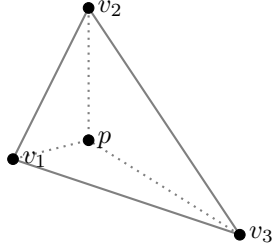


1 Barycentric Coordinates

Barycentric coordinates enable you to evenly interpolate between three values among the vertices of a triangle.



$$U = (u_1, u_2, u_3) \quad (1)$$

$$v_1 = (x_1, y_1) \quad (2)$$

$$v_2 = (x_2, y_2) \quad (3)$$

$$v_3 = (x_3, y_3) \quad (4)$$

$$p = (x_p, y_p) \quad (5)$$

The Barycentric coordinates can be defined in terms of the following relationships:

$$\begin{cases} u_1 + u_2 + u_3 = 1 \\ u_1x_1 + u_2x_2 + u_3x_3 = x_p \\ u_1y_1 + u_2y_2 + u_3y_3 = y_p \end{cases} \quad (6)$$

Let's reduce the amount of variables in these equations:

$$u_3 = 1 - u_1 - u_2 \quad (7)$$

$$\begin{cases} u_1(x_1 - x_3) + u_2(x_2 - x_3) = x_p - x_3 \\ u_1(y_1 - y_3) + u_2(y_2 - y_3) = y_p - y_3 \end{cases} \quad (8)$$

Now we can turn the system of equations into matrix form:

$$T = \begin{bmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \end{bmatrix} \quad (9)$$

$$U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (10)$$

$$R = \begin{bmatrix} x_p - x_3 \\ y_p - y_3 \end{bmatrix} \quad (11)$$

$$T \cdot U = R \quad (12)$$

So the solution is

$$U = T^{-1} \cdot R \quad (13)$$

The main effort goes towards finding T^{-1}

$$T^{-1} = \frac{\text{adj}(T)}{\det(T)} \quad (14)$$

$$\det(T) = (x_1 - x_3)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_3) \quad (15)$$

$$\text{adj}(T) = \begin{bmatrix} y_2 - y_3 & x_3 - x_2 \\ y_3 - y_1 & x_1 - x_3 \end{bmatrix} \quad (16)$$

$$T^{-1} = \frac{1}{\det(T)} \cdot \begin{bmatrix} y_2 - y_3 & x_3 - x_2 \\ y_3 - y_1 & x_1 - x_3 \end{bmatrix} \quad (17)$$

$$T^{-1} \cdot R = \frac{1}{\det(T)} \cdot \begin{bmatrix} (y_2 - y_3)(x_p - x_3) + (x_3 - x_2)(y_p - y_3) \\ (y_3 - y_1)(x_p - x_3) + (x_1 - x_3)(y_p - y_3) \end{bmatrix} \quad (18)$$

And the final formula you need to find (u_1, u_2, u_3) given the points v_1, v_2, v_3, p is

$$u_1 = \frac{(y_2 - y_3)(x_p - x_3) + (x_3 - x_2)(y_p - y_3)}{(x_1 - x_3)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_3)} \quad (19)$$

$$u_2 = \frac{(y_3 - y_1)(x_p - x_3) + (x_1 - x_3)(y_p - y_3)}{(x_1 - x_3)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_3)} \quad (20)$$

$$u_3 = 1 - u_2 - u_1 \quad (21)$$