

**Listings of equations, figures, tables and  
sections of the article 'Musical elements in  
the discrete-time representation of  
sound' and of the scripts in the MASS  
toolbox**

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The article is the main document of the MASS toolbox. Being it of considerable length and complexity, this document contains listings of its elements to facilitate its navigation, apprehension and general usage.

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## SI-2 Equations

Table 0.1: Equation numbers and their descriptions. All these equations are implemented in file `src/sections/2.py`.

Number	Description
1	relation between number of samples and duration
2	power of the wave
3	decibels by difference by means of the power of each wave
4	double amplitude implies $\approx 6dB$
5	double power implies $\approx 3dB$
6	double volume ( $10dB$ ) implies a factor of $\approx 3.16$ for amplitude
7	direct relation between variations in amplitude and decibels
8	equivalences in a periodic sound with respect to wavelength, frequency and sample rate
10	sample amplitudes in a sinusoid
11	sample amplitudes in a sawtooth wave
12	sample amplitudes in a triangular wave
13	sample amplitude in a square wave
9	samples in a sound derived from a sampled wave period
14	reconstruction of samples from the Fourier components
15	reconstruction of real samples (e.g. for audio) from Fourier components
16	number of pairs of Fourier coefficients which are related to the same frequency
17	indexes of equivalent frequencies and coefficients for real signals
19	equal modules between samples of real signals
20	equivalence in phases between samples of real signals
21	complete reconstruction of the signal using Fourier components and previous equations
22	sample sequence related to a basic note
23	sample sequence of a period of an arbitrary waveform
24	note samples derived from a sampled waveform
25	distance of a (sound) source to each ear given the distance between the ears and an (x,y) position of the source
26	Interaural Time Difference (ITD), the time difference of a sound reaching each ear
27	Interaural Intensity Difference (IID), the intensity difference (in decibels) of a sound reaching each ear
28	ITD and IID in terms of sample delays and their amplitudes
29	azimuthal angle of a (x, y) source
30	samples that result from mixing sounds
31	samples that result from concatenating sounds

Table 0.2: Equation numbers and their descriptions. All these equations are implemented in file `src/sections/3.py`.

Number	Description
32	sample sequence generated by means of a lookup table (LUT)
33	frequency at each sample in a linear transition of frequency
34	indices of a LUT in a linear transition of frequency
35	sample sequence obtained through a LUT in a linear transition of frequency
36	frequency at each sample in an exponential transition of frequency (linear pitch)
37	indices of a LUT in an exponential transition of frequency (linear pitch)
38	sample sequence obtained through a LUT in an exponential transition of frequency
39	amplitude factors at each sample in an exponential transition of amplitude ( $\approx$ linear volume)
40	sample sequence with an exponential transition of amplitude ( $\approx$ linear volume)
42	amplitude factors at each sample in a linear transition of amplitude
41	sample sequence in an exponential transition of amplitude ( $\approx$ linear volume) with difference given in decibels
43	sample sequence obtained through the convolution of two other sequences (e.g. for applying FIR filters)
44	difference equation (e.g. for applying IIR filters)
45	IIR coefficients for a simple, useful and well-behaved low-pass filter
46	IIR coefficients for a simple, useful and well-behaved high-pass filter
47	auxiliary variables for the following band-pass and band-reject filters
48	IIR coefficients for a simple, useful and well-behaved band-pass filter
49	IIR coefficients for a simple, useful and well-behaved band-reject filter
50	Fourier coefficients of a white noise
51	Fourier coefficients of a pink noise
52	Fourier coefficients of a brown noise
53	Fourier coefficients of a blue noise
54	Fourier coefficients of a violet noise
55	Fourier coefficients of a black noise
56	indices for a vibrato given its frequency and using a LUT
57	samples for applying a vibrato
58	frequency at each sample of a sound with vibrato
59	indices for LUT in a sound with vibrato
60	sample sequence of a sound with vibrato
61	amplitude at each sample for a tremolo
62	sample sequence of a sound with tremolo
63	components in FM synthesis when both modulator and carrier are sines
64	the Bessel function
65	components in AM synthesis when both modulator and carrier are sines
66	indices for LUT in modulator of an FM synthesis
67	sample sequence of a modulator in an FM synthesis
68	frequency at each sample of a sound derived from FM synthesis
69	indices for the final signal in FM synthesis using LUT
70	sample sequence of a sound generated through FM and using LUT
71	amplitude at each sample in a sound generated through AM
72	sample sequence of a sound generated through AM and using LUT

Table 0.3: Equation numbers and their descriptions. All these equations are implemented in file `src/sections/3.py`.

Number	Description
73	an example of bonds between musical characteristics
74	relation between frequencies and speed in the Doppler effect
??	relation between position, speed and amplitude in the Doppler effect
??	relation between position, speed and amplitude in the Doppler effect
75	samples of a FIR filter for the first period of a reverberation
76	samples of a FIR filter for the second period of a reverberation
77	samples of the FIR filter for a reverberation (considering both first and second periods)
78	amplitude factors for each sample in an ADSR envelope
79	sample sequence of a sound with an ADSR envelope

Table 0.4: Equation numbers and their descriptions. All these equations are implemented in file `src/sections/4.py`.

Number	Description
80	pitchs represented in two divisions of the octave
81	perfectly symmetric scales in each octave with the twelve semitones
82	diatonic scales
83	the succession of tones and semitones of a diatonic scale
84	sequences of semitones for the three types minor scales
85	harmonic series in terms of semitones
86	triads (chords constituted by thirds)
87	a convention to specify a unit of rhythmic division or agglomeration
88	definition of algebraic groups



## SI-3 Figures

Table 0.5: Figure numbers and their descriptions. All these equations are implemented in files `src/aux/*`.

Number	Description
1	PCM audio (discrete and digital) samples
2	synthetic and sampled waveforms
4	spectrum of basic waveforms
3	spectrum of a real note and of one derived from one sampled period
5	sinusoid represented by two samples
6	sinusoid represented by three samples
7	sinusoids in 4 samples
8	basic waveforms within 4 samples
9	sinusoids in 6 samples
10	basic waveforms withing 6 samples
11	ITD and IID (spatial localization cues)
12	mixing PCM audio
13	concatenation/juxtaposition of PCM audio
14	Lookup table
15	transitions of intensity
16	convolution
17	convolution yielding time shifting, multiple time delays, sound amalgam
18	frequency response of useful IIR filters
19	spectrum and waveform of noises
20	spectrogram of a vibrato
21	waveform of a tremolo
22	ADSR envelope
23	counterpoint movements
24	musical metric in terms of divisions of temporal units
25	distinctions of musical climax by localization

## SI-4 Tables

Table 0.6: Table numbers and their descriptions.

Number	Description
1	musical intervals, their notations and qualities
2	tonal harmonic functions in the major scale
3	duration scales yielding perception of pitch and rhythm
4	music by permutation of units (change ringing)

## SI-5 Scripts

### SI-5.1 For all equations and relations in each section.

Table 0.7: Script files and their descriptions.

Filename	Description
<code>src/sections/2.py</code>	implementation of all the equations for the basic note in PCM audio in Section 2
<code>src/sections/3.py</code>	implementation of all the equations for variations within a note described in Section 3
<code>src/sections/4.py</code>	implementation of techniques for assembling notes into music described in Section 4

## SI-5.2 To render musical pieces

Table 0.8: Piece names, script files and the concepts they exemplify from Section 2. All files are in the directory `src/pieces2/`.

Name	Filename	Description
Reduced-fi	<code>reduced-fi.py</code>	concatenation of simple notes
Sonic pictures	<code>quadrosSonoros.py</code>	mixing of simple notes

Table 0.9: Piece names, script files and the concepts they exemplify from Section 3. All files are in the directory `src/pieces3/`.

Name	Filename	Description
ADa and SaRah	<code>ADaandSaRah.py</code>	ADSR envelope
Tremolos, Vibratos and Frequency	<code>bonds.py</code>	bonds between tremolos, vibratos and frequency
Bella Rugosi	<code>bellaRugosi.py</code>	rugosity achieved through frequencies between 15 and 30 Hz
Children Choir	<code>childChoir.py</code>	achieving choir sonorities by small variations of the same basic note
Noisy Band	<code>noisyBand.py</code>	using various noises
ParaMeter Transitions	<code>paraMeter.py</code>	gradual changes of parameters within one note
Little Train of Impulsive Hill-billies <sup>1</sup>	<code>littleTrain.py</code>	the use of convolution with impulses to achieve rhythm
Shakes and Wiggles	<code>shakesWiggles.py</code>	tremolos and vibratos

Table 0.10: Piece names, script files and the concepts they exemplify from Section 4. All files are in the directory `src/pieces4/`.

Name	Filename	Description
Acorde cedo	<code>acordeCedo.py</code>	chord successions and modulation
Count point	<code>contaPonto.py</code>	melodic lines conducted within the rules of counterpoint
Crystals	<code>crystals.py</code>	symmetric musical scales
Dirracional	<code>dirracional.py</code>	directional arcs
Intervals	<code>intervals.py</code>	musical intervals
MicroTone	<code>microTone.py</code>	microtonality (the use of intervals smaller than the semitone)
Poly-Hit-My	<code>polyHitMy.py</code>	polyrhythm (multiple metrics at once)

### **SI-5.3 To render the figures used in the article**

The files `src/aux/*` render each of the figures (listed above in Section [SI-3](#)).

### **SI-5.4 Scripts related to FIR and IIR filters**

The files `src/aux/filters/*` render the figures related to the filters and noises (listed above in Section [SI-3](#)) and hold some other routines, such a biquad filter recipe.

## SI-6 Other documents

The files in `latex/` render the article PDF and this Supporting Information file.

The script `src/aux/iso226.py` can generate the Equal Loudness Contour as described in its latest revision [\[2\]](#).

The article is largely based on the MSc dissertation [\[1\]](#).

## Bibliography

- [1] Fabbri (2013). Música no áudio digital : descrição psicofísica e caixa de ferramentas. MSc dissertation. Available at: [http://www.teses.usp.br/teses/disponiveis/76/76132/tde-19042013-095445/publico/RenatoFabbri\\_ME\\_corrigida.pdf](http://www.teses.usp.br/teses/disponiveis/76/76132/tde-19042013-095445/publico/RenatoFabbri_ME_corrigida.pdf)
- [2] ISO: 226. (2003). Normal Equal-Loudness Level Contours.