Decision Tree

LING 572

Fei Xia

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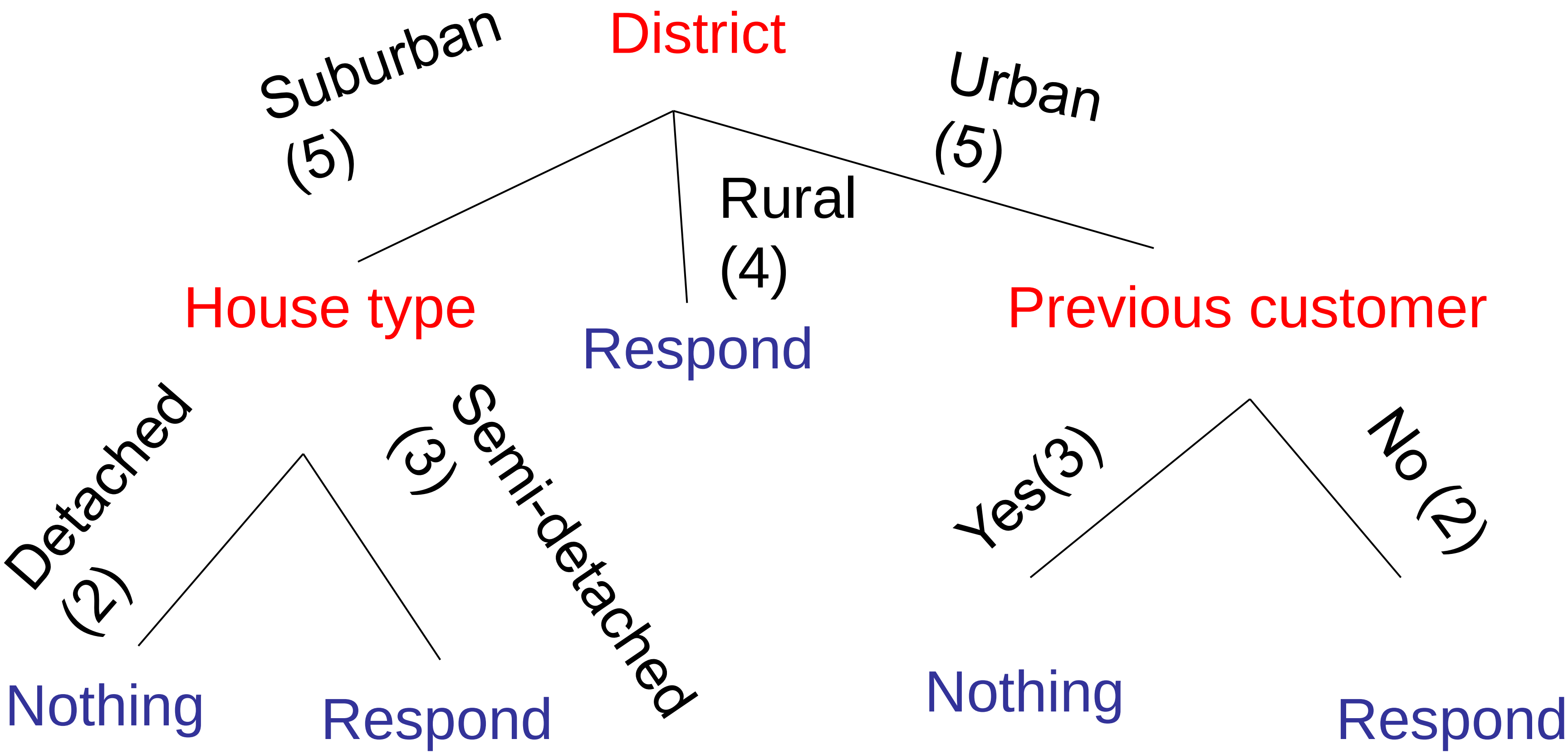
Main idea

- Build a tree ➔ decision tree
  - Each node represents a test
  - Training instances are split at each node
- Greedy algorithm

A classification problem

District	House type	Income	Previous Customer	Outcome (target)
Suburban	Detached	High	No	Nothing
Suburban	Semi-detached	High	Yes	Respond
Rural	Semi-detached	Low	No	Respond
Urban	Detached	Low	Yes	Nothing
…				

Decision tree



Decision tree representation

- Each internal node is a test:
  - Theoretically, a node can test multiple features
  - In most systems, a node tests exactly one feature
- Each branch corresponds to test results
  - A branch corresponds to a feature value or a range of feature values
- Each leaf node assigns
  - a class: decision tree
  - a real value: regression tree

What’s the best decision tree?

- “Best”: We need a bias (e.g., prefer the “smallest” tree):
  - Smallest depth?
  - Fewest nodes?
  - Which trees are the best predictors of unseen data?

- Occam's Razor: we prefer the simplest hypothesis that fits the data.

➔ Find a decision tree that is as small as possible and fits the data

Finding a smallest decision tree

- The space of decision trees is too big for systemic search for a smallest decision tree.

- Solution: greedy algorithm

Basic algorithm: top-down induction

1. Find the “best” feature, A, and assign A as decision feature for the node

2. For each value (or a range of values) of A, create a new branch, and divide up training examples

3. Repeat the process 1-2 until the gain is small enough

Major issues

Q1: Choosing best feature: what quality measure to use?

Q2: Determining when to stop splitting: avoid overfitting

Q3: Handling features with continuous values

Q1: What quality measure

- Any suggestions?

- Information gain
- Gain Ratio
- χ^2
- Mutual information
- ….

Entropy of a training set

- S is a sample of training examples
- Entropy is one way of measuring the impurity of S
- $P(c_i)$ is the proportion of examples in S whose category is c_i.

$$H(S)=-\sum_i p(c_i) \log p(c_i)$$

Information gain

- InfoGain(Y | X): We must transmit Y. How many bits on average would it save us if both ends of the line knew X?

- Definition:

  InfoGain (Y | X) = H(Y) – H(Y|X)

- Also written as InfoGain (Y, X)

Information Gain

- InfoGain(S, A): expected reduction in entropy due to knowing A.

$$
\begin{aligned}
InfoGain(S, A) &= H(S) - H(S \mid A) \\
&= H(S) - \sum_{a} p(A = a) H(S \mid A = a) \\
&= H(S) - \sum_{a \in Values(A)} \frac{|S_a|}{|S|} H(S_a)
\end{aligned}
$$

- Choose the A with the max information gain.

(a.k.a. choose the A with the min average entropy)

An example

S=[9+,5-]
E=0.940

Income

High | Low

[3+,4-] E=0.985

[6+,1-] E=0.592

InfoGain (S, Income)
=0.940-(7/14)*0.985-(7/14)*0.592
=0.151

S=[9+,5-]
E=0.940

PrevCustom

Yes | No

[6+,2-] E=0.811

[3+,3-] E=1.00

InfoGain(S, PrevCustom)
=0.940-(8/14)*0.811-(6/14)*1.0
=0.048

Other quality measures

- Problem of information gain:
  - Information Gain prefers attributes with many values.

- An alternative: Gain Ratio

$$GainRatio(S,A) = \frac{InfoGain(S,A)}{SplitInfo(S,A)}$$

$$SplitInfo(S,A) = H_S(A) = -\sum_{a \in Values(A)} \frac{|S_a|}{|S|} \log_2 \frac{|S_a|}{|S|}$$

Where S_a is subset of S for which A has value a.

Q2: Avoiding overfitting

- Overfitting occurs when our decision tree characterizes too much detail, or noise in our training data.

- Consider error of hypothesis h over
  - Training data: ErrorTrain(h)
  - Entire distribution D of data: ErrorD(h)

- A hypothesis h overfits training data if there is an alternative hypothesis h’, such that
  - ErrorTrain(h) < ErrorTrain(h’), and
  - ErrorD(h) > errorD(h’)

How to avoiding overfitting

- Stop growing the tree earlier. E.g., stop when
  - InfoGain < threshold
  - Size of examples in a node < threshold
  - Depth of the tree > threshold
  - …

- Grow full tree, then post-prune

➔ In practice, both are used. Some people claim that the latter works better than the former.

Post-pruning

- Split data into training and validation sets
- Do until further pruning is harmful:
  - Evaluate impact on validation set of pruning each possible node (plus those below it)
  - Greedily remove the ones that don’t improve the performance on validation set

➔Produces a smaller tree with the best performance

Performance measure

- Accuracy:
  - on validation data
  - K-fold cross validation

- Misclassification cost: Sometimes more accuracy is desired for some classes than others.

- MDL: size(tree) + errors(tree)

Rule post-pruning

- Convert the tree to an equivalent set of rules
- Prune each rule independently of others
- Sort final rules into a desired sequence for use
- Perhaps most frequently used method (e.g., C4.5)

Q3: handling numeric features

- Continuous feature ➔ discrete feature
- Example
  - Original attribute: Temperature = 82.5
  - New attribute: (temperature > 72.3) = t, f

➔ Question: how to choose split points?

Choosing split points for a continuous attribute

- Sort the examples according to the values of the continuous attribute.
- Identify adjacent examples that differ in their target labels and attribute values ➔ a set of candidate split points
- Calculate the gain for each split point and choose the one with the highest gain.

Summary of Major issues

Q1: Choosing best attribute: different quality measures.

Q2: Determining when to stop splitting: stop earlier or post-pruning

Q3: Handling continuous attributes: find the breakpoints

Strengths of decision tree

- Simplicity (conceptual)
- Efficiency at testing time
- Interpretability: Ability to generate understandable rules
- Ability to handle both continuous and discrete attributes.

Weaknesses of decision tree

- Efficiency at training: sorting, calculating gain, etc.
- Theoretical validity: greedy algorithm, no global optimization
- Predication accuracy: trouble with <u>non-rectangular</u> regions
- Stability and robustness
- Sparse data problem: split data at each node.

Hw2

The task

- Class labels: three newsgroups (politics, mideast, and misc)
- Training data: 2700 instances (900 for each class)
- Test data: 300 instances (100 for each class)
- Features: words
- Task:
  - (Q1-Q3) Run Mallet DT learner
  - (Q4-Q5) Build your own DT learner

Q4: build a DT learner

- Each node checks exactly one feature

- Features are all binary; that is, a feature is either present or non-present

  ➔ The DT is a binary tree

- Quality measure: Information gain

Efficiency issue

- To select the best feature, you will need to calculate the info gain for each feature
- Therefore, you will need to calc the counts of (c, f) and (c, not f) for each class label c and each feature f.
- Try to do this efficiently.
- Report “wall clock time” in Tables 2 and 3.
  - When you start a job, write down the time
  - When the job is finished, look at the timestamp of the files
  - Report the difference between the two

Patas usage

- When testing your code, use small data sets and small depth values first.

- If your code runs more than 5 minutes, use condor submit.

- Always monitor your jobs.

Additional slides

Addressing the weaknesses

- Used in classifier ensemble algorithms:
  - Bagging
  - Boosting

- Decision tree stump: one-level DT

Other issues

Q4: Handling training data with missing feature values

Q5: Handing features with different costs

– Ex: features are medical test results

Q6: Dealing with y being a continuous value

Q4: Unknown attribute values

Possible solutions:

- Assume an attribute can take the value “blank”.
- Assign most common value of A among training data at node n.
- Assign most common value of A among training data at node n which have the same target class.
- Assign prob p_i to each possible value v_i of A
  - Assign a fraction (p_i) of example to each descendant in tree
  - This method is used in C4.5.

Q5: Attributes with cost

- Ex: Medical diagnosis (e.g., blood test) has a cost
- Question: how to learn a consistent tree with low expected cost?
- One approach: replace gain by
  - Tan and Schlimmer (1990)

$$\frac{Gain^2(S,A)}{Cost(A)}$$

Q6: Dealing with continuous target attribute ➔ Regression tree

- A variant of decision trees
- Estimation problem: approximate real-valued functions: e.g., the crime rate
- A leaf node is marked with a real value or a linear function: e.g., the mean of the target values of the examples at the node.
- Measure of impurity: e.g., variance, standard deviation, …

Summary of other issues

Q4: Handling training data with missing attribute values: blank value, most common value, or fractional count

Q5: Handing attributes with different costs: use a quality measure that includes the cost factors.

Q6: Dealing with continuous goal attribute: various ways of building regression trees.

Summary

- Basic case:
  - Discrete input attributes
  - Discrete target attribute
  - No missing attribute values
  - Same cost for all tests and all kinds of misclassification.
- Extended cases:
  - Continuous attributes
  - Real-valued target attribute
  - Some examples miss some attribute values
  - Some tests are more expensive than others.

Common algorithms

- ID3
- C4.5
- CART

ID3

- Proposed by Quinlan (so is C4.5)
- Can handle basic cases: discrete attributes, no missing information, etc.
- Information gain as quality measure

C4.5

- An extension of ID3:
  - Several quality measures
  - Incomplete information (missing attribute values)
  - Numerical (continuous) attributes
  - Pruning of decision trees
  - Rule derivation
  - Random mood and batch mood

CART

- CART (classification and regression tree)
- Proposed by Breiman et. al. (1984)
- Constant numerical values in leaves
- Variance as measure of impurity