Nuclear Momentum Distributions

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Overview

One body momentum distribution

Chance of finding a particle with momentum $[k, k + dk]$ is $n_1(k)k^2 dk$

$$n_1(\vec{k}) = \frac{1}{(2\pi)^3} \int d\vec{r}_1 \int d\vec{r}_1{}' e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_1{}')} \rho_1(\vec{r}_1, \vec{r}_1{}').$$

$\rho_1(\vec{r}_1, \vec{r}_1{}')$ is the one-body non-diagonal density matrix

$$\rho_1(\vec{r}_1, \vec{r}_1{}') = \int \{d\vec{r}_{2-A}\} \Psi_A^*(\vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_A) \Psi_A(\vec{r}_1{}', \vec{r}_2, \vec{r}_3, ..., \vec{r}_A).$$

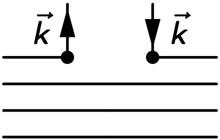
One body momentum distribution

In the second quantization formalism

$$n_1(\vec{k}) = \frac{1}{A} \langle \Psi_A | \psi^\dagger(\vec{k}) \psi(\vec{k}) | \Psi_A \rangle$$

and

$$\rho_1(\vec{r}_1, \vec{r}_1{}') = \frac{1}{A} \langle \Psi_A | \psi^\dagger(\vec{r}_1) \psi(\vec{r}_1{}') | \Psi_A \rangle$$



OBMD for Independent Particle Model

Particles move independent in potential $\rightarrow$ Nuclear wave function = Slater determinant of one-particle wave functions

$$\Psi_A(\vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_A) = \frac{1}{\sqrt{A!}} \sum_{i_1 i_2 \ldots i_A} \varepsilon_{i_1 i_2 \ldots i_A} \; \phi_{i_1}(\vec{r}_1)\phi_{i_2}(\vec{r}_2)...\phi_{i_A}(\vec{r}_A).$$

One obtains for the one-body momentum distribution

$$n_1(\vec{k}) = \frac{1}{A} \sum_i \tilde{\phi}_i^*(\vec{k})\tilde{\phi}_i(\vec{k}).$$

OBDM for Spherical Harmonic Oscillator potential

Time independent Schrodinger equation for the nucleons

$$\left(-\frac{\hbar^2}{2M_N}\nabla^2 + \frac{1}{2}M_N\omega^2 r^2\right)\phi_{nlm}(\vec{r}) = E\phi_{nlm}(\vec{r}).$$

with solutions

$$\phi_{nlm}(\vec{r}) = \left[\frac{2n!}{\Gamma(n+l+\frac{3}{2})}\nu^{l+\frac{3}{2}}\right]^{\frac{1}{2}} r^l e^{-\frac{\nu r^2}{2}} L_n^{l+\frac{1}{2}}(\nu r^2) \times Y_{lm}(\Omega)$$

with

$$\nu \equiv \frac{M_N\omega}{\hbar}.$$

The parameter $\hbar\omega$ can be parameterized

$$\hbar\omega(MeV) = 45A^{-1/3} - 25A^{-2/3}$$

OBDM for Spherical Harmonic Oscillator potential

In momentum space Schrodinger equation has same form, so solutions have same form

$$\begin{aligned}\tilde{\phi}_{nlm}(\vec{k}) &= K_{nl}(k) Y_{lm}(\Omega_k) \\ &= \left[\frac{2n!}{\Gamma(n+l+\frac{3}{2})}\nu'^{l+\frac{3}{2}}\right]^{\frac{1}{2}} k^l e^{-\frac{\nu' k^2}{2}} L_n^{l+\frac{1}{2}}(\nu' k^2) \times Y_{lm}(\Omega_k)\end{aligned}$$

with

$$\nu' \equiv \frac{\hbar}{M_N \omega}.$$

L_n^{α} : generalized Laguerre polynomals
Y_{lm} : Spherical harmonics

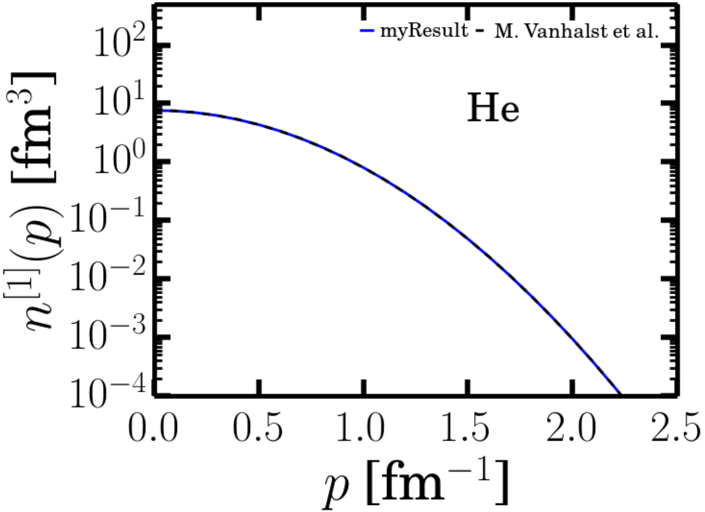
One can now calculate the one-body momentum distribution

$$\tilde{\phi}_{nlm}(\vec{k}) \equiv \langle \vec{k} | nlm \rangle = K_{nl}(k) Y_{lm}(\Omega_k)$$

$$\begin{aligned} n_1(k) &= \frac{2}{A} \sum_{nlm} K_{nl}^2(k) \int d\Omega_k \, Y_{lm}^*(\Omega_k) Y_{lm}(\Omega_k) \\ &= \frac{2}{A} \sum_{nl} (2l+1) K_{nl}^2(k). \end{aligned}$$

The sum goes over all n, l of the occupied states.

Results



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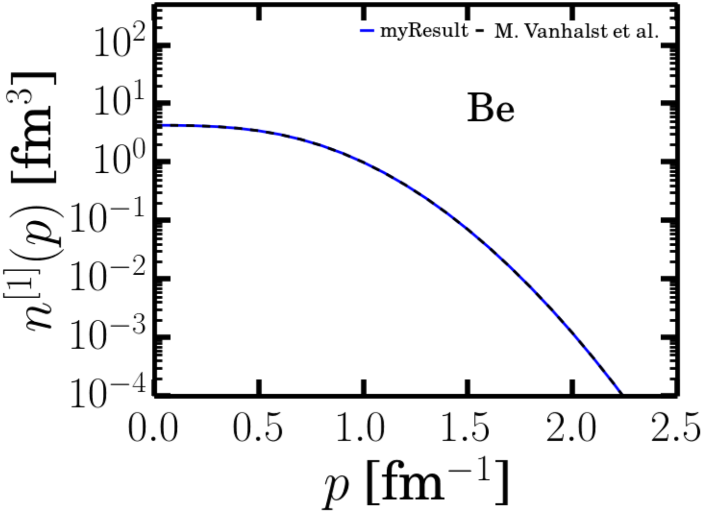

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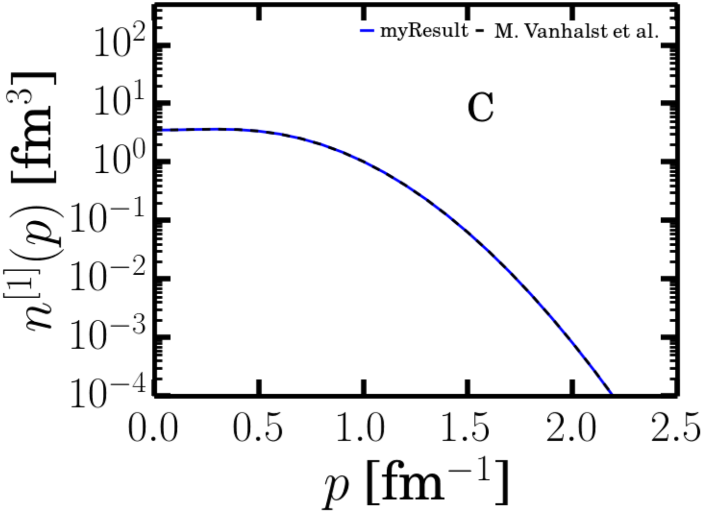

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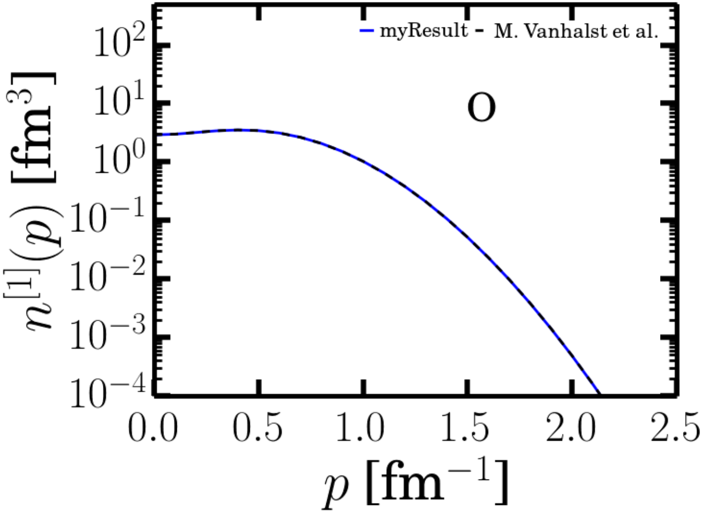

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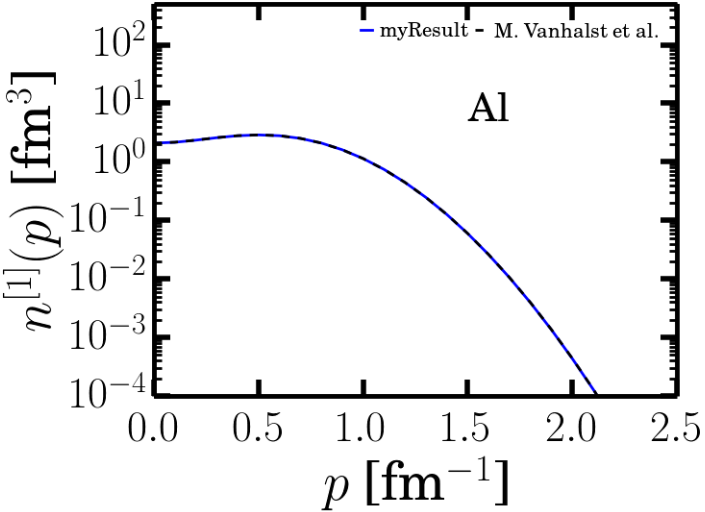

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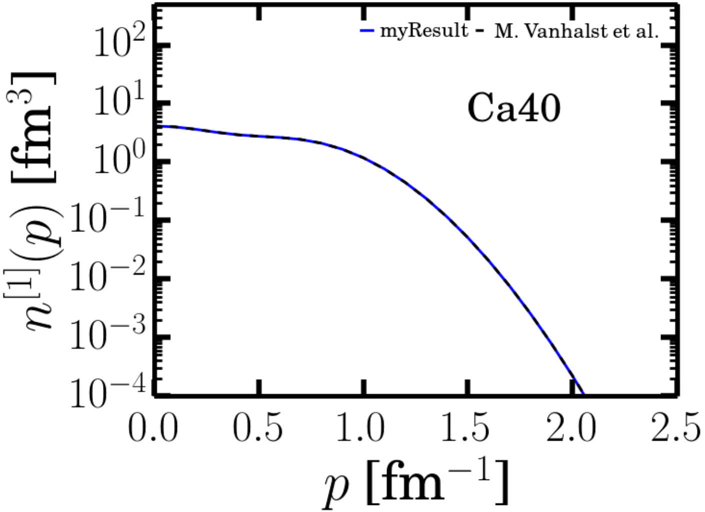

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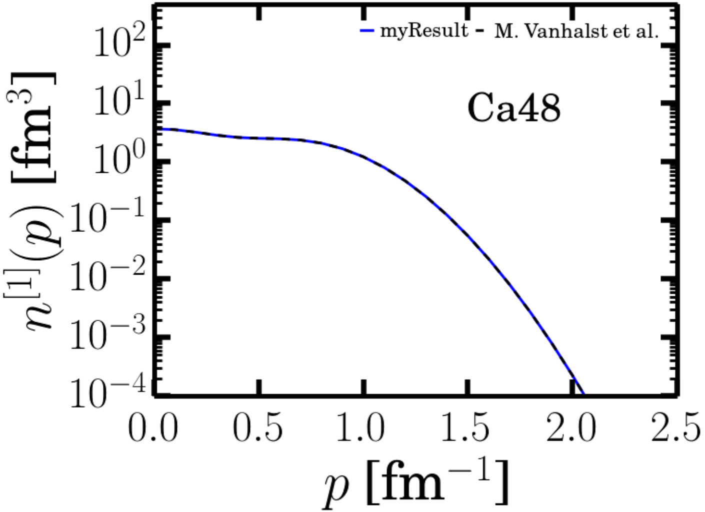

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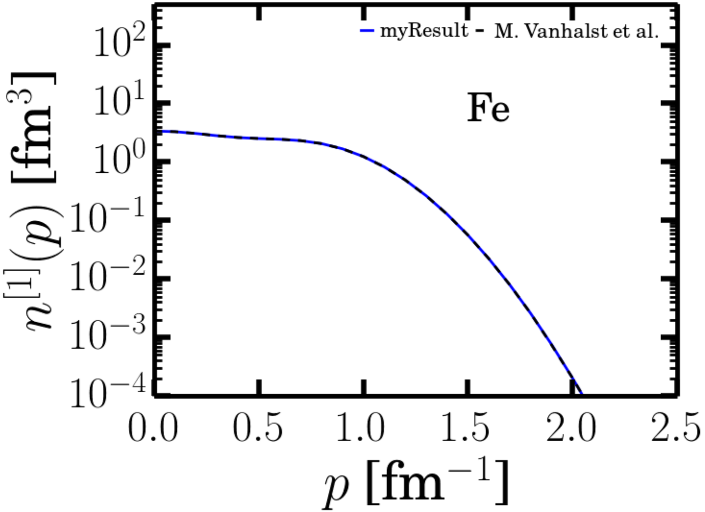

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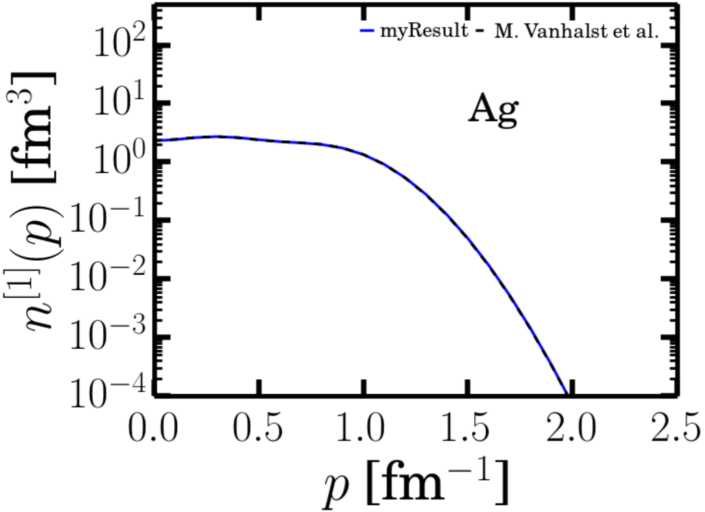

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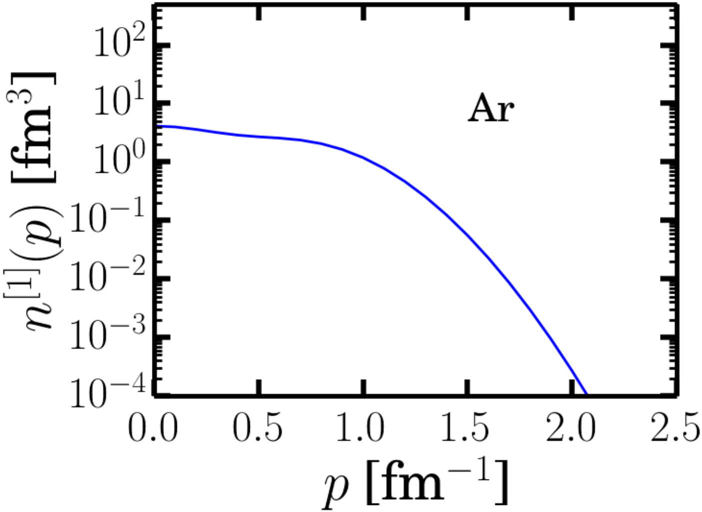

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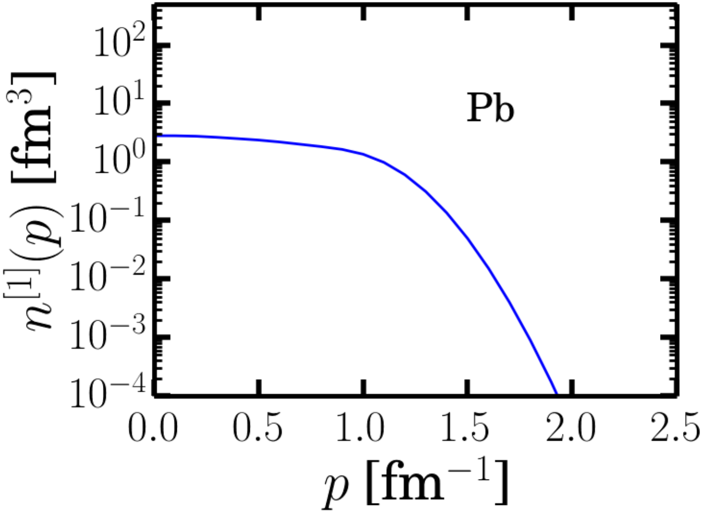
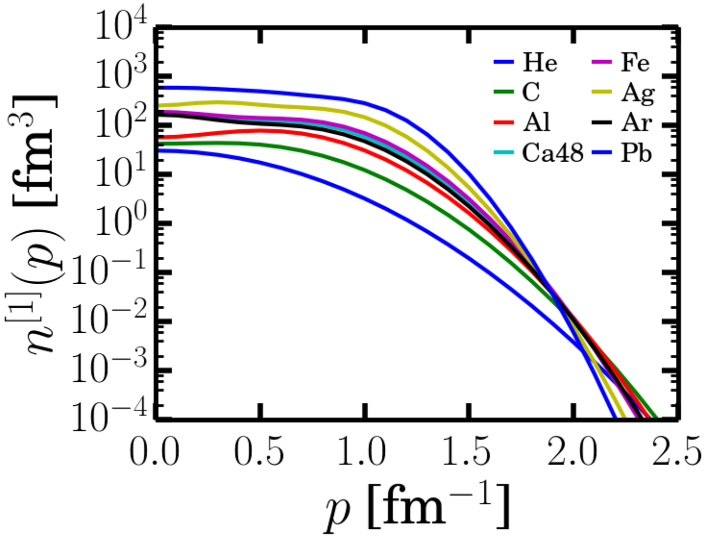


Norm = 1

Results

Normalisation = A



Results: Discussion

- $n_1(k)$ very small for p$\rightarrow \infty$

Outlook

- Two-body momentum distribution
- Try Wood-Saxon potential
- Introduce correlations