Computational Physics - Project 4

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Contents

1 Results

In figure 1, 2, 3 and 4, the expectation values $< E >$ and $< |M| >$, the specific heat C_V and the susceptibility χ for lattice sizes $L = 20, 40, 60, 80$ can be seen as functions of T close to the critical temperature. It is the easiest to estimate the critical temperature by looking at figure 4, because it is at the temperature where C_V is the highest. In figure 5 and 6 we increased the number of Monte Carlo cycles and decreased the temperature step length and interval to determine the temperature of the maximum of C_V more precisely. In table 1 the measured critical temperatures can be seen. While it is possible to measure T_C even more precisely by increasing the Monte Carlo cycles and decreasing the Temperature step length, it is not possible aim at any precision, due to the quickly increasing calculation time. In figure 3 we can see that near T_C the measured values for the susceptibility are very unstable. At T_C it peak down to a very low value and then increases again to go against zero. In figure 1 is can be observed that at T_C the slope of the curve $< E(T) >$ starts to decrease again. As already mentioned in figure 4 we can clearly see a peak of the graph at T_C. In this graph it can also be seen, that the critical temperature decreases with bigger lattice sizes.

Figure 2 shows, that while $< |M| >$ stays nearly stable at a non-zero value and decreases very slow for temperatures much smaller than T_C, near the critical temperature it decreases quickly and goes against zero. This is a shows that a phase transition is taking place. For a temperature lower than T_C the lattice has a general magnetization. Hence it is ferromagnetic. For temperatures higher than T_C the lattice has nearly no magnetization. Hence...

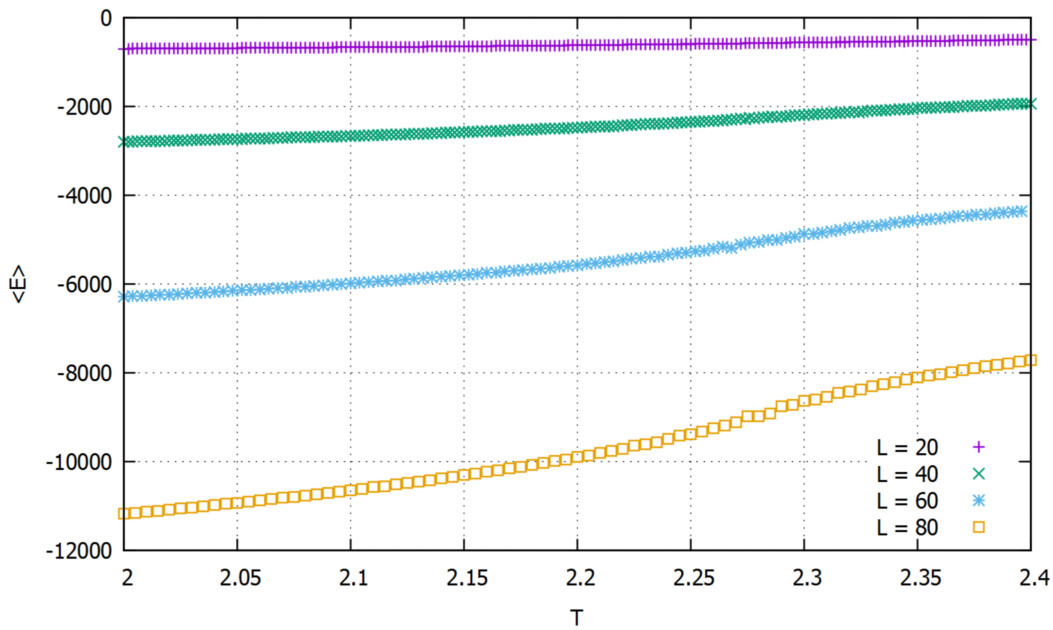

Figure 1: Expectation value of the energy against temperature for different lattice sizes (10^6 Monte Carlo cycles)

L	T_C
20	(2.33 ± 0.01)
40	(2.295 ± 0.007)
60	(2.285 ± 0.005)
80	(2.280 ± 0.007)

Table 1: Critical temperature for different lattice sizes L

The connection between different lattice sizes and the critical temperature is given by the following formula

$$T_C(L) - T_C(L = \infty) = a \cdot L^{-\frac{1}{\nu}} \tag{1}$$

We have $\nu = 1$, so we get

$$T_C(L) = T_C(L = \infty) + a \cdot \frac{1}{L} \tag{2}$$

If we now plot T_C against $\frac{1}{L}$ we should get a line. By measuring the value at which the line crosses the y-axis we can determine $T_C(L = \infty)$. At figure 7 we have plotted our measured values for the critical temperature (which can be seen in table 1) against $\frac{1}{L}$ and signed in a fit line and an error line. The fit line crosses the y-axis at $T_C = 2.263$ while the error line crosses it at $T_C = 2.273$. Thus our value for the critical temperature for an infinite sized lattice is

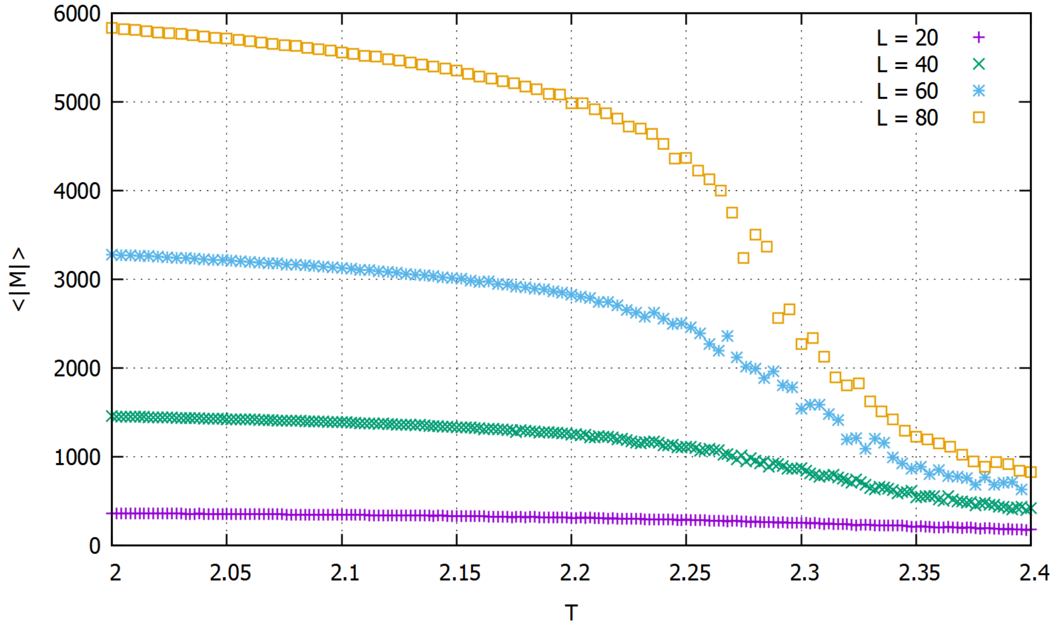

Figure 2: Expectation value of the absolute magnetisation against temperature for different lattice sizes (10^6 Monte Carlo cycles)

$T_C(L=\infty) = (2.26 \pm 0.01)$. The analytical solution $T_C^a = 2.269$ is in the first error interval of this value. Hence our value and the analytical solution match.

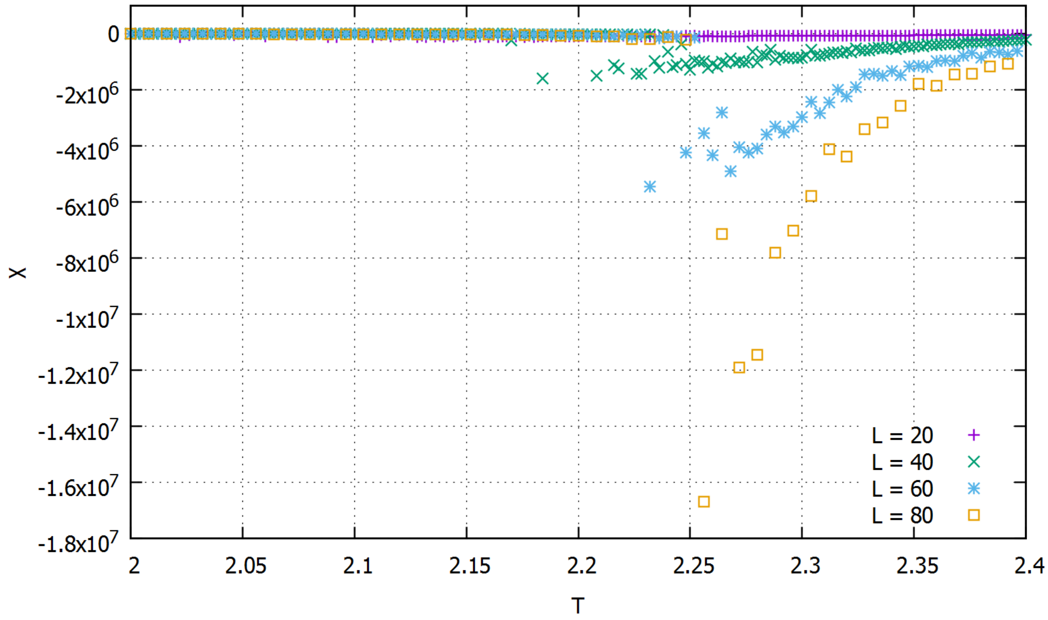

Figure 3: Susceptibility against temperature for different lattice sizes (10^6 Monte Carlo cycles)

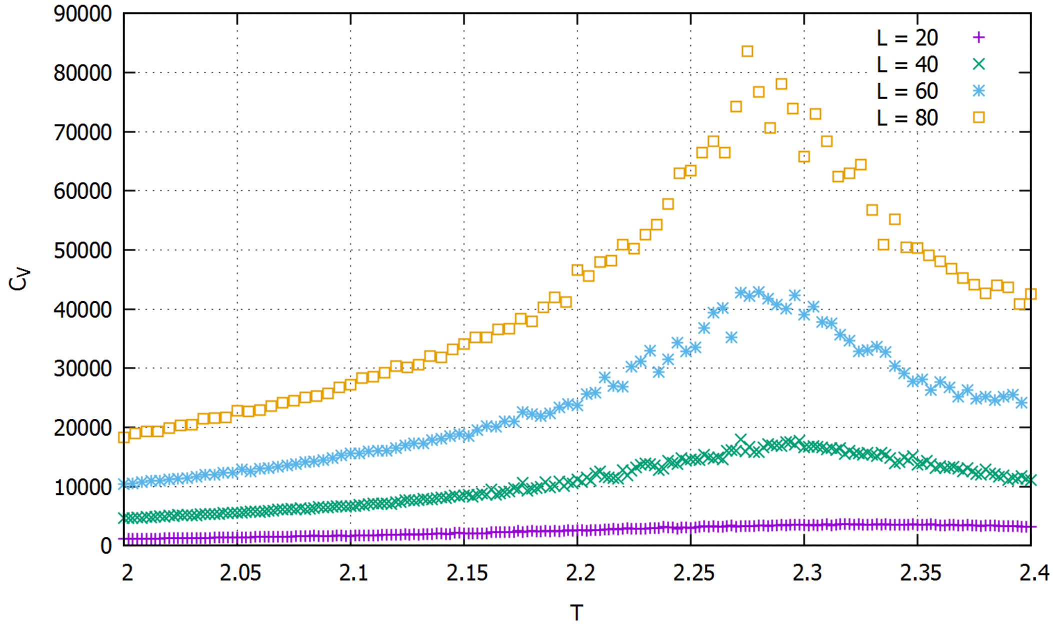

Figure 4: Expectation value of the specific heat against temperature for different lattice sizes (10^6 Monte Carlo cycles)

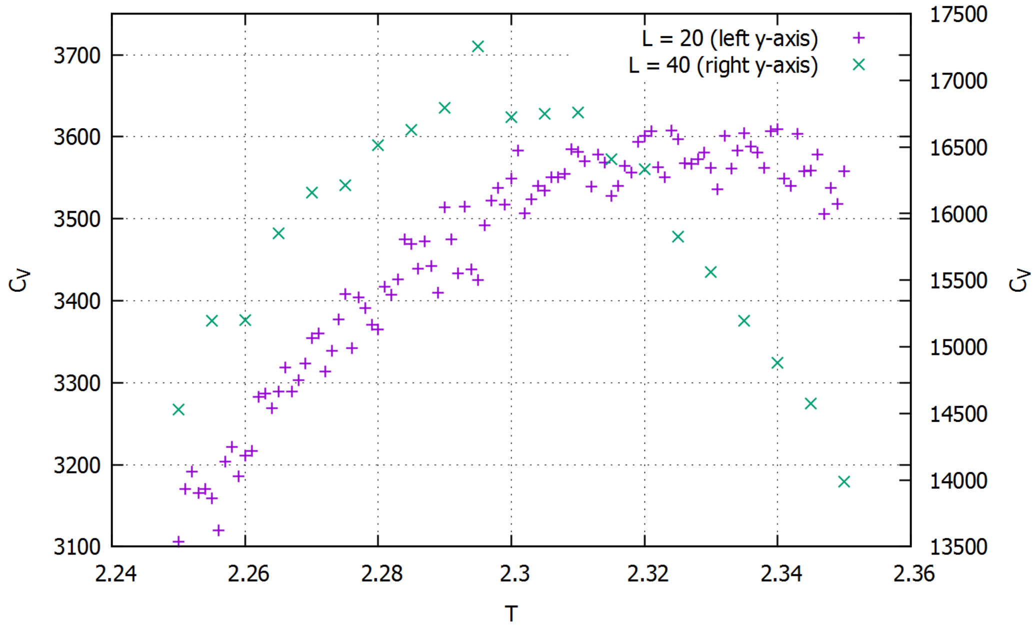

Figure 5: Expectation value of the specific heat against temperature for lattice sizes L = 20 and L = 40 ($5 \cdot 10^6$ Monte Carlo cycles)

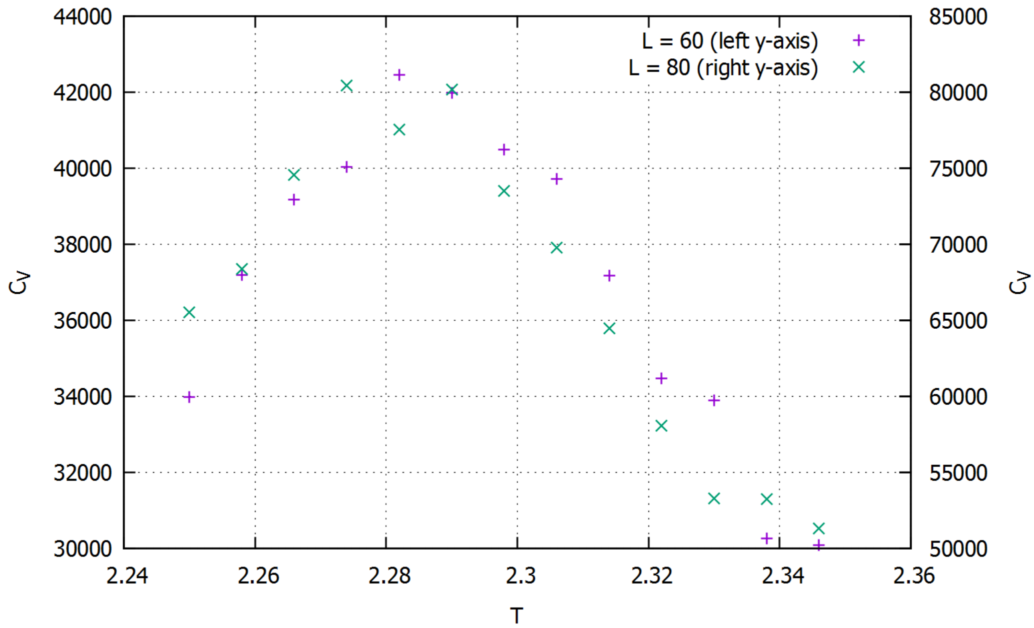

Figure 6: Expectation value of the specific heat against temperature for lattice sizes L = 60 and L = 80 ($5 \cdot 10^6$ Monte Carlo cycles)

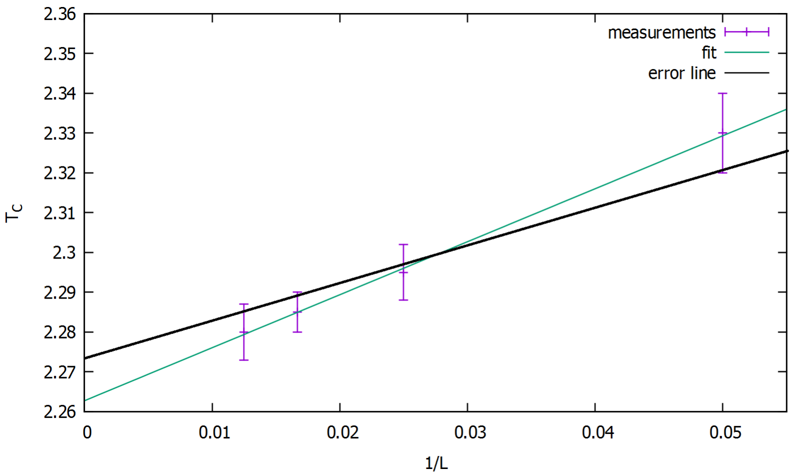

Figure 7: Critical temperature T_C against $\frac{1}{L}$