Computational Physics: Error Estimates and Error Propagation

Ujwal Kumar

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1 Introduction

In this report, we build on last week's work providing estimates for the gravitational constant, g, in the context of a cannonball flying through air. We now work on providing error estimates for the estimate of g.

We use a bootstrapping method to repeatedly sample through data plotting the 'y' position against the time and generate a range of estimates of g. With these range of estimates, we can thus predict a value of g with errors.

2 Problems

2.1 Problem 1

In this problem, we use the `drand48` routine to generate 100,000 random numbers and compare the expected pdf to a histogram of this data.

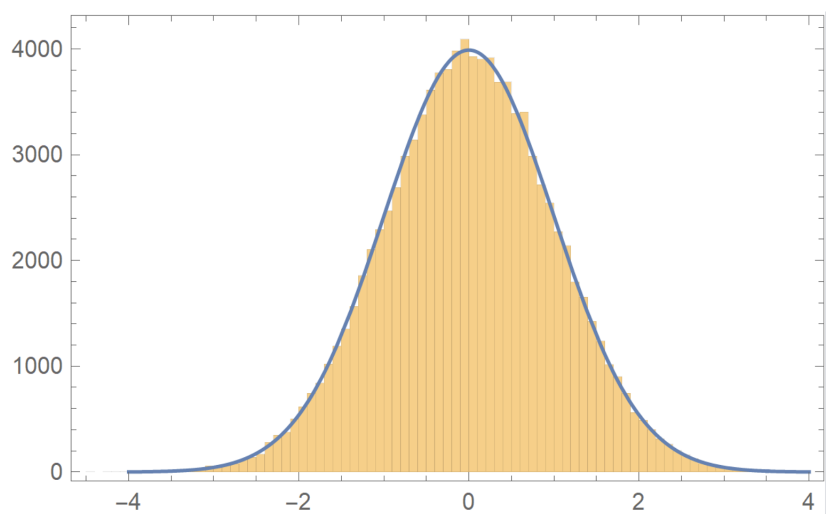

Figure 1: PDF of 100,000 randomly distributed numbers generated using `drand48` fit with a Gaussian

In Figure 1, we see that the PDF used to fit the histogram fits it well as expected with a PDF peaking around 0.

2.2 Problem 2

We now use a Monte Carlo method to compute the distribution of $\log(x^2 + y^2)$ where x and y are two variables with Gaussian distributions. We also know that $\bar{x} = 1$ and $\bar{y} = 4$.

$$\sigma_f^2 = \left(\frac{\delta f}{\delta x}\right)^2 \sigma_x^2 + \left(\frac{\delta f}{\delta y}\right)^2 \sigma_y^2 \tag{1}$$

The analytical distribution of this function can be given by the variance in Equation 1.

The Monte Carlo histogram in comparison to the analytical calculation using Formula 1 shows us that the variance we should expect is:

$$\frac{16\bar{x}^2}{(\bar{x}^2+\bar{y}^2)^2} + \frac{4\bar{y}^2}{(\bar{x}^2+\bar{y}^2)^2} = 0.277$$

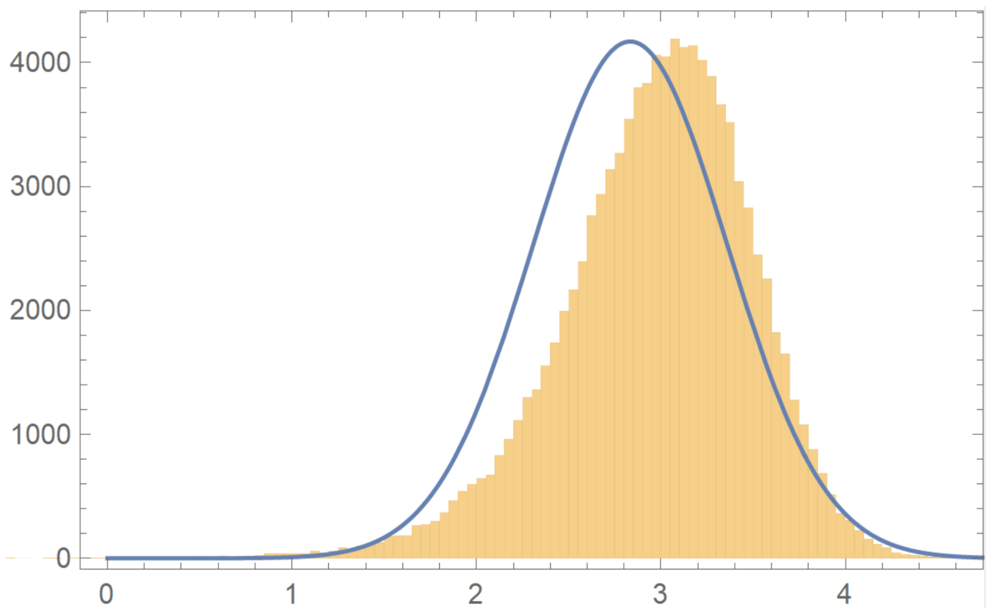

Figure 2: Distribution of $\log(x^2+y^2)$ centered around 3 with an overlaid PDF fit to the histogram. Logarithmic distribution of two Gaussian variables affects the normality of the entire function resulting in the offset seen in the fitted PDF.

2.3 Problem 3

We now use the Monte Carlo method to determine the uncertainty on last week's estimate of g using 100,000 replicas of the `cannonball.dat` file. Through this histogram, we find $g = 9.33\ m/s^2$ with a σ of 0.04 as from Terminal's output of the code's results but I was unable to make a histogram of the same due to this error that I was unable to resolve: `Skeleton is not a Graphics primitive or directive.`

2.4 Problem 4

We now turn our attention to the datafile `bootstrap.dat` which contains two columns. The first two entries in the first row are N_{students} and $N_{\text{time steps}}$ and subsequent rows are filled with pairs of [t, y(t)].

*Couldn't get to this question as I was really struggling to get a working code in the first place and taking time with some of these assignments would be helpful to learning some C before proceeding with more difficult assignments as I don't think I could reproduce any of the code the professor writes in class nowadays.

3 Conclusion

I found this assignment to be understandable entirely in concept but I struggled to get my code working in time and had to use a lot of help from my peers to generate a working code. With that said, my biggest takeaway was the power of bootstrapping and how repeated sampling as a general concept in nature is quite interesting. It reminds of how a circle drawn onto a square dart board with repeated, random throws of darts at the entire square yields a relationship for π (https://www.youtube.com/watch?v=M34TO71SKGk).