

**Listings of equations, figures, tables and
sections of the article 'Musical elements in
the discrete-time representation of
sound' and of the scripts in the MASS
toolbox**

Renato Fabbri (USP),
Vilson Vieira da Silva Junior (Cod.ai),
Antônio Carlos Silvano Pessotti (Unimep),
Débora Cristina Corrêa (UWA),
Osvaldo N. Oliveira Jr. (USP)

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The article is the main document of the MASS toolbox. Being it of considerable length and complexity, this document contains listings of its elements to facilitate its navigation, apprehension and general usage.

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SI-C-2 Equations

Table 0.1: Equation numbers and their descriptions. All these equations are implemented in file `src/sections/2.py`.

Number	Description
1	relation between number of samples and duration
2	power of the wave
3	decibels by difference by means of the power of each wave
4	double amplitude implies $\approx 6dB$
5	double power implies $\approx 3dB$
6	double volume ($10dB$) implies a factor of ≈ 3.16 for amplitude
7	direct relation between variations in amplitude and decibels
8	equivalences in a periodic sound with respect to wavelength, frequency and sample rate
10	sample amplitudes in a sinusoid
11	sample amplitudes in a sawtooth wave
12	sample amplitudes in a triangular wave
13	sample amplitude in a square wave
9	samples in a sound derived from a sampled wave period
SI-B-1	reconstruction of samples from the Fourier components
SI-B-2	reconstruction of real samples (e.g. for audio) from Fourier components
SI-B-3	number of pairs of Fourier coefficients which are related to the same frequency
SI-B-4	indexes of equivalent frequencies and coefficients for real signals
SI-B-6	equal modules between samples of real signals
SI-B-7	equivalence in phases between samples of real signals
SI-B-8	complete reconstruction of the signal using Fourier components and previous equations
14	sample sequence related to a basic note
15	sample sequence of a period of an arbitrary waveform
16	note samples derived from a sampled waveform
17	distance of a (sound) source to each ear given the distance between the ears and an (x,y) position of the source
18	Interaural Time Difference (ITD), the time difference of a sound reaching each ear
19	Interaural Intensity Difference (IID), the intensity difference (in decibels) of a sound reaching each ear
20	ITD and IID in terms of sample delays and their amplitudes
21	azimuthal angle of a (x, y) source
22	samples that result from mixing sounds
23	samples that result from concatenating sounds

Table 0.2: Equation numbers and their descriptions. All these equations are implemented in file `src/sections/3.py`.

Number	Description
24	sample sequence generated by means of a lookup table (LUT)
25	frequency at each sample in a linear transition of frequency
26	indices of a LUT in a linear transition of frequency
27	sample sequence obtained through a LUT in a linear transition of frequency
28	frequency at each sample in an exponential transition of frequency (linear pitch)
29	indices of a LUT in an exponential transition of frequency (linear pitch)
30	sample sequence obtained through a LUT in an exponential transition of frequency
31	amplitude factors at each sample in an exponential transition of amplitude (\approx linear volume)
32	sample sequence with an exponential transition of amplitude (\approx linear volume)
34	amplitude factors at each sample in a linear transition of amplitude
33	sample sequence in an exponential transition of amplitude (\approx linear volume) with difference given in decibels
35	sample sequence obtained through the convolution of two other sequences (e.g. for applying FIR filters)
36	difference equation (e.g. for applying IIR filters)
37	IIR coefficients for a simple, useful and well-behaved low-pass filter
38	IIR coefficients for a simple, useful and well-behaved high-pass filter
39	auxiliary variables for the following band-pass and band-reject filters
40	IIR coefficients for a simple, useful and well-behaved band-pass filter
41	IIR coefficients for a simple, useful and well-behaved band-reject filter
42	Fourier coefficients of a white noise
43	Fourier coefficients of a pink noise
44	Fourier coefficients of a brown noise
45	Fourier coefficients of a blue noise
46	Fourier coefficients of a violet noise
47	Fourier coefficients of a black noise
48	indices for a vibrato given its frequency and using a LUT
49	samples for applying a vibrato
50	frequency at each sample of a sound with vibrato
51	indices for LUT in a sound with vibrato
52	sample sequence of a sound with vibrato
53	amplitude at each sample for a tremolo
54	sample sequence of a sound with tremolo
55	components in FM synthesis when both modulator and carrier are sines
56	the Bessel function
57	components in AM synthesis when both modulator and carrier are sines
58	indices for LUT in modulator of an FM synthesis
59	sample sequence of a modulator in an FM synthesis
60	frequency at each sample of a sound derived from FM synthesis
61	indices for the final signal in FM synthesis using LUT
62	sample sequence of a sound generated through FM and using LUT
63	amplitude at each sample in a sound generated through AM
64	sample sequence of a sound generated through AM and using LUT

Table 0.3: Equation numbers and their descriptions. All these equations are implemented in file `src/sections/3.py`.

Number	Description
65	an example of bonds between musical characteristics
66	relation between frequencies and speed in the Doppler effect
67	samples of a FIR filter for the first period of a reverberation
68	samples of a FIR filter for the second period of a reverberation
69	samples of the FIR filter for a reverberation (considering both first and second periods)
70	amplitude factors for each sample in an ADSR envelope
71	sample sequence of a sound with an ADSR envelope

Table 0.4: Equation numbers and their descriptions. All these equations are implemented in file `src/sections/4.py`.

Number	Description
SI-A-1	pitches represented in two divisions of the octave
SI-A-2	perfectly symmetric scales in each octave with the twelve semitones
SI-A-3	diatonic scales
SI-A-4	the succession of tones and semitones of a diatonic scale
SI-A-5	sequences of semitones for the three types minor scales
SI-A-6	harmonic series in terms of semitones
SI-A-7	triads (chords constituted by thirds)
SI-A-8	a convention to specify a unit of rhythmic division or agglomeration
SI-A-9	definition of algebraic groups

SI-C-3 Figures

Table 0.5: Figure numbers and their descriptions. All these equations are implemented in files `src/aux/*`.

Number	Description
1	PCM audio (discrete and digital) samples
2	synthetic and sampled waveforms
4	spectrum of basic waveforms
3	spectrum of a real note and of one derived from one sampled period
SI-B-1	sinusoid represented by two samples
SI-B-2	sinusoid represented by three samples
SI-B-3	sinusoids in 4 samples
SI-B-4	basic waveforms within 4 samples
SI-B-5	sinusoids in 6 samples
SI-B-6	basic waveforms withing 6 samples
5	ITD and IID (spatial localization cues)
6	mixing PCM audio
7	concatenation/juxtaposition of PCM audio
8	Lookup table
9	transitions of intensity
10	convolution
11	convolution yielding time shifting, multiple time delays, sound amalgam
12	frequency response of useful IIR filters
13	spectrum and waveform of noises
14	spectrogram of a vibrato
15	waveform of a tremolo
16	ADSR envelope
SI-A-1	counterpoint movements
SI-A-2	musical metric in terms of divisions of temporal units
SI-A-3	distinctions of musical climax by localization

SI-C-4 Tables

Table 0.6: Table numbers and their descriptions.

Number	Description
SI-A-1	musical intervals, their notations and qualities
SI-A-2	tonal harmonic functions in the major scale
SI-A-3	duration scales yielding perception of pitch and rhythm
SI-A-4	music by permutation of units (change ringing)

SI-C-5 Scripts

SI-C-5.1 For all equations and relations in each section.

Table 0.7: Script files and their descriptions.

Filename	Description
<code>src/sections/2.py</code>	implementation of all the equations for the basic note in PCM audio in Section 2
<code>src/sections/3.py</code>	implementation of all the equations for variations within a note described in Section 3
<code>src/sections/SIA.py</code>	implementation of techniques for assembling notes into music described in the Supporting Information dedicated to the organization of notes in music [3]

SI-C-5.2 To render musical pieces

Table 0.8: Piece names, script files and the concepts they exemplify from Section 2. All files are in the directory `src/pieces2/`.

Name	Filename	Description
Reduced-fi	<code>reduced-fi.py</code>	concatenation of simple notes
Sonic pictures	<code>quadrosSonoros.py</code>	mixing of simple notes

Table 0.9: Piece names, script files and the concepts they exemplify from Section 3. All files are in the directory `src/pieces3/`.

Name	Filename	Description
ADa and SaRah	<code>ADaandSaRah.py</code>	ADSR envelope
Bonds	<code>bonds.py</code>	bonds between tremolos, vibratos and frequency
Bella Rugosi	<code>bellaRugosi.py</code>	rugosity achieved through frequencies between 15 and 30 Hz
Children Choir	<code>childChoir.py</code>	achieving choir sonorities by small variations of the same basic note
Noisy Band	<code>noisyBand.py</code>	using various noises
Parameter Transitions	<code>parameter.py</code>	gradual changes of parameters within one note
Little Train of Impulsive Hill-billies ¹	<code>littleTrain.py</code>	the use of convolution with impulses to achieve rhythm
Shakes and Wiggles	<code>shakesWiggles.py</code>	tremolos and vibratos

Table 0.10: Piece names, script files and the concepts they exemplify the Supporting Information dedicated to the organization of notes in music [3]. All files are in the directory `src/piecesSIA/`.

Name	Filename	Description
Acorde cedo	<code>acordeCedo.py</code>	chord successions and modulation
Count point	<code>contaPonto.py</code>	melodic lines conducted within the rules of counterpoint
Crystals	<code>crystals.py</code>	symmetric musical scales
Dirracional	<code>dirracional.py</code>	directional arcs
Intervals	<code>intervals.py</code>	musical intervals
MicroTone	<code>microTone.py</code>	microtonality (the use of intervals smaller than the semitone)
Poly-Hit-My	<code>polyHitMy.py</code>	polyrhythm (multiple metrics at once)

SI-C-5.3 To render the figures used in the article

The files `src/aux/*` render each of the figures (listed above in Section [SI-C-3](#)).

SI-C-5.4 Scripts related to FIR and IIR filters

The files `src/aux/filters/*` render the figures related to the filters and noises (listed above in Section [SI-C-3](#)) and hold some other routines, such a biquad filter recipe.

SI-C-6 Other documents

The files in `latex/*` render the article PDF and this Supporting Information file.

The script `src/aux/iso226.py` can generate the Equal Loudness Contour as described in its latest revision [\[2\]](#).

The article is largely based on the MSc dissertation [\[1\]](#).

Bibliography

- [1] Fabbri (2013). Música no áudio digital : descrição psicofísica e caixa de ferramentas. MSc dissertation. Available at: http://www.teses.usp.br/teses/disponiveis/76/76132/tde-19042013-095445/publico/RenatoFabbri_ME_corrigida.pdf
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