

# A new MATLAB interface to MueLu

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October 27, 2015

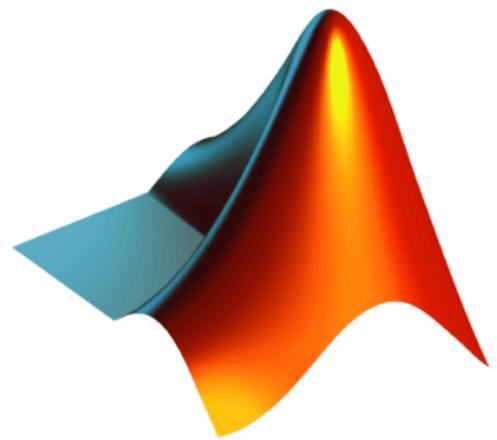
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MATLAB



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Chris Siefert



MueMex = MATLAB interface for MueLu



Brian Kelley

# What is MueLu?

MueLu is . . .

. . . the next-generation multigrid framework package in Trilinos.

- provides AMG methods to solve large linear systems of equations
- can be understood as successor but not replacement for ML
- supports both Epetra and Tpetra as linear algebra framework
- is *the* typical Trilinos package



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  - runs on usual laptops, super-computers and (soon) next-generation HPC systems
- MueLu is *international*: we can give support in



english (since 2010)



german (since 2011)



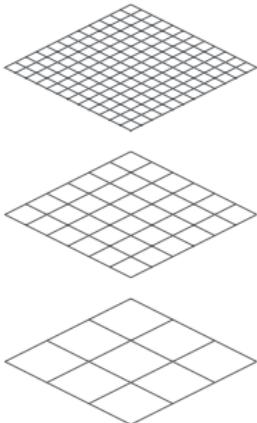
russian (since 2012)



french (discontinued in 2012)



# Basic concept of Algebraic Multigrid (AMG)

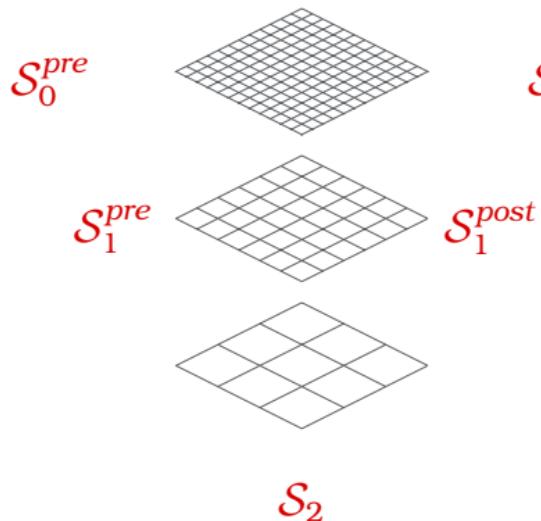


## Main idea

Capture errors at multiple resolutions.



# Basic concept of Algebraic Multigrid (AMG)



## Two main components

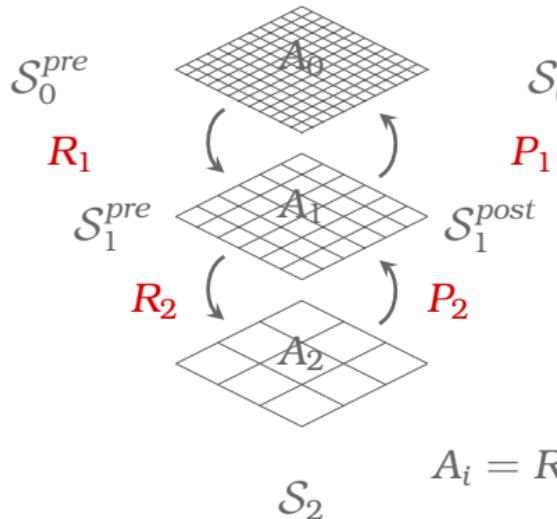
- Smoothers
  - Approximate solve on each level
  - “Cheap” reduction of oscillatory error (high energy)
  - $\mathcal{S}_L \approx A_L^{-1}$  on the coarsest level  $L$

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# Basic concept of Algebraic Multigrid (AMG)



## Two main components

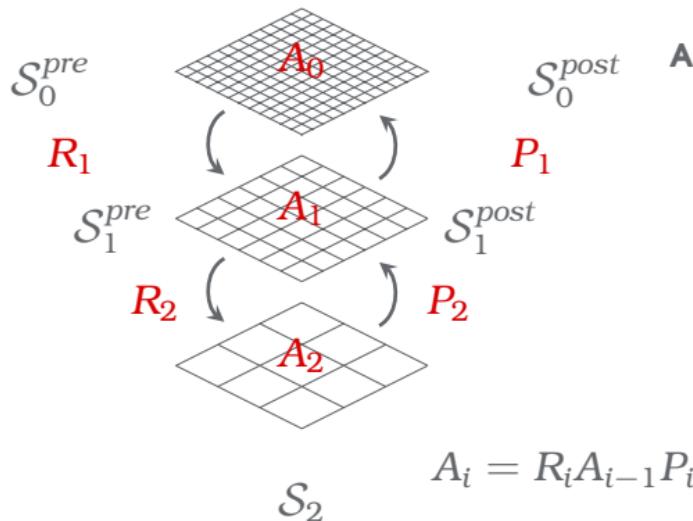
- **Smoothers**
  - Approximate solve on each level
  - “Cheap” reduction of oscillatory error (high energy)
  - $S_L \approx A_L^{-1}$  on the coarsest level  $L$
- **Grid transfers (prolongators and restrictors)**
  - Data movement between levels
  - Definition of coarse level matrices.

Main idea

Capture errors at multiple resolutions.



# Basic concept of Algebraic Multigrid (AMG)



## Algorithmic phases

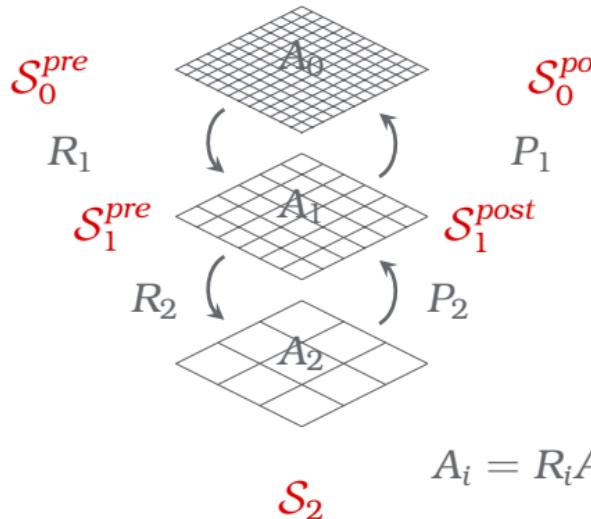
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  - Build transfer operators to determine coarse level matrices
  - Initialize level smoothers

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# Basic concept of Algebraic Multigrid (AMG)



## Algorithmic phases

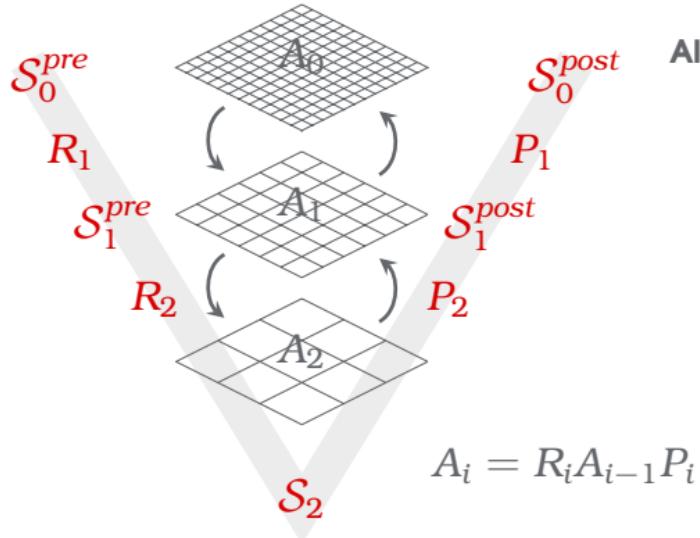
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Main idea

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# Basic concept of Algebraic Multigrid (AMG)



## Algorithmic phases

- Setup phase
  - Build transfer operators to determine coarse level matrices
  - Initialize level smoothers
- Solving phase
  - Run through multigrid cycle (e.g. V-cycle)
  - Iteratively solve linear system or apply some sweeps with multigrid as preconditioner within an iterative linear solver

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  - Access to all preconditioners from Trilinos through MueLu



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  - Run MueLu multigrid setup once
  - Use multigrid hierarchy for solving several (similar) linear systems with varying linear operators and/or right hand sides.



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## Why is MATLAB useful for MueLu?

- Analyze and tweak multigrid methods using the full functionality of MATLAB.
- Perform basic research on multigrid methods for specific problems.



# 1. How to use MueMex

# Basic Laplace example – Setup

Define problem:

```
1 >> [A, coords] = laplacianfun([50, 50]);
```



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Minimal setup: Use default parameters defined by MueLu

```
1 >> [problemID, oc] = muelu('setup', A);
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Multigrid parameters: Provide user parameters (see MueLu user guide)

```
1 >> [problemID, oc] = muelu('setup', A, 'coarse: max  
size', 50);
```



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1 >> [A, coords] = laplacianfun([50, 50]);
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Multigrid parameters: Provide user parameters (see MueLu user guide)

```
1 >> [problemID, oc] = muelu('setup', A, 'coarse: max  
size', 50);
```

XML parameter file: Provide user parameters through xml file

```
1 [problemID, oc] = muelu('setup', A, 'xml parameter  
file', 'myParams.xml');
```

MATLAB 7.11.0 (R2010b)

File Edit Debug Parallel Desktop Window Help

Current Folder /home/tobias/promotion/trilinos-mex/packages/muelu/matlab/bin

Shortcuts How to Add What's New

Current Folder

matlab bin Name ↗ CMakefiles Tests cmake\_install.cmake constructAggregates.m CTestTestfile.cmake driver1b.xml Makefile matlab muelu.m muemex.mexa64 mymatlab setup.m

Command Window

```

I: active processes: 1/2
Ptent # rows per proc : avg = 5.40e+01, dev = 0.0%, min = +0.0%, max = +0.0%
Ptent # nnz per proc : avg = 5.40e+01, dev = 0.0%, min = +0.0%, max = +0.0%
Eigenvalue estimate
Calculating max eigenvalue estimate now (max iters = 10)
Prolongator damping factor = 0.97 (1.33 / 1.38)
Fused (I-omega*D(-1) A)*Ptent
P size = 54 x 8, nnz = 127
P Load balancing info
P # active processes: 1/1
P # rows per proc : avg = 5.40e+01, dev = 0.0%, min = +0.0%, max = +0.0%
P # nnz per proc : avg = 1.27e+02, dev = 0.0%, min = +0.0%, max = +0.0%
Transpose P (MueLu::TransPFactory)
R size = 8 x 54, nnz = 127
R Load balancing info
R # active processes: 1/1
R # rows per proc : avg = 8.00e+00, dev = 0.0%, min = +0.0%, max = +0.0%
R # nnz per proc : avg = 1.27e+02, dev = 0.0%, min = +0.0%, max = +0.0%
Computing Ac (MueLu::RAPFactory)
M0M: A x P
Matrix product nnz per row estimate = 9
M0M: R x (AP) (explicit)
Matrix product nnz per row estimate = 30
Ac size = 8 x 8, nnz = 44
Ac Load balancing info
Ac # active processes: 1/1
Ac # rows per proc : avg = 8.00e+00, dev = 0.0%, min = +0.0%, max = +0.0%
Ac # nnz per proc : avg = 4.40e+01, dev = 0.0%, min = +0.0%, max = +0.0%
Max coarse size (<= 50) achieved
Setup Smoother (MueLu::Amesos2Smoothen{type = Superlu})
```

---

```

--- Multigrid Summary ---
Number of levels = 4
Operator complexity = 1.33
```

level	rows	nnz	nnz/row	c ratio	procs
0	2500	12300	4.92		1
1	425	3591	8.45	5.88	1
2	54	404	7.48	7.87	1
3	8	44	5.50	6.75	1

```

Smoother (level 0) both : "Ifpack2::Relaxation": {Initialized: true, Computed: true, Type: Symmetric Gauss-Seidel}
Smoother (level 1) both : "Ifpack2::Relaxation": {Initialized: true, Computed: true, Type: Symmetric Gauss-Seidel}
Smoother (level 2) both : "Ifpack2::Relaxation": {Initialized: true, Computed: true, Type: Symmetric Gauss-Seidel}
Smoother (level 3) pre : SuperLU solver interface, direct solve
Smoother (level 3) post : no smoother
```

Set up problem #0

```

A>> [problemID, oc] = muelu('setup', A, 'coarse: max_size', 50);
```

# Basic Laplace example – Solve

Solve problem:

```
1 >> b = ones(2500,1);  
2 >> [x, numIters] = muelu(problemID, b);
```

MATLAB output:

```
>> [x, numIters] = muelu(problemID, b);  
***** Belos Iterative Solver: Pseudo Block Gmres  
***** Maximum Iterations: 1000  
***** Block Size: 1  
***** Residual Test:  
***** Test 1 : Belos::StatusTestImpResNorm<>: (2-Norm Res Vec) / (2-Norm Prec Res0), tol = 1e-08  
*****  
Iter 0, [ 1] : 1.00000e+00  
Iter 1, [ 1] : 5.608530e-01  
Iter 2, [ 1] : 1.953063e-02  
Iter 3, [ 1] : 1.303521e-03  
Iter 4, [ 1] : 7.361411e-05  
Iter 5, [ 1] : 4.277384e-06  
Iter 6, [ 1] : 2.581678e-07  
Iter 7, [ 1] : 1.169750e-08  
Iter 8, [ 1] : 7.920252e-10  
Success, Belos converged!
```



# Basic Laplace example – Analysis

Visualize solution and error:

```
1 >> plot3(coords(:,1), coords(:,2), x, 'r.')
2 >> plot3(coords(:,1), coords(:,2), x-A\b, 'r.')
```



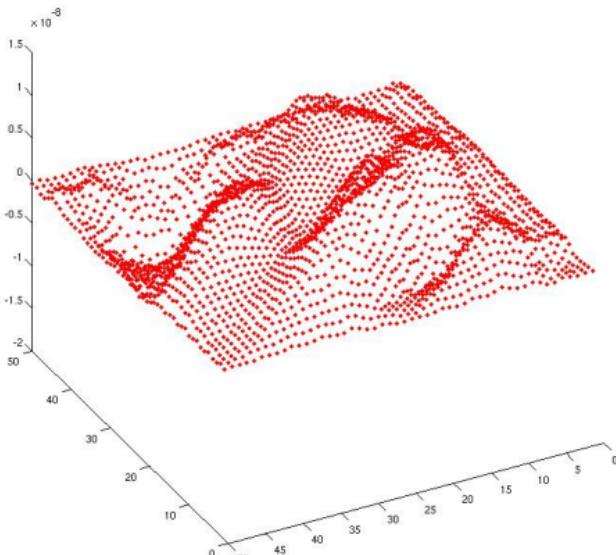
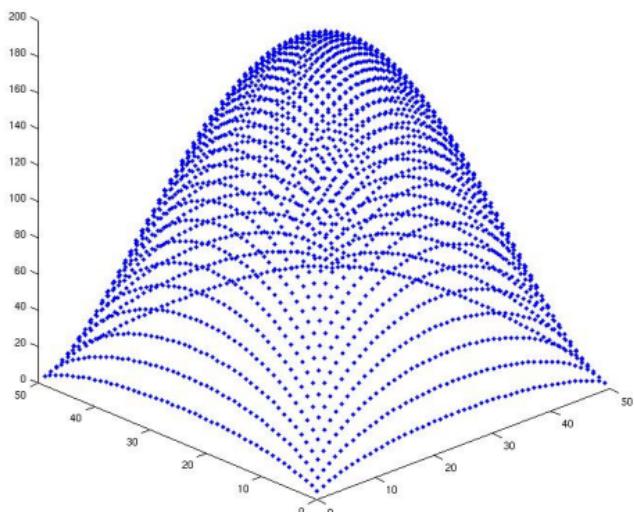
# Basic Laplace example – Analysis

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```

1 >> plot3(coords(:,1), coords(:,2), x, '.')
2 >> plot3(coords(:,1), coords(:,2), x-A\b, 'r.')

```



# Complex scalars

MueLu also works with complex scalars:

```
1 >> A = gallery('tridiag',2500,-1.0,2.0,-1.0) + gallery('tridiag'
   ' ,2500,-100.0,200.0,-100.0) * i;
2 >> [p,oc] = muelu('setup',A);
```

MATLAB output:

```
-----
---                                     Multigrid Summary
-----
Number of levels      = 2
Operator complexity = 1.33
level    rows     nnz   nnz/row  c ratio   procs
  0      2500    7498      3.00          1
  1       834    2500      3.00      3.00          1
Smoother (level 0) both : "Ifpack2::Relaxation":
  {Initialized: true, Computed: true,
   Type: Symmetric Gauss-Seidel,
   sweeps: 1, damping factor: (1, 0),
   Global matrix dimensions: [2500, 2500],
   Global nnz: 7498}
Smoother (level 1) pre  : SuperLU solver interface, direct solve
Smoother (level 1) post : no smoother
Set up problem #0
```



# Multiple right-hand sides

MueLu can solve multiple right-hand sides:

```
1 >> b = ones(2500,2);
2 >> for j = 1:2500, b(j,2) = 1/2500*j + 1/(2500*2500)*j.*j*i; end
3 >> [x,numIters] = muelu(p,b);
```

MATLAB output:

```
*****
***** Belos Iterative Solver: Pseudo Block Gmres
***** Maximum Iterations: 1000
***** Block Size: 1
***** Residual Test:
*****   Test 1 : Belos::StatusTestImpResNorm<>: (2-Norm Res Vec) / (2-Norm Prec Res0), tol = 1e-08
*****
Iter    0, [ 1] : 1.000000e+00  Iter    0, [ 2] : 1.000000e+00
Iter    2, [ 1] : 1.592409e-01  Iter    2, [ 2] : 1.414063e-01
Iter    4, [ 1] : 8.602037e-05  Iter    4, [ 2] : 7.553543e-05
Iter    6, [ 1] : 4.425200e-08  Iter    6, [ 2] : 3.866241e-08
Iter    7, [ 1] : 1.070092e-09  Iter    7, [ 2] : 9.321384e-10
Success, Belos converged!
```



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1 >> b = ones(2500,2);
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Success, Belos converged!
```

## Attention:

If the hierarchy is built with a complex operator  $A$ , the RHS vector has to contain at least one imaginary value!

# Results

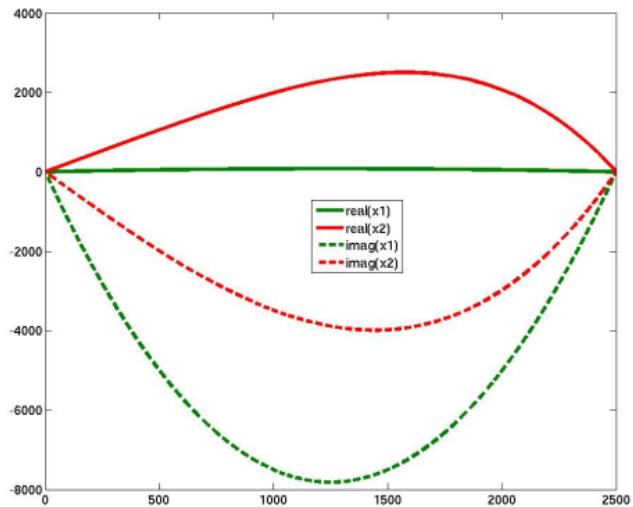
Visualization of results in MATLAB:

```

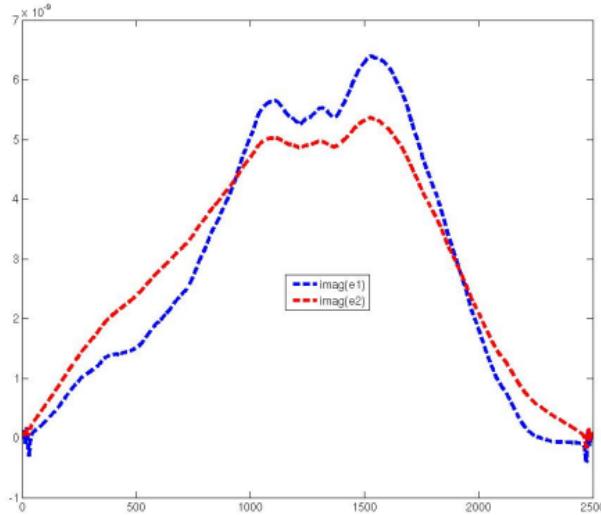
1 >> plot(real(x(:,1))); hold on; plot(real(x(:,2)));
2 >> plot(imag(x(:,1))); plot(imag(x(:,2))); hold off;
3 >> plot(imag(A\b-x));

```

Plot of solution vector:



Plot of imaginary part of error:



## 2. How to access MueLu data

# Study multigrid methods

**Example:** Study the multigrid effect on a 1d example:

```
1 >> A = gallery('tridiag', 600, -1, 2, -1);  
2 >> b=ones(600,1);  
3 >> [problemID,oc] = muelu('setup',A,'coarse: max size',50,  
multigrid algorithm','unsmoothed');
```

MATLAB output:

```
---  
                         Multigrid Summary  
---  
  
Number of levels      = 4  
Operator complexity = 1.48  
level   rows    nnz   nnz)row   c ratio   procs  
  0     600    1798      3.00          1  
  1     200     598      2.99      3.00          1  
  2      67     199      2.97      2.99          1  
  3      23      67      2.91      2.91          1
```

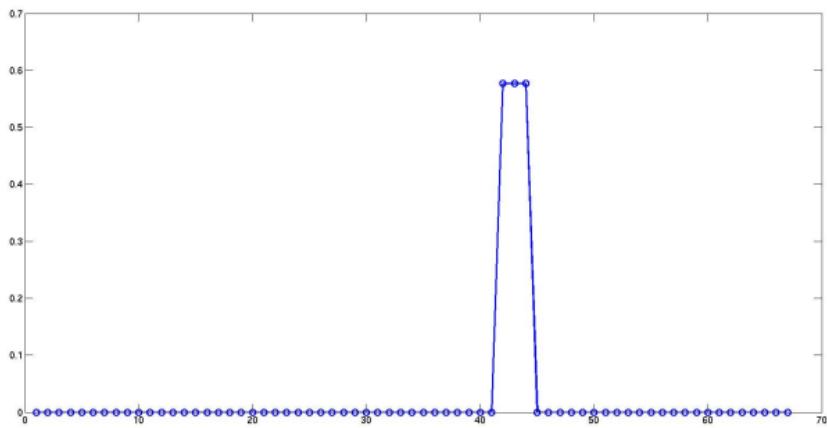


# How to access transfer operators?

Extract (non-smooth) prolongation operators:

```
1 >> Ptent1 = muelu('get', problemID, 1, 'P');
2 >> Ptent2 = muelu('get', problemID, 2, 'P');
3 >> Ptent3 = muelu('get', problemID, 3, 'P');
4 >> plot(Ptent3(:,15)); hold on; plot(Ptent3(:,15), 'o');
```

Plot of non-smooth basis function #15 of Ptent3:



# How to access coarse level operators?

Extract coarse level operator on level 3 and solve coarse level problem using MATLAB:

```

1 >> b1=Ptent1'*b;
2 >> b2=Ptent2'*b1;
3 >> b3=Ptent3'*b2;
4 >>
5 >> A3 = muelu('get',
   problemID, 3, 'A');
6 >>
7 >> x3 = A3 \ b3;

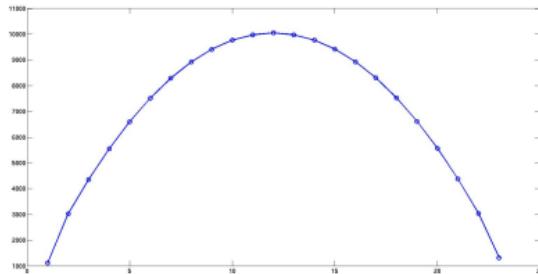
```

Plot of coarse level solution (level 3):

```

1 >> plot(x3); hold on;
2 >> plot(x3,'o'); hold off;

```



Exact coarse level solution of fine level problem.



# Fine level solution and level smoothing

Plot prolonged solution on level 2:

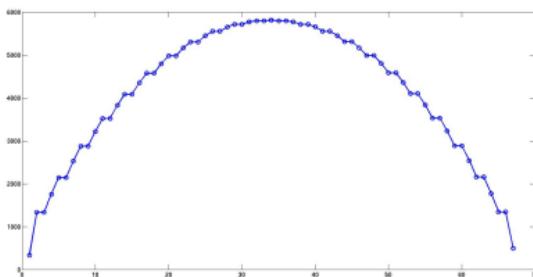
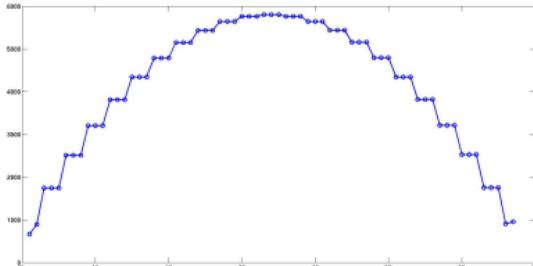
```

1 >> x2p = Ptent3 * x3;
2 >> plot(x2p); hold on;
3 >> plot(x2p,'o'); hold off;
```

Apply one sweep with Jacobi to prolonged solution:

```

1 >> A2 = muelu('get',
2     problemID, 2, 'A');
3 >> T = inv(D) * (tril(-A2
4     ,-1)+triu(-A2,1));
5 >> x2s = T * x2p + inv(D) *
6     b2;
7 >> plot(x2s); hold on; plot(
8     xs2,'o'); hold off;
```



# 2D example – default parameters

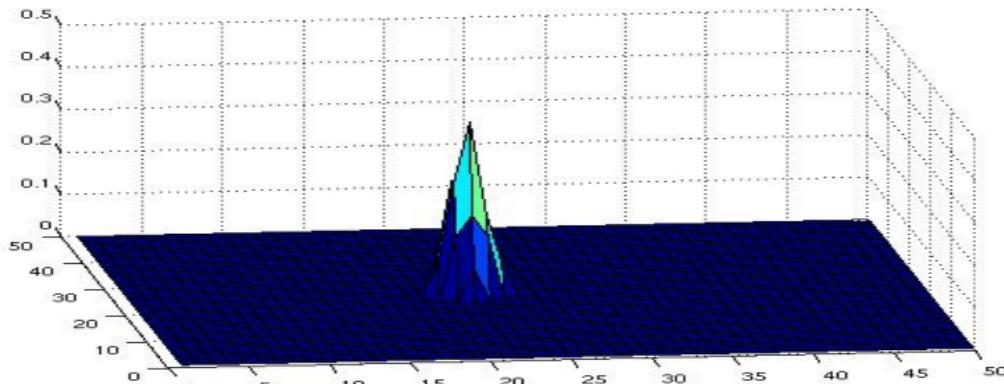
Visualize transfer operator basis functions of 2D Laplace problem:

```

1 >> [A, coords] = laplacianfun([50 50]);
2 >> [problemID] = muelu('setup', A);
3 >> P1 = muelu('get', problemID, 1, 'P');
4 >> [X, Y]=meshgrid(coords(:,1),coords(:,2));
5 >> Z = griddata(coords(:,1),coords(:,2),full(P1(:,212)),X, Y);
6 >> surf(X, Y, Z);

```

Plot of *smooth* prolongator basis function (associated with aggregate 212):



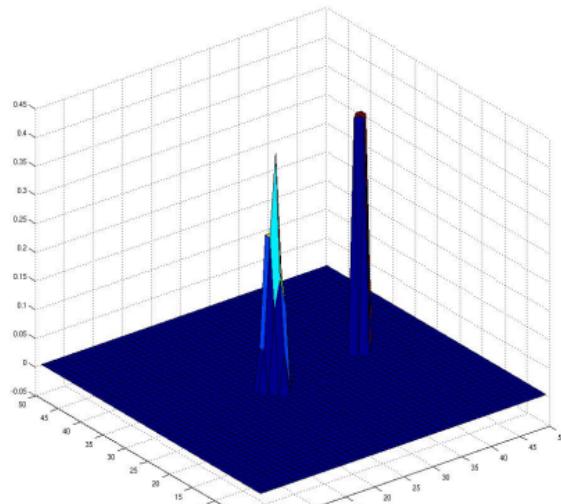
# 2D example – non-smooth transfers

Comparison of *smooth* prolongator basis function (associated with aggregate 212) and *non-smooth* basis function (associated with aggregate 234):

```

1 >> [problem2, oc] = muelu('setup'
2   'A,'multigrid algorithm','
3   'unsmoothed');
4 >> Ptent1 = muelu('get',
5   problem2, 1, 'P');
6 >> Z2 = griddata(coords(:,1),
7   coords(:,2),full(Ptent1
8   (:,234)),X,Y);
9 >> surf(X,Y,Z); hold on; surf(X,
10  Y,Z2); hold off;

```



3. Where can i learn more about MueLu?

# MueLu resources

- The MueLu user guide
  - can be found here: <https://trilinos.org/packages/muelu/muelu-documentation/>
  - serves as reference handbook
  - provides an overview of all available user parameters and basic examples
- Examples come with the MueLu sources in the examples folder
- Doxygen



# The MueLu tutorial

- can be found here: [www.trilinos.org/packages/muelu/muelu-tutorial](http://www.trilinos.org/packages/muelu/muelu-tutorial)
- comes with an **interactive GUI** for individual experiments
- **no Trilinos installation necessary:** we provide a VirtualBox image and a docker container
- The MueLu tutorial is divided into
  - **The beginners tutorial:** Chapters 1-5 meant for absolute multigrid beginners. No programming skills necessary. Explains basic usage for standard problems.
  - **The advanced tutorial:** Chapters 6-11 meant for intermediate users of MueLu. Explains design concepts of MueLu and focuses on advanced use concepts.
  - **Expert topics:** Chapters 12-end cover expert topics primarily for developers.



# Conclusion

- MueMex: MATLAB interface for MueLu
- allows to use MueLu as solver within MATLAB
- provides easy access to MueLu internals for further analysis
- works also for complex problems and multiple RHS
- ideal tool for research in context of multigrid
  - no C++ knowledge necessary
  - rapid development (no compilation necessary)
  - perfect tool for quick experiments and parameter studies
- MueMex is still under heavy development

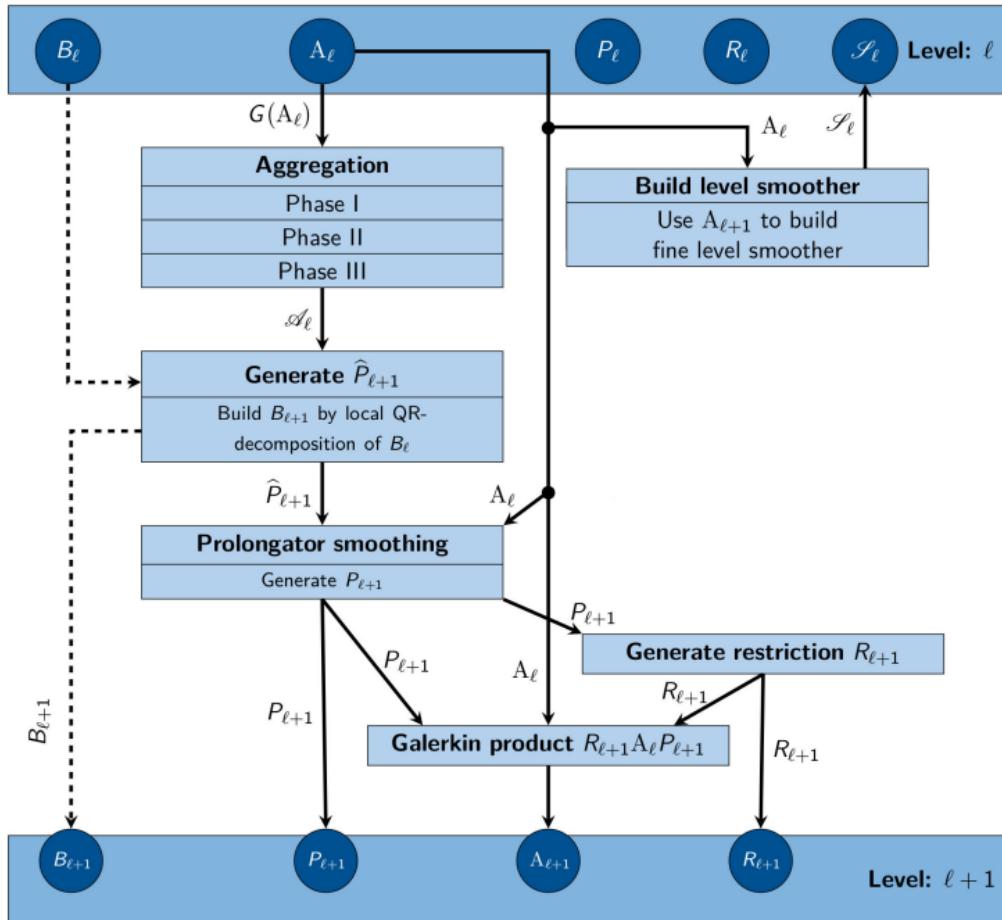


One more thing. . .

# MueMex – MATLAB extensions for MueLu

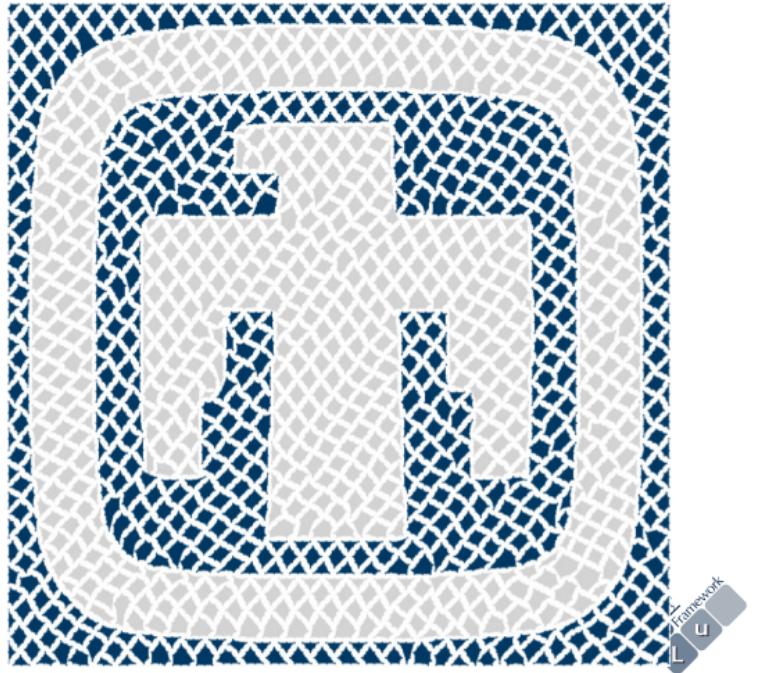
- Advanced software design principles of MueLu
  - Flexibility through modularity: multigrid framework
  - Strict splitting of algorithms and data
  - Algorithms are implemented in *factories* which use some input data to calculate/generate some other output data
- MueMex fully integrates in MueLu framework
  - Use callback mechanism to allow to plug in new factories in MueLu written in MATLAB
  - MueMex factories have full access to MueLu framework
  - MueMex factories can use full power of MATLAB

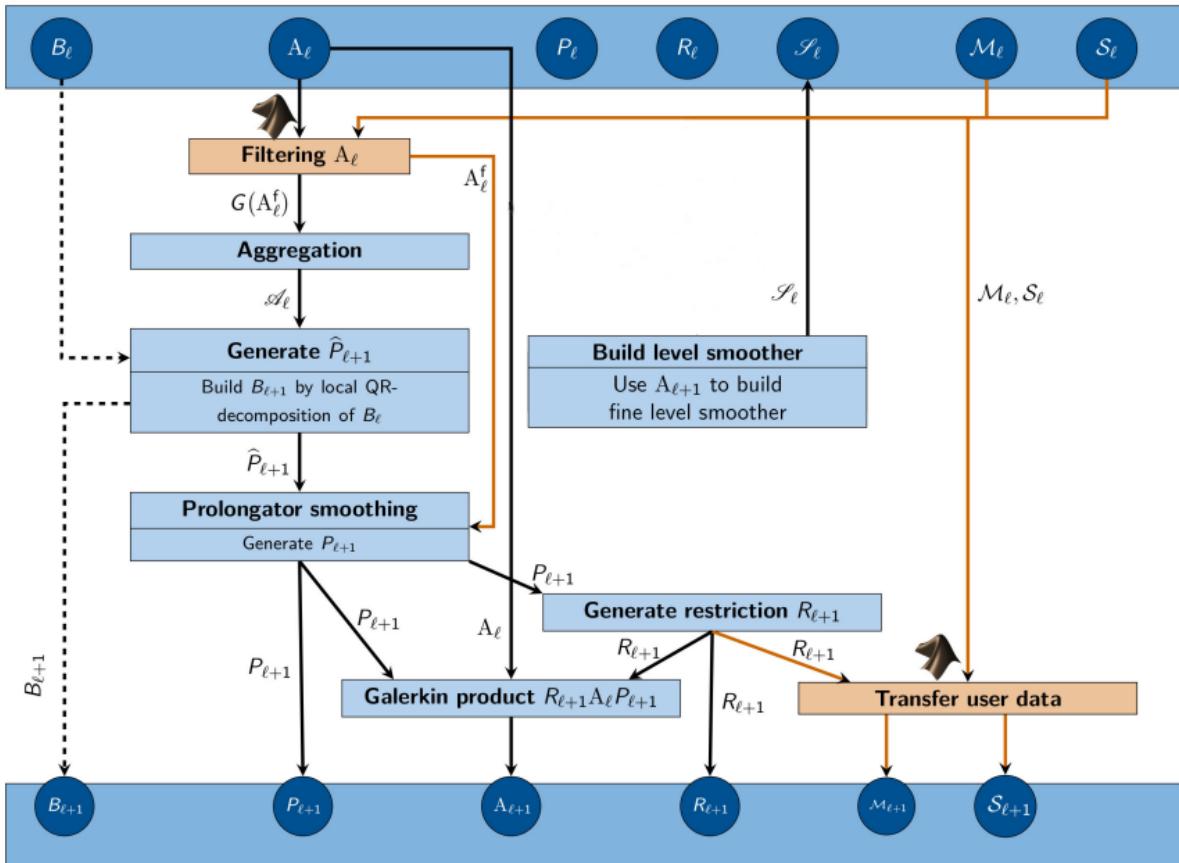




# Demonstration

Use monochrome picture data to drop entries in input graph of aggregation factory to enforce user-specified aggregates:





# Demonstration

- Write a factory in MATLAB which uses
  - the non-filtered matrix  $A$  as input
  - drops all off-diagonal entries in  $A$  which represent connections between color 1 (e.g. blue) and color 2 (e.g. white).
  - stores the filtered matrix  $A$  for being used in aggregation algorithm
- transfer monochrome picture data accordingly to coarse level (this is optional if only two-level method is used)
- XML file controls the interconnection and dependencies of factories
  - MATLAB factories fully integrate in existing **MueLu framework** with all factories written in C++/MATLAB
  - optimal **flexibility**
  - **recombination and reuse** of factories without recompilation of source code
  - perfect tool to design new **application-specific preconditioning strategies**



One last thing. . .

# Thanks

Thank you for your attention

Special thanks go to

- the MueMex developers, especially
  - Brian Kelley
  - Chris Siefert
- the MueLu developers, especially
  - Jonathan Hu
  - Andrey Prokopenko
  - Jeremie Gaidamour
- the developers of
  - Amesos and Amesos2
  - Ifpack and Ifpack2
  - Zoltan and Zoltan2
  - Epetra and Tpetra
- all Trilinos developers in general



