11. Multi-criteria Decision-Making Algorithms

PYTHON COURSE

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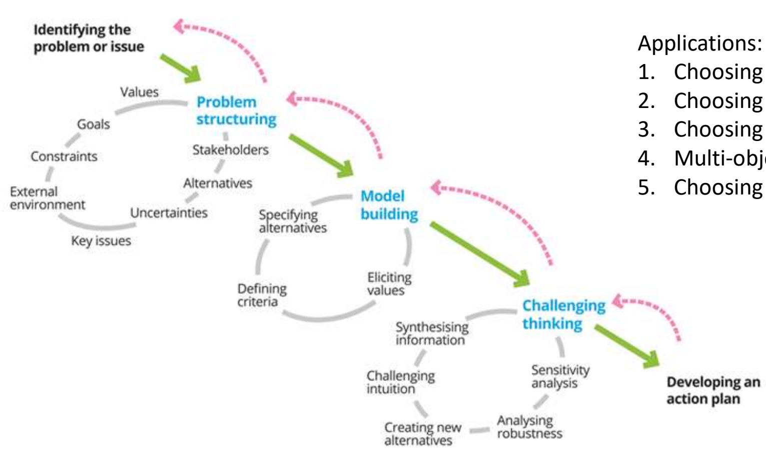
Multi-Criteria Decision-Making (MCDM) Algorithms Concept



Applications:
1. Choosing technology
2. Choosing process pathway
3. Choosing debottlenecking schedule
4. Multi-objective optimization
5. Choosing stocks, location, items, etc.

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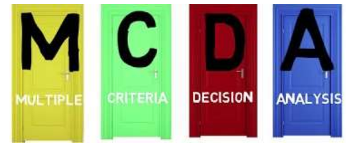
Algorithmic Steps of MCDM



1. Define criteria that are important and to be compared
2. Obtain alternative solutions to be ranked or chosen between
3. Weights representing the relative importance of the criteria
4. Normalize the data carry out the decision-making algorithm

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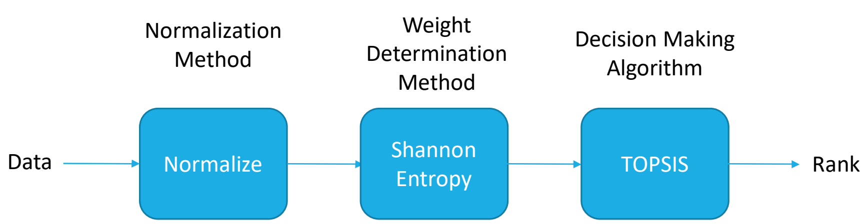
What we will do



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Step 1: Load the file and normalize

Environmental Impact	Energy Consumption	Profit	Product Yield	Product Quality
103310.01	106376.27	53042	39.805	48.228
22640.76	24951.52	657998	18.51	48.3
20103.3	22358.18	648282	17.915	48.3
8974.44	11137.39	455246.1	17.915	48.084
8514.9	10667.73	455246.1	17.915	48.248
3420	5430	455246.1	10.1	46.756
20113.29	107530	779539.2	60.1	48.228
20113.29	25380.34	1122898	22.13	48.3
20113.29	22787	1103099	21.265	48.3
8974.44	11331.38	1058369	14.465	48.084
8514.9	11331.38	996223.4	13.93	48.248
11451.96	11331.38	1281286	12.84	47.576
17445.96	10871.93	1761034	18.145	24.968
16836.57	10330.8	1886242	18.4	24.968
11262.15	5430	1532068	14.22	8.3

1. You are given a file called “technology.csv”.
2. Prepare it as a ndarray.
3. Perform column min-max normalization. $x_{ij} = \frac{y_{ij} - {y_{ij}}^{min}}{{y_{ij}}^{max} - {y_{ij}}^{min}}$
4. Save it as “data_norm.csv”.

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Step 2: Define the weights (Shannon Entropy)

1. Load the normalized data:

$$G = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix}$$

```
[[1.924e-01 9.080e-01 1.912e-01 3.300e-01 1.682e-01 1.000e+00]
 [1.670e-01 9.075e-01 1.658e-01 3.247e-01 1.563e-01 1.000e+00]
 [5.560e-02 6.585e-01 5.590e-02 2.968e-01 4.420e-02 9.946e-01]
 [5.100e-02 6.583e-01 5.130e-02 2.951e-01 3.980e-02 9.987e-01]
 [0.000e+00 6.014e-01 0.000e+00 2.194e-01 0.000e+00 9.614e-01]
 [1.000e+00 6.860e-01 1.000e+00 3.963e-01 1.000e+00 9.982e-01]
 [1.924e-01 9.978e-01 1.954e-01 5.836e-01 2.406e-01 1.000e+00]
 [1.671e-01 1.000e+00 1.700e-01 5.728e-01 2.233e-01 1.000e+00]
 [5.560e-02 6.585e-01 5.780e-02 5.484e-01 8.730e-02 9.946e-01]
 [5.100e-02 6.583e-01 5.310e-02 5.145e-01 7.660e-02 9.987e-01]
 [8.040e-02 6.039e-01 1.600e-03 6.700e-01 5.480e-02 9.819e-01]
 [1.404e-01 0.000e+00 5.330e-02 9.317e-01 1.609e-01 4.167e-01]
 [1.343e-01 3.000e-04 4.800e-02 1.000e+00 1.660e-01 4.167e-01]
 [7.850e-02 1.260e-02 0.000e+00 8.068e-01 8.240e-02 0.000e+00]]
```

2. Calculate the entropy

$$H_j = -\frac{1}{\ln m} \sum_{i=1}^{m} x_{ij} \ln(x_{ij}) \qquad Here: x_{ij} = \frac{x_{ij}}{\sum_{j=1}^{m} x_{ij}}$$

3. Calculate the weight

$$w_j = \frac{1 - H_j}{n - \sum_{j=1}^{n} H_j}$$

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[0.26536135 0.38851388 0.08670789 0.21241096 0.04700592]
```

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Step 3: TOPSIS

Technique for Order Preference by Similarity to Ideal Solution

1. Multiply the normalized data with the weights:

$$z_{ij} = x_{ij} w_j$$

2. Calculate the Euclidian distance between each data and best solution/worst solution

$$S_i^+ = \sqrt{\sum_{j=1}^{n} \left(z_{ij} - z_j^+\right)^2}$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} \left(z_{ij} - z_j^-\right)^2}$$

	Environmental Impact	Energy Consumption	Profit	Product Yield	Product Quality
Best Possible	0	0	1	1	1
Worst Possible	1	1	0	0	0

3. Calculate the closeness of each points, bigger C_i is better.

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-}$$

4. Rank each solutions based on C_i

$$Rank = argsort(C_i)$$

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Challenge 1 : TOPSIS for Buying Best Engineering Machine

You are given 9 machines below.

Machine Speed	Machine Efficiency	Machine Quality	Machine Yield	Machine Energy Consumption
210	330	54.5	0.00111	150
212	632.5	46	0.00117	355
212	655	87.5	0.000515	305
206.5	1575	38	0.00026	483
206.5	360	111.5	0.00089	190
187.5	1825	80	0.00071	532.5
210	1930	21	0.00002055	771
593	4405	14.05	0.00135	1250
212.5	1655	120	0.00113	448.5

Each criteria has a weight of:

0.291	0.079	0.206	0.188	0.098

1. Find the best machine by ranking using TOPSIS with the given weight.
2. Find the best machine by ranking using TOPSIS with Shannon Entropy weight.

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Pure Grey Relation Analysis Ranking

Material Strength	Material Toughness
0.054	0.95
0.066	0.31
0.071	0.48
0.092	1.05
0.072	0.29
0.058	0.45
0.131	0.87
0.119	1.15
0.091	0.22
0.143	0.61
0.075	0.52
0.104	1.02
0.152	0.34
0.12	0.41
0.125	0.47
0.112	1.23

1. Min-Max Normalization
2. Calculate Difference from Ideal of Every Data
   $\Delta_{ij}^{+} = |x_{0j} - x_{ij}|$
3. Calculate Grey Relational Coefficient (GRC)
   $\xi_{ij}^{+} = \frac{\Delta_{ij}^{+min} + \varsigma \Delta_{ij}^{+max}}{\Delta_{ij}^{+} + \varsigma \Delta_{ij}^{+max}}$
4. Calculate Grey Relational Grade (GRG)
   $\gamma_{i}^{-} = \frac{1}{n} \sum_{j=1}^{n} \xi_{ij}^{-}$
5. Rank solution by GRG.
6. How does it compare to Shannon Entropy-TOPSIS?

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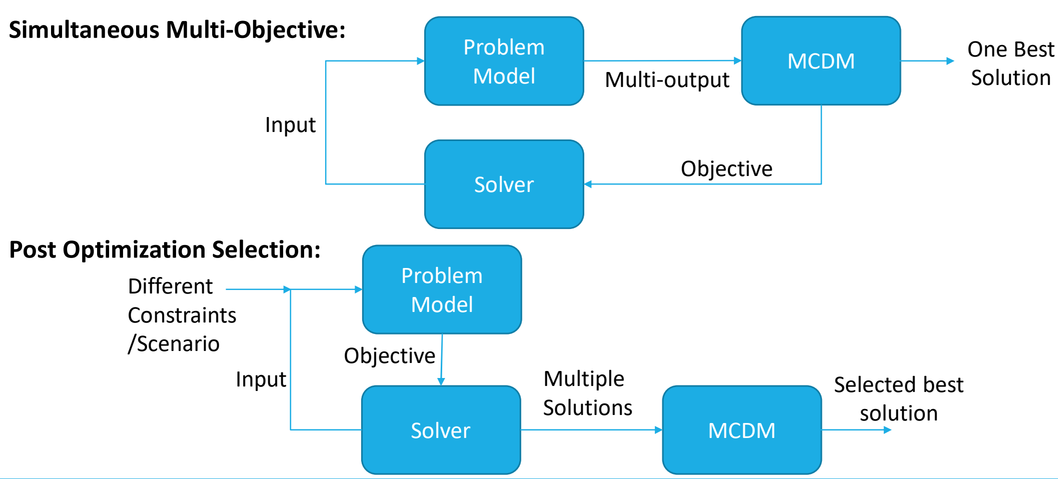
Concept of MCDM for Optimization



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Homework: Open-ended MCDM

You are required to find a real-life problem of your own. This can be a research problem.

Apply TOPSIS or any other MCDM algorithms on it to make better decision.