Automatic Model Selection with Large Language Models for Reasoning

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Abstract

Chain-of-Thought and Program-Aided Language Models represent two distinct reasoning methods, each with its own strengths and weaknesses. We demonstrate that it is possible to combine the best of both worlds by using different models for different problems, employing a large language model (LLM) to perform model selection. Through a theoretical analysis, we discover that the performance improvement is determined by the differences between the combined methods and the success rate of choosing the correct model. On eight reasoning datasets, our proposed approach shows significant improvements. Furthermore, we achieve new state-of-the-art results on GSM8K and SVAMP with accuracies of 96.5% and 93.7%, respectively. Our code is publicly available at https://github.com/XuZhao0/Model-Selection-Reasoning.

1 Introduction

Large language models (LLMs) have made impressive progresses in numerous fields (Devlin et al., 2019; Brown et al., 2020; OpenAI, 2023; Chowdhery et al., 2022; Bubeck et al., 2023; Wei et al., 2022a) and are often powerful enough to solve problems through a single unified method. While convenient, this approach tends to ignore the distinct structures and variations among the problems, which would benefit from using different methods. On the other hand, in human society, individuals with different skill sets excel in various roles, leading to a thriving world as a whole.

In the case of reasoning, Chain-of-Thought (CoT) (Wei et al., 2022b) and Program-Aided Language Models (PAL) (Gao et al., 2022) have emerged as two effective methods that offer different strengths and weaknesses. Essentially, CoT decomposes a reasoning problem into a series of intermediate steps using natural language. In contrast, PAL represents the solution to a reasoning

problem through a Python function. CoT, which relies on natural language, is more general and offers clearer explanations, reducing ambiguity. Meanwhile, PAL, employing a programming language, enjoys guaranteed computation correctness. Intuitively, choosing between CoT and PAL based on their reasoning results to a specific problem would prove beneficial. However, without access to the ground-truth, choosing a better method itself becomes a machine learning problem.

In order to select among multiple results, previous studies have suggested training a ranker (Uesato et al., 2022). While training a dedicated model generally results in improved accuracy, it can also be somewhat cumbersome and entail significant costs. Conversely, large language models (LLMs) have demonstrated good calibration and have been used to assess the accuracy of their own outputs (Guo et al., 2017; Shinn et al., 2023; Xie et al., 2023). Therefore, in this study, we propose utilizing LLMs to perform model selection with few-shot learning. Additionally, we ask LLMs to provide explanations for their choices, ensuring they consider deliberate comparisons in the process.

Through a theoretical analysis, we discover that the effectiveness of the proposed method is mainly influenced by two factors: (1) the significance of the difference between two models, and (2) the probability of selecting the correct model. More specifically, we can attain a higher overall performance when there is a substantial difference between the models being considered and when the correct model is more likely to be chosen. However, we also observe that even without an exceptional model selection, we can still achieve improvement in certain cases. This validates our decision to simply employ an LLM for the purpose of model selection.

We evaluate the proposed method on eight reasoning tasks, with CoT and PAL serving as

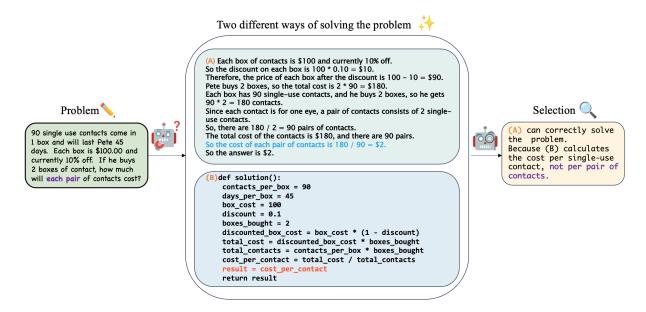


Figure 1: We propose to perform model selection to combine two distinct methods, <code>CoT</code> and <code>PAL</code>. The figure illustrates an example where PAL makes mistakes about crucial information and therefore fails to answer the question correctly. In contrast, CoT manages to correctly answer the same question. Our selection model successfully chooses the correct solution and provides a brief explanation to support its choice.

the baseline methods. On these eight reasoning benchmarks, our method demonstrates substantial performance improvements when employing Codex, ChatGPT, and GPT-4 as the backbone LLMs. Moreover, our approach attains new state-of-the-art accuracies of 96.5% and 93.7% on GSM8K (Cobbe et al., 2021) and SVAMP (Patel et al., 2021), respectively.

2 Automatic Model Selection with Large Language Models

2.1 Method

In this study, we examine reasoning tasks using two baseline models: CoT and PAL. To tackle complex reasoning tasks, CoT employs an LLM to generate several intermediate reasoning steps before arriving at a final conclusion. This approach has proven to be highly effective for various tasks. Due to its use of natural language, the reasoning steps are clearly explained, resulting in less ambiguity. Furthermore, natural language supports a broad range of reasoning that may involve common sense and confidence of the reasoning steps. In contrast, PAL breaks down a reasoning problem into computations using Python. Naturally, employing Python ensures a high level of computational accuracy. However, similar to how Python code can be challenging to comprehend, this approach also introduces ambiguity regarding the meaning of the

program.

Considering the unique strengths and weaknesses of these two approaches, it would be advantageous to select between the two methods based on the problem at hand. Furthermore, model selection could be more precise if we incorporate the reasoning chains from CoT and PAL, as the model could potentially identify errors, gaps, or ambiguities within the chains and determine the correct one.

We employ an LLM to make the model selection. Specifically, we present the LLM with a few examples of choosing between PAL or CoT based on their reasoning chains. In all the included examples, either CoT or PAL is correct, i.e., we do not include instances where both are correct or both are incorrect. In our prompt, we present the choices as two options: (A) or (B), with the results of (A) and (B) being generated by PAL and CoT, respectively. By providing a few model selection examples in the context, an LLM learns to output either (A) or (B) to make a decision. However, there are instances where the model fails to generate (A) or (B); in these cases, we randomly choose a method.

Owing to the in-context learning capabilities of LLMs, we find that they exhibit reasonable accuracy in selecting the appropriate method. Additionally, as shown in our theoretical analysis, a highly accurate method selection is not actually necessary

for the algorithm to perform well. We discover that the performance improvement is jointly determined by the significance of the difference between the two combined methods and the effectiveness of the model selection. Furthermore, there are situations where the model selection might be poor, yet the overall improvement remains substantial. In addition to requesting that the model selection identifies the correct method, we also ask it to provide explanations, which proves to be effective in our experiments. The proposed algorithm is illustrated in Figure 1.

2.2 Theoretical Analysis

In this section, we conduct a theoretical analysis to determine under which condition the proposed method could work (and fail).

Quantifying error rates Let us denote the error rates of the two base methods, m_1 and m_2 , by err_1 and err_2 , respectively. Without loss of generality, let m_1 be a better base method in the overall performance: i.e., $err_1 \leq err_2$. For a given question x, we define ρ_x as the probability of choosing a more accurate method, either m_1 or m_2 , for the given x using the proposed approach. Define $R(x) = p(\operatorname{correct} \mid x, m_2) - p(\operatorname{correct} \mid x, m_1)$ where $p(\operatorname{correct} \mid x, m_i)$ represents the probability of outputting a correct prediction given input x with method m_i . Then we can quantify the final error rate err of the proposed method as follows:

Proposition 1. For any methods m_1 and m_2 with any combining probability function ρ_x ,

$$err = err_1 - \mathbb{E}_x [|R(x)| (\rho_x - \mathbb{1}\{R(x) < 0\})]$$

We refer readers to Appendix B for the full proof. Proposition 1 decomposes the possible improvement (or deterioration) over base methods in terms of R(x) and ρ_x . It quantitatively shows when and how we can expect improvement (or deterioration) based on these two factors. For example, to improve over the best base method m_1 , Proposition 1 suggests us to choose another base method m_2 such that |R(x)| is not too small and ρ_x is high when |R(x)| is large. In other words, it discourages us from choosing too similar methods as base methods, because for similar methods, |R(x)| tends to be small and it is challenging to increase ρ_x even when |R(x)| is large due to the similarity. This provides a theoretical motivation for us to use CoT and PAL, instead of combining CoT with another CoT.

On the accuracy of selection Define ρ to be the overall probability of selecting a better method: $\rho = \mathbb{E}_x[\rho_x]$. Theorem 1 shows that ρ can be much worse than that of a random guess to achieve the improvement over the base methods m_1 and m_2 ; i.e., $\operatorname{err} < \operatorname{err}_1$ and $\operatorname{err}_1 \le \operatorname{err}_2$ can happen with $\rho < 0.5$:

Theorem 1. For any $\epsilon > 0$, there exist data distribution over x, two base methods (m_1, m_2) , and combining probability (ρ_x) such that $\operatorname{err} < \operatorname{err}_1$, $\operatorname{err}_1 < \operatorname{err}_2$, and $\rho < \epsilon$.

We provide a stronger version of Theorem 1 in Appendix A (that implies Theorem 1) and its proof in Appendix B.

Theorem 1 supports our proposition that, despite not training a new model for the selection process and with the in-context learning limited to a few-shot prompt, it is possible to achieve improvement, even if we do not achieve $\rho_x>0.5$ in some instances. This theoretical analysis offers deeper insights into the conditions and strategies for effective performance improvement with the proposed methodology.

3 Experiments

3.1 Setup

Datasets and backbones We conduct experiments on eight datasets that span a range of arithmetic and symbolic reasoning tasks. 7 of these datasets, including GSM8K (Cobbe et al., 2021), SVAMP (Patel et al., 2021), ASDIV (Miao et al., 2020), SingleOP, SingleEQ, AddSub and MultiArith (Koncel-Kedziorski et al., 2016), are about arithmetic reasoning, while Date Understanding (Srivastava et al., 2022) focuses on symbolic reasoning. To comprehensively evaluate the effectiveness of our approach, we employ three LLMs as backbone systems: Codex (code-davinci-002), ChatGPT (gpt-3.5-turbo) and GPT-4 (gpt-4). In our experiments, we always use the same LLM for the two base models and the model selection LLM. ¹.

Prompt design To effectively exploit the incontext learning abilities of the LLMs, we create a set of few-shot examples by manually creating an error in one model's reasoning chain. For arithmetic reasoning tasks, we employ an 8-shot example for Codex and 5-shot example for ChatGPT and GPT-4. For date understanding task, we use 6-shot

¹Codex results are obtained in February and March 2023, ChatGPT in April and May, and GPT-4 in May 2023.

Backbones	Methods	GSM8K	SVAMP	ASDIV	Arithmeti SingleOP	c SingleEQ	AddSub	MultiArith	Symbolic Date
Codex	CoT	64.4	77.6	80.2	92.7	93.5	88.4	95.7	64.5
	PAL	71.5	79.6	79.1	95.4	96.5	91.9	99.7	77.5
	Ours	74.7	82.2	81.6	96.3	96.9	91.6	99.7	79.4
ChatGPT	CoT	80.8	83	89.3	94.8	97.4	90.4	98.7	69.1
	PAL	79.2	80.3	83	90.7	97.6	89.4	96.3	68.3
	Ours	82.6	84.3	89.4	94.8	97.8	90.6	98.7	70.2
GPT-4	CoT	94.6	91.9	92.7	97.2	97.2	93.9	98	90
	PAL	94.0	92.2	90.2	95.2	98.8	94.9	98.5	88.1
	Ours	95.6	93.7	93.5	97.3	98.6	95.7	99	90.5

Table 1: Results comparison (Accuracy %) on 7 arithmetic datasets and 1 symbolic dataset with greedy decoding. We evaluate our methods on three backbone LLMs.

Backbones	Methods	SC@5	SC@10
ChatGPT	CoT	85.4	86.7
	PAL	80.9	82.6
	Ours	88.2 (+2.8)	88.9 (+2.2)
GPT-4	CoT	95.6	95.5
	PAL	94.7	95.4
	Ours	96.5 (+0.9)	96.5 (+1.0)

Table 2: Results comparison (Accuracy %) on GSM8K with the integration of the Self-Consistency (SC). SC@5 and SC@10 represents 5 and 10 sampled paths respectively. The previous state-of-the-art on GSM8K is 95.5, achieved by Zheng et al. (2023).

for all backbones. The used prompts can be found in Appendix C.

Hyperparameters In the solution generation and selection process, we set the temperature at 0 for greedy decoding. Some experiments also use self-consistency (Wang et al., 2022b) to further improve performance. In this context, we set the temperature for CoT at 0.5 and PAL at 0.8 during answer generation. For model selection, the temperature is set to 0 so that we always select the method that has a higher probability of being correct. However, it is worth noting that ChatGPT and GPT-4 do not always follow the few-shot instructions. As such, when these models output "Both" or "Neither" to a task, we randomly select one of the two choices as the final result for simplicity. In our initial experiments, we discovered that our method is highly robust with respect to hyperparameters. For instance, simply having the model selection LLM output the selection result leads to significant improvements, and there is no need to fine-tune the threshold for the selection probability.

3.2 Main Results

The results of our experiments with greedy decoding are shown in Table 1. First, we find that our proposed method effectively and robustly enhances performance on most settings with different datasets of different difficulties and different backbone LLMs, simply by combining two base models. For example, with GPT-4, we achieve an accuracy of 95.6% on GSM8K and 93.7% on SVAMP without self-consistency.

Second, our results show a considerable improvement even when one of the base models performs much worse than the other. For instance, we observe a significant 3.2% improvement over PAL's 71.5% accuracy on GSM8K, even though CoT has a lower accuracy of 64.4%.

Third, our model's general applicability is further underscored by its 1.9% improvement on the symbolic date understanding task when we use Codex, illustrating its applicability not only to mathematical reasoning but also to other reasoning tasks. In fact, on this task, the accuracy difference between two base models is as large as 13%, our proposed method still improves the accuracy from 77.5% to 79.4%.

It is worth noting that the improvement on Codex and GPT-4 is larger than the improvement on Chat-GPT. As we will show in the analysis, this is due to ChatGPT has limited in-context learning capabilities and cannot choose the correct method based on the presented reasoning chains.

Experiments with self-consistency We aim to investigate the relationship between self-consistency and model selection and whether they complement each other. To do this, we run our algorithm 5 or 10 times and use self-consistency to perform majority voting, arriving at the final answer.

Backbones	Metrics	GSM8K	SVAMP	ASDIV	Arithmeti SingleOP	c SingleEQ	AddSub	MultiArith	Symbolic Date
Codex	$\Delta_{\mathrm{UpperBound}}$	10	8.1	6.5	1.6	0.9	2.0	0.3	4.6
	Success Rate	74.8	72.5	63.7	87.9	88	70	92.9	87.8
	Improvement	+3.2	+2.6	+1.4	+0.9	+0.4	-0.3	0	+1.9
ChatGPT	$\Delta_{\mathrm{UpperBound}}$	8.6	6.0	3.4	2.4	1.4	2.5	0.6	9.8
	Success Rate	60.4	66.4	69.8	58	57.1	60.9	75	53.6
	Improvement	+1.8	+1.3	+0.1	0	+0.4	+0.2	0	+1.1
GPT-4	$\Delta_{\mathrm{UpperBound}}$	2.5	3.6	2.2	2.4	0.6	1.3	0.5	1.1
	Success Rate	72.6	68.7	64.2	57	69.2	85.7	100	86.7
	Improvement	+1.0	+1.8	+0.8	+0.1	-0.2	+0.8	+0.5	+0.5

Table 3: We define the following terms: $\Delta_{\rm UpperBound} = {\rm Acc}_{\rm Upper\ Bound} - {\rm Acc}_{m_1}$, where ${\rm Acc}_{\rm Upper\ Bound}$ is the upper bound accuracy where we assume a perfect model selection and m_1 is the stronger one of the two base models. $\Delta_{\rm UpperBound}$ reflects the expected performance difference between the two base models. Success Rate calculates the correct selection rates when either CoT is correct or PAL is correct, i.e., we ignore the cases where both methods are either correct or wrong. Improvement is the performance improvement achieved over the performance of the stronger base model.

As demonstrated in Table 2, we can achieve substantial improvements over self-consistency alone.

With GPT-4, even though both base models already score around 95%, combining them leads to a new state-of-the-art performance on GSM8K at 96.5%. The performance of using 5 samples and 10 samples turn out to be the same with GPT-4. Additionally, using ChatGPT, we attain an accuracy of 88.9% when sampling 10 times.

3.3 Influencing Factors

We have shown that our model performs well across different datasets. To better understand the reasons for the performance improvement on different datasets and backbone LLMs, we present the performance improvement, and the associated influencing factors in Table 3. We can find that there is a high expected performance difference between the two base methods, reflected in the high $\Delta_{\rm Upper\ Bound}$, which means how different the two base models behave across questions. A larger $\Delta_{\text{Upper Bound}}$ indicates a larger room for potential improvement. Specifically, notice that on GSM8K, with ChatGPT, $\Delta_{\text{Upper Bound}}$ is 8.6% although the accuracy of CoT and PAL are very similar (80.8% and 79.2% respectively). Similarly, with GPT-4, $\Delta_{\rm Upper\ Bound}$ is 2.5% while both the accuracy of the two base models are close (94.6% vs 94.0%).

In addition, we find that the success rate of model selection is relatively high with Codex and GPT-4. For example, with GPT-4, the success rate is 72.6% on GSM8K. In contrast, ChatGPT suffers on the success rate, which explains the relatively weaker performance improvement on ChatGPT.

This echoes with the theoretical analysis shown in Section 2.2. Essentially, to achieve better performance, we need a different behavior of model results, i.e., a high |R(x)| and a high success rate ρ , which jointly contribute more substantial improvements. Indeed, the study on $\Delta_{\rm Upper\ Bound}$ and success rate explains the significant performance improvement on these datasets.

4 Analysis

In this section, we provide a few analysis to see when and how the method works.

4.1 Combination between Similar Methods

Backbone m_1	ChatGPT CoT		GPT-4 CoT		
m_2	PAL	CoT'	PAL	CCoT	
Acc_{m_1}	80.8	80.8	94.6	94.6	
Acc_{m_2}	79.2	79.2	94.0	95.1	
Ours	82.6	80.8	95.6	95.2	
Improvement	(+1.8)	(+0)	(+1.0)	(+0.1)	
$\Delta_{\mathrm{Upper\ Bound}}$	8.6	7.5	2.5	1.7	
Success Rate	60.4	52.2	72.6	58.8	

Table 4: Other model combinations results on GSM8K. CoT' denotes the base CoT model when the temperature is set to 0.1. CCoT denotes ComplexCoT (Fu et al., 2022).

We choose CoT and PAL as our two base models due to the motivation of combining different strengths of distinct models. We conduct experiments to examine whether the performance improves when we combine two similar base models. We use two variants of CoT in this experiment: CoT' where we set the temperature at 0.1,

ComplexCoT (Fu et al., 2022) where we use more complex examples in the prompt. Both of these methods' accuracies are similar or higher than the accuracy of PAL. The results are shown in Table 4. We have the following observations:

- In our experiments, we found that model selection between CoT and CoT', or CoT and ComplexCoT, does not lead to substantial performance gains, even though the accuracy of CoT' and ComplexCoT is on par with PAL. On the other hand, model selection between CoT and PAL results in consistent performance improvements. To understand the reasons behind these outcomes, we further investigate the $\Delta_{\rm Upper\ Bound}$ and the success selection rate.
- $\Delta_{\mathrm{Upper\ Bound}}$ of CoT-PAL exceeds that of other combinations, CoT-CoT' and ComplexCoT-CoT, despite their employing two stronger or equivalent two base models. This observation suggests a larger absolute value of the accuracy difference per question for CoT-PAL. It indicates that CoT and PAL perform more dissimilarly than other model combinations. Theoretically, it represents a larger |R(x)|. As Proposition 1 highlights, without a substantial |R(x)|, it is unlikely to achieve significant performance gain since the improvement component is factored by |R(x)|.
- The success selection rate of CoT-PAL surpasses that of other model combinations. It means that the selection model is more likely to select the correct choice when one solution derives from CoT and the other from PAL. In theory, this higher success rate implies that when |R(x)| is high for a given question x, the success selection probability ρ_x for CoT-PAL is higher than others.

These findings support our initial motivation and hypothesis. We choose CoT and PAL as our two base models because they represent distinct reasoning approaches using natural language and programming language. We expect these models to exhibit a significant difference in errors and accuracies, indicated by a high $\Delta_{\rm Upper\ Bound}.$ Moreover, the considerable disparity in errors for a particular question makes it easier for large language models (LLMs) to select the correct option, leading to a

Backbone	Explanation	Acc	Success Rate	
Codex	w/o exp w/ exp	74.7 74.6	74.9 74.2	
ChatGPT	w/o exp	81.8	55.9	
	w/ exp	82.6	60.4	
GPT-4	w/o exp	95.5	69.9	
	w/ exp	95.6	72.6	

Table 5: Accuracy and success rate with and without explanation on GSM8K.

higher success rate compared to selecting between two similar base models like CoT-CoT'. This holds true even when different prompts or temperature settings are used.

4.2 Ablation Study on Explanation

To perform model selection, we provide explanations in the prompt and also ask the model to generate explanations after making a choice, as we expect to improve the model's selection ability by pointing out why the other choice is incorrect. To investigate the potential role of explanations in enhancing the model's selection capability, we conduct experiments on GSM8K by excluding explanations from the answer.

The results shown in Table 5 reveal that for backbone LLMs with more powerful in-context learning abilities, such as Codex and GPT-4, the inclusion of explanations within the answer seems to have negligible influence on the model's performance. In other words, for Codex and GPT-4, they have a relatively high success selection rate even when no explanation is given. However, for ChatGPT, which possesses relatively low in-context learning capability, the integration of explanations yields a larger improvement in the model's success selection rate. This observation suggests that for backbones with limited in-context learning abilities, explanations can serve as a valuable tool to improve the success selection rate.

4.3 Qualitative Studies

We are also interested in gaining an intuitive understanding of the algorithm, so we conduct some qualitative analysis. As displayed in Table 6, in the first example, both ChatGPT and GPT-4 can accurately identify that PAL is the correct solution. However, ChatGPT provides an incorrect

Question: Courtney attended a concert and reported that the audience was 48 in number. However, Kelly went to the same concert and said that Courtney had made the mistake of overstating the number of people in attendance by 20%. If Kelly was right, how many people really attended the concert?

```
(A)
```

Answer:

Courtney reported 48 people in attendance.

If this number is overstated by 20%, then we need to find the actual number of people.

First, we need to find 20% of 48.

20% of 48 is (20/100) * 48 = 9.6.

Now, we subtract this number from Courtney's reported number to find the actual number of people.

```
48 - 9.6 = 38.4
```

Since we cannot have a fraction of a person, we round the number to the nearest whole number.

So, the actual number of people in attendance was approximately 38.

So the answer is 38.

```
def solution():
    courtney_report = 48
    overstatement_percentage = 0.20
    actual_attendance =
        courtney_report /(1 +
        overstatement_percentage)
    result = int(actual_attendance)
    return result
```

Which of the above two choices can **correctly** answer the math problem?

ChatGPT: (B) can correctly answer the math problem. Because (A) rounds up the result to the nearest whole number, which is not necessary.

GPT-4: (B) can correctly answer the math problem. Because (A) incorrectly calculated the actual number of people by subtracting the overstatement percentage from the reported number instead of dividing it.

Question: How much does it cost you for lunch today at Subway if you pay \$40 for a foot-long fish sub and thrice as much for a six-inch cold-cut combo sub?

```
(A)
```

Answer:

If the foot-long fish sub costs \$40, then the six-inch cold-cut combo sub costs 3 * \$40 = \$120.

So the total cost for both subs is \$40 + \$120 = \$160.

Therefore, it costs \$160 for lunch today at Subway.

```
(B)

def solution():
    cost_footlong_fish = 40
    cost_sixinch_coldcut =
        cost_footlong_fish / 3
    total_cost = cost_footlong_fish +
        cost_sixinch_coldcut
    result = total_cost
    return result
```

Which of the above two choices can **correctly** answer the math problem?

ChatGPT: (A) can correctly answer the math problem. Because (B) calculates the cost of the six-inch cold-cut combo sub as one-third of the cost of the foot-long fish sub instead of three times the cost of the foot-long fish sub.

GPT-4: (A) can correctly answer the math problem. Because (B) incorrectly calculates the cost of the six-inch cold-cut combo sub as one-third of the foot-long fish sub instead of thrice as much.

Table 6: In the first example, ChatGPT performs model selection correctly, but gives a wrong explanation, while GPT-4 gives a correct explanation. The second example shows where both ChatGPT and GPT-4 select correctly and give the correct explanation.

explanation, while GPT-4 offers a valid one. In the second example, which is relatively simpler, both ChatGPT and GPT-4 successfully perform accurate model selection and provide valid explanations. In the first example, we can see that GPT-4 actually possesses exceptional reasoning capabilities and provide reliable explanations.

5 Related Work

Ensemble learning. In machine learning, the strategy of combining various models to address a single problem is exemplified in techniques such as bagging (Breiman, 1996), boosting (Freund and Schapire, 1997; Chen and Guestrin, 2016; Ke et al., 2017), and random forest (Ho, 1995; Breiman, 2001). The underlying idea in these methods is that a group of weak learners can collectively manifest

as a strong learner. This concept has also found its place in deep learning through the use of ensembles. For reasoning, Self-consistency samples diverse reasoning paths and chooses the most consistent answer through majority voting, which can be regarded as ensembling (Wang et al., 2022b). Wang et al. (2022a) takes it a step further by introducing rationale-augmented ensembles, emphasizing rationale sampling in the output space as a critical component for enhancing performance robustly. However, typically, ensembling places equal weights on models through majority voting, which may restrict the full potential of the diverse strengths that each model offers.

Reasoning. The research community has made tremendous progress in the field of reasoning. Apart from CoT (Wei et al., 2022b) and PAL (Gao et al., 2022; Chen et al., 2022), Zhou et al. (2022) simplifies complex problems by breaking them down into a series of subproblems solved in sequence. It has also been shown that prompts with higher complexity lead to superior performance on multi-step reasoning tasks (Fu et al., 2022). Kojima et al. (2022) shows that by simply adding "Let's think step by step" before each answer, LLMs can be competent zero-shot reasoners. Selection-Inference (Creswell et al., 2022) alternates between selection and inference stages, generating causal reasoning steps to the final answer. Kazemi et al. (2022) proposes a backward chaining algorithm that breaks reasoning down into four sub-models. REFINER (Paul et al., 2023) generates intermediate reasoning steps under the scrutiny of a critic model. Xie et al. (2023) delves into the exploration of the reasoning search space and generates reasoning chains by a self-evaluation guided stochastic beam search. Yao et al. (2023) introduces Tree of Thoughts (ToT), enabling exploration over "thoughts". It is important to mention that the contributions of these methods are distinct from our approach, and the progress made by them could potentially be seamlessly integrated using the method we propose.

Self-Evaluation. LLM calibration studies reveal that the probabilistic predictions made by current LLMs closely align with the actual frequencies of token occurrences, hence producing well-calibrated predictions for certain tasks (Guo et al., 2017; Kadavath et al., 2022; Jiang et al., 2020). As LLMs exhibit reliable calibration, there is a growing number of research emphasizing the use of self-evaluation

for verification. Shinn et al. (2023) proposes an approach to provide an agent with dynamic memory and self-reflection capabilities. Madaan et al. (2023) proposes a method to generate outputs from LLMs and refine its previously generated output given its own feedback. Different from these works where the underlying method is the same, in this work, we are interested in combining systems with different strengths and weaknesses.

6 Conclusion

This study introduces a method that effectively combines two distinct base models, CoT and PAL, by enabling an LLM to make the correct selection. We provide a theoretical analysis that supports the feasibility of such a model combination, which is further validated through empirical results. Our method achieves performance improvements across eight datasets with various backbone LLMs. This research represents a significant step towards tapping into the potential of diversity and collaboration among models in LLMs.

In our future work, we aim to expand this framework to other domains. Another intriguing concept involves exploring the use of diverse system instructions to elicit varying model behaviors for model combinations. For example, we could prompt a model to provide reasoning steps in as much detail as possible or as succinctly as possible.

7 Limitation

- We focus on reasoning tasks in this work, but we posit that exploring model selection in other domains also presents a worthwhile direction for future research.
- As our approach solely relies on in-context learning capabilities of LLMs, it is sensitive to the prompts, which is a common issue with in-context learning.
- Our approach combines different models through a selection mechanism, which may not wholly leverage the unique strengths of each model. Future work can explore alternate methodologies for model collaboration.

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A A detailed version of Theorem 1

In this appendix, we provide a detailed version of Theorem 1. Whereas Theorem 1 only states the existence of problem instances, Theorem 2 constructs such instances concretely: i.e., Theorem 2 implies Theorem 1. Define $\mu_x[\mathcal{X}]$ to be the distribution for the expected errors: i.e., an expected error can be written by $\mathbb{E}_{x \sim \mu_x[\mathcal{X}],y,f}[\mathbb{1}[y \neq f(x)]]$ for some function f. Define $S[\mathcal{X}] = \{x \in \mathcal{X} : R(x) < 0\}$. Let us denote $U[\mathcal{X}]$ as the uniform distribution over \mathcal{X} . Given any \mathcal{X} , we write $n = |\mathcal{X}|, T = |S[\mathcal{X}]|, \alpha = T/n$. Assume that $1 \leq T < n$.

Theorem 2. Let $\mu_x[\mathcal{X}] = U[\mathcal{X}]$ and \mathcal{X} be given such that $|\mathcal{X}| < \infty$. Let $\epsilon, \delta \in (0,1)$ and $\lambda \in (0,1]$ such that $\beta = \frac{\epsilon T}{n-T} \in (0,1)$ and $\lambda \geq 1 - \frac{\beta}{\epsilon T}(n-T-\delta)$. Let R and ρ_x be set such that $R(x) = -\epsilon$ for $x \in S[\mathcal{X}]$, $R(x) = \beta$ for $x \in \mathcal{X} \setminus S[\mathcal{X}]$, $(1/T) \sum_{x \in S[\mathcal{X}]} \rho_x = \lambda$, and $(1/(n-T)) \sum_{x \in \mathcal{X} \setminus S[\mathcal{X}]} \rho_x = \epsilon (T/(n-T))(1-\lambda)\beta^{-1} + \delta/(n-T)$. Then, we have that $\operatorname{err} < \operatorname{err}_1$, $\operatorname{err}_1 \leq \operatorname{err}_2$, and

$$\rho = 1 - \alpha + \lambda [2\alpha - 1] + \frac{\delta}{n}.$$

In particular, when $\alpha \geq 0.5$, we have $\rho \to 0$ as $\alpha \to 1$ and $(\delta/(n-T)) \to 0$ (with $\lambda = 1 - \frac{\beta}{T\epsilon}(n-T-\delta)$); when $\alpha < 0.5$, we have $\rho \to 0$ as $\alpha \to 0$ and $(\delta/n) \to 0$ (with $\lambda = 1$).

The proof of Theorem 2 is presented in Appendix B. Theorem 2 shows that the overall success probability of the selection process can be much worse than a random guess to achieve the improvement over the base methods m_1 and m_2 ; i.e., $\operatorname{err} < \operatorname{err}_1$ and $\operatorname{err}_1 \le \operatorname{err}_2$ can happen with $\rho < 0.5$. Indeed, it is possible to have $\rho \to 0$ with the improvement ($\operatorname{err} < \operatorname{err}_1$ and $\operatorname{err}_1 \le \operatorname{err}_2$) when the size of $\mathcal X$ is large: when $\alpha \ge 0.5$, we can choose $\lambda = 1 - \frac{\beta}{T\epsilon}(n-T-\delta)$ with which $\operatorname{err} < \operatorname{err}_1$, $\operatorname{err}_1 \le \operatorname{err}_2$, and $\rho \to 0$ as $\alpha \to 1$ and $(\delta/(n-T)) \to 0$. When $\alpha < 0.5$, we can choose $\lambda = 1$ with which $\operatorname{err} < \operatorname{err}_1$, $\operatorname{err}_1 \le \operatorname{err}_2$, and $\rho \to 0$ as $\alpha \to 0$ and $(\delta/n) \to 0$. This supports our proposition that, despite not training a new model for the selection process and with the in-context learning limited to a few-shot prompt, it is possible to achieve improvement, even if we do not achieve $\rho_x > 0.5$ in some instances.

Theorem 2 also suggests that if the overall performance of two base methods is similar, captured by ϵ , the overall selection process can be weak to achieve some improvement, as long as the success selection probability is relatively high when the two methods have very different expected errors (or accuracies) for a given question. In essence, Theorem 2 suggests a trade-off: we want |R(x)| to be larger when deciding which two base methods m_1 and m_2 to choose, implying that we prefer base methods to perform dissimilarly on \mathcal{X} . On the other hand, if two base methods exhibit a substantial expected accuracy difference, then the selection process needs to be stronger to improve the performance (i.e., ρ needs to be larger). However, if the expected accuracy difference between two base methods is relatively small, increasing the power of the selection process is not that necessary to boost performance.

B Proofs

B.1 Proof of Proposition 1

Proof. Define acc = 1 - err and $acc_i = 1 - err_i$ for $i \in \{1, 2\}$. Since expected error $= \mathbb{E}[\mathbb{I}[\text{incorrect prediction}]] = P(\text{incorrect prediction}) = 1 - P(\text{correct prediction})$, we have that

$$acc_i = p(correct|m_i) = \mathbb{E}_x[p(correct|x, m_i)]$$

where correct represents the event of the correct prediction. Similarly,

$$acc = \mathbb{E}_x \left[\sum_{i=1}^2 p(m_i|x) p(correct|x, m_i) \right]$$

where $p(m_i|x)$ represents the probability of selecting method m_i given x via the proposed method. Thus,

$$acc - acc_1 = \mathbb{E}_x \left[p(m_1|x)p(correct|x, m_1) + p(m_2|x)p(correct|x, m_2) - p(correct|x, m_1) \right]$$

$$= \mathbb{E}_x \left[(p(m_1|x) - 1)p(\operatorname{correct}|x, m_1) + p(m_2|x)p(\operatorname{correct}|x, m_2) \right]$$

= $\mathbb{E}_x \left[(p(m_2|x)p(\operatorname{correct}|x, m_2) - (1 - p(m_1|x))p(\operatorname{correct}|x, m_1) \right]$

Since $1 - p(m_1|x) = p(m_2|x)$,

$$acc - acc_1 = \mathbb{E}_x \left[(p(m_2|x)p(correct|x, m_2) - p(m_2|x)p(correct|x, m_1) \right]$$
$$= \mathbb{E}_x \left[(p(m_2|x)R(x)) \right].$$

Here, we notice that

$$p(m_2|x) = \begin{cases} \rho_x & \text{if } R(x) \ge 0\\ 1 - \rho_x & \text{if } R(x) < 0 \end{cases}$$
$$= \mathbb{1}\{R(x) \ge 0\}\rho_x + \mathbb{1}\{R(x) < 0\}(1 - \rho_x).$$

By plugging this into the above equation,

$$acc - acc_1 = \mathbb{E}_x \left[(\mathbb{1}\{R(x) \ge 0\}\rho_x + \mathbb{1}\{R(x) < 0\}(1 - \rho_x))R(x) \right]$$

= $\mathbb{E}_x [R(x)\mathbb{1}\{R(x) \ge 0\}\rho_x] + \mathbb{E}_x [R(x)\mathbb{1}\{R(x) < 0\}(1 - \rho_x)]$

Since $R(x)\mathbb{1}\{R(x) \ge 0\} = |R(x)|\mathbb{1}\{R(x) \ge 0\}$ and $R(x)\mathbb{1}\{R(x) < 0\} = -|R(x)|\mathbb{1}\{R(x) < 0\}$, we have that

$$\operatorname{acc} - \operatorname{acc}_{1} = \mathbb{E}_{x}[|R(x)|\mathbb{1}\{R(x) \ge 0\}\rho_{x}] - \mathbb{E}_{x}[|R(x)|\mathbb{1}\{R(x) < 0\}(1 - \rho_{x})]$$
$$= \mathbb{E}_{x}[|R(x)|((\mathbb{1}\{R(x) \ge 0\} + \mathbb{1}\{R(x) < 0\})\rho_{x} - \mathbb{1}\{R(x) < 0\})]$$

Since $(\mathbb{1}\{R(x) \ge 0\} + \mathbb{1}\{R(x) < 0\}) = 1$ for any x,

$$acc - acc_1 = \mathbb{E}_x[|R(x)|(\rho_x - \mathbb{1}\{R(x) < 0\})].$$

B.2 Proof of Theorem 2

Proof. We first confirm that R(x) and ρ_x define valid probabilities under the condition of this statement. For R(x), since $\epsilon \in (0,1)$ and $\beta \in (0,1)$, it defines valid probabilities for methods m_1 and m_2 . For ρ_x , since $\lambda \in [0,1]$, it also defines valid probabilities for the case of $x \in S[\mathcal{X}]$. For the case of $x \in \mathcal{X} \setminus S[\mathcal{X}]$, since $\epsilon(T/(n-T))(1-\lambda)\beta^{-1} + \delta/n \geq 0$, we need to show that $\epsilon(T/(n-T))(1-\lambda)\beta^{-1} + \delta/n \leq 1$. That is,

$$\epsilon (T/(n-T))(1-\lambda)\beta^{-1} + \delta/(n-T) \le 1$$

$$\iff \epsilon T(1-\lambda)\beta^{-1} \le n - T - \delta$$

$$\iff 1 - \frac{\beta(n-T-\delta)}{\epsilon T} \le \lambda,$$

which is satisfied by the condition on λ that $\lambda \geq 1 - \frac{\beta}{\epsilon T}(n - T - \delta)$. Thus, the condition on ρ_x defines the valid probabilities for both cases of $x \in S[\mathcal{X}]$ and $x \in \mathcal{X} \setminus S[\mathcal{X}]$.

We now show that $err < err_1$. Invoking Proposition 1,

$$err = err_1 - \mathbb{E}_x [|R(x)| (\rho_x - \mathbb{1}\{R(x) < 0\})].$$

Thus, we have err $< \operatorname{err}_1$ if $\mathbb{E}_x \left[|R(x)| \left(\rho_x - \mathbb{1} \{ R(x) < 0 \} \right) \right] > 0$. This condition can be rewritten as

$$\mathbb{E}_{x} [|R(x)| (\rho_{x} - \mathbb{1}\{R(x) < 0\})] > 0$$

$$\iff \frac{1}{n} \sum_{x \in \mathcal{X}} [|R(x)| (\rho_{x} - \mathbb{1}\{R(x) < 0\})] > 0$$

$$\iff \sum_{x \in \mathcal{X} \setminus S[\mathcal{X}]} |R(x)| \rho_x + \sum_{x \in S[\mathcal{X}]} |R(x)| \rho_x > \sum_{x \in S[\mathcal{X}]} |R(x)|$$

$$\iff \sum_{x \in \mathcal{X} \setminus S[\mathcal{X}]} |R(x)| \rho_x + \epsilon T\lambda > \epsilon T$$

$$\iff \beta \sum_{x \in \mathcal{X} \setminus S[\mathcal{X}]} \rho_x > \epsilon T - \epsilon T\lambda = \epsilon T(1 - \lambda)$$

$$\iff \sum_{x \in \mathcal{X} \setminus S[\mathcal{X}]} \rho_x > \frac{\epsilon T(1 - \lambda)}{\beta}$$

This is satisfied by the condition on ρ that $(1/(n-T))\sum_{x\in\mathcal{X}\setminus S[\mathcal{X}]}\rho_x=\epsilon(T/(n-T))(1-\lambda)\beta^{-1}+\delta/(n-T)$ for some $\delta>0$: i.e.,

$$\sum_{x \in \mathcal{X} \setminus S[\mathcal{X}]} \rho_x = \frac{\epsilon T(1-\lambda)}{\beta} + \delta.$$

Therefore, we have that $err < err_1$.

We now show that $\operatorname{err}_1 \leq \operatorname{err}_2$. Similarly to the proof of Proposition 1, we define $\operatorname{acc}_i = 1 - \operatorname{err}_i$ for $i \in \{1, 2\}$. Then, the inequality $\operatorname{err}_1 \leq \operatorname{err}_2$ holds if $\operatorname{acc}_1 \geq \operatorname{acc}_2$. By using correct to represent the event of the correct prediction, this condition can be rewritten as

$$acc_1 \ge acc_2$$

$$\iff \sum_{x \in \mathcal{X}} p(\operatorname{correct}|x, m_1)] \ge \sum_{x \in \mathcal{X}} p(\operatorname{correct}|x, m_2)]$$

$$\iff 0 \ge \sum_{x \in \mathcal{X}} R(x) = (n - T)\beta - \epsilon T$$

$$\iff \frac{\epsilon T}{n - T} \ge \beta$$

This is satisfied by $\beta = \frac{\epsilon T}{n-T}$. Thus, we have that $\operatorname{err}_1 \leq \operatorname{err}_2$. Using these, we now compute the ρ as

$$\begin{split} \rho &= \mathbb{E}_{x \sim \mu_x[\mathcal{X}]}[\rho_x] \\ &= \frac{1}{n} \sum_{x \in \mathcal{X} \backslash S[\mathcal{X}]} \rho_x + \frac{1}{n} \sum_{x \in S[\mathcal{X}]} \rho_x \\ &= \frac{1}{n} \left(\frac{\epsilon T(1-\lambda)}{\beta} + \delta \right) + \alpha \lambda \\ &= \frac{\epsilon T(1-\lambda)}{\beta n} + \alpha \lambda + \frac{\delta}{n} \\ &= \frac{\epsilon T(1-\lambda)(n-T)}{\epsilon T n} + \alpha \lambda + \frac{\delta}{n} \\ &= (1-\lambda) - \alpha(1-\lambda) + \alpha \lambda + \frac{\delta}{n} \\ &= 1 - \alpha + \lambda [2\alpha - 1] + \frac{\delta}{n}. \end{split}$$

Finally, we prove the asymptotic behavior using this equation. When $\alpha < 0.5$, by setting $\lambda = 1$, we have that

$$\rho = 1 - \alpha + \lambda [2\alpha - 1] + \frac{\delta}{n} = 1 - \alpha + 2\alpha - 1 + \frac{\delta}{n} = \alpha + \frac{\delta}{n} \to 0$$

as $\alpha \to 0$ and $(\delta/n) \to 0$. When $\alpha \ge 0.5$, by setting $\lambda = 1 - \frac{\beta}{T\epsilon}(n - T - \delta)$, we have that

$$\rho = 1 - \alpha + \left(1 - \frac{\beta}{T\epsilon}(n - T - \delta)\right)[2\alpha - 1] + \frac{\delta}{n}$$

$$= 1 - \alpha + 2\alpha - 1 - [2\alpha - 1]\frac{\beta}{T\epsilon}(n - T - \delta) + \frac{\delta}{n}$$
$$= \alpha - [2\alpha - 1]\frac{\beta}{T\epsilon}(n - T - \delta) + \frac{\delta}{n}$$

By defining $Q = \frac{\beta}{T\epsilon}(n - T - \delta)$, we have

$$\rho = \alpha - [2\alpha - 1]Q + \frac{\delta}{n}.$$

Here,

$$Q = \frac{\beta}{T\epsilon}(n-T-\delta) = \frac{\epsilon T}{n-T}\frac{1}{T\epsilon}(n-T-\delta) = \frac{1}{n-T}(n-T-\delta) = 1 - \frac{\delta}{n-T}$$

Thus,

$$\begin{split} \rho &= \alpha - [2\alpha - 1] \left(1 - \frac{\delta}{n - T} \right) + \frac{\delta}{n} \\ &= \alpha - 2\alpha + 1 + \delta \frac{2\alpha - 1}{n - T} + \frac{\delta}{n} \\ &= 1 - \alpha + \delta \left(\frac{2\alpha - 1}{n - T} + \frac{1}{n} \right) \to 0 \end{split}$$

as $\alpha = T/n \to 1$ and $(\delta/(n-T)) \to 0$: e.g., by setting $\delta = \zeta(n-T)$ and take $\zeta \to 0$, with which $(\delta/(n-T)) = \zeta \to 0$.

C Prompts

We show examples for model selection prompts used on different tasks with different backbones. We only show a few examples for each case. Full prompts can be found in our code.

Math Problem: Olivia has \$23. She bought five bagels for \$3 each. How much money does she have left? **Question**: Which of the following two choices can **correctly** answer the math problem?

```
(A)
def solution():
                                                  Answer:
    money_initial = 23
                                                  Olivia had 23 dollars.
    bagels = 5
                                                  5 bagels for 3 dollars each will be 5 * 3 = 15 dollars.
    bagel_cost = 3
                                                  So she has 23 - 5 = 18 dollars left.
    money_spent = bagels * bagel_cost
                                                  The answer is 18.
    money_left = money_initial -
         money_spent
     result = money_left
     return result
Answer: (A)
```

Table 7: An example of 8-shot model selection prompts used on 7 arithmetic datasets with Codex.

Date Understanding Problem: 2015 is coming in 36 hours. What is the date one week from today in MM/DD/YYYY? **Question**: Which of the following two choices can **correctly** answer the date understanding problem?

```
(A)
                                               (B)
def solution():
    # If 2015 is coming in 36 hours,
                                               If 2015 is coming in 36 hours, then it is coming in 2 days.
         then today is 36 hours before.
                                               2 days before 01/01/2015 is 12/30/2014, so today is
    today = datetime(2015, 1, 1) -
                                               12/30/2014.
         relativedelta(hours=36)
                                               So one week from today will be 01/06/2015.
    # One week from today,
                                               So the answer is 01/06/2015.
    one_week_from_today = today +
         relativedelta(weeks=1)
    # The answer formatted with %m/%d
         /%Y is
    result = one_week_from_today.
         strftime('%m/%d/%Y')
    return result
Answer: (A)
```

Table 8: An example of 6-shot model selection prompts used on Date Understanding task with Codex.

There are two choices to the same math problem. One uses natural language to answer the question, while the other uses Python program to answer it. Either of them can correctly answer the math problem. You need to identify which choice can correctly answer the math problem. Here is one example how to do it,

Math problem: Olivia has \$23. She bought five bagels for \$3 each. How much money does she have left?

```
(A)
                                                  (B)
                                                   def solution():
Answer:
                                                        money_initial = 23
Olivia had 23 dollars.
                                                        bagels = 5
5 bagels for 3 dollars each will be 5 * 3 = 15 dollars.
                                                        bagel_cost = 3
So she has 23 - 15 = 8 dollars left.
                                                        money_spent = bagels + bagel_cost
So the answer is 8.
                                                        money_left = money_initial -
                                                            money_spent
                                                        result = money_left
                                                        return result
```

Which of the above two choices can **correctly** answer the math problem?

(A) can correctly answer the math problem. Because (B) adds the number of bagels to the cost of each bagel instead of multiplying them.

Now it's your turn. Here is another math problem and two choices.

Math Problem: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

```
(A)
                                                 (B)
                                                 def solution():
Answer:
                                                      golf_balls_initial = 58
Michael started with 58 golf balls.
                                                      golf_balls_lost_tuesday = 23
Then after losing 23 on tuesday, he had 58 - 23 = 35.
                                                      golf_balls_lost_wednesday = 2
After losing 2 more, he had 35 + 2 = 37 golf balls.
                                                      golf_balls_left =
So the answer is 37.
                                                          golf_balls_initial - \
                                                           golf_balls_lost_tuesday
                                                               \golf_balls_lost_wednesday
                                                      result = golf_balls_left
                                                      return result
```

Which of the above two choices can **correctly** answer the math problem?

(B) can correctly answer the math problem. Because (A) adds 2 more balls after losing 2 more on Wednesday instead of subtracting them.

Table 9: Two examples of 5-shot model selection prompts used on 7 arithmetic datasets with ChatGPT.

There are two choices to the same date understanding problem. One uses natural language to answer the question, while the other uses Python program to answer it. Either of them can correctly answer the date understanding problem. You need to identify which choice can correctly answer the problem. Here is one example how to do it,

Date Understanding Problem: 2015 is coming in 36 hours. What is the date one week from today in MM/DD/YYYY?

(A)

Answer:

If 2015 is coming in 36 hours, then it is coming in 2 days. And 2 days before 01/01/2015 is 12/30/2014, so today is 12/30/2014.

So one week from today will be 01/05/2015.

So the answer is 01/05/2015.

Which of the above two choices can **correctly** answer the date understanding problem?

(A) can correctly answer the date understanding problem. Because (B) incorrectly calculates the date 36 hours later instead of 36 hours before.

Now it's your turn. Here is another date understanding problem and two choices.

Date Understanding Problem: The first day of 2019 is a Tuesday, and today is the first Monday of 2019. What is the date today in MM/DD/YYYY?

(A)

Answer:

If the first day of 2019 was Tuesday, then 01/01/2019 was a Tuesday.

And today is the first monday, would be 5 days later.

So today is 01/06/2019.

So the answer is 01/06/2019.

```
(B)
```

```
def solution():
    # If the first day of 2019 is a
        Tuesday, and today is the first
            Monday of 2019, then today is
            6 days later.
    today = datetime(2019, 1, 1) +
            relativedelta(days=6)
    # The answer formatted with %m/%d
            /%Y is
    result=today.strftime('%m/%d/%Y')
    return result
```

Which of the above two choices can **correctly** answer the date understanding problem?

(B) can correctly answer the problem. Because (A) missed the fact that there are 6 days between the first day of 2019 and the first Monday of 2019.

Table 10: Two examples of 6-shot model selection prompts used on Date Understanding task with ChatGPT and GPT-4.

There are two choices to the same math problem. One uses natural language to answer the question, while the other uses Python code to answer it. Either of them can correctly answer the math problem. You need to identify which choice can correctly answer the math problem. Here is one example how to do it,

Math problem: There were nine computers in the server room. Five more computers were installed each day, from monday to thursday. How many computers are now in the server room?

```
(A)
                                                   (B)
                                                   def solution():
                                                        computers_initial = 9
There were originally 9 computers.
                                                        computers\_added = 5
For each of 4 days from monday to thursday, 5 more
                                                        computers_total =
computers were added.
                                                            computers_initial +
So 5 * 4 = 20 computers were added.
                                                            computers_added
So there are 9 + 20 = 29 computers now.
                                                        result = computers_total
So the answer is 29.
                                                        return result
```

Which of the above two choices can **correctly** answer the math problem?

(A) can correctly answer the math problem. Because (B) missed the fact that computers were added each day from monday to thursday.

Now it's your turn. Here is another math problem and two choices.

Math Problem: A piece of square paper has a perimeter of 32 centimeters. Nicky's dog, Rocky, tore off 1/4 of the paper. What is the area of the remaining paper?

```
(B)
(A)
                                                      def solution():
Answer:
                                                           perimeter = 32
A square has 4 equal sides.
                                                           fraction\_torn = 1 / 4
The perimeter of the square paper is 32 centimeters.
                                                           area_total = (perimeter / 4) ** 2
So each side of the square is 32 / 4 = 8 centimeters.
                                                           area_remaining = (1 -
The area of the whole square paper is side * side = 8 * 8 =
                                                                fraction_torn) * area_total
64 square centimeters.
                                                           result = area_remaining
Rocky tore off 1/4 of the paper.
                                                           return result
So The area of the remaining paper is 1/4 * 64 = 16 square
centimeters.
So the answer is 16.
```

Which of the above two choices can **correctly** answer the math problem?

(B) can correctly answer the math problem. Because (A) incorrectly calculated the area of the torn-off portion instead of the remaining portion.

Table 11: Two examples of 5-shot model selection prompts used on 7 arithmetic datasets with GPT-4.