CS2180: Artificial Intelligence Lab 8: Linear and Logistic Regression

March 2, 2020

Q1) Gradient Descent in 2D: Let $w, x \in \mathbb{R}^2$. Consider the functions $f_1(w) = \frac{1}{2}w(1)^2 + \frac{1}{2}w(2)^2$, $f_2(w) = \frac{10}{2}w(1)^2 + \frac{1}{2}w(2)^2$, $f_3(w) = \frac{1}{2}w(1)^2 + \frac{10}{2}w(2)^2$, $f_4(w) = \frac{1}{2}w(1)^2 + \frac{1}{2}w(2)^2 + 5w(1) - 3w(2) - 2$. For the functions $f_i, i = 1, \ldots, 4$

a) Show the gradient and contour plots. [25Marks]

b) Perform gradient descent to find the minima and show the trajectories of the gradient descent algorithm. Use different step-sizes to demonstrate, one-sided, oscillatory and divergent modes of convergence behavior. [25 Marks]

Q2)[30 Marks] Data in the file *linear* is given in the form $(x_i, y_i)_{i=1}^n$, where $x_i \in \mathbb{R}$, and $y_i \in \mathbb{R}$. Let $w = (w(1), w(0)) \in \mathbb{R}^2$. Learn the optimal w_* for loss function $L(w) = \sum_i L_i(w)$, where $L_i(w) = (w(1)x^i + w(0) - y^i)^2$.

Q3)[20 Marks] Data in the file *logistic* is given in the form $(x_i, y_i)_{i=1}^n$, where $x_i \in \mathbb{R}^2$, and $y_i \in \{-1, +1\}$. Let $w = (w(2), w(1), w(0)) \in \mathbb{R}^3$, $\phi(z) = \frac{1}{1+\exp(-z)}$, and $z_k = y_k w^\top x_k = y_k(w(2)x(2) + w(1)x(1) + w(0))$. Define the loss function to be $L(w) = \sum_{k=1}^n - \ln \phi(z_k)$. The logistic regression update is given by:

$$w_{t+1} = w_t + \alpha \sum_{k=1}^{n} (1 - \phi(z_k)) y_k x_k \tag{1}$$