# System Simulation Midterm Exam

Due: 01 April 2019

## Problem 01: Predictor-Corrector Design

Design a two-step predictor-corrector pair.

(A) Design the most accurate explicit two-step predictor $H_p(z)$ with a zero at $z = \frac{4}{7}$.

(B) Design the most accurate implicit two-step corrector $H_c(z)$ with a zero at $z = \frac{4}{7}$.

## Problem 02: Predictor Analysis

Analyze the predictor, $H_p(z)$ that you designed in Problem 01. Please make sure that your answer for Problem 01, part (A) is correct before attempting this problem.

(A) What is the order of accuracy of the predictor?

(B) What is the principle local truncation error for the predictor?

(C) Obtain the detailed stability region for the predictor using the primary domain.

(D) If the secondary domain is relatively easy to obtain, give the detailed stability region for the predictor using the secondary domain.

(E) Based on the stability region, determine the regions in $\mathbb{C}_z$ for which the associated simulations are unstable (and inaccurate), for which the associated simulations are stable but inaccurate, and for which the associated simulations are stable and accurate.

## Problem 03: Explicit Simulation

Consider the system whose state-space representation is

$$\begin{aligned}
\dot{x} &= \begin{pmatrix} -4.7 & -1.55 & -0.55 \\ 0.3 & -2.75 & -0.35 \\ 1.1 & 1.85 & -2.55 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} u \\
y &= \begin{pmatrix} 2 & 1 & 1 \end{pmatrix} x.
\end{aligned}$$

(A) Determine the eigenvalues of the state-space representation.

(B) On a copy of the detailed stability region for your predictor, plot the lines $z = \lambda T$ for appropriate values of $T > 0$.

(C) Select three values of $T$, one for which the associated simulation is unstable (and inaccurate), for which the associated simulation is stable but inaccurate, and for which the associated simulation is stable and accurate.

(D) For the three $T$ selected in Part C, determine the corresponding closed-loop simulation poles.

(E) Simulation the system twice, once using the stable and accurate value of $T$ and once using the stable but inaccurate value of $T$. Compare and contrast the results.

## Problem 04: Corrector Analysis

Analyze the corrector, $H_c(z)$ that you designed in Problem 01. Please make sure that your answer for Problem 01, part (B) is correct before attempting this problem.

(A) What is the order of accuracy of the corrector?

(B) What is the principle local truncation error for the corrector?

(C) Obtain a detailed stability region for the corrector using the primary domain.

(D) Determine a value of $T$ that results in a stable and accurate simulation.

(E) Compare and contrast the stability region for the corrector with the stability region obtained for the predictor.

## Problem 05: Predictor-Corrector Simulation

Consider the system whose state-space representation is

$$\begin{aligned}\dot{x} &= \begin{pmatrix} -4.7 & -1.55 & -0.55 \\ 0.3 & -2.75 & -0.35 \\ 1.1 & 1.85 & -2.55 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} u \\ y &= \begin{pmatrix} 2 & 1 & 1 \end{pmatrix} x.\end{aligned}$$

(A) Using the $T$ obtained in Problem 03 that resulted in a stable and accurate simulation, simulate the system using the predictor-corrector pair that you designed in Problem 01. You may correct as many times as you wish[1].

(B) Using the $T$ obtained in Problem 04, simulate the system using the predictor-corrector pair that you designed in Problem 01. You may correct as many times as you wish[2].

(C) Compare and contrast the results using just the predictor (in Problem 03) to the results using the corrector.

## Problem 06: The Periodically Forced Van Der Pol System

While the Van Der Pol system is a second-order system, and therefore the zero-input response cannot exhibit chaos, when the Van Der Pol System is periodically forced, it is in essence a fourth-order system[3] and chaos can appear. The forced Van Der Pol system is

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \epsilon(1 - x_1^2)x_2 - x_1 + u\end{aligned}$$

[1] Although you must correct at least once!

[2] Although once again you must correct at least once.

[3] If

$$u(t) = A\cos(\omega t),$$

then the Van Der Pol system is augmented by the additional states required to implement

$$\ddot{u} = \omega^2 u,$$

say

$$\begin{aligned}\dot{u} &= x_3 \\ \dot{x}_3 &= \omega^2 u.\end{aligned}$$

The amplitude $A$ contributes to the initial conditions.

where $u$ is the forcing function. Obtain three phase-plane plots for the forced Van Der Pol system when $\epsilon \in \{6, 8.53, 10\}$ for the forcing function

$$u(t) = 1.2\cos\left(\frac{2\pi t}{10}\right)$$

using the Adams-Bashforth-2 method. Remember to remove the transient. Characterize each of resulting phase plots to the best of your ability.