A geometric discretisation scheme applied to the Abelian Chern-Simons theory

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Abstract

We give a detailed general description of a recent geometrical discretisation scheme and illustrate, by explicit numerical calculation, the scheme's ability to capture topological features. The scheme is applied to the Abelian Chern-Simons theory and leads, after a necessary field doubling, to an expression for the discrete partition function in terms of untwisted Reidemeister torsion and of various triangulation dependent factors. The discrete partition function is evaluated computationally for various triangulations of S^3 and of lens spaces. The results confirm that the discretisation scheme is triangulation independent and coincides with the continuum partition function.

1 INTRODUCTION

A very useful way to regularize a quantum field theory is provided by the lattice formulation introduced by Wilson [1]. (See for example, [2] for a detailed treatment). However, this formulation has difficulty in capturing topological features of a field theory, for example, the topological theta-term in QCD. It is therefore of interest to investigate alternative discretisation schemes. In this paper we describe an alternative scheme which is applicable to antisymmetric tensor field theories including Abelian gauge theories and fermion field theory in the Kähler–Dirac framework [3], and which is well-suited for capturing the topological features of such theories. It is based on developing analogies between the different types of fields and the way they appear in a quantum field theory with a corresponding list of discrete variables and operators. The method is valid for any arbitrary compact 3-manifold without boundary. We illustrate the scheme by applying it to the pure Abelian Chern-Simons gauge theory in 3 dimensions, and to a doubled version, the so-called Abelian BF gauge theory. In the latter case the topological features of the theory are completely reproduced by our discretisation scheme, even before taking the continuum limit. We illustrate