Nuclear Momentum Distributions

Jarrick Nys

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1 Definitions

1.1 One-particle momentum distribution

The One-particle momentum distribution gives the chance of finding a particle with a momentum in the interval $[\vec{k}, \vec{k} + d\vec{k}]$. It is given by the following expression

$$n_1(\vec{k}) = \frac{1}{(2\pi)^3} \int d\vec{r}_1 \int d\vec{r}_1' e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_1')} \rho_1(\vec{r}_1, \vec{r}_1') \tag{1}$$

where $\rho_1(\vec{r}_1, \vec{r}_1')$ is the one-body non-diagonal density matrix

$$\rho_1(\vec{r}_1, \vec{r}_1') = \int \{d\vec{r}_{2-A}\} \Psi_A^*(\vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_A) \Psi_A(\vec{r}_1', \vec{r}_2, \vec{r}_3, ..., \vec{r}_A). \tag{2}$$

Here, $\Psi_A(\vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_A)$ is the ground state wave function of the nucleus A and with the notation

$$\{d\vec{r}_{i-A}\} = d\vec{r}_i d\vec{r}_{i+1} ... \vec{r}_A. \tag{3}$$

For $\langle \Psi_A \,|\, \Psi_A \rangle = 1$, one has that

$$\int d\vec{k} n_1(\vec{k}) = 1 \tag{4}$$

In the second quantization of quantum field theory one can express the one-particle momentum distribution as

$$n_1(\vec{k}) = \langle \Psi_A | \, c_k^\dagger c_k \, | \Psi_A \rangle \tag{5}$$

and the one ony-body non-diagonal density matrix as

$$\rho_1(\vec{r}_1, \vec{r}_1') = \int \{d\vec{r}_{2-A}\} \Psi_A^*(\vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_A) \Psi_A(\vec{r}_1', \vec{r}_2, \vec{r}_3, ..., \vec{r}_A) \tag{6}$$

$$= \int \{d\vec{r}_{2-A}\} \langle \Psi_A \,|\, \vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_A \rangle \langle \vec{r}_1', \vec{r}_2, \vec{r}_3, ..., \vec{r}_A \,|\, \Psi_A \rangle \tag{7}$$

$$= \int \{d\vec{r}_{2-A}\} \langle \Psi_A | \, \psi^\dagger(\vec{r}_1)\psi^\dagger(\vec{r}_2) \dots \psi^\dagger(\vec{r}_A) \, |0\rangle \langle 0| \, \psi(\vec{r}_1)\psi(\vec{r}_2) \dots \psi(\vec{r}_A) \, |\Psi_A\rangle \tag{8}$$

1.2 Two-particle momentum distribution

The two-particle momentum distribution gives the chance of finding a particle with momentum in the interval $[\vec{k}_1, \vec{k}_1 + d\vec{k}]$ when there is another particle with a momentum in the interval $[\vec{k}_2, \vec{k}_2 + d\vec{k}]$. It is given by the following expression

$$n(\vec{k}_1, \vec{k}_2) = \frac{1}{(2\pi)^6} \int d\vec{r}_1 \int d\vec{r}_2 \int d\vec{r}_1' \int d\vec{r}_2' \mathrm{e}^{i\vec{k}_1 \cdot (\vec{r}_1 - \vec{r}_1')} \mathrm{e}^{i\vec{k}_2 \cdot (\vec{r}_2 - \vec{r}_2')} \rho_2(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') \tag{9}$$

where $\rho_2(\vec{r}_1, \vec{r}_2, \vec{r}_1', \vec{r}_2')$ is the two-body non-diagonal density matrix

$$\rho_1(\vec{r}_1, \vec{r}_2, \vec{r}_1', \vec{r}_2') = \int \{d\vec{r}_{3-N}\} \Psi_A^*(\vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_A) \Psi_A(\vec{r}_1', \vec{r}_2', \vec{r}_3, ..., \vec{r}_A). \tag{10}$$

One can also define the two-particle momentum distribution in the relative and centre of mass (rcm) coordinates instead of the centre well (cw) coordinates

$$\vec{r}_{12} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2) \tag{11}$$

$$\vec{R}_{12} = \frac{1}{\sqrt{2}} (\vec{r}_1 + \vec{r}_2) \tag{12}$$

$$\vec{p} = \frac{1}{\sqrt{2}} \left(\vec{k}_1 - \vec{k}_2 \right) \tag{13}$$

$$\vec{P} = \frac{1}{\sqrt{2}} \left(\vec{k}_1 + \vec{k}_2 \right) \tag{14}$$

$$n(\vec{p},\vec{P}) = \frac{1}{(2\pi)^6}\int d\vec{r}_{12}\int d\vec{R}_{12}\int d\vec{r}'_{12}\int d\vec{R}'_{12}\mathrm{e}^{i\vec{p}\cdot(\vec{r}_{12}-\vec{r}'_{12})}\mathrm{e}^{i\vec{P}\cdot(\vec{R}_{12}-\vec{R}'_{12})}\rho_2(\vec{r}_{12},\vec{R}_{12};\vec{r}'_{12},\vec{R}'_{12}) \tag{15}$$

where

$$\rho_2(\vec{r}_{12},\vec{R}_{12};\vec{r}'_{12},\vec{R}'_{12}) = \rho_2\left(\vec{r}_1 = \frac{\vec{r}_{12}+\vec{R}_{12}}{\sqrt{2}}, \vec{r}_2 = \frac{-\vec{r}_{12}+\vec{R}_{12}}{\sqrt{2}}, \vec{r}'_1 = \frac{\vec{r}'_{12}+\vec{R}'_{12}}{\sqrt{2}}, \vec{r}'_2 = \frac{-\vec{r}'_{12}+\vec{R}'_{12}}{\sqrt{2}}\right) \tag{16}$$

In the second quantization formalism one can write the two-patricle momemtum distribution as

$$n_2(\vec{k_1},\vec{k_2}) = \langle\Psi_A|\, c^{\dagger}_{k_1}c^{\dagger}_{k_2}c_{k_1}c_{k_2}\,|\Psi_A\rangle \tag{17}$$

2 One-particle momentum distribution for IPM

In an independent particle model the total wave function of the nucleus is a slater determinant of the one-particle wave functions. A nucleon moves independent in a sort of mean field potential created by all the other nucleons.

$$\Psi_A(\vec{r}_1,\vec{r}_2,\vec{r}_3,...,\vec{r}_A) = \frac{1}{\sqrt{A!}}\sum_P (-1)^P \psi_{P1}(\vec{r}_1)\psi_{P2}(\vec{r}_2)...\psi_{PA}(\vec{r}_A) \tag{18}$$

where the sum is over all permutations of the indices of the one-particle wave functions. We also have

$$\int d\vec{r}_i\psi^*_l(\vec{r}_i)\psi_m(\vec{r}_i) = \delta_{lm} \tag{19}$$

The one-particle non-diagonal density matrix becomes

$$\rho_1(\vec{r}_1, \vec{r}_1') = \frac{1}{A!} \sum_P \sum_L (-1)^{P+L} \int d\vec{r}_2 d\vec{r}_3 ... d\vec{r}_A \psi^*_{P_1}(\vec{r}_1) \psi^*_{P_2}(\vec{r}_2) ... \psi^*_{P_A}(\vec{r}_A) \psi_{L_1}(\vec{r}_1') \psi_{L_2}(\vec{r}_2) ... \psi_{L_A}(\vec{r}_A) \tag{20}$$

$$= \frac{1}{A!} \sum_P \sum_L (-1)^{P+L} \psi^*_{P_1}(\vec{r}_1) \psi_{L_1}(\vec{r}_1') \delta_{P_2,L_2} \delta_{P_3,L_3} ... \delta_{P_A,L_A} \tag{21}$$

$$= \sum_i \psi^*_i(\vec{r}_1) \psi_i(\vec{r}_1'). \tag{22}$$

We can plug this into (1)

$$n_1(\vec{k}) = \frac{1}{(2\pi)^3} \sum_i \int d\vec{r}_1 \int d\vec{r}_1' e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_1')} \psi^*_i(\vec{r}_1) \psi_i(\vec{r}_1') \tag{23}$$

$$= \sum_i \psi^*_i(\vec{k}) \psi_i(-\vec{k}) \tag{24}$$