Code in the MASS framework

Renato Fabbri (USP),
Vilson Vieira da Silva Junior (Cod.ai),
Antônio Carlos Silvano Pessotti (Unimep),
Débora Cristina Corrêa (UWA),
Osvaldo N. Oliveira Jr. (USP)

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Abstract

This document displays the Python code in the MASS framework. The code is accessible as Python scripts in the main repository [1], and this PDF is made available because it might facilitate browsing the implementations. Check the final consideration in this document for further directions.

SI-D-1 Sections

Here is the code related to each section of the MASS article [2].

Python implementation of equations in Section 2

```
import numpy as n
from scipy.io import wavfile as w

# auxiliary functions __n and __s.
# These only normalize the sonic vectors and
# write them as 16 bit, 44.1kHz WAV files.
def __n(sonic_array):
    """Normalize sonic_array to have values only between -1 and 1"""

    t = sonic_array
    if n.all(sonic_array==0):
        return sonic_array
    else:
        return ( (t-t.min()) / (t.max() -t.min()) )*2.-1.

def __s(sonic_array=n.random.uniform(size=100000), filename="asound.wav", f_s=44100):
    """A minimal approach to writing 16 bit WAVE files.
```

```
    One can also use, for example:
        import sounddevice as S
        S.play(array) # the array must have values between -1 and 1"""

    # to write the file using XX bits per sample
    # simply use s = n.intXX(__n(sonic_array)*(2**(XX-1)-1))
    s = n.int16(__n(sonic_array)*32767)
    w.write(filename, f_s, s)


############ Sec. 2.1 Duration
# relation between the number of samples and the sound duration
f_s = 44100  # sample rate
Delta = 3.7   # duration of Delta in seconds

Lambda = int(f_s*Delta)  # number of samples
# Eq. 1
T = n.zeros(Lambda)  # silence with ~Delta, in seconds

# write as a PCM file (WAV)
__s(T, 'silence.wav')

############ Sec. 2.2 Loudness
Lambda = 100  # 100 samples
T = n.random.random(Lambda)  # 100 random samples

# Eq. 2 Power of wave
pow1 = (T**2.).sum()/Lambda

T2 = n.random.normal(size=Lambda)
pow2 = (T2**2.).sum()/Lambda  # power of another wave

# Eq. 3 Volume difference, in decibels, given the powers
V_dB = 10.*n.log10(pow2/pow1)

# Eq. 4 double the amplitude => gains 6 dB
T2 = 2.*T
pow2 = (T2**2.).sum()/Lambda
V_dB = 10.*n.log10(pow2/pow1)
is_6db = abs(V_dB - 6) < .05  # is_6db is True

# Eq. 5 double the power => gains 3 dB
pow2 = 2.*pow1
V_dB = 10.*n.log10(pow2/pow1)
is_3dB = abs(V_dB - 3) < .05  # is_3dB is True

# Eq. 6 double the volume => gains 10 dB => amplitude * 3.16
V_dB = 10.
A = 10.**(V_dB/20.)
T2 = A*T  # A ~ 3.1622776601

# Eq. 7 Decibels to amplification conversion
A = 10.**(V_dB/20.)
```

```
############ Sec. 2.3 Pitch
f_0 = 441
lambda_0 = f_s//f_0
cycle = n.arcsin(n.random.random(lambda_0))  # random samples
# Eq. 8 Sound with fundamental frequency f_0
Tf = n.array(list(cycle)*1000)  # 1000 cycles

# normalizing to interval [-1, 1]
__s(Tf,'f_0.wav')


############ Sec. 2.4 Timbre
L = 100000.  # sample number of sequences (Lambda)
ii = n.arange(L)
f = 220.5
lambda_f = f_s/f
# Eq. 9 Sinusoid
Sf = n.sin(2.*n.pi*f*ii/f_s)
# Eq. 10 Sawtooth
Df = (2./lambda_f)*(ii % lambda_f)-1
# Eq. 11 Triangular
Tf = 1.-n.abs(2.-(4./lambda_f)*(ii % lambda_f))
# Eq. 12 Square
Qf = ((ii % lambda_f) < (lambda_f/2))*2-1

Rf = w.read("22686__acclivity__oboe-a-440_periodo.wav")[1]
# Eq. 13 Sampled period
Tf = Rf[n.int64(ii) % len(Rf)]


############ Sec. 2.5 The spectrum of sampled sound
Lambda = 50
T = n.random.random(Lambda)*2.-1.
C_k = n.fft.fft(T)
A_k = n.real(C_k)
B_K = n.imag(C_k)
w_k = 2.*n.pi*n.arange(Lambda)/Lambda

# Eq .14 Spectrum recomposition in time
def t(i):
    return (1./Lambda)*n.sum(C_k*n.e**(1j*w_k*i))

# Eq. 15 Real recomposition
def tR(i):
    return (1./Lambda)*n.sum(n.abs(C_k)*n.cos(w_k*i-n.angle(C_k)))

# Eq. 16 Number of paired spectrum coefficients
tau = int( (Lambda - Lambda % 2)/2 + Lambda % 2-1 )

# Eq. 17 Equivalent coefficients
F_k = C_k[1:tau+1]
F2_k = C_k[Lambda-tau:Lambda][::-1]
```

```python

# Eq. 18 Equivalent modules of coefficients
ab = n.abs(F_k)
ab2 = n.abs(F2_k)
MIN = n.abs(ab-ab2).sum()  # MIN ~ 0.0

# Eq, 19 Equivalent phases of coefficients
an = n.angle(F_k)
an2 = n.angle(F2_k)
MIN = n.abs(an+an2).sum()  # MIN ~ 0.0

# Eq. 20 Components combination in each sample
w_k = 2*n.pi*n.arange(Lambda)/Lambda

def t_(i):
    return (1./Lambda)*(A_k[0]+2.*n.sum(n.abs(C_k[1:tau+1]) *
                        n.cos(w_k*i-n.angle(C_k)) + A_k[Lambda/2] *
                        (1-Lambda % 2)))


############ Sec. 2.6 The basic note
f = 220.5  # Herz
Delta = 2.5  # seconds
Lambda = int(2.5*f_s)
ii = n.arange(Lambda)

# Eq. 21 Basic note (preliminary)
ti_ = n.random.random(int(f_s/f))  # arbitrary sequence of samples
TfD = ti_[ii % len(ti_)]

# Eq. 22 Choose any waveform
Lf = [Sf, Qf, Tf, Df, Rf][1]  # We already calculated these sequences

# Eq. 23 Basic note
TfD = Lf[ii % len(Lf)]


############ Sec. 2.7 Spatialization: localization and reverberation
zeta = 0.215  # meters
# considering any (x,y) localization
x = 1.5  # meters
y = 1.  # meters
# Eq. 24 Distances from each ear
d = n.sqrt((x-zeta/2)**2+y**2)
d2 = n.sqrt((x+zeta/2)**2+y**2)
# Eq. 25 Interaural Time Difference
ITD = (d2-d)/343.2  # seconds
# Eq. 26 Interaural Intensity Difference
IID = 20*n.log10(d/d2)  # dBs

# Eq. 27 ITD and IID application in a sample sequence (T)
Lambda_ITD = int(ITD*f_s)
IID_a = d/d2
T = 1-n.abs(2-(4./lambda_f)*(ii % lambda_f))  # triangular
```

```
T2 = n.hstack((n.zeros(Lambda_ITD), IID_a*T))
T = n.hstack((T, n.zeros(Lambda_ITD)))

som = n.vstack((T2, T)).T
fun.WS('stereo.wav', f_s, som)
# mirrored
som = n.vstack((T, T2)).T
fun.WS('stereo2.wav', f_s, som)

# Eq. 28 Object angle
theta = n.arctan2(y, x)

# Reverberation is implemented in 3.py
# because it makes use of knowledge of the next section


############ Sec. 2.8 Musical uses
Delta = 3.  # 3 seconds
Lambda = int(Delta*f_s)
f1 = 200.  # Hz
foo = n.linspace(0., Delta*f1*2.*n.pi, Lambda, endpoint=False)
T1 = n.sin(foo)  # sinusoid of Delta seconds and freq  =  f1

f2 = 245.  # Hz
lambda_f2 = int(f_s/f2)
T2 = (n.arange(Lambda) % lambda_f < (lambda_f2/2))*2-1  # square

f3 = 252.  # Hz
lambda_f3 = f_s/f3
T3 = n.arange(Lambda) % lambda_f3  # sawtooth
T3 = (T3/T3.max())*2-1

# Eq. 29 mixing
T = T1+T2+T3
# writing file
__s(T, 'mixed.wav')

# Eq. 30 concatenation
T = n.hstack((T1, T2, T3))
# writing file
__s(T, 'concatenated.wav')
```

Python implementation of equations in Section 3

```python
import numpy as n
from scipy.io import wavfile as w

# auxiliary functions __n and __s.
# These only normalize the sonic vectors and
# write them as 16 bit, 44.1kHz WAV files.
def __n(sonic_array):
    """Normalize sonic_array to have values only between -1 and 1"""

    t = sonic_array
    if n.all(sonic_array==0):
        return sonic_array
    else:
        return ( (t-t.min()) / (t.max() -t.min()) )*2.-1.

def __s(sonic_array=n.random.uniform(size=100000), filename="asound.wav", f_s=44100):
    """A minimal approach to writing 16 bit WAVE files.

    One can also use, for example:
        import sounddevice as S
        S.play(array) # the array must have values between -1 and 1"""

    # to write the file using XX bits per sample
    # simply use s = n.intXX(__n(sonic_array)*(2**(XX-1)-1))
    s = n.int16(__n(sonic_array)*32767)
    w.write(filename, f_s, s)


f_s = 44100  # Hz, sample rate

############ Sec. 3.1 Lookup table (LUT)
# at least 1024 samples in the table
Lambda_tilde = Lt = 1024

# Sinusoid
foo = n.linspace(0, 2*n.pi, Lt, endpoint=False)
S = n.sin(foo)  # a sinusoidal period with T samples

# Square:
Q = n.hstack((n.ones(Lt/2)*-1, n.ones(Lt/2)))

# Triangular:
foo = n.linspace(-1, 1, Lt/2, endpoint=False)
Tr = n.hstack((foo, foo*-1))

# Sawtooth:
D = n.linspace(-1, 1, Lt)

# real sound, import period and
# use the number of samples in the period
Rf = w.read("22686__acclivity__oboe-a-440_periodo.wav")[1]

f = 110.  # Hz
Delta = 3.4  # seconds
Lambda = int(Delta*f_s)
```

```python

# Samples:
ii = n.arange(Lambda)

# Eq. 31 LUT
Gamma = n.array(ii*f*Lt/f_s, dtype=n.int)
# It is possible to use S, Q, D or any other period of a real sound
# with a sufficient length
L = Tr
TfD = L[Gamma % Lt]


############ Sec. 3.2 Incremental variations of frequency and intensity
# == FREQUENCY VARIATIONS ==
f_0 = 100.  # initial freq in Hz
f_f = 300.  # final freq in Hz
Delta = 2.4  # duration

Lambda = int(f_s*Delta)
ii = n.arange(Lambda)
# Eq. 32 linear variation
f_i = f_0+(f_f-f_0)*ii/(float(Lambda)-1)
# Eq. 33 coefficients for LUT
D_gamma = f_i*Lt/f_s
Gamma = n.cumsum(D_gamma)
Gamma = n.array(Gamma, dtype=n.int)
# Eq. 34 resulting sound
Tf0ff = L[Gamma % Lt]

# Eq. 35 exponential variation
f_i = f_0*(f_f/f_0)**(ii/(float(Lambda)-1))
# Eq. 36 coefficients for the LUT
D_gamma = f_i*Lt/f_s
Gamma = n.cumsum(D_gamma)
Gamma = n.array(Gamma, dtype=n.int)
# Eq. 37 resulting sound
Tf0ff = L[Gamma % Lt]


# == INTENSITY VARIATIONS ==
# First, make/have an arbitrary sound to
# apply the variations in amplitude
f = 220.  # Hz
Delta = 3.9  # seconds
Lambda = int(Delta*f_s)

# Sample indexes:
ii = n.arange(Lambda)

# (as in Eq. 31)
Gamma = n.array(ii*f*Lt/f_s, dtype=n.int)
L = Tr
T = TfD = L[Gamma % Lt]

a_0 = 1.  # starting fraction of the amplitude
```

```
a_f = 12.  # ending fraction of the amplitude
alpha = 1.  # index of transition smoothing

# Eq. 38 exponential transition of amplitude
A = a_0*(a_f/a_0)**((ii/float(Lambda))**alpha)
# Eq. 39 applying envelope A to the sound
T2 = A*T

# Eq. 40 linear transition of amplitude
A = a_0+(a_f-a_0)*(ii/float(Lambda))

# Eq. 41 exponential transition of V_dB decibels
V_dB = 31.
T2 = T*((10*(V_dB/20.))**((ii/float(Lambda))**alpha))


############ Sec 3.3 Application of digital filters
# See src/aux/delays.py for generating Fig. 17
# See src/aux/filters/iir.py for generating Fig. 18

# synthetic impulse response (for a "reverb", a better reverb is bellow in: Reverberation)
H = (n.random.random(10)*2-1)*n.e**(-n.arange(10))

# Eq. 42 Convolution (application of a FIR filter)
T2 = n.convolve(T, H)  # T from above

# Eq. 43  difference equation
A = n.random.random(2)  # arbitrary coefficients
B = n.random.random(3)  # arbitrary coefficients

def applyIIR(signal, A, B):
    signal_ = []
    for i in range(len(signal)):
        samples_A = signal[i::-1][:len(A)]
        A_coeffs = A[:i+1]
        A_contrib = (samples_A*A_coeffs).sum()

        samples_B = signal_[-1:-1-i:-1][:len(B)-1]
        B_coeffs = B[1:i+1]
        B_contrib = (samples_B*B_coeffs).sum()
        t_i = (A_contrib + B_contrib)/B[0]
        signal_.append(t_i)
    return signal_

fc = .1
# Eq. 44 low-pass IIR filter with a single pole
x = n.e**(-2*n.pi*fc)  # fc => cutoff frequency where the resulting signal has -3dB
# coefficients
a0 = 1-x
b1 = x
# applying the filter
T2 = [T[0]]
for t_i in T[1:]:
    T2.append(t_i*a_0+T2[-1]*b1)
```

```
# Eq. 45 high-pass filter with a single pole
x = n.e**(-2*n.pi*fc)  # fc => cutoff frequency where the resulting signal has -3dB
# coefficients
a0 = (1+x)/2
a1 = -(1+x)/2
b1 = x

# applying the filter
T2 = [a0*T[0]]
last = T[0]
for t_i in T[1:]:
    T2 += [a0*t_i + a1*last + b1*T2[-1]]
    last = n.copy(t_i)


fc = .1  # now fc is the center frequency
bw = .05
# Eq. 46 Auxiliary variables for the notch filters
r = 1-3*bw
k = (1-2*r*n.cos(2*n.pi*fc)+r**2)/(2-2*n.cos(2*n.pi*fc))

# Eq. 47 band-pass filter coefficients
a0 = 1-k
a1 = -2*(k-r)*n.cos(2*n.pi*fc)
a2 = r**2 - k
b1 = 2*r*n.cos(2*n.pi*fc)
b2 = -r**2

# applying the filter
T2 = [a0*T[0]]
T2 += [a0*T[1]+a1*T[0]+b1*T2[-1]]
last1 = T[1]
last2 = T[0]
for t_i in T[2:]:
    T2 += [a0*t_i+a1*last1+a2*last2+b1*T2[-1]+b2*T2[-2]]
    last2 = n.copy(last1)
    last1 = n.copy(t_i)

# Eq. 48 band-reject filter coefficients
a0 = k
a1 = -2*k*n.cos(2*n.pi*fc)
a2 = k
b1 = 2*r*n.cos(2*n.pi*fc)
b2 = -r**2

# applying the filter
T2 = [a0*T[0]]
T2 += [a0*T[1]+a1*T[0]+b1*T2[-1]]
last1 = T[1]
last2 = T[0]
for t_i in T[2:]:
    T2 += [a0*t_i+a1*last1+a2*last2+b1*T2[-1]+b2*T2[-2]]
    last2 = n.copy(last1)
    last1 = n.copy(t_i)
```

```
############ Sec. 3.4 Noise
# See src/filters/ruidos.py for rendering Figure 19
Lambda = 100000  # Use an even Lambda for compliance with the following snippets
# Separation between frequencies of neighbor spectral coefficients:
df = f_s/Lambda

# Eq. 49 White noise
# uniform moduli of spectrum and random phase
coefs = n.exp(1j*n.random.uniform(0, 2*n.pi, Lambda))

f0 = 15.  # minimum frequency which we want in the sound
i0 = n.floor(f0/df)  # first coefficient to be considered
coefs[:i0] = n.zeros(i0)

# coefficients have real part even and imaginary part odd
coefs[Lambda/2+1:] = n.real(coefs[1:Lambda/2])[::-1] - 1j * \
    n.imag(coefs[1:Lambda/2])[::-1]
coefs[Lambda/2] = 1.  # max freq is only real (as explained in Sec. 2.5)

# Achievement of the temporal samples of the noise
ruido = n.fft.ifft(coefs)
r = n.real(ruido)
__s(r, 'white.wav')

# auxiliary variables to all the following noises
fi = n.arange(coefs.shape[0])*df # frequencies related to the coefficients
f0 = fi[i0] # first frequency to be considered

# Eq. 50 Pink noise
# the volume decreases by 3dB at each octave
factor = 10.**(-3/20.)
alphai = factor**(n.log2(fi[i0:]/f0))

c = n.copy(coefs)
c[i0:] = coefs[i0:]*alphai
# real is even, imaginary is odd
c[Lambda/2+1:] = n.real(c[1:Lambda/2])[::-1] - 1j * \
    n.imag(c[1:Lambda/2])[::-1]

ruido = n.fft.ifft(c)
r = n.real(ruido)
__s(r, 'pink.wav')


# Eq. 51 Brown(ian) noise
# the volume decreases by 6dB at each octave
fator = 10.**(-6/20.)
alphai = fator**(n.log2(fi[i0:]/f0))
c = n.copy(coefs)
c[i0:] = c[i0:]*alphai

# real is even, imaginary is odd
c[Lambda/2+1:] = n.real(c[1:Lambda/2])[::-1] - 1j * \
    n.imag(c[1:Lambda/2])[::-1]

ruido = n.fft.ifft(c)
```

```python
r = n.real(ruido)
__s(r, 'brown.wav')

ruido_marrom = n.copy(r) # it will be used for reverberation


# Eq. 52 Blue noise
# the volume increases by 3dB at each octave
fator = 10.**(3/20.)
alphai = fator**(n.log2(fi[i0:]/f0))
c = n.copy(coefs)
c[i0:] = c[i0:]*alphai

# real is even, imaginary is odd
c[Lambda/2+1:] = n.real(c[1:Lambda/2])[::-1] - 1j * \
    n.imag(c[1:Lambda/2])[::-1]

ruido = n.fft.ifft(c)
r = n.real(ruido)
__s(r, 'blue.wav')


# Eq. 53 Violet noise
# the volume increses by 6dB at each octave
fator = 10.**(6/20.)
alphai = fator**(n.log2(fi[i0:]/f0))
c = n.copy(coefs)
c[i0:] = c[i0:]*alphai

# real is even, imaginary is odd
c[Lambda/2+1:] = n.real(c[1:Lambda/2])[::-1] - 1j * \
    n.imag(c[1:Lambda/2])[::-1]

ruido = n.fft.ifft(c)
r = n.real(ruido)
__s(r, 'violet.wav')

# Eq.54 Black noise
# the volume decreases more than 6dB at each octave
fator = 10.**(-12/20.)
alphai = fator**(n.log2(fi[i0:]/f0))
c = n.copy(coefs)
c[i0:] = c[i0:]*alphai

# real is even, imaginary is odd
c[Lambda/2+1:] = n.real(c[1:Lambda/2])[::-1] - 1j * \
    n.imag(c[1:Lambda/2])[::-1]

ruido = n.fft.ifft(c)
r = n.real(ruido)
__s(r, 'black.wav')


############ Sec. 3.5 Tremolo e vibrato, AM e FM
# See src/aux/vibrato.py and src/aux/tremolo.py for rendering Figures 20 and 21
f = 220.
```

```
Lv = 2048  # size of the table for the vibrato
fv = 1.5  # vibrato frequency
nu = 1.6  # maximum semitone deviation (vibrato depth)
Delta = 5.2  # sound duration
Lambda = int(Delta*f_s)

# Vibrato table
x = n.linspace(0, 2*n.pi, Lv, endpoint=False)
tabv = n.sin(x)  # sinusoidal vibrato

ii = n.arange(Lambda)  # çndices
# Eq. 55 indexes of the LUT for the vibrato
Gammav = n.array(ii*fv*float(Lv)/f_s, n.int)
# Eq. 56 samples of the oscillatory pattern of the vibrato
Tv = tabv[Gammav % Lv]
# Eq. 57 frequency at each sample
F = f*(2.**(Tv*nu/12.))
# Eq. 58 indexes of the LUT for the sound
D_gamma = F*(Lt/float(f_s))  # displacement in the table for each sample
Gamma = n.cumsum(D_gamma)  # total displacement at each sample
Gamma = n.array(Gamma, dtype=n.int)  # final indexes
# Eq. 59 the samples of the sound
T = Tr[Gamma % Lt]  # Lookup

__s(T, "vibrato.wav")


Tt = n.copy(Tv)  # same oscillatory pattern from the vibrato
# Eq. 60 Envelope of the tremolo
V_dB = 12.  # decibels variation involved in the tremolo (tremolo depth)
A = 10**((V_dB/20)*Tt)  # amplitude multiplicative factors for each sample
# Eq. 61 Application of the amplitude envelope to the original sample sequence T
Gamma = n.array(ii*f*Lt/f_s, dtype=n.int)
T = Tr[Gamma % Lt]
T = T*A
__s(T, "tremolo.wav")


# the following equations are not used to synthesize sounds,
# but only to express the spectrum resulting from FM and AM synthesis
# Eq. 62 - FM spectrum, implemented in Eqs. 65-69
# Eq. 63 - Bessel function
# Eq. 64 - AM spectrum, implemented in Eqs. 70,71

fv = 60.  # typically, fv > 20Hz (otherwise one might want to use the equations above for the vibra
# Eq. 65 indexes of the LUT for the FM modulator
Gammav = n.array( ii*fv*float(Lv)/f_s, n.int )
# Eq. 66 oscillatory pattern (sample-by-sample) of the modulator
Tfm = tabv[Gammav % Lv]
f = 330.
mu = 40.
# Eq. 67 frequency at each sample
F = f+Tfm*mu
```

```
# Eq. 68 indexes of the LUT
D_gamma = f_i*(Lt/float(f_s))  # displacement in the lookup between each sample
Gamma = n.cumsum(D_gamma)  # total displacement in the lookup at each sample
Gamma = n.array(Gamma, dtype=n.int)  # indexes
# Eq. 69 FM
T = S[Gamma % Lt]  # final samples

# writing the sound file
__s(T, "fm.wav")


# AM
Tam = n.copy(Tfm)
V_dB = 12.  # am depth in decibels
alpha = 10**(V_dB/20.)  # AM depth in amplitude
# 2.71 AM envelope
A = 1+alpha*Tam
Gamma = n.array(ii*f*Lt/f_s, dtype=n.int)
# 2.70 AM
T = Tr[Gamma % Lt]*A
__s(T, "am.wav")


############# Sec. 3.6 Musical usages
# Eq. 72 Relations between characteristics
# See the musical piece Tremolos, vibratos and the frequency
# in src/pieces3/bonds.py TremolosVibratosEaFrequencia.py

# Doppler effect
v_r= 10 # receptor moves in the direction of the source with velocity v_r m/s
v_s=-80. # source moves in the direction of receptor with velocity v_s m/s
v_som=343.2
f_0=1000 # frequency of the source

# Eq. 73 Frequency resulting from the Doppler effect
f=((v_som + v_r) / (v_som + v_s)) * f_0
# after crossing of source and receptor:
f_=((v_som - v_r) / (v_som - v_s)) * f_0

# initial distances:
x_0=0 # source at front of x_0
y_0=200 # height of y_0 metros

Delta=5. # duration in seconds
Lambda=Delta*f_s # number of samples
# posiçges ao longo do tempo, X_i=n.zeros(Lambda)
Y=y_0 - ((v_r-v_s)*Delta) * n.linspace(0,1,Lambda)

# At each sample, calculating ITD and IID as explained in the last section
# In this case, ITD e IID are == 0 because the source is centered
# Eq. 74 Amplitude resulting from the Doppler effect
# Assume z_0 meters above receptor:
z_0=2.
D=( z_0**2+Y**2  )**0.5 # distance at each PCM sample
# Amplitude of sound related to the distance:
```

```
A_=z_0/D
# Amplitude change factor resulting from the Doppler effect:
A_DP=( (v_r-v_s)/343.2+1 )**0.5
A_DP_=( (-v_r+v_s)/343.2+1 )**0.5
A_DP=(Y>0)*A_DP+(Y<0)*A_DP_
A=A_ * A_DP

# Upon crossing, the velocities change sign:
# Eq. 75 Frequency progression
coseno=(Y)/((Y**2+z_0**2)**0.5)
F=( ( 343.2+v_r*coseno ) / ( 343.2+v_s*coseno ) )*f_0
# coefficients of the LUT
D_gamma = F*Lt/f_s
Gamma = n.cumsum(D_gamma)
Gamma = n.array(Gamma, dtype=n.int)

L = Tr  # Triangular wave
# Resulting sound:
Tdoppler = L[Gamma % Lt]
Tdoppler*=A

# normalizing and writing sound
__s(Tdoppler, 'doppler.wav')


###### Reverberation
# First reverberation period:
Delta1 = 0.15 # typically E [0.1,0.2]
Lambda1= int(Delta1*f_s)
Delta = 1.9 # total duration of reverberation
Lambda=int(Delta*f_s)

# Sound reincidence probability probability in the first period:
ii=n.arange(Lambda)
P = (ii[:Lambda1]/float(Lambda1))**2.
# incidences:
R1_=n.random.random(Lambda1)<P
A=10.**((-50./20)*(ii/Lambda))
# Eq. 76 First period of reverberation:
R1=R1_*A[:Lambda1]  # first incidences

# Brown noise with exponential decay (of amplitude) for the second period:
# Eq. 77 Second period of reverberation:
Rm=ruido_marrom[Lambda1:Lambda]
R2=Rm*A[Lambda1:Lambda]
# Eq. 78 Impulse response of the reverberation
R=n.hstack((R1,R2))
R[0]=1.

# Making an arbitrary sound to apply the reverberation:
f_0 = 100.  # starting freq (Hz)
f_f = 700.  # final freq (Hz)
Delta = 2.4  # duration
Lambda = int(f_s*Delta)
ii = n.arange(Lambda)
```

```python

# (using Eq. 35 for exponential variation)
F = f_0*(f_f/f_0)**(ii/(float(Lambda)-1))
# (using Eq. 36 for the LUT indexes)
D_gamma = F*Lt/f_s
Gamma = n.cumsum(D_gamma)
Gamma = n.array(Gamma, dtype=n.int)
# (using Eq. 2.37 for making the sound)
Tf0ff = L[Gamma % Lt]

# Applying the reverberation
T_=Tf0ff
T=n.convolve(T_,R)
__s(T, "reverb.wav")


# Eq. 79 ADSR - linear variation
Delta = 5.  # total duration in seconds
Delta_A = 0.1  # Attack
Delta_D = .3  # Decay
Delta_R = .2  # Release
a_S = .1  # Sustain level

Lambda = int(f_s*Delta)
Lambda_A = int(f_s*Delta_A)
Lambda_D = int(f_s*Delta_D)
Lambda_R = int(f_s*Delta_R)

# Achievement of the ADRS envelope: A_
ii = n.arange(Lambda_A, dtype=n.float)
A = ii/(Lambda_A-1)
A_ = A
ii = n.arange(Lambda_A, Lambda_D+Lambda_A, dtype=n.float)
D = 1-(1-a_S)*((ii-Lambda_A)/(Lambda_D-1))
A_ = n.hstack((A_, D))
S = a_S*n.ones(Lambda-Lambda_R-(Lambda_A+Lambda_D), dtype=n.float)
A_ = n.hstack((A_, S))
ii = n.arange(Lambda-Lambda_R, Lambda, dtype=n.float)
R = a_S-a_S*((ii-(Lambda-Lambda_R))/(Lambda_R-1))
A_ = n.hstack((A_, R))

# Eq. 80 Achievement of a sound with the ADSR envelope
ii = n.arange(Lambda, dtype=n.float)
Gamma = n.array(ii*f*Lt/f_s, dtype=n.int)
T = Tr[Gamma % Lt]*(A_)

__s(T, "adsr.wav")


# Eq. 79 ADSR - exponential variation
xi = 1e-2  # -180dB for starting fade in and ending in the fade out
De = 2*100.  # total duration
DA = 2*20.  # attack duration
DD = 2*20.  # decay duration
DR = 2*20.  # release duration
SS = .4  # fraction of amplitude in which sustain occurs
```

```
Lambda = int(f_s*De)
Lambda_A = int(f_s*DA)
Lambda_D = int(f_s*DD)
Lambda_R = int(f_s*DR)

A = xi*(1./xi)**(n.arange(Lambda_A)/(Lambda_A-1))  # attack samples
A = n.copy(A)
D = a_S**((n.arange(Lambda_A, Lambda_A+Lambda_D)-Lambda_A)/(Lambda_D-1)) # decay samples
A = n.hstack((A, D))
S = a_S*n.ones(Lambda-Lambda_R-(Lambda_A+Lambda_D))  # sustain samples
A = n.hstack((A, S))
R = (SS)*(xi/SS)**((n.arange(Lambda-Lambda_R, Lambda)+Lambda_R-Lambda)/(Lambda_R-1)) # release
A = n.hstack((A,  R))

# Eq. 80 Achievement of sound with ADSR envelope
ii = n.arange(Lambda, dtype=n.float)
Gamma = n.array(ii*f*Lt/f_s, dtype=n.int)
T = Tr[Gamma % Lt]*(A)

__s(T, "adsr_exp.wav")
```

Python implementation of equations in the "Organization of notes in music" Supporting Information [5]

```python
import numpy as n

# auxiliary functions __n and __s.
# These only normalize the sonic vectors and
# write them as 16 bit, 44.1kHz WAV files.
def __n(sonic_array):
    """Normalize sonic_array to have values only between -1 and 1"""

    t = sonic_array
    if n.all(sonic_array==0):
        return sonic_array
    else:
        return ( (t-t.min()) / (t.max() -t.min()) )*2.-1.

def __s(sonic_array=n.random.uniform(size=100000), filename="asound.wav", f_s=44100):
    """A minimal approach to writing 16 bit WAVE files.

    One can also use, for example:
        import sounddevice as S
        S.play(array) # the array must have values between -1 and 1"""

    # to write the file using XX bits per sample
    # simply use s = n.intXX(__n(sonic_array)*(2**(XX-1)-1))
    s = n.int16(__n(sonic_array)*32767)
    w.write(filename, f_s, s)

f_s = 44100.  # Hz, sample rate
Lambda_tilde = Lt = 1024.
foo = n.linspace(0, 2*n.pi, Lt, endpoint=False)
S_i = n.sin(foo)  # a sinusoid period of T samples

H = n.hstack

# using the content from the previous sections,
# this is a very simple synthesizer of notes
def v(f=200, d=1., tab=S_i, fv=2., nu=2., tabv=S_i):
    Lambda = n.floor(f_s*d) ii = n.arange(Lambda)
    Lv = float(len(T))

    Gammav_i = n.floor(ii*fv*Lv/f_s)  # indexes for LUT
    Gammav_i = n.array(Gammav_i, n.int)
    # variation pattern of vibrato for each sample
    Tv_i = tabv[Gammav_i % int(Lv)]

    # frequency in Hz for each sample
    F_i = f*(2.**(Tv_i*nu/12.))
    # movement inside table for each sample
    D_gamma_i = F_i*(Lt/float(f_s))
    Gamma_i = n.cumsum(D_gamma_i)  # movement in the total table
    Gamma_i = n.floor(Gamma_i)  # the indexes
    Gamma_i = n.array(Gamma_i, dtype=n.int)  # the indexes
    return tab[Gamma_i % int(Lt)]  # looking for indexes in table
```

```
############ Sec. S-M-1.1 Tuning, intervals, scales and chords
just_ratios = [1, 9/8, 5/4, 4/3, 3/2, 5/3, 15/3, 2]
pythagorean_ratios = [1, 9/8, 81/64, 4/3, 3/2, 27/16, 243/128, 2]
equal_temperament_ratios = [2**(i/12) for i in range(13)]

f = 220  # an arbitrary frequency
just_scale = [i*f for i in just_intonations]
pythagorean_scale = [i*f for i in pythagorean_ratios]
equal_temperament_scale = [i*f for i in equal_temperament_ratios]

js = H([v(i) for i in just_scale])
__s(js, "just_scale.wav")
ps = H([v(i) for i in pythagorean_scale])
__s(js, "pythagorean_scale.wav")
es = H([v(i) for i in equal_temperament_scale])
__s(js, "equal_temperament_scale.wav")

# Microtonality
# quarter tones
epslon = 2**(1/12.)
s1 = [0., 1.25, 1.75, 2., 2.25, 4., 5., 5.25]
factors = [epslon**i for i in s1]
scale = H([v(f*i) for i in factors])
__s(scale, "quarter_tones1.wav")

epslon = 2**(1/24.)
factors = [epslon**i for i in range(24)]
scale = H([v(f*i) for i in factors])
__s(scale, "quarter_tones2.wav")

# Octave sevenths
epslon_ = 2**(1/7.)
s2 = [0., 1., 2., 3., 4., 5., 6., 7.]
factors = [epslon_**i for i in s2]
scale = H([v(f*i) for i in factors])
__s(scale, "octave_sevenths.wav")

# Eq. S-M-1 relating note grids
# expressing octave sevenths in the quarter tone grid:
s2_ = [i*24/7 for i in s2]

# Table 1: Intervals
# using epsilon = 2**(1/12)
I1j = 0.
I2m = 1.
I2M = 2.
I3m = 3.
I3M = 4.
I4J = 5.
ITR = 6.
I5J = 7.
I6m = 8.
I6M = 9.
I7m = 10.
I7M = 11.
I8J = 12.
```

```
I_i = n.arange(13.)

perfect_consonances = [0, 7, 12]
imperfect_consonances = [3, 4, 8, 9]
weak_dissonances = [2, 10]
strong_dissonances = [1, 11]
special_cases = [5, 6]

# the interval sums nine for inversion by traditional nomenclature
# fifth is inverted into a fourth (5+4 = 9)
# but always sums 12
# at inversions of semitones
# fifth (7) is inverted into a fourth (5) (7+5 = 12)
def inv(I):
    """Returns inversed interval of I: 0< =  I < = 12"""
    return 12-I


# harmonic interval
def intervaloHarmonico(f, I):
    return (v(f)+v(f*2.**(I/12.)))*0.5


# melodic interval
def intervaloMelodico(f, I):
    return n.hstack((v(f), v(f*2.**(I/12.))))

# Eq. SI-A-2 Symmetric scales
Ec = [0., 1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11.]
Ewt = [0., 2., 4., 6., 8., 10.]
Etm = [0., 3., 6., 9.]
EtM = [0., 4., 8.]
Ett = [0., 6.]

# Eq. SI-A-3 Diatonic scales
Em = [0., 2., 3., 5., 7., 8., 10.]
Emlo = [1., 3., 5., 6., 8., 10.]
EM = [0., 2., 4., 5., 7., 9., 11.]
Emd = [0., 2., 3., 5., 7., 9., 10.]
Emf = [0., 1., 3., 5., 7., 8., 10.]
Eml = [0., 2., 4., 6., 7., 9., 11.]
Emmi = [0., 2., 4., 5., 7., 8., 10.]

# Eq. SI-A-4 Diatonic pattern
E_ = n.roll(n.array([2.,2.,1.,2.,2.,2.,1.]), n.random.randint(7.))
E = n.cumsum(E_)-E_[0.]

# Eq. SI-A-5 Harmonic and melodic minor scales
Em = [0., 2., 3., 5., 7., 8., 10.]
Emh = [0., 2., 3., 5., 7., 8., 11.]
Emm = [0.,2.,3.,5.,7.,9.,11.,12.,10.,8.,7.,5.,3.,2.,0.]

# Eq. SI-A-6 Harmonic series
H = [ 0, 12, 19+0.02, 24, 28-0.14, 31+0.2, 34-0.31,
      36, 38+0.04, 40-0.14, 42-0.49, 43+0.02,
      44+0.41, 46-0.31, 47-0.12,
```

```python
      48, 49+0.05, 50+0.04, 51-0.02, 52-0.14 ]

# Eq. SI-A-7 Triads
AM = [0., 4., 7.]
Am = [0., 3., 7.]
Ad = [0., 3., 6.]
Aa = [0., 4., 8.]

def withMinorSeventh(A): return A+[10.]
def withMajorSeventh(A): return A+[11.]


############ Sec. SI-A-2 Atonal and tonal harmonies, harmonic expansion and modulation
# Table SI-A-2
def dominant(TT):
    """
    Returns the dominant chord of another chord

    It suposes TT complete and
    in fundamental and closed position.
    And fundamental represented as zero.

    """
    T = n.copy(TT)
    T[0] = -1
    T[1] = 2
    return T


def subDominant(TT):
    """
    Returns the sub-dominant chord of another chord

    It suposes TT complete and
    in fundamental and closed position.
    And fundamental represented as zero.

    """
    T = n.copy(TT)
    T[1] = 5
    if TT[1] == 4:
        T[2] = 9
    else:
        T[2] = 8
    return T

def relativa(TT):
    """Returns the relative chord.

    TT is a major or minor triad at a closed and fundamental position."""
    T = n.copy(TT)
    if T[1]-T[0] == 4:  # TT is major
        T[2] = 9.  # returns minor chord a minor third bellow
    elif T[1]-T[0] == 3:  # TT is minor
        T[0] = 10.  # returns major chord a major third above
    else:
```

```python
        print("send me only minor or major perfect triads")
    return T


def antiRelativa(TT):
    """Returns the anti-relative chord."""
    T = n.copy(TT)
    if T[1]-T[0] == 4.:  # major
        T[0] = 11.  # returns up minor
    if T[1]-T[0] == 3.:  # menor
        T[2] = 8.  # returns down major
    return T

# Mediants
def sup(TT):
    T = n.copy(TT)
    if T[1]-T[0] == 4.:  # major
        T[0] = 11.
        T[2] = 8.  # returns major
    if T[1]-T[0] == 3.:  # minor
        T[0] = 10.
        T[2] -= 1.  # returns minor
    return T

def inf(TT):
    T = n.copy(TT)
    if T[1]-T[0] == 4.:  # major
        T[2] = 9
        T[0] = 1.  # returns major
    if T[1]-T[0] == 3.:  # minor
        T[2] = 8.
        T[0] = 11.  # returns minor
    return T

def supD(TT):
    T = n.copy(TT)
    if T[1]-T[0] == 4.:  # major
        T[0] = 10.
        T[1] = 3.  # returns major
    if T[1]-T[0] == 3.:  # minor
        T[0] = 11.
        T[1] = 4.  # returns minor
    return T

def infD(TT):
    T = n.copy(TT)
    if T[1]-T[0] == 4.:  # major
        T[1] = 3.
        T[2] = 8.  # returns major
    if T[1]-T[0] == 3.:  # minor
        T[1] = 4.
        T[2] = 9.  # returns minor
    return T

# Main tonal functions
tonicM = [0., 4., 7.]
tonicm = [0., 3., 7.]
```

```python
subM = [0., 5., 9.]
subm = [0., 5., 8.]
dominant = [2., 7., 11.]
Vm = [2., 7., 10.]  # minor chord is not dominant


############ Sec. SI-A-3 Counterpoint
def contraNotaNotaSup(alturas=[0,2,4,5,5,0,2,0,2,2,2,0,7,\
                                     5,4,4,4,0,2,4,5,5,5]):
    """Returns a melody given input melody

    Limited in 1 octave"""
    first_note = alturas[0]+(7,12)[n.random.randint(2)]
    contra = [first_note]

    i=0
    cont=0 # parallels counter
    reg=0 # interval register where the parallel was done
    for al in alturas[:-1]:
        mov_cf=alturas[i:i+2]
        atual_cf,seguinte_cf=mov_cf
        if seguinte_cf-atual_cf>0:
            mov="asc"
        elif seguinte_cf-atual_cf<0:
            mov="asc"
        else:
            mov="obl"

        # possibilities by consonances
        possiveis=[seguinte_cf+interval for interval in\
                                    [0,3,4,5,7,8,9,12]]
        movs=[]
        for pos in possiveis:
            if pos -contra[i] < 0:
                movs.append("desc")
            if pos - contra[i] > 0:
                movs.append("asc")
            else:
                movs.append("obl")

        movt=[]
        for m in movs:
            if 'obl' in (m,mov):
                movt.append("obl")
            elif m==mov:
                movt.append("direto")
            else:
                movt.append("contrario")
        blacklist=[]
        for nota,mt in zip(possiveis,movt):

            if mt == "direto": # direct movement
                # does not accept perfect consonances
                if nota-seguinte_cf in (0,7,8,12):
                    possiveis.remove(nota)
        ok=0
        while not ok:
```

```python
            nnota=possiveis[n.random.randint(len(possiveis))]
            if nnota-seguinte_cf==contra[i]-atual_cf: # parallel
                intervalo=contra[i]-atual_cf
                novo_intervalo=nnota-seguinte_cf
                if abs(intervalo-novo_intervalo)==1: # same 3 or 6 type
                    if cont==2: # if already had 2 parallels
                        pass # another interval
                    else:
                        cont+=1
                        ok=1
            else: # oblique or opposite movement
                cont=0 # make parallels equal to zero
                ok=1
        contra.append(nnota)
        i+=1
    return contra


############ Sec. SI-A-4 Rhythm
# See Poli Hit Mia musical piece


############ Sec SI-A-5 Repetition and variation: motifs and larger units
# Ubiquitous concepts
# examples
S = [1, 2, 1.5, 3]  # a sequence of parameters, e.g. durations
S1 = S[::-1]  # reversion
S2 = [i*4 for i in S]  # expansion
S3 = [i*.5 for i in S]  # contraction
S4 = S[2:] + S[:2]

# now suppose that S is a sequence of pitches
S5 = [i+7 for i in S]  # transposition
S6 = [i-12 for i in S]  # interval inversion


############ Sec SI-A-6 Directional structures
# See Dirracional musical piece


############ Sec. SI-A-7 Cyclic structures
# See 3 Trios musical pieces
# and the PPEPPS
```

SI-D-2 Musical pieces

The code for the musical pieces are omitted from this PDF because it would make the document lengthy. Please see [3] and [1] to know what are the available scripts for rendering musical pieces and reach them.

SI-D-3 Auxiliary files

The code for the auxiliary files (e.g. to render the figures in the article [2] are omitted from this PDF because it would make the document lengthy. Please see [3] and [1] to know what are the available auxiliary scripts and reach them.

SI-D-4 Final considerations

This document exhibits the code that implements the relations in the sections of [2]. All this scripts are available in [1] with other documentations and further scripts e.g. to render musical pieces and the figures in [2]. One should also reach [3] to know about the resources in the MASS framework. This document should be available at [4].

References

[1] R. Fabbri (2019). Música no áudio digital : descrição psicofísica e caixa de ferramentas. MSc dissertation. Available at: `http://www.teses.usp.br/teses/disponiveis/76/76132/tde-19042013-095445/publico/RenatoFabbri_ME_corrigida.pdf`

[2] R. Fabbri et al. Musical elements in the discrete-time representation of sound. 2019. `https://github.com/ttm/mass/raw/master/doc/article.pdf`

[3] R. Fabbri et al. Equations, scripts, figures, tables and documents in the MASS framework. 2019. `https://github.com/ttm/mass/raw/master/doc/listings.pdf`

[4] R. Fabbri et al. PDF presentation of the Python implementations in the MASS framework. 2019. `https://github.com/ttm/mass/raw/master/doc/code.pdf`.

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