System Simulation Homework

March 6, 2019

General Homework Guidelines

Submission of homework will be done electronically via the course's Brightspace page. Each problem will have a submission folder. The required pieces for each simulation are

(A) a .pdf file containing all plots for the simulations,

(B) an .m file containing the code used to perform the simulation, and

(C) a .pdf file of a scan of any work done by hand.

Problem 01: Simulation of a Difference Equation

Simulate the difference equation given by

$$\begin{aligned} x[0] &= 0.11 \\ x[k+1] &= \alpha\,(1 - x[k])\,x[k], \end{aligned}$$

for at least seven values of gain α. Six of the values must be $\alpha = 0.80$, $\alpha = 1.35$, $\alpha = 2.75$, $\alpha = 3.20$, $\alpha = 3.52$, and $\alpha = 4.0$. Then choose a value of $\alpha \in (3.57, 4.0)$. Create a plot of $x[k]$ versus k that includes at least 100 samples, but not so many that the system dynamics are obscured.

Please upload to the course Brightspace the required documents.

It is highly recommended that you use MATLAB for this assignment, as you are expected to use MATLAB for the assignments for the rest of the semester. If you wish to use another program, please discuss it with me before doing so.

The purpose of this assignment is for you to refresh yourself on how to write an m-file in MATLAB and plot the results. You will be doing this for at least two problems a week for the rest of the semester.

The MATLAB commands that I used are

- Assignment and arithmetic commands
- $zeros$
- $linspace$
- for
- $plot$
- $title$, $xlabel$, and $ylabel$

Problem 02: SR-71 Supersonic Inlet

The SR-71 was a supersonic aircraft that could fly at speeds in excess of $Mach$ 3.2. Its propulsion system used two Pratt & Whitney J-58 turbo-ramjet engines. Essential to the performance of these engines is the supersonic inlet on their front. These inlets convert fast cold air into slow hot air through a normal shock wave that sits just downstream of the inlet throat. The position of the shock wave determines the efficiency of the energy conversion. The closer the shock is to the throat, the more efficient the conversion, however, if the shock moves up past the throat, the engine will unstart in milliseconds, potentially causing a catastrophic failure. The shock position is controlled by linearly opening and closing downstream bypass doors. The transfer function for $Mach$ 2.5 operation is

$$H(s) = \frac{X_s(s)}{W_{BP}(s)} = \frac{50e^{-0.015s}}{s + 2 + 50e^{-0.02s}}, \tag{1}$$

which can be approximated by

$$H(s) \approx \frac{50(-s^2 + 33.3333s + 13333.3)}{s^3 + 185.333s^2 + 12133.3s + 693333}. \tag{2}$$

Simulate the approximated transfer function given in Equation 2 using the explicit z-transform substitution method. Provide an impulse response and a step response from your simulation, along with your analysis, your programs, and the plots. You may wish to compare your results with the *impulse* and *step* commands in MATLAB.

Problem 03: Buck Converter - Transfer Function Approach

A DC-DC Buck Converter is a circuit that is used to convert voltages and/or currents. Currently, buck converters are seeing use as regulators for photovoltaic cell outputs. An ideal Buck converter is shown in Figure 1. The switch on the input voltage makes this a non-linear system; additionally, both the input voltage and the switching duty cycle can be considered inputs to the system. A technique called state-space averaging is used to obtain transfer functions from each input v_{in}(t) and $s(t)$ to the output $v_{out}(t)$ by assuming that $s(t)$ is a constant equal to its duty cycle and $V_{in}(t)$ is a constant voltage, respectively. These transfer functions are

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{4}{3} \times 10^7}{s^2 + 250s + 3.33 \times 10^7}$$

and

$$\frac{V_{out}(s)}{S(s)} = \frac{4 \times 10^8}{s^2 + 250s + 3.33 \times 10^7}.$$

(A) Simulate the response of $v_{out}(t)$ if $v_{in}(t) = 12\,\mathcal{U}_s\,(t)$ using backwards Euler operational substitution technique to obtain a difference equation.

(B) Simulate the response of $v_{out}(t)$ if $s(t) = 0.4\,\mathcal{U}_s\,(t)$ using backwards Euler operational substitution technique to obtain a difference equation.

Be sure to include your analysis, your programs, and the plots.

Problem 04: Buck Converter - State Space

In Problem 03 transfer functions for a buck converter using a time-averaging approach were used to simulate the converter under ideal (no noise) conditions. However, that model is difficult to interpret.

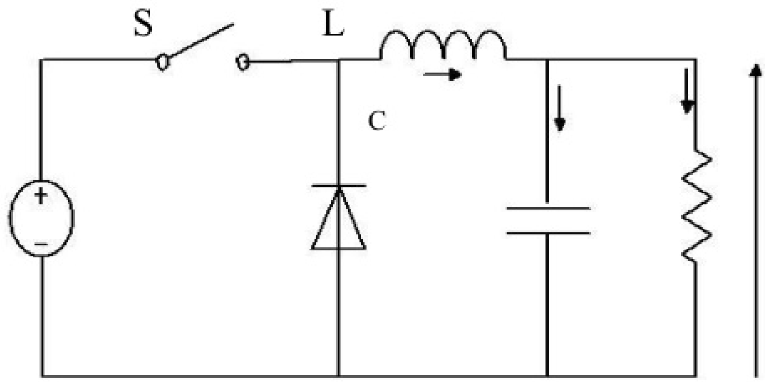

Figure 1: Ideal DC-DC Buck Converter.

A state-space representation for the converter is given by

$$\begin{aligned}
\dot{x}_1(t) &= -\frac{1}{L}x_2(t) + \frac{1}{L}\ s(t)\ v_{in}(t) \\
\dot{x}_2(t) &= \frac{1}{C}x_1(t) - \frac{1}{RC}x_2(t) \\
v_{out}(t) &= x_2(t),
\end{aligned}$$

where $x_1(t)$ is the current through the inductor, $x_2(t)$ is the voltage across the capacitor, R, L, and C are the component values, $v_{in}(t)$ is the input voltage, $v_{out}(t)$ is the output voltage, and $s(t)$ represents the state of the switch: $s(t) = 1$ when the switch is closed and $s(t) = 0$ when the switch is open. Use $R = 400\ \Omega$, $L = 3\ mH$, and $C = 10\ \mu F$.

(A) Explore the MATLAB commands $tf2ss$ and $ss2tf$. Use $ss2tf$ to verify the transfer functions $\frac{V_{out}(s)}{V_{in}(s)}$ and $\frac{V_{out}(s)}{S(s)}$ by setting $s(t) = 0.4$ and $v_{in}(t) = 12$ respectively.

(B) Simulate this buck converter if the input voltage is $v_{in}(t) = 12\mathcal{U}_s(t)$ and $s(t)$ has frequency $f = 55\ kHz$ and a duty cycle of 0.4 using Euler's method as a linear multistep method on the state space equations. Be sure to include your analysis, your programs, and the plots.

Problem 05: State-Space Representation for an RLC Circuit

Consider the circuit shown in Figure A.

(A) Obtain a state-space representation for the RLC circuit using the following steps. Note that your state-space representation should be an expression in terms of the component values R_1, R_2, R_3, C_1, C_2, C_3, and L.

  (a) Assign each capacitor voltage and the inductor current to a state.
  (b) Write the sum of the currents into each of the nodes at v_1, v_2, and v_3 in terms of the state variables.
  (c) Write the sum of the voltages in the loop that includes the input and capacitor C_1 in terms of the state variables.
  (d) Combine the resulting equations into a single matrix equation for the state-update equation.

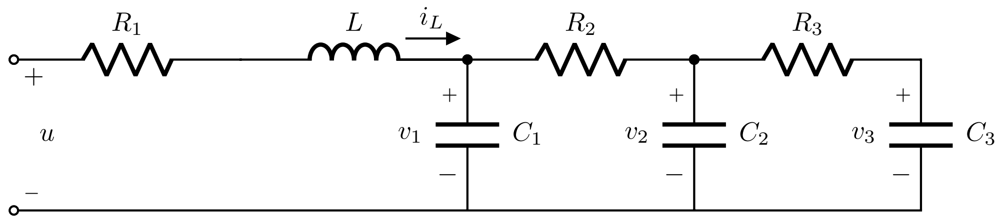

Figure 2: RLC Circtuit for Problem 05.

Component	Value
R_1	500 Ω
R_2	1 $k\Omega$
R_3	1 $k\Omega$
C_1	4.7 μF
C_2	4.7 μF
C_3	4.7 μF
L	2 H

Table 1: Component Values for RLC Circuit.

(e) If each of the capacitor voltage are be considered an output. Explain why

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

(B) Using the component values shown in Table , write the numeric state-space representation.

(C) Assume that only the voltage across capacitor C_3 is the output and write the numeric state-space representation using the component values in Table . Then determine the associated transfer function (you may use the MATLAB command $ss2tf$). Verify that the eigenvalues of A and the poles of the transfer function are the same.

Problem 06: The Rössler Attractor

The Rössler attractor is a chaotic attractor solution to the system

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c). \end{aligned}$$

Rössler studied the chaotic attractor with $a = 0.2$, $b = 0.2$, and $c = 5.7$. More recent studies have used $a = 0.1$, $b = 0.1$, and $c = 14$. Typically, phase plane plots of x versus y after the transient has vanished are studied.
For several situations simulate the Rossler system using the Adams-Bashforth second-order (AB2) linear multistep method applied to the state variable description with time-step $T = 100\ \mu s$. Use

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_0 = \begin{pmatrix} 5 \\ 5 \\ 10 \end{pmatrix}$$

as the initial conditions.

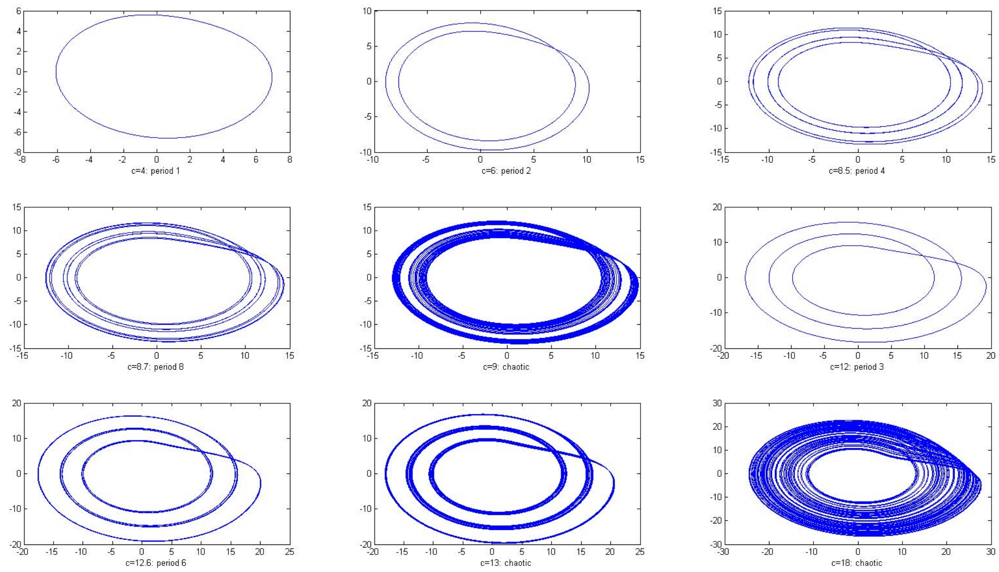

Figure 3: Rössler Attracter Phase Plane Portraits.

(A) Verify three of the behaviors in Figure 3. These phase plane plots are of x versus y. Be sure to include program and plots. Start the method correctly.

(B) Use *plot*3 in MATLAB to plot the simulation results of the system with

$$\begin{aligned} a &= b = 0.3 \\ c &= 8.0 \end{aligned}$$

(C) Finally, look over the paper "Power spectral analysis of a dynamical system"[1] and verify two of the phase plane plots presented there. Note that the parameters have different names in this paper.

To plot the system after the transient has died, simulate the system for N steps from the initial conditions, and plot the points from steps M to N. For example, to generate the plots in Figure 3, I used $N = 10,000,001$, and plotted the system from $M = 5,000,000$ to N using

```
plot(x(5000000:N),y(5000000:N))
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Problem 07: The Lorenz System

The Lorenz system is given by

$$\begin{aligned} \dot{x} &= 10(y-x) \\ \dot{y} &= -xz + Rx - y \\ \dot{z} &= xy - \frac{8}{3}z \end{aligned} \tag{3}$$

This system is a simplified approximation to the Earths weather, and also describes boiling convection in a pot, and a fluid loop thermosiphon as shown in Figure 4.

[1] Crutchfield, J., Farmer, D., Packard, N. H., Shaw, R., Jones, G., & Donnelly, R. J. (1980). Power spectral analysis of a dynamical system. Physics Letters A, 76(1), 1-4.

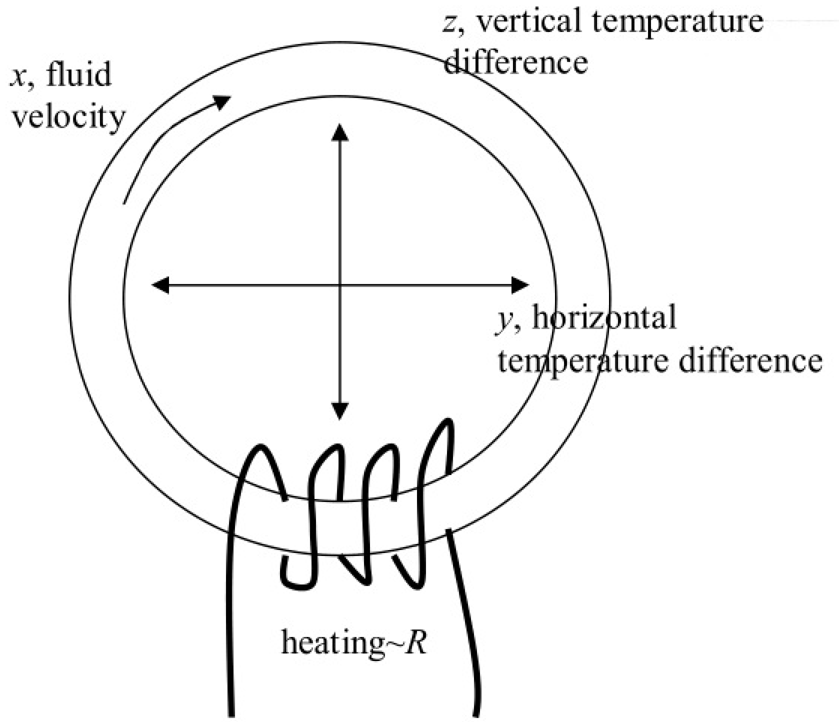

Figure 4: Fluid Loop Thermosiphon.

(A) Determine the equilibrium points for the Lorenz system using $R = 28$.

(B) For $R = 28$ and

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_0 = \begin{pmatrix} 1 \\ 2 \\ 10 \end{pmatrix},$$

simulate the Lorenz system using AB-4. Be sure to start the simulation properly. Include your programs, time responses, and $3-D$ phase space plots. If you have time, play a little, and use other values for R.

Problem 08: Detailed Stability Regions

Obtain a detailed stability region plots for the AB-3 integrator.

Problem 09: Determining the Most Accurate Integration Method

Use Lambert's equations to determine the most accurate, three step, explicit integration method with a z-plane poles constrained to be at $z = 0.15$. Plot the stability region for this method. Locate relatively stable, relatively unstable, and absolutely unstable regions.

Problem 10: A Chebyshev Filter

A Chebyshev filter provides improved cutoff properties for low- pass filtering in exchange for some ripple in the passband. A 1 Hz, fourth-order, Chebyshev lowpass filter allowing 5 dB ripple in the passband is given by

$$G(s) = \frac{0.0850}{s^4 + 0.4174s^3 + 1.0871s^2 + 0.2805s + 0.1512}.$$

(A) Determine the s-plane poles of $G(s)$.

(B) Plot a frequency response for $G(s)$.

(C) Plot the AB-2 region of absolute stability and plot the detailed stability region from the primary domain. These can come from adapting the MATLAB code provided on the course Brightspace.

(D) Plot all of the λT-products for this system on the stability region for a relatively stable and accurate value of T, for a relatively unstable and inaccurate T, and for unstable T.

(E) For the three T selected in part (D), determine the corresponding z-plane poles.

(F) Implement an AB-2 simulation to obtain a step response for this filter using both of your relatively stable and relatively unstable values of T. Use $tf2ss$ in MATLAB to get a state representation.

Problem 11: An RLC Circuit Revisited

Consider the system given in Figure A.

(A) Determine the eigenvalues of the A matrix in the state-space representation.

(B) Plot the AB-3 region of absolute stability, and plot the detailed stability region mapped from the primary domain.

(C) Plot all of the λT-products for this system on the stability region for a relatively stable and accurate value of T, and for a relatively unstable and inaccurate T.

(D) For the two T selected in part (D), determine the corresponding z-plane poles, labeling the principle and the spurious poles.

(E) Implement an AB-3 simulation to obtain a step response for this filter using both of your relatively stable and relatively unstable values of T. Use $tf2ss$ in MATLAB to get a state representation.

Problem 12: Sprott's Simple Chaotic System

Use the AB-4 method to simulate the system given by

$$\begin{aligned}\dot{x} &= 0.4x + z \\ \dot{y} &= xz - y \\ \dot{z} &= -x + y,\end{aligned}$$

with

$$x_0 = y_0 = z_0 = 0.1.$$

Plot time responses and a 3-dimensional plot of the attractor, verifying the phase plot given by Sprott[2], and be sure to include your code.

[2] Sprott, J. C. "Some simple chaotic flows." Physical review E 50.2 (1994): R647.

Problem 13: Chua's System

The equations for Chua's system are

$$\begin{aligned}
\dot{x} &= \alpha\left(y + \frac{x - 2x^3}{7}\right) \\
\dot{y} &= x - y + z \\
\dot{z} &= -\frac{100}{7}y,
\end{aligned}$$

where α can typically vary from 0 to about 10.6. Simulate this sytem using the forward Euler-Trapezoidal Rule predictor-corrector pair in PECE mode or PECECE mode. Use some small initial conditions and several values of α. Submit plots of the state variables vs. t as well as a three-dimensional phase plot. A bifurcation diagram for this system is given in Figure 5

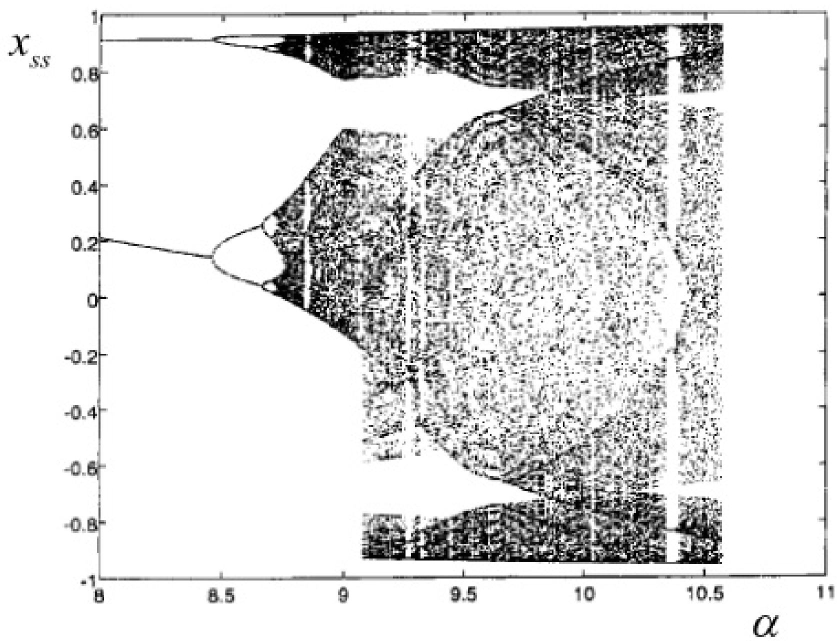

Figure 5: Bifurcation Diagram for Chua's System.

Problem 14: An Elliptic Filter Revisited

An elliptic filter provides steep rolloff, while maintaining equiripple in the passband and the stopband. A 1 Hz, fifth-order elliptic filter allowing 0.001 dB ripple in the passband, and 20 dB ripple in the stopband is given by

$$G(s) = \frac{0.6042s^4 + 4.1572s^2 + 6.1591}{s^5 + 3.4630s^4 + 7.1704s^3 + 10.2787s^2 + 8.6926s + 6.159}.$$

(A) Determine the s-plane poles and zeroes of $G(s)$.

(B) Plot a frequency response for $G(s)$.

(C) Get an ABM2 PECE and PECECE stability region from the appendix of the book.

(D) by hand, plot all of the λT-products for this system on the stability regions for a relatively stable and accurate value of T, and for a relatively unstable and inaccurate T.

(E) implement an ABM2 PECE or PECECE simulation to obtain a step response for this filter using both of your relatively stable and relatively unstable values of T.

Problem 15: Mosquito-Borne Disease

Simulate the mosquito-borne disease system using modified Euler. The Ross-MacDonald mosquito-borne disease system model is

$$\begin{aligned} \dot{Y} &= abI\left(\frac{H-Y}{H}\right) - \xi Y \\ \dot{I} &= ac(V-I)\frac{Y}{H} - \delta I, \end{aligned}$$

where V is the total mosquito population, I is the number of infected mosquitoes, H is the total human population, Y is the number of infected humans, a is the mosquito bite rate, b is the mosquito-to-human transmission probability, c is the human-to-mosquito transmission probability, ξ is the human recovery rate, and δ is the mosquito death rate. Assume that the numbers of humans H and the numbers of mosquitoes V are constant. Recall that average duration of mosquito infection is $D_M = \frac{1}{\delta}$ and that the average duration of human infection is $D_H = \frac{1}{\xi}$. The basic reproductive number R_0 is the average number of secondary infections that result if a single infectious individual is introduced into an entirely susceptible population and is given by

$$R_0 = ma^2bcD_HD_M = (abD_M) \times (macD_H) = R_0^{VH} \times R_0^{HV},$$

where $m = \frac{V}{H}$ is the ratio of the number of mosquitoes to the number of humans. population. If $R_0 > 1$, then the infection will invade and persist. Assume a human population of $H = 100$. Table 2 gives three sets of parameters for possible outbreaks. For each of the cases,

Parameter	Case 1	Case 2	Case 3
a	0.3	0.1	0.5
b	0.2	0.03	0.4
c	0.5	0.275	0.4
δ	0.03302	0.03304	0.1
ξ	0.01	0.0035	0.05

Table 2: Parameters for Mosquito Outbreaks.

(A) Determine the ratio of mosquitoes to humans m_0 under which an outbreak will not invade or persist.

(B) Simulate the system for three values of mosquito population $V = mH$, one with $m < m_0$, one with m slightly larger than m_0, and one with m much larger than m_0.

(C) Draw conclusions about each of the cases, discussing the ease of transmission of the disease.

Problem 16: Dynamical Models of Happiness

Sprott[3] considers several dynamical systems to model happiness. Using a fourth-order Runge-Kutta method, simulate the model given by Equation (4) in the paper using the parameters shown below Figure 7 in the paper. Use small initial conditions. Submit plots of R vs t, H vs. t, and H vs R.

[3] Sprott, J. C. "Dynamical models of happiness." Nonlinear Dynamics, Psychology, and Life Sciences 9.1 (2005): 23-36.

Problem 17: Folded Torus System

A relatively easily realized circuit that generates Torus breakdown is given by Matsumoto et al.[4] Equation (3) gives a simplified dimensionless form. Fix the parameters $a = 0.07$, $b = 0.1$, and $\beta = 1$. Simulate the folded torus system for the parameter $\alpha = 2$, and $\alpha = 15$, using the standard fourth-order Runge-Kutta method as a variable timestep method with an embedded error estimate. Be sure to plot error vs time, and timestep vs time, as well as, the state variables vs time, and x vs y to reproduce the phase planes of the paper. Note that the variable e in the paper is not the error estimate!

Problem 18: Stiff System I

To Be Determined

Problem 19: Sludge

Consider the convection-conduction equation

$$\frac{\partial u(x,t)}{\partial t} + \frac{\partial u(x,t)}{\partial x} = \frac{\partial^2 u(x,t)}{\partial x^2} + f(x,t),$$

with $u(x,0) = 1$, $f(x,t) = 0$, $u(0,t) = v_1(t)$, $u(L,t) = v_2(t)$, and $L = 4$. Let

$$v_1(t) = 1 + \mathcal{U}_s\,(t-10) - \mathcal{U}_s\,(t-12)\,,$$

and $v_2(t) = 1$.
Among other things, this system models the flow of some diffusing material (or heat), $u(x,t)$, in a moving media. An example of this would be some type of waste being dumped into a stream, or heat diffusing in a shower flow.

(A) Determine the free-space transfer function for this system, $\frac{U(p,s)}{F(p,s)}$.

(B) Map the boundary of stability in the p-plane into the s-plane. Discuss the behavior of the spatial frequencies.

(C) Spatially lump this system to obtain a set of ordinary differential equations.

(D) Find the free-space transfer function for the spatially lumped system, that is, get $\frac{U(z_x,s)}{F(z_x,s)}$.

(E) Map the boundary of stability in the z_x-plane into the s-plane. Discuss the behavior of the spatial frequencies and the accuracy of the approximation relative to the mapping in part (B).

(F) Approximate the time derivative in part (C) by an appropriate simulation method to obtain a set of difference equations in time and space.

(G) Find the free-space transfer function for the discrete-time-spatially-lumped system, that is, get $\frac{U(z_x,z_t)}{F(z_x,z_t)}$.

(H) Map the boundary of stability in the z_x-plane into the z_t-plane. Discuss the behavior of the spatial frequencies and the accuracy of the approximation relative to the exact mapping in part (I).

(I) Determine the exact discrete-time mapping of the map in part (B), that is, plot $z_t = e^{sT}$, with s given by the denominator of $\frac{U(p,s)}{F(p,s)}$ equal to zero.

[4]Matsumoto, Takashi, L. Chua, and Ryuji Tokunaga. "Chaos via torus breakdown." IEEE Transactions on Circuits and Systems 34.3 (1987): 240-253.

(J) Determine some reasonable values for T and H from the maps in parts (H) and (I), and simulate the discrete-time-discrete-space system you derived. Plot the results on a waterfall plot using the MATLAB mesh, surf, contour, or some other appropriate command.

Bonus

(K) From part (C), create a state space realization, $\dot{u} = Au + Bv$, and pick any point in the medium as an output point. Find the eigenvalues of the A-matrix for your chosen value of H.

(L) From part (F), create a state space realization, $u[k+1] = Au[k] + Bv[k]$, and pick any point in the medium as an output point. Find the eigenvalues of the A-matrix for your chosen values of H and T.