Simulating the inner planets of the Solar System using Beeman Algorithm

Introduction

In this project I have created a simulation of the inner plants of the solar system. The aim was to create a realistic simulation in order to perform experiments and see if the results match the real world. The experiments I implemented were “Satellite to Mars” and “Asteroids and doomsday”.

There are multiple ways to get a satellite to Mars, but I decided on using a ‘Hohmann Transfer Orbit’ in which a satellite is put into an elliptical orbit fired from Earth such that the aphelion of the orbit crosses the path of Mars. However the satellite has to be launched when the Earth and Mars are at the correct angle, so that the satellite encounters Mars. The aim of the experiment therefore was to try launching satellites at Mars at different angles within a range of the theoretical angle and calculate a closest approach that the satellite was to Mars.

For the Asteroid simulation I wanted to see how likely it is for an asteroid to encounter Earth, by randomly placing asteroids into the simulation, and I would track the distances from Earth to see if any could collide with the Earth.

The so-called ‘doomsday’ scenario occurs when all planets are aligned in a row from the Sun. In this simulation I wanted to see how often the planets would align to within a given tolerance. To measure the time that the planets would align I ran the simulation with different tolerances to see if the value would change.

To make sure the simulation was also working correctly tracked the total energy of the system, by summing all the kinetic energy of the planets as well as their gravitational potential energy at each timestep to see if it remained constant.

Methods

Planet Class

The Idea behind this class is to create objects to represent any object within the simulation. It contains all necessary fields to hold all the data related to planets i.e. mass, position and velocity.

Solar Class

This class is used to hold the entire simulation. I decided to separate the simulation itself from any experiments that could be run on it. This meant that I could instantiate a simulation any number of times. The idea of this meant that each experiment could be run in its own environment which would allow for a greater control over variables. For example I didn’t want asteroids flying around while I was trying to get the satellite to Mars. Therefore within this class I have all necessary methods required to calculate the next positions and velocities of the planets using the Beeman Algorithm as well as for executing an entire timestep. This class also includes the methods for animating the simulation. The animation is completed ‘on the fly’, meaning that the a timestep is executed and immediately the results are displayed on screen as opposed to animating precompiled results. The advantage of this is that the simulation can run indefinitely, but it meant that I had to remove the animation when running the experiments, otherwise they would take too long to complete.

Within thin class I also included two class methods to convert between radians and degrees. This means that they can be used by other programs without the Solar class being instantiated.

Experiments

Satellite to Mars

In this experiment I am investigating the ideal angle at which to launch a satellite to Mars using the Hohmann transfer orbit. Firstly I needed to calculate the orbital parameters of the orbit. I have shown the formulas I used in Appendix 1. In order for the experiment to work properly I placed the satellite on the opposite side of the Earth to ensure that gravity from Earth wasn't a factor. The ideal angle calculated was Mars being 44.3 degrees in from of Earth. To calculate the angle between the planets. I thought simply using the dot product formula would suffice, however this would not show if Mars was in front or Earth or not. Therefore I had calculate their clockwise angles relative to the x-axis to fix the issue. Due to the precision of floating point numbers I rounded the angles to 3 decimal places so that the values could be compared for equality. The data for the experiment was recorded by appending data to a text log file, and a csv file so that the data could be graphed.

I ran the simulation for two years, then continued to run the simulation until the angle between Mars and Earth is equal to the desired angle, offset by 180 degrees since I'm launching the probe on the opposite side. Once the angle is the same I create and new Planet object to represent the probe and append it to self.sim.planets so that it is part of the simulation. Next I continue running the simulation and calculate the distance between Mars and the probe and the value of 'Closest Approach' which is then logged to the data files. The simulation is run for one complete orbit of the probe to ensure that the closest approach is contained within the simulation.

Asteroids

This experiment was designed to investigate how likely it is for Earth to be hit by an asteroid. Every timestep there is a probability that an asteroid is created. Asteroids are created using the Planet class and appended to the list of other Planet objects. Random values for position, velocity and mass are generated within the accepted range. The distances between each asteroid and earth are calculated and if any are within a given distance, this would be logged as a close encounter. The output of the simulation is a text document which contains the log of the entire experiment showing each asteroid creation as well as if there were any close encounters.

The experiment is then run through the 'runAsteroidSimulation' method, which requires as parameters an integer for the number of years for which the simulation is run as well as a probability for the likelihood of an asteroid to be created, which is a float between 0-1.

A close encounter is defined as an asteroid that is closer than 10^{18} meters. This was chosen as it means that a close encounter is quite likely and can be seen on the log file as well as being a distance such that it reasonably close to earth to be of interest.

Due the random positions and velocities of the asteroid they have quite eccentric orbits around the sun, meaning that encounters with the Earth are more likely since they cross the earth elliptical multiple times.

Doomsday

For the doomsday scenario I wanted to see how often the planets would come to one and other within a tolerance of a certain angle in order to see how often the so called 'doomsday' scenario occurs. The method isAligned is used to check if the planets are aligned, returning True/False. The angle that each planet makes with each other relative to the Sun has to be less than the tolerance in order for them to be classed as aligned.

The runSimulation() method is responsible for running each individual case. In this method I run the simulation for a year to misalign the planets initially. Then continue running timesteps until they are aligned. I then then log the result from the experiment to a log file. The experiment was run with multiple different angles to see if a narrower alignment angle was less likely to happen.

Orbital Periods

In this experiment I want to check that the orbital periods of the planets in my simulation match the orbital periods of planets in real life. To do this I use Kepler's 3rd law of motion to calculate the orbital period from it's distance from the sun. The output of the experiment was again written to a log file.

Energy Conservation

Here I want to check that my simulation is working correctly by calculating the total energy of the simulation at each timestep. Since energy is conserved, if I calculate the total energy of the simulation by summing the kinetic and gravitational energy for every planet I should see that it remains constant.

Results

Hohmann Transfer (Mars Experiment)

To calculate the parameters of the Hohmann Orbit I used the formulas in Appendix 1. Calcuating the orbital period of the transfer was 518 days. Taking half of this number since we want to know the time to get to Mars is 259 days. According to NASA's website the Viking Probe was launched August 20th and was put into a circular obit around Mars on 19th June 1976; resulting in a total flight time of 305 days. Similarly calculating for the ideal launch angle was 44.3 degrees.

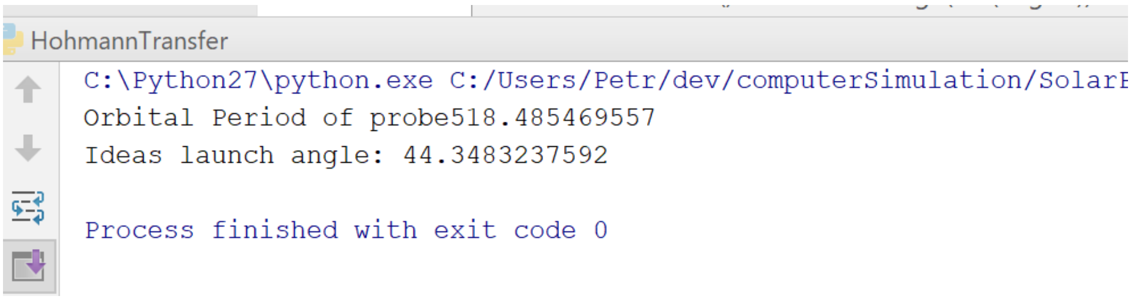
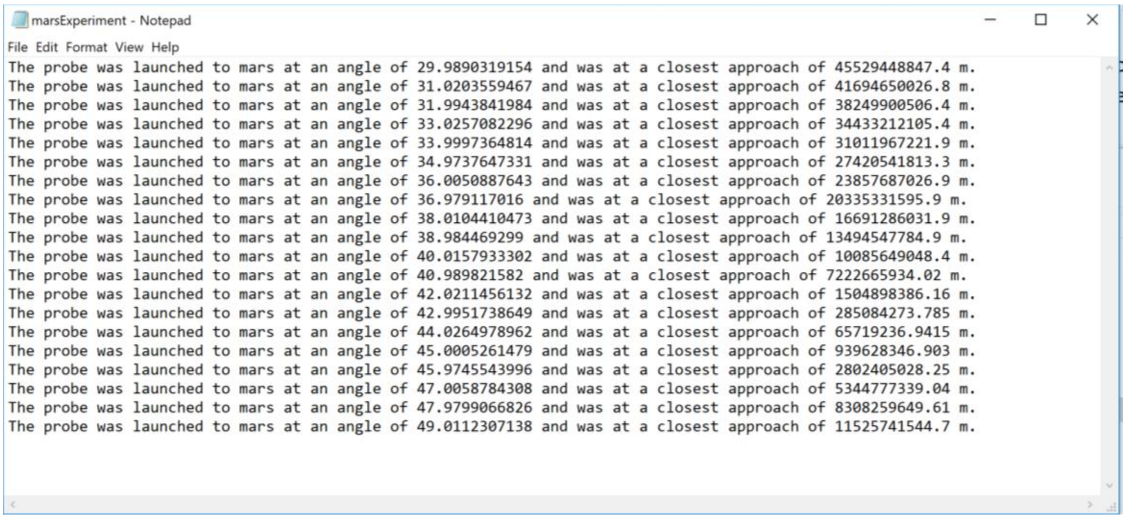


Figure 1 - Orbital Parameters returned from Simulation

Running the experiment for angles of 30 – 50 degrees with 1 degeee increments yielded the following resuls.



```
marsExperiment - Notepad
File Edit Format View Help
The probe was launched to mars at an angle of 29.9890319154 and was at a closest approach of 45529448847.4 m.
The probe was launched to mars at an angle of 31.0203559467 and was at a closest approach of 41694650026.8 m.
The probe was launched to mars at an angle of 31.9943841984 and was at a closest approach of 38249900506.4 m.
The probe was launched to mars at an angle of 33.0257082296 and was at a closest approach of 34433212105.4 m.
The probe was launched to mars at an angle of 33.9997364814 and was at a closest approach of 31011967221.9 m.
The probe was launched to mars at an angle of 34.9737647331 and was at a closest approach of 27420541813.3 m.
The probe was launched to mars at an angle of 36.0050887643 and was at a closest approach of 23857687026.9 m.
The probe was launched to mars at an angle of 36.979117016 and was at a closest approach of 20335331595.9 m.
The probe was launched to mars at an angle of 38.0104410473 and was at a closest approach of 16691286031.9 m.
The probe was launched to mars at an angle of 38.984469299 and was at a closest approach of 13494547784.9 m.
The probe was launched to mars at an angle of 40.0157933302 and was at a closest approach of 10085649048.4 m.
The probe was launched to mars at an angle of 40.989821582 and was at a closest approach of 7222665934.02 m.
The probe was launched to mars at an angle of 42.0211456132 and was at a closest approach of 1504898386.16 m.
The probe was launched to mars at an angle of 42.9951738649 and was at a closest approach of 285084273.785 m.
The probe was launched to mars at an angle of 44.0264978962 and was at a closest approach of 65719236.9415 m.
The probe was launched to mars at an angle of 45.0005261479 and was at a closest approach of 939628346.903 m.
The probe was launched to mars at an angle of 45.9745543996 and was at a closest approach of 2802405028.25 m.
The probe was launched to mars at an angle of 47.0058784308 and was at a closest approach of 5344777339.04 m.
The probe was launched to mars at an angle of 47.9799066826 and was at a closest approach of 8308259649.61 m.
The probe was launched to mars at an angle of 49.0112307138 and was at a closest approach of 11525741544.7 m.
```

Figure 2 - Log File of Mars Experiment

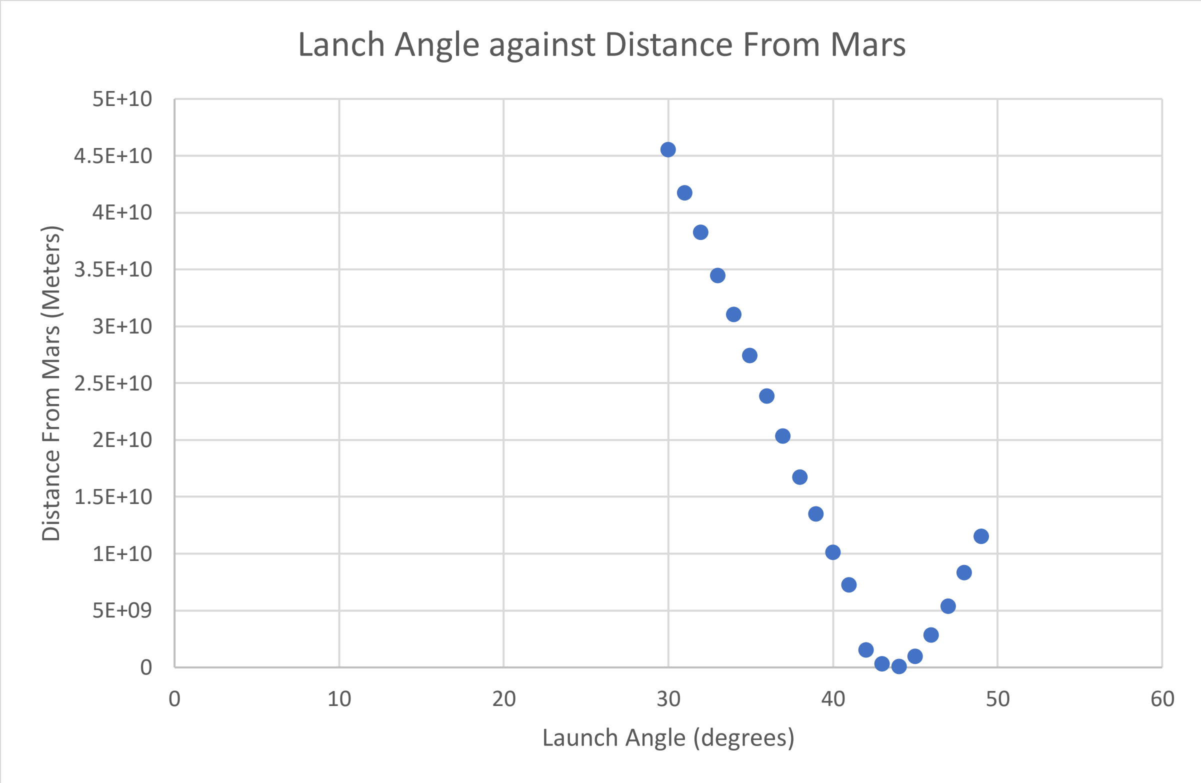


Figure 3 - Graph of results from Mars Experiment

From Figure 3 you can clearly see that the lowest value for closest approach was when the angle was 44 degrees. At this angle the probe came to a closest approach of 65,700 km of Mars. A geostationary orbit around mars is at a distance of 17,000 km[1], therefore a value of 65,700 is very respectable for a flypast. From the graph you can see, that even a few degrees difference in launch angle results in a huge distance from Mars. This suggests that the launch window is extremely narrow. It is also highly likely that if a probe is to be launched to Mars, that it performs several corrective manoeuvres along it's journey to correct its trajectory in order to get itself as close as possible to Mars to perform a flypast. Corrective burns will also be necessary since Mars' orbit is elliptical, not circular as was the assumption for calculating the Hohmann Transfer.

Asteroids

The Asteroid simulation was run for four years, with a probability asteroid creation of 0.1 (10%). The experimental log is quite long so I have included a snapshot.

[1] https://en.wikipedia.org/wiki/Areostationary_orbit

```
asteroidExperimentLog - Notepad
File Edit Format View Help
New Experiment Started for 4 Years!
An asteroid was randomly created with a position of [82932282695 226645839144] and a velocity of [-41680  38067]. There are now 1
An asteroid was randomly created with a position of [259186656512 -18984918637] and a velocity of [11688 14229]. There are now 2
An asteroid was randomly created with a position of [120792738919 182665783735] and a velocity of [  -218 -25403]. There are now 3
An asteroid was randomly created with a position of [-172292425707 -243788446855] and a velocity of [-1364 -3109]. There are now 4
An asteroid was randomly created with a position of [189595070797 -299360109604] and a velocity of [-31777   8189]. There are now 5
An asteroid was randomly created with a position of [84441864898 -10012367450] and a velocity of [ -952 -3852]. There are now 6
An asteroid was randomly created with a position of [214134782536 -322505991598] and a velocity of [39383   914]. There are now 7
An asteroid was randomly created with a position of [91190180539 -146998217541] and a velocity of [ -1445 -21755]. There are now 8
An asteroid was randomly created with a position of [-167287538584 35829711956] and a velocity of [-34235  35649]. There are now 9
An asteroid was randomly created with a position of [-226565972012 211903659795] and a velocity of [38983 24863]. There are now 10
An asteroid was randomly created with a position of [-1252058218269 615341493155] and a velocity of [34712 33603]. There are now 11
An asteroid was randomly created with a position of [-919364782123 816155538823] and a velocity of [32672 21748]. There are now 12
An asteroid was randomly created with a position of [-811456257933 -343128382717] and a velocity of [ 10098 -21410]. There are now 13
An asteroid was randomly created with a position of [-887433154881 494702812101] and a velocity of [-17114   8800]. There are now 14
An asteroid was randomly created with a position of [-709522272384 859978589504] and a velocity of [  4823 -35127]. There are now 15
An asteroid was randomly created with a position of [-1840938373230 1966139484826] and a velocity of [ 6895 47469]. There are now 16
An asteroid was randomly created with a position of [407657109974 -257052635296] and a velocity of [31093  7980]. There are now 17
There was a close encouter with an asteroid at time: 12300000 and the asteroid had a mass of 2860000000000000000000. The asteroid was 8144144999.29m  from earth
There was a close encouter with an asteroid at time: 12400000 and the asteroid had a mass of 2860000000000000000000. The asteroid was 6925091180.55m  from earth
There was a close encouter with an asteroid at time: 12500000 and the asteroid had a mass of 2860000000000000000000. The asteroid was 9607813194.08m  from earth
An asteroid was randomly created with a position of [-2235915953304 -951315978188] and a velocity of [-11602  -4609]. There are now 18
An asteroid was randomly created with a position of [172120773965 -2265534735609] and a velocity of [-27303 -10993]. There are now 19
An asteroid was randomly created with a position of [779433516946 3100833484437] and a velocity of [-22982  30006]. There are now 20
An asteroid was randomly created with a position of [-2735599966021 2677896670100] and a velocity of [33177 -5354]. There are now 21
An asteroid was randomly created with a position of [-3582388351147 3621536287428] and a velocity of [ 27162 -26829]. There are now 22
An asteroid was randomly created with a position of [1528736122020 2163306373720] and a velocity of [ 21153 -16772]. There are now 23
An asteroid was randomly created with a position of [112358369094 4152379521712] and a velocity of [22413  8498]. There are now 24
An asteroid was randomly created with a position of [-3874904046951 -503026732022] and a velocity of [-14561 -12434]. There are now 25
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Figure 4 - Asteroid Experiment Log

The results show that two encounters did occur. The first was an encounter of an asteroid that came to a closest approach of 6.9 million kilometres, and the second had a close encounter of 1.7 million kilometres. In total over the course of the simulation 136 asteroids were created.

Even if we take the closest value of the approach and compare it to the diameter of the Earth which is 12,700 km, this asteroid is still 134 times the radius of the Earth distance away. This means that there is still an extremely low likelihood for an impact to occur.

Due to the inefficiencies of this simulation, it does not provide enough data to allow for a good approximation of the likelihood of impact. The probability that any asteroid will actually hit Earth is the fraction of the asteroid orbits that would lead to an impact over the total number of asteroids. Say if there are one million total asteroids, and one of those leads to an impact, then we can conclude that the odds of the asteroid hitting the Earth are one in a million. But because the simulation struggles with even 100 asteroids at once, it is unfeasible to achieve such a result without modifying the code.

Planet Alignment

I ran the experiment with a range of tolerance angles from 10 degrees to 20 degrees, with 1 degree increments.

```
planetAlignment - Notepad
File Edit Format View Help
The Planets Have Aligned to within 10 degrees after 141.696409778 years.
The Planets Have Aligned to within 11 degrees after 115.547677042 years.
The Planets Have Aligned to within 12 degrees after 115.547677042 years.
The Planets Have Aligned to within 13 degrees after 115.547677042 years.
The Planets Have Aligned to within 14 degrees after 115.547677042 years.
The Planets Have Aligned to within 15 degrees after 26.1582183714 years.
The Planets Have Aligned to within 16 degrees after 26.1550564932 years.
The Planets Have Aligned to within 17 degrees after 26.151894615 years.
The Planets Have Aligned to within 18 degrees after 26.1550564932 years.
The Planets Have Aligned to within 19 degrees after 26.151894615 years.
```

Figure 5 - Log Data from Planet Alignment Experiment

From the results you can clearly see that there are 3 distinct time scales on which alignments happen (142, 115 and 26 years). Although these alignments are uncommon, they do however happen regularly and within reasonable timescales.

Although the idea that a seemingly coincidental alignment of planets can possibly have some sort of influence of doom is false, it is interesting to find that these planetary alignment do happen more frequently than first expected. Also alignments between a few planets are very common and are used by Astronomers to get higher detailed pictures of planets because they are relatively closer to the Earth.

Relative Orbital Periods

Running the simulation experiment yielded the following results written to a log file.

relativeOrbitalPeriods - Notepad

File Edit Format View Help

```
Mercury has period 0.242675105485 Years
Sun has period 0.0 Years
Venus has period 0.613028571429 Years
Mars has period 1.87950207469 Years
Earth has period 1.0 Years
```

Figure 6 - Orbital Periods of planets in Earth Years

Planets - Data Table

Like 5

Dwarf Planets are listed in a separate table below.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
diameter (Earth=1)	0.382	0.949	1	0.532	11.209	9.44	4.007	3.883
diameter (km)	4,878	12,104	12,756	6,787	142,800	120,000	51,118	49,528
mass (Earth=1)	0.055	0.815	1	0.107	318	95	15	17
mean distance from Sun (AU)	0.39	0.72	1	1.52	5.20	9.54	19.18	30.06
orbital period (Earth years)	0.24	0.62	1	1.88	11.86	29.46	84.01	164.8

Figure 7 - Real World Orbital Periods (https://www.windows2universe.org/our_solar_system/planets_table.html)

As you can see the results obtained are as expected. This is not altogether unexpected as I used real world data to initialise the position and velocity of planets in the simulation. However the simulation still yields real world results.

Energy Conservation

I logged the total energy of the simulation by summing the kinetic energy of all the planets and the gravitational potential energy from the sun for each planet. Figure 6 shows how the total energy oscillates over time, however taking the average of this function shows that it remains constant.

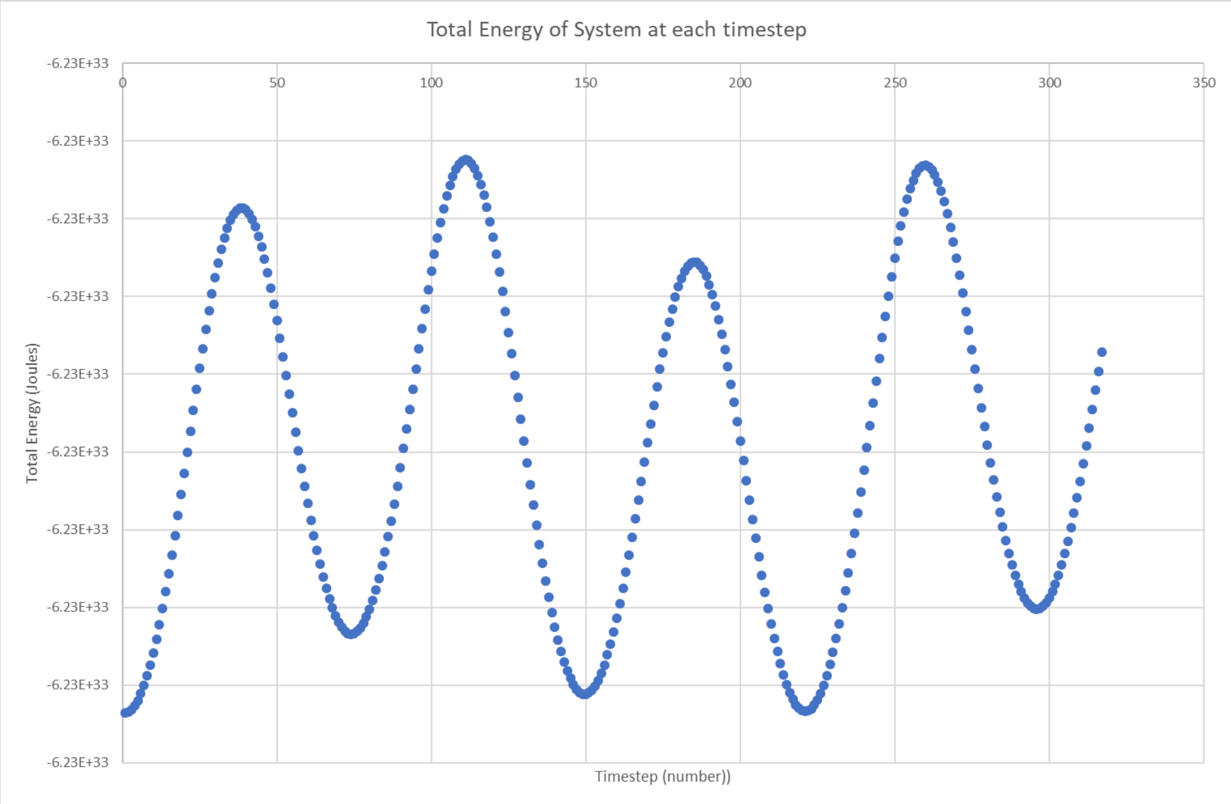


Figure 8 - Total Energy of Simulation at Each Timestep

Discussion

Hohmann Transfer

The Hohmann Transfer can also include a secondary burn to put the probe into a circular orbit around the Sun when it reaches Mars. This manoeuvre could also be added to the simulation to try and simulate a full mission to the planet Mars as well as a manoeuvre to bring the probe back to Earth. The results from the experiment clearly show the minimum distance occurring at 44 degrees. However since the simulation takes some time to computer I decided to use integer values for the angle. Ideally I would use angles with a greater precision in order to fully show that the ideal angle is correct at 44.3 degrees.

Asteroids

Due to the compute time of this simulation running large data sets for this simulation is impractical. The experiment was only run for 4 years with only 139 asteroid created and it still took a few minutes to complete. To really get a accurate idea of how likely an asteroid impact is, would require many million asteroid paths to be simulated. However the data still shows that asteroids can come close to Earth, but the likelihood of any of them hitting the earth are extremely small. This because even smaller if we consider the real world which is three dimensional. Meaning that the likelihood of impact becomes even smaller.

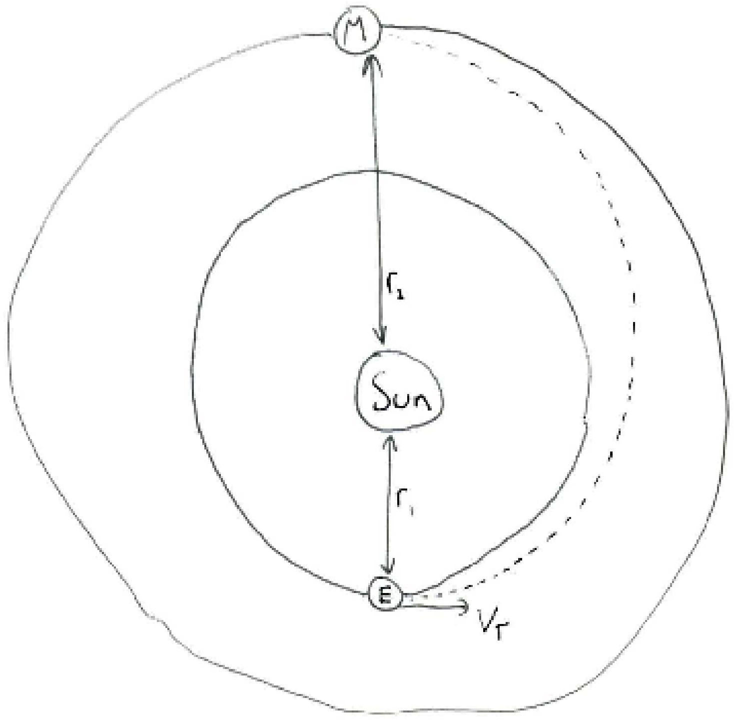
Doomsday

From the data you can see that the greater the tolerance angle, the shorter the time taken for alignment to happen. The results obtained are reliable, however the simulation was only run for once occurrence of alignment. The issue is that alignment may not be evenly distributed. For example you could have 2 alignments happen within a short time, but the next alignment might take a lot longer to occur. To calculate a mean value for when alignments can happen, I would run the simulation until maybe 10 alignments have happened. From this I can then extrapolate the mean value for alignments occurring. However due to the necessary compute time with this simulator, this isn't possible.

Conclusion

In conclusion the simulation of the Solar System proved to be realistic, as the results from the experiments match real world data. Probably the most successful experiment, the Hohmann Transfer, verifying a very close result and showing a very close encounter to Mars of just 65,000 km. Also the simulation showed the same relative orbital times of the planets around the Sun as the real world. Calculating the total energy of the system shows that it remains constant through the experiment, although with some small fluctuations, showing that Conergy was conserved as you would expect in the real world. The asteroid simulation proved the most challenging to get a realistic result, simply due to the rarity of asteroid impacts with the Earth and the length of compute time it would take to see such an occurrence.

Appendix 1



semi-major axis $= \dfrac{r_1 + r_2}{2}$

$\Rightarrow$ From Keplers 3^{rd} Law

$$p^2 = a^3$$
$$p = \sqrt{a^3} = \sqrt{\left(\frac{r_1+r_2}{2}\right)^3}$$

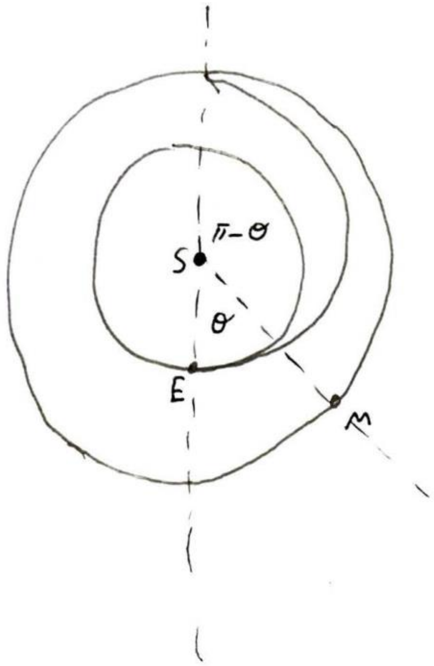
, This is the period of the entire eliptical orbit, we will use half of this to calculate for when we are at aphelion.

The velocity of the transfer is given by:

$$V_T = \sqrt{2GM_s\left(\frac{r_2}{r_1(r_1+r_2)}\right)}$$

, the direction is in the same direction as the Earth.

To calculate the angle at which to launch probe, we take a ratio between the periods of the orbits of Mars and Mars



$$\frac{\frac{1}{2}P}{P_M} = \frac{(\pi - \theta)}{2\pi}$$

where P is the period of our eliptical transfer orbit,