Introduction to Statistics

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Content

- Definitions
- Sample and Population
- Type of Variables: categorical, Ordinal, ratio
- Data Collection: sampling
- Summary measure
- Data Representation: frequency table, histograms, Bar Chart and Frequency polygons, Box-plot, …
- Data Collection: Observational studies and Experiments

Objective of this lecture

- the objective of this introduction is to cover basic statistical knowledge about the collection and interpretation of data :
  - display and summarise large amounts of quantitative information, before undertaking a more sophisticated analysis.

What is statistical analysis?

- Statistics:

  "… determination of the probable from the possible"

  Davis, Statistics and data analysis in geology, p. 6

- Inferential statistics:
  - from samples to populations → what could have been or will be observed if we have analysed the entire population
- Descriptive statistics:
  - numerical summaries of samples → what was observed

Why use statistical analysis?

- Descriptive statistics: summarize some data in a **shorter** form
- Inferential statistics: understand some process and possible predict based the outcome
  - We need a conceptual representation (model) from which we infer the process.
  - We need to know if the model is "**correct**"?
    - Are we imagining relations where there are none?
    - Are there true relations we haven't found?

Statistical analysis gives a way to quantify the confidence we can have in our inferences.

What is "statistical analysis"?

This term refers to a wide range of techniques to...

- Describe
- Explore
- Understand
- Prove
- Predict

...based on sample datasets collected from populations, using some sampling strategy.

Populations and samples

- Population: a **set** of all elements (individuals)
  - Finite vs. "infinite"
- Sample: a **subset** of elements taken from a population

We make inferences about a population from a sample taken from it.

- If it is feasible to examine the entire population; then there is no inference from a sample.
  - Example: all pixels in an image.
- Questions: **is the sample** Representative or biased?

Populations and samples

Representative vs. biased

- For example, if we are interested in conducting a survey of the amount of *physical exercise* undertaken by the general public.

- surveying persons entering and leaving a **gym** would provide:
  - a biased **sample** of the **population**,
  - and the results obtained cannot generalise to the population

Populations and samples

Representative vs. biased

An Example of a non-representative sample: The *Literary Digest* Poll on the 1936 U.S. Presidential elections

Literary digest magazine sent survey to 10 million people who where subscribers or owned car or telephones

- ✓ 2.3 million people responded
- ✓ The results incorrectly predicted a landside for the republican candidate

Populations and samples

An Example of a could be non-representative sample: The change in Canada in 2010 of the National Household Survey (NHS).

- *long-form census* was given to a random sample of 1/5 of household and was mandatory.
- NHS is given to 1/3 of household and is voluntary

- How might voluntary response affect the quality of data received

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Types of Bias of a sample

- **Selection Bias**: Occurs when the sample is selected in such away that it systematically excludes on under-represent part of the population
- **Measurement or Response Bias**: Occurs when the data are collected in such a way that it tends to result in observed values that are different from the actual value in some systematic way
- **Nonresponse Bias**: Occurs when responses are not obtained from all individuals selected for inclusion in a sample

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How to select a representative Sample

Some method of non-random sample selection

- Systematic sampling:
  - select every k^{th} individual from a list of the population,
  - where the position of the first person chosen is randomly selected form the first k individuals.

Note: This will give a non-representative sample if there is a structure in the list

- Convenience or Volunteer Sampling:
  - Use the first n individuals that are available or the individuals who volunteer to participate.

Note: This is almost sure to give a non-representative sample which cannot be generalised to the population

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How to select a representative Sample

Use Randomisation

Some methods of random sample selection:

- **Sample Random Sampling** (SRS): Each possible sample of size n from the populations is equally likely to be the sample that is chosen.
- **Stratified sampling**: Divide the population into non-overlapping subgroups called starts and choose a SRS within each subgroup.
- **Cluster Sampling**: dived the population into non-overlapping subgroups called clusters, select clusters at random, and include all individuals in the chosen cluster in the sample

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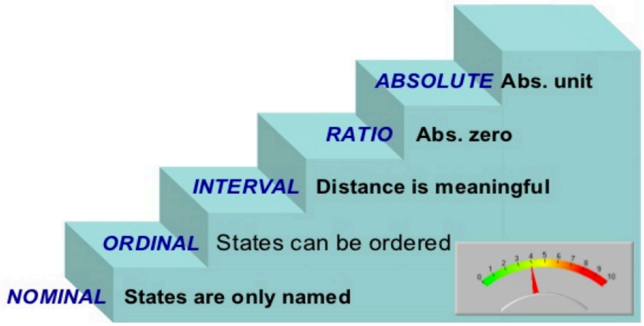
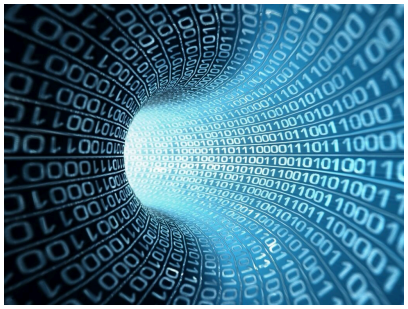
Variables, Observations, and data

- Variables are quantities measured on a sample
- Observation a particular outcome.
- Data is a collection of Several observations.





Type of Variables

- Qualitative variables (Nominal or Categorical) have:
  - non-numeric outcomes,
  - with no **natural** ordering.

  For example, gender, disease status, and type of car

- Quantitative variables (ordinal, Interval, Ratio) have
  - numeric outcomes
  - can be **discrete** or **continuous**.

  For example, survival time, height, age, number of children, and number of faults

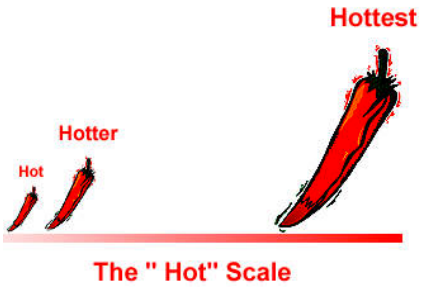
Qualitative variables: Nominal (Categorical)


Sorting into categories...

- Values are from a set of classes with **no natural** ordering
  - Example: Land uses (agriculture, forestry, residential . . .)
- Can determine **equality**, but **not rank**
- Meaningful sample statistics:
  - mode (class with most observations);
  - frequency distribution (how many observations in each class)

- **Note**:
  - Numbers may be used to label the classes but these are arbitrary and have no numeric meaning (the "first" class could just as well be the "third");
  - ordering is by convenience (e.g. alphabetic)

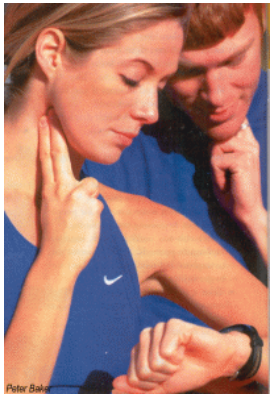
Quantitative variables: Ordinal



- Values are from a set of naturally ordered classes with no meaningful units of measurement
  - Can determine rank (greater, less than)
  - This ordering is an intrinsic part of the class definition
  - Example: Soil structural grade (0 = structureless, 1 = very weak, 2 = weak, 3 =medium, 4 = strong, 5= very strong)
- Meaningful sample statistics:
  - mode;
  - frequency distribution

Note: Numbers may be used to label the classes; their order is meaningful, but not the intervals between adjacent classes are not defined (e.g. the interval from 1 to 2 vs. that from 2 to 3)

Quantitative variables: Interval



- Values are measured on a **continuous** scale with well-defined units of measurement
  - no **natural origin** of the scale,
  - i.e. the zero is arbitrary,
- Example: Temperature in C.
  - "**It is twice as warm yesterday as today**" is meaningless, even though "Today it is 20C and yesterday it was 10C" may be true.
- Meaningful statistics: quantiles, mean, variance

Quantitative variables: Ratio

- Values are measured on a continuous scale with well-defined units of measurement
  - a **natural origin** of the scale, i.e. the zero is meaningful
- Examples: Temperature in K; concentration of a chemical in solution
  - "There is twice a much heat in this system as that" is meaningful, if one system is at 300K and the other at 150K
- Meaningful statistics:
  - Quantile, mean, variance;
  - the coefficient of variation. (CV = SD / Mean; is a ratio).

Summary measures

- A set of data on its own is very hard to interpret it is often useful to obtain **quantitative summaries** of certain aspects of the data.

- Summary measurements are divided in two types:
  - quantities which are "**typical**" of the data, known as **measures of location**
  - quantities which **summarise** the **variability** of the data, Known as **measures of spread**.

Summary measures

- Measures of location
  - Sample mean
  - Sample median
  - Sample mode
- Measures of spread
  - Range
  - Mean absolute deviation (M.A.D.)
  - Sample variance and standard deviation
  - Quartiles and the interquartile range
  - Coefficient of variation

Summary measures: Measures of location

- Sample **mean**:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

- Sample **median**: is the **middle** observation
  when the data are **ranked** in increasing order.

$$\text{SampleMedian} = \begin{cases} x_{\left(\frac{n+1}{2}\right)}, & n \text{ odd}, \\ \frac{1}{2}x_{\left(\frac{n}{2}\right)} + \frac{1}{2}x_{\left(\frac{n}{2}+1\right)}, & n \text{ even}. \end{cases}$$

- Sample **mode**: is the value which occurs with the **greatest frequency**

$$\text{SampleMode} = \{y_k | f_k = \max_i \{f_i\}\}.$$

Summary measures: Measures of spread

- **Range**: **difference** between the **largest** and **smallest** observation

$$\text{Range} = x_{(n)} - x_{(1)}$$

- **Mean absolute deviation** (M.A.D.): average **absolute deviation** from the **sample mean**

$$\text{M.A.D.} = \frac{|x_1 - \bar{x}| + \cdots + |x_n - \bar{x}|}{n} = \frac{1}{n}\sum_{i=1}^{n} |x_i - \bar{x}|$$

- Sample **variance** & **standard deviation**: **average squared distance** of the observations from their mean value

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\left\{\left(\sum_{i=1}^{n} x_i^2\right) - n\bar{x}^2\right\}.$$

Summary measures: Measures of spread

- **Quartiles** and the **interquartile range**
  - **lower quartile** (LQ) has a quarter of the data less than it→ (n+1)/4th
  - **upper quartile** (UQ) has a quarter of the data above it.→ 3(n+1)/4th
  - **inter-quartile** (IRQ) range is the difference between the upper and lower quartiles

Calculating lower quartiles

$n = 15$	LQ at $(15+1)/4 = 4$	LQ is $x_{(4)}$
$n = 16$	LQ at $(16+1)/4 = 4\frac{1}{4}$	LQ is $\frac{3}{4}x_{(4)} + \frac{1}{4}x_{(5)}$
$n = 17$	LQ at $(17+1)/4 = 4\frac{1}{2}$	LQ is $\frac{1}{2}x_{(4)} + \frac{1}{2}x_{(5)}$
$n = 18$	LQ at $(18+1)/4 = 4\frac{3}{4}$	LQ is $\frac{1}{4}x_{(4)} + \frac{3}{4}x_{(5)}$
$n = 19$	LQ at $(19+1)/4 = 5$	LQ is $x_{(5)}$

$$\text{IQR} = \text{UQ} - \text{LQ}.$$

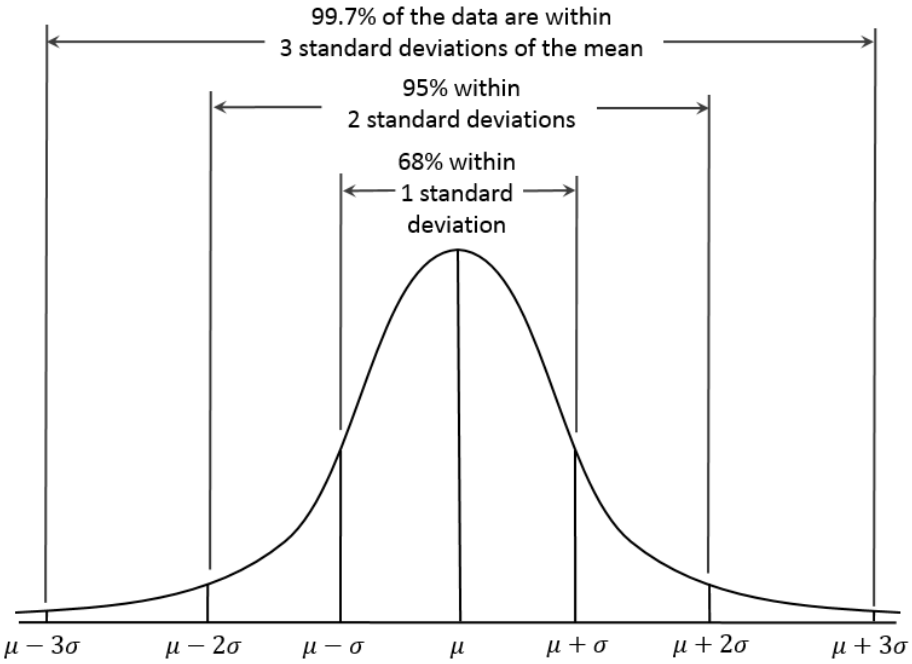
- **Coefficient of variation**: is the ratio of the standard deviation to the mean

$$\text{Coefficient of variation} = \frac{s}{\bar{x}},$$

The empirical rule

- If the data distribution is unimodal and symmetric → There is empirical rule

The empirical rule states that for a normal distribution, nearly all of the data will fall within three standard deviations of the mean.

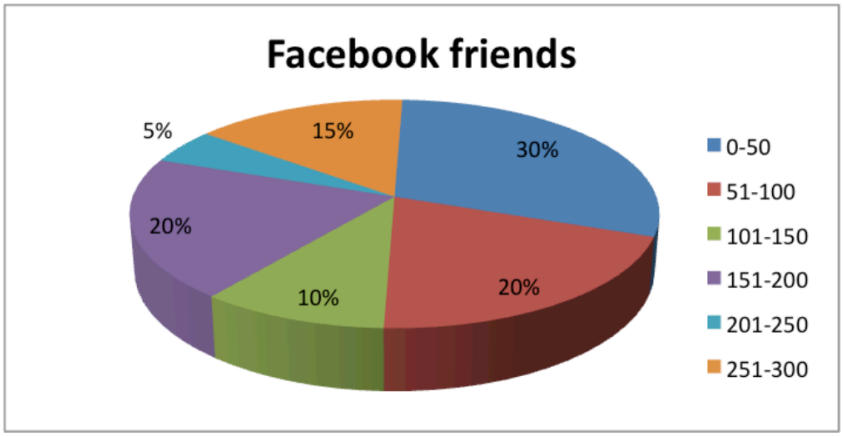


Data presentation

- Frequency tables
- Histogram
- Bar Chart and Frequency polygons
- Box-and-whisker plots
- …

Pie chart

- Suitable to represent categorical data;
- Used to show percentages;
- Areas are proportional to value of category



Data presentation: Frequency tables

Frequency table shows a tally of the number of data observations in different categories

- For **discrete** **qualitative** & **quantitative** data, we use all of the observed values as categories.
  - if the number of observations is large, consecutive observations may be grouped to form **combined categories**
- For **continuous** data, the choice of **categories** is more arbitrary.
  - Usually 8 to 12 non-overlapping consecutive intervals of equal width are used

Data presentation: Frequency tables

The number of calls from motorists per day for roadside service was recorded for the month of December

Class interval	Tally	Frequency
0 - 39	I	1
40 - 79	𝍸	5
80 - 119	𝍸 𝍸 II	12
120 - 159	𝍸 III	8
160 - 199	IIII	4
200 - 239	I	1
	Sum =	31

(germinating seeds), we can construct the following frequency table.

No. germinating	85	86	87	88	89	90	91	92	93	94
Frequency	3	1	5	2	3	6	11	4	4	1

$n = 40$

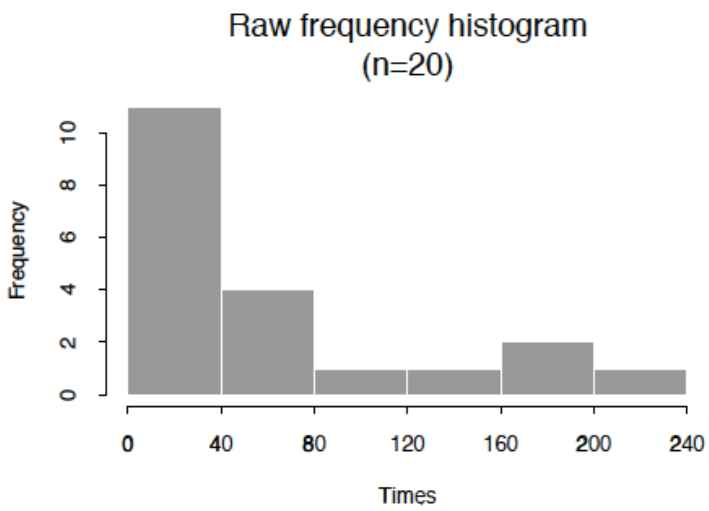
Data presentation: Histogram

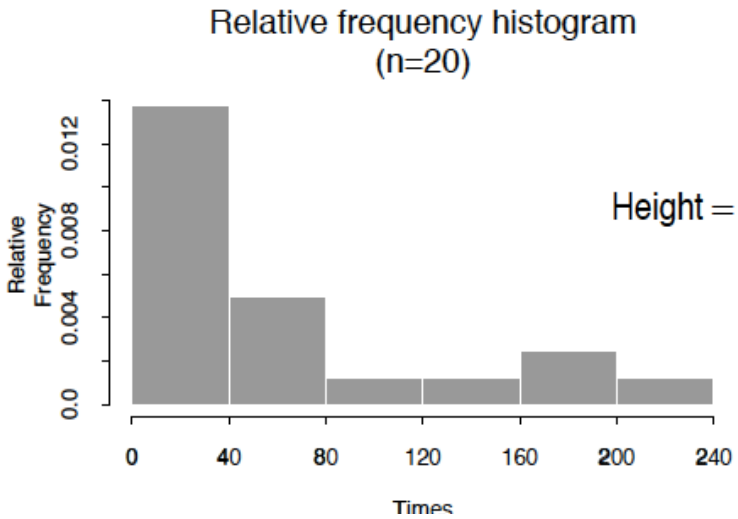
Once the frequency table has been constructed, pictorial representation can be considered.

For most **continuous** data sets, the best diagram to use is a **histogram**

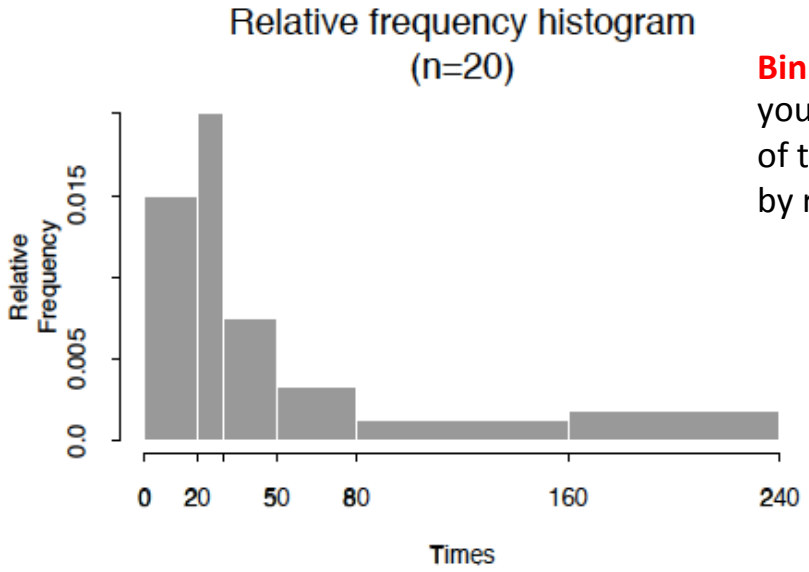
- Rectangles are drawn on this base with their areas proportional to the frequencies
  - classification intervals are represented to scale on the abscissa (x-axis)
  - heights of the rectangles are proportional to the frequencies iff. class intervals are of equal width

Data presentation: Histogram





$$\text{Height} = \frac{\text{Frequency}}{n \times \text{BinWidth}}.$$



Bin widths should be chosen so that you get a good idea of the distribution of the data, without being swamped by random variation.

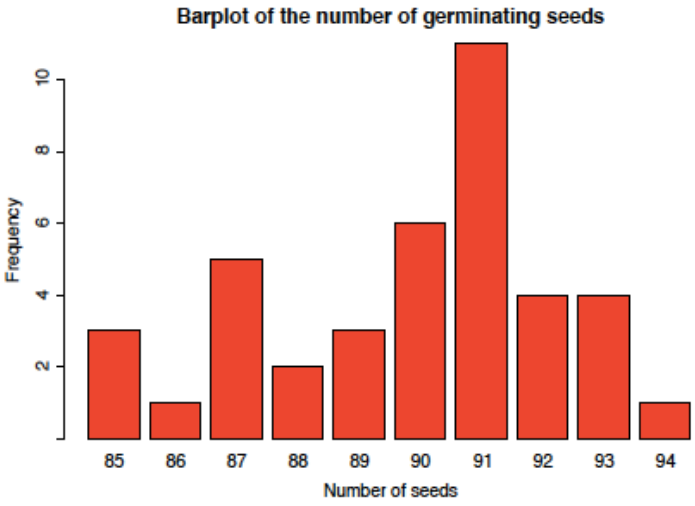
Data presentation: Bar Chart and Frequency polygons

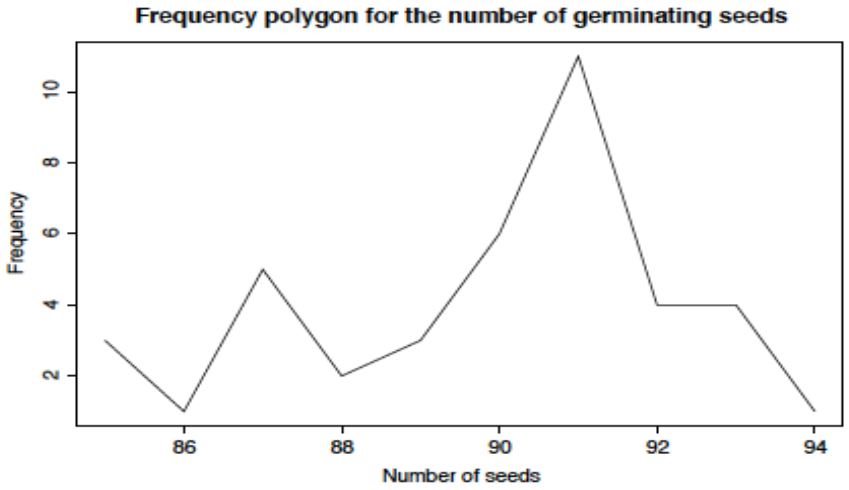
When the data are **discrete** and the frequencies refer to **individual** values, we can display them using:

- **Bar chart** with **heights** of bars representing frequencies
- **Frequency polygon** in which only the **tops** of the bars are marked, and then these points are joined by straight lines.

Data presentation: Histogram

Using the frequency table constructed earlier, we can construct a Bar Chart and Frequency Polygon as follows.

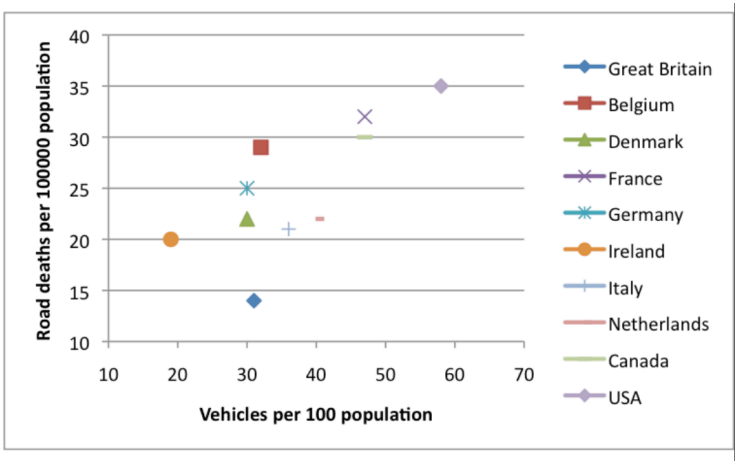




$$n = 40$$

Scatter Plot

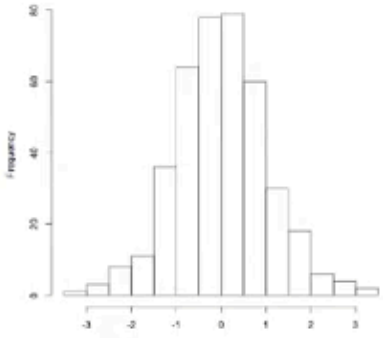
- Displays values for two variables for a set of data;
- The independent variable is plotted on the horizontal axis, the dependent variable on the vertical axis;
- It allows to determine correlation
  - Positive (bottom left -> top right)
  - Negative (top left -> bottom right)
  -



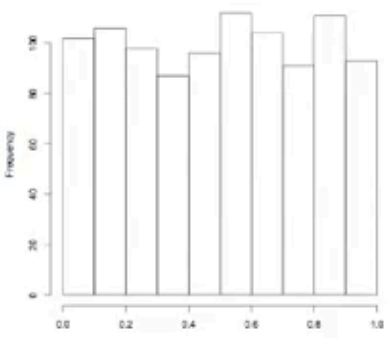
Data presentation: Box-and-whisker plots

- description of the main **features** of a the observations.
- There are many variations on the box plot. The simplest form is constructed a fellows:

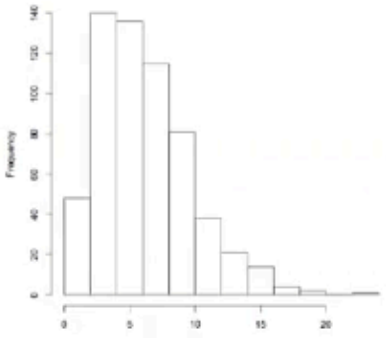
1. draw a rectangular box which stretches from the **lower quartile** to the **upper quartile**,
2. divide in two at the **median**.
3. From each end of the box, a line is drawn to the **maximum** and **minimum** observations (these lines are sometimes called whiskers, hence the name)



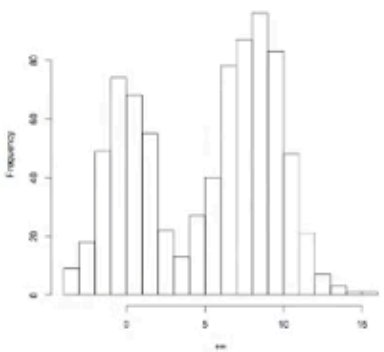
Symmetric
Unimodal



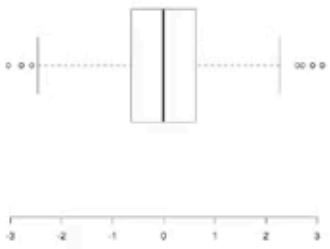
Symmetric
Uniform

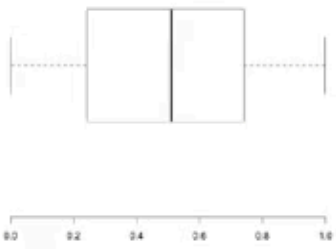


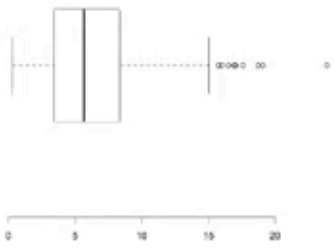
Right-skewed
Unimodal

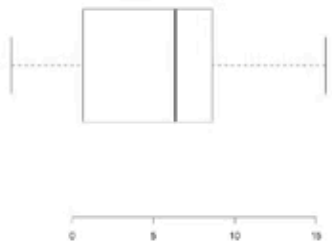


Bimodal









Data Collection: Observational Studies & Experiments

- Anecdotes
- Observations Studies
- Experiments

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Observation vs. Experiments

- **Observation** of the characteristics of an existing sample
  - The goal is either to draw conclusion:
    - about the population
    - differences between two or more groups
    - The relationships between variables
  - The investigator has **no control** over which individuals are in which group any other of their characteristics.

- **Experiments**:
  - **Interventions** are imposed by the investigator in the subjects.

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Observational studies

observational study is the mechanisms that can result in an **observed association** between **an explanatory variable** and the out come:

- Changes in the explanatory variable cause the outcome to change

- Reverse causation causes the explanatory variable to change

1. The association is coincidence
2. There is a common cause that causes both the explanatory and the outcome to change
3. A **confounding** variable is associated with the explanatory variable and causes the outcome to change

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Confounding variables: lurking variable

- Lurking variables are variables that are not considered in the study but may affect the nature of the relationship between the **explanatory variable** and the **outcome**

- A lurking variable can be a confounding variable or the source of a common response or another variable that, when considered, changes the nature of the association

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Experimental studies

- Response variable (or dependent variables) the outcome of interest measured each subject, on entity participating in the study
- Explanatory variables (predictor or Independent variable): A variable that we think that might explain the value of the response variable
- experiments: the experimenter manipulate the explanatory variable to see the effect on the response