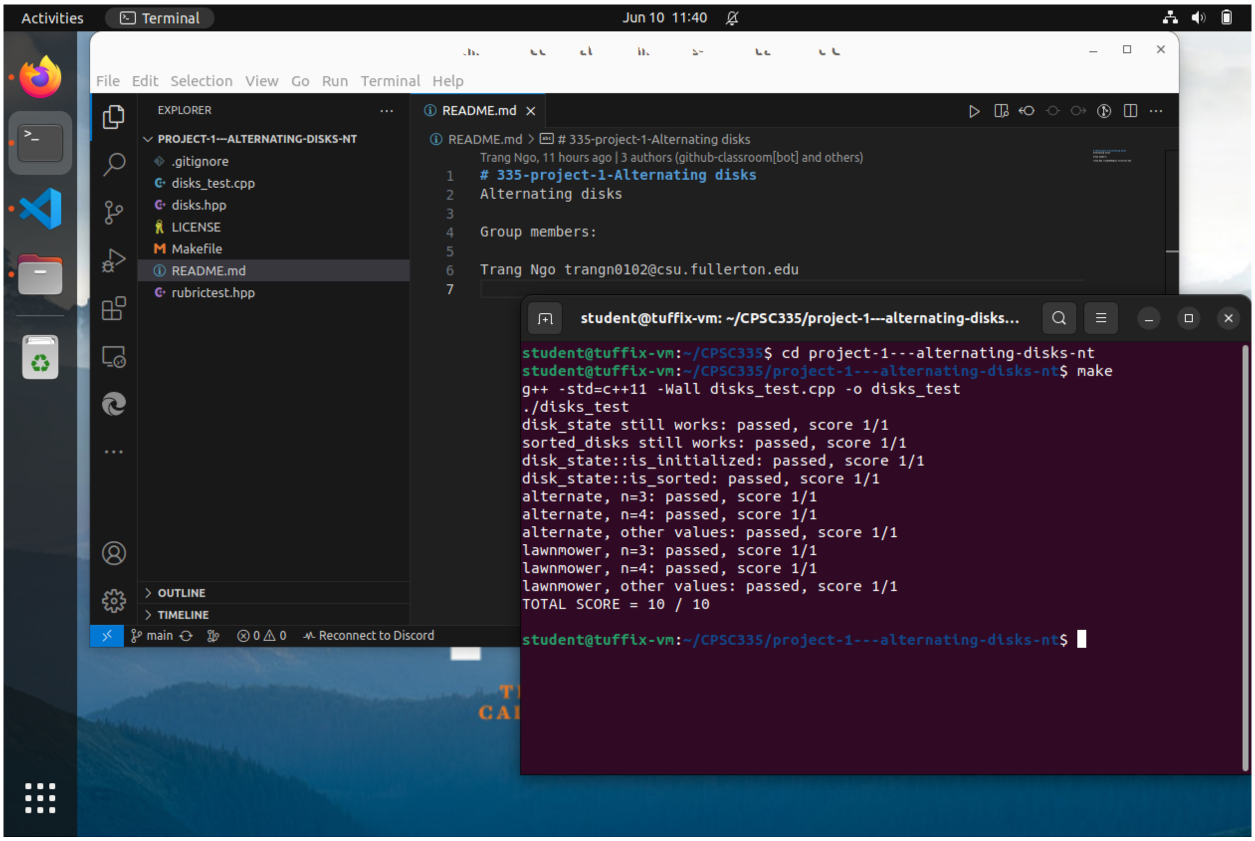
This is my submission for Project 1: Alternating Disks

Tuffix Screenshot:



Pseudocode

Psudocode

Lawnmower algorithms

```
def lawnmower (disk)
 disk= total length of disk   // 1
  swap-count = 0   // 1
for i from 0 to (n+1)/2 do   // ((n+1)/2 - 0)+1 =
  for a from 0 to 2n-2 do   // 2n-2-0+1 = 2n-1
    if (disk at index a == light and disk at index a+1 ==dark)   // 3
      swap (disk at index a, disk at index a+1)   // 1
      increment of swap-count by 1   // 1
    endif   // 0
  endfor   // 0
                                   -(1-2n+1)= -1 +2n-1
  for b from 2n-1 down to 1 do // (((1-(2n-1))/-1)+1) = (-1(1-(2n-1))+1) = 2n-2 +1 = 2n-1
    if (disk at index b== dark and disk at index b-1 == light)   // 3
      swap (disk at index b, disk at index b-1)   // 1
      increment of swap-count by 1   // 1
    endif   // 0
  endfor   // 0
endfor   // 0
return lawnmower of the disk and swap-count   // 0
```

Step count

$$SC = 1 + 1 + \left(\frac{n+1}{2} + 1\right) * \left[(2n-1) * (3 + \max(1,1)\right] + \left[(2n-1) * 3 + \max(1,1)\right]$$

$$= 2 + \left(\frac{n+1}{2} + \frac{2}{2}\right) * \left[(2n-1) * 4\right] + \left[(2n-1) * 4\right]$$

$$= 2 + \frac{n+3}{2} * \left[8n - 4 + 8n - 4\right]$$

$$= 2 + \frac{n+3}{2} * \left[16n - 8\right]$$

$$= 2 + \frac{(16n^2 - 8n + 48n - 24)}{2}$$

$$= 2 + \frac{(16n^2 + 40n - 24)}{2}$$

$$= 2 + 8n^2 + 20n - 12$$

$$= 8n^2 + 20n - 10$$

Time complexity $= O(n^2)$

Proof 1 : $8n^2 + 20n - 10 \in O(n^2)$

By definition, you need to find $c > 0$ and $n_0 \geq 0$ such that
$8n^2 + 20n - 10 \leq c * n^2$
Choose $c = |8| + |20| + |-10| = 38$
So $8n^2 + 20n - 10 \leq 38n^2 \quad \forall n \geq n_0$
and $n_0 = 0$

Proof 2 : $8n^2 + 20n - 10 \in O(n^2)$ by limit theorem

$$\lim_{n \to \infty} \frac{8n^2 + 20n - 10}{n^2}$$

$$= \lim_{n \to \infty} \frac{(8n^2 + 20n - 10)'}{(n^2)'}$$

$$= \lim_{n \to \infty} \frac{16n + 20}{2n}$$

$$= \lim_{n \to \infty} \frac{(16n + 20)'}{(2n)'}$$

$$= \lim_{n \to \infty} \frac{16}{2}$$

$= 8 \geq 0$ and a constant

Conclude $8n^2 + 20n - 10 \in O(n^2)$

Alternate algorithm

```
def alterate ( disk)
 disk = total length of disk   // 1
  swap_ count = 0        // 1
for i = 0 to n        // (n - 0) + 1  = n+1
  if (i % 2 == 0) do     // 2
  // start with leftmost disk
    for a from 0 to 2n-1 skip 2 do  //((2n-1-0)/2)+1) = (2n-1)/2 + 1
      if ( disk at index a == light and disk at index a+1 == dark) //3
        swap( disk at index a , disk at index a+1)  //1
        increment of swap_count by 1      //1
      endif    //0
    endfor  //0
  else
   // start with second leftmost disk
   for b from 1 to 2n-2 skip 2 do //((2n-2-1)/2)+1) = (2n-3)/2 + 1
      if ( disk at index b == light and disk at index b+1 == dark) //3
        swap (disk at index b, disk at index b+1)   // 1
        increment of swap- count by 1   // 1
      endif //0
    endfor //0
endfor   //0
return alternate of the disk and swap-count   // 0
```

Step Count

$$S.C = 1+1 + (n+1) * \left[2 + \max\left(\left(\frac{2n-1}{2} + 1\right) * (3 + \max(1,1)), \left(\frac{2n-3}{2} + 1\right) * (3 + \max(1,1)) \right) \right]$$

$$= 2 + (n+1) * \left[2 + \max\left(\left(\frac{2n-1}{2} + \frac{2}{2}\right) * 4, \left(\frac{2n-3}{2} + \frac{2}{2}\right) * 4 \right) \right]$$

$$= 2 + (n+1) * \left[2 + \max\left(\left(\frac{2n+1}{2}\right) * 4, \left(\frac{2n-1}{2}\right) * 4 \right) \right]$$

$$= 2 + (n+1) * \left[2 + \max\left(\left(\frac{8n+4}{2}\right), \left(\frac{8n-4}{2}\right) \right) \right]$$

$$= 2 + (n+1) * \left[2 + \max(4n+2, 4n-2) \right]$$

$$= 2 + (n+1) * [2 + 4n + 2]$$

$$= 2 + (n+1) * [4n + 4]$$

$$= 2 + 4n^2 + 4n + 4n + 4$$

$$= 2 + 4n^2 + 8n + 4$$

$$= 4n^2 + 8n + 6$$

Time complexity $= O(n^2)$

Proof 1 : $4n^2 + 8n + 6 \in O(n^2)$

By definition, you need to find $c > 0$ and $n_0 \geq 0$ such that

$4n^2 + 8n + 6 \leq c * n^2$

Choose $c = |4| + |8| + |6| = 18$

So $4n^2 + 8n + 6 \leq 18n^2 \quad \forall\, n \geq n_0$

and $n_0 = 0$

Proof 2 : $4n^2 + 8n + 6 \in O(n^2)$ by limit theorem

$$\lim_{n \to \infty} \frac{4n^2 + 8n + 6}{n^2}$$

$$= \lim_{n \to \infty} \frac{(4n^2 + 8n + 6)'}{(2n)'}$$

$$= \lim_{n \to \infty} \frac{8n + 8}{2n}$$

$$= \lim_{n \to \infty} \frac{(8n + 8)'}{(2n)'}$$

$$= \lim_{n \to \infty} \frac{8}{2}$$

$= 4 \geq 0$ and a constant

Conclude $4n^2 + 8n + 6 \in O(n^2)$