

Conditional random field

LING 572

Fei Xia

Week 6: 2/11/2010

Highlights

- CRF is a form of undirected graphical model
- Proposed by Lafferty, McCallum and Pereira in 2001
- Used in many NLP tasks: e.g., Named-entity detection
- Types:
 - Linear-chain CRF
 - Skip-chain CRF
 - General CRF

Outline

- Graphical models
- Linear-chain CRF
- Skip-chain CRF

Graphical models

Graphical model

- A graphical model is a probabilistic model for which a graph denotes the conditional independence structure between random variables:
 - Nodes: random variables
 - Edges: dependency relation between random variables
- Types of graphical models:
 - Bayesian network: directed acyclic graph (DAG)
 - Markov random fields: undirected graph

Bayesian network

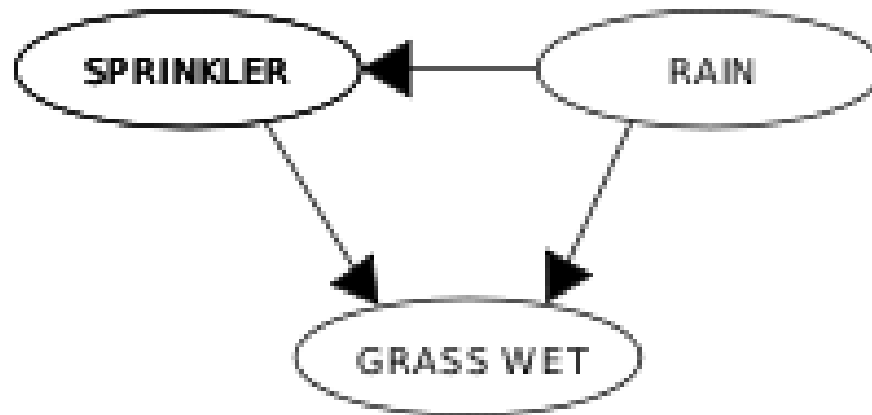
Bayesian network

- Graph: directed acyclic graph (DAG)
 - Nodes: random variables
 - Edges: conditional dependencies
 - Each node X is associated with a probability function $P(X \mid \text{parents}(X))$
- Learning and inference: efficient algorithms exist.

An example

(from http://en.wikipedia.org/wiki/Bayesian_network)

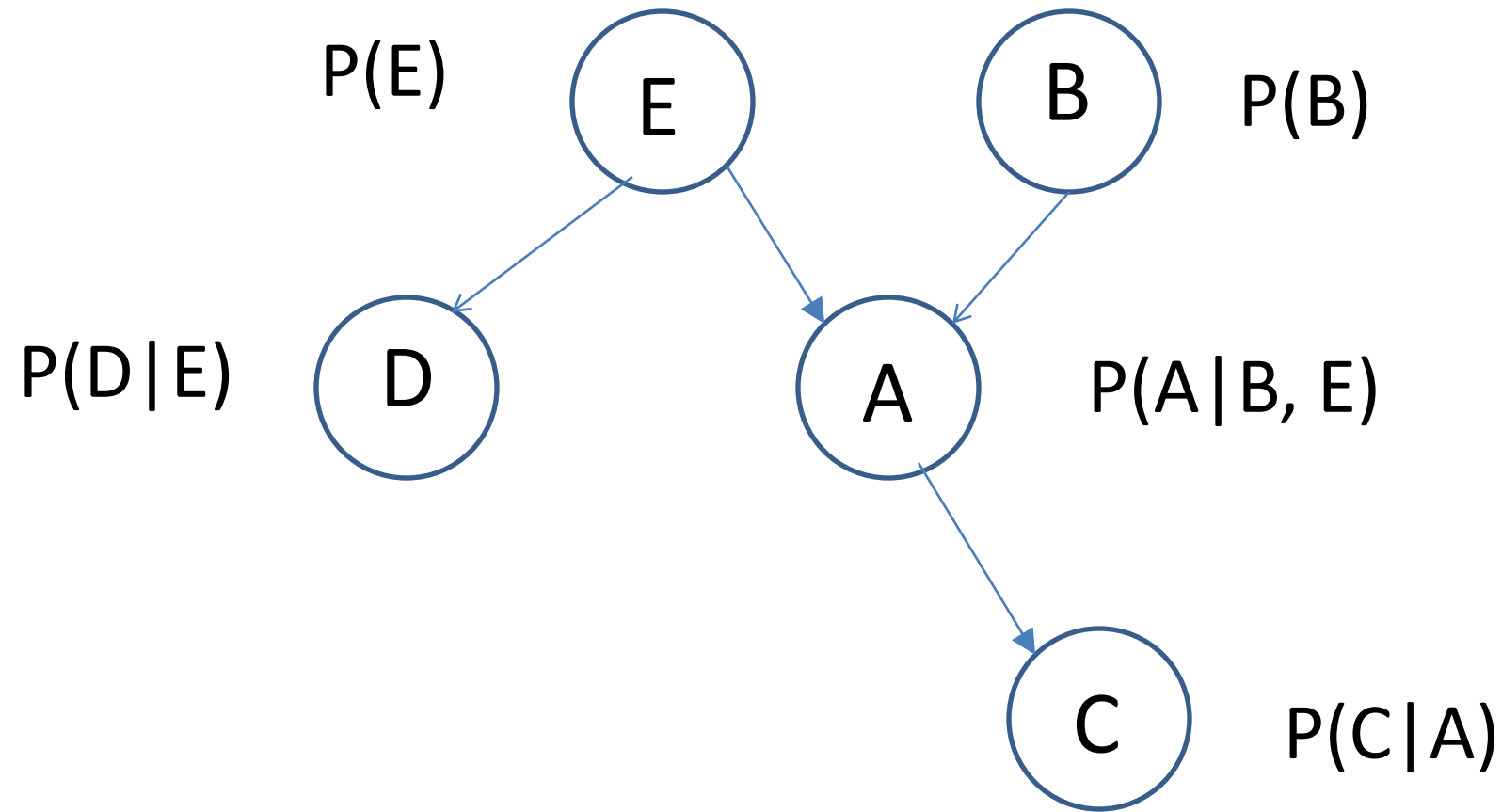
RAIN	SPRINKLER	
	T	F
F	0.4	0.6
T	0.01	0.99



	RAIN	
	T	F
	0.2	0.8

		GRASS WET	
SPRINKLER	RAIN	T	F
F	F	0.0	1.0
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.99	0.01

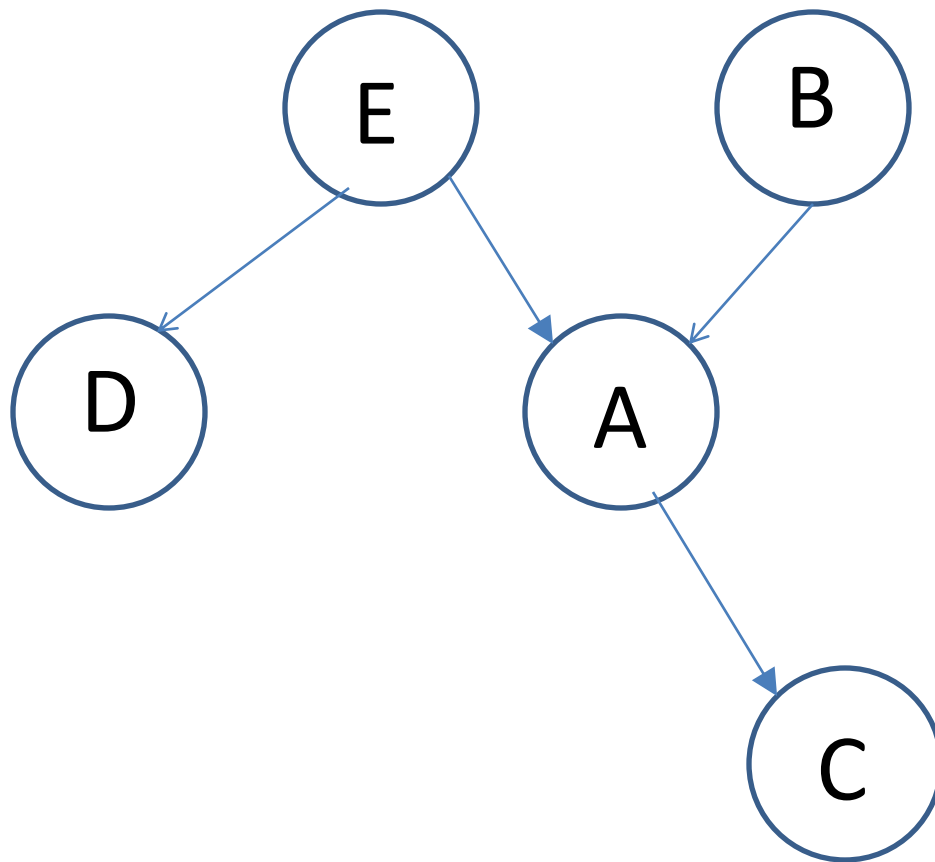
Another example



Bayesian network: properties

Local Markov property: each variable X_i is conditionally independent of its nondecendants given its parents variables.

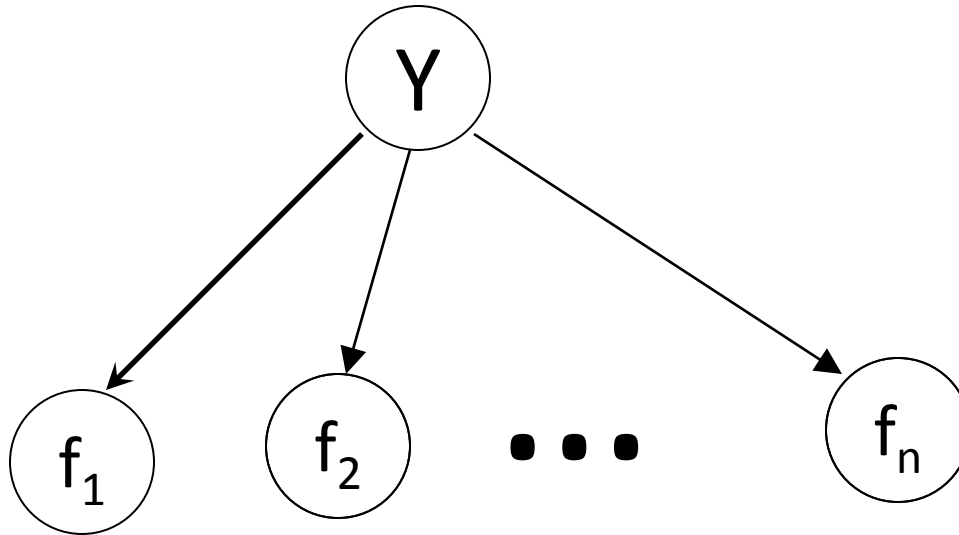
$$\begin{aligned} &P(X_1, \dots, X_n) \\ &= \prod_{i=1}^n P(X_i | X_1, \dots, X_{i-1}) \\ &= \prod_{i=1}^n P(X_i | \textit{parents}(X_i)) \end{aligned}$$



$$P(A, B, C, D, E) = P(B)P(E)P(A|B, E)P(C|A)P(D|E)$$

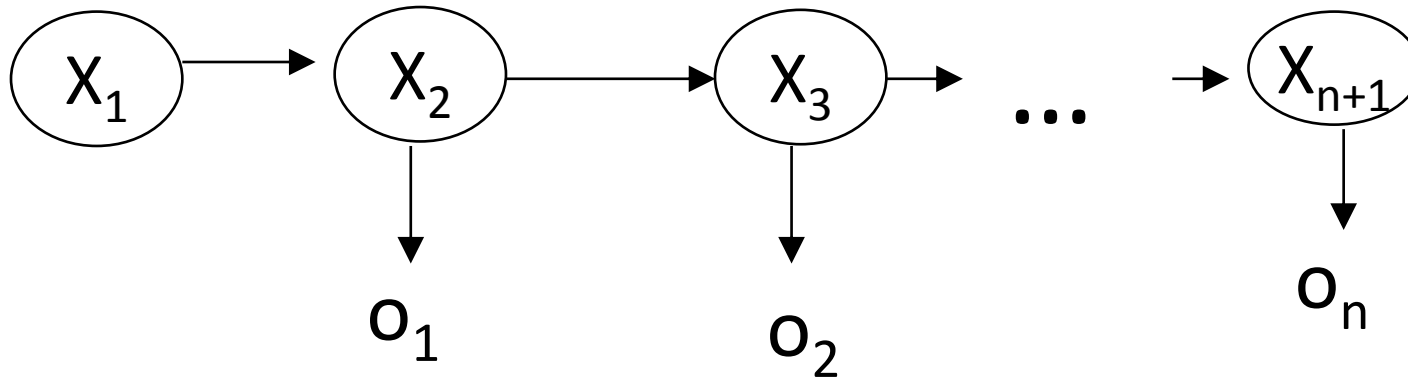
$$P(B, E|C, D) = \frac{P(B, E, C, D)}{P(C, D)} = \frac{\sum_A P(A, B, C, D, E)}{\sum_A \sum_B \sum_E P(A, B, C, D, E)}$$

Naïve Bayes Model



$$\begin{aligned} P(X, Y) &= P(f_1, f_2, \dots, f_n, Y) \\ &= P(Y)P(f_1|Y) \dots P(f_n|Y) \\ &= P(Y) \prod_{k=1}^n P(f_k|Y) \end{aligned}$$

HMM



- State sequence: $X_{1,n+1}$
- Output sequence: $O_{1,n}$

$$P(O_{1,n}, X_{1,n+1}) = \pi(X_1) \prod_{i=1}^n (P(X_{i+1} | X_i) P(O_i | X_{i+1}))$$

Generative model

- It is a directed graphical model in which the output (i.e., what to be predicted) topologically precede the input (i.e., what is given as observation).
- Naïve Bayes and HMM are generative models.

Markov random field

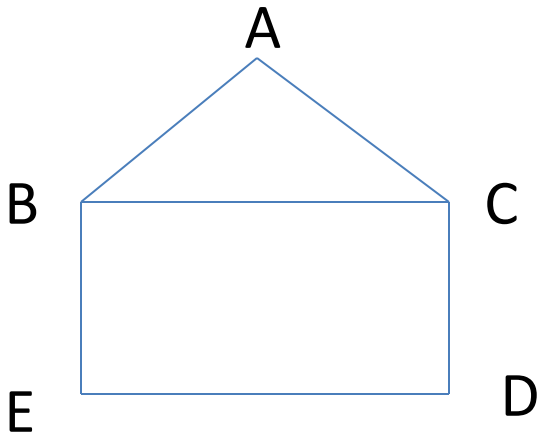
Markov random field

- Also called “Markov network”
- It is a graphical mode in which a set of random variables have a Markov property:
 - Local Markov property: A variable is conditionally independent of all other variables given its neighbors.

$$P(X_i | X_j, ne(X_i)) = P(X_i | ne(X_i))$$

Cliques

- A **clique** in an undirected graph is a subset of its vertices such that every two vertices in the subset are connected by an edge.
- A **maximal clique** is a clique that cannot be extended by adding one more vertex.
- A **maximum clique** is a clique of the largest possible size in a given graph.



clique:

maximal clique:

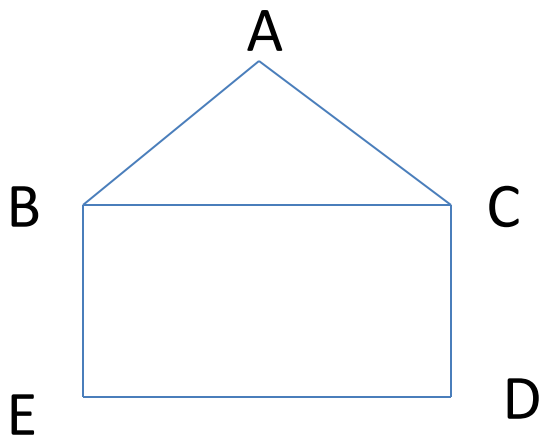
maximum clique:

Clique factorization

$G = (V, E)$ be an undirected graph.

$cl(G)$ be the set of cliques of G .

$$P(X) = \frac{1}{Z} \prod_{C \in cl(G)} \phi_C(X_C)$$



$$\begin{aligned} P(A, B, C, D, E) \\ = \frac{1}{Z} \phi_{ABC}(A, B, C) \phi_{BE}(B, E) \phi_{DE}(D, E) \phi_{C,D}(C, D) \end{aligned}$$

Conditional Random Field

Definition. Let $G = (V, E)$ be a graph such that $Y = (Y_v)_{v \in V}$, so that Y is indexed by the vertices of G . Then (X, Y) is a conditional random field in case, when conditioned on X , the random variables Y_v obey the Markov property with respect to the graph: $p(Y_v | X, Y_w, w \neq v) = p(Y_v | X, Y_w, w \sim v)$, where $w \sim v$ means that w and v are neighbors in G .

A CRF is a random field globally conditioned on the observation X .

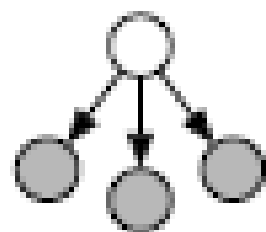
$$p(y|x) = \frac{1}{Z(x)} \prod_{\Phi_A \in G} \exp \left\{ \sum_{k=1}^{K(A)} \lambda_{Ak} f_{Ak}(y_A, x_A) \right\}$$

Linear-chain CRF

Motivation

- Sequence labeling problem: e.g., POS tagging
 - HMM: Find best sequence, but cannot use rich features
 - MaxEnt: Use rich features, but may not find the best sequence
- Linear-chain CRF: HMM + MaxEnt

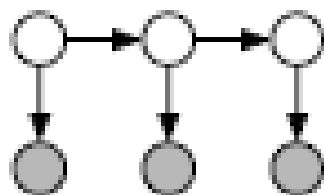
Relations between NB, MaxEnt, HMM, and CRF



Naive Bayes



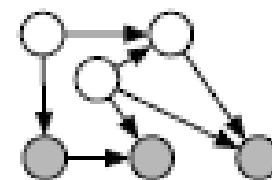
SEQUENCE



HMMs



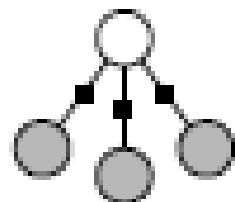
**GENERAL
GRAPHS**



Generative directed models



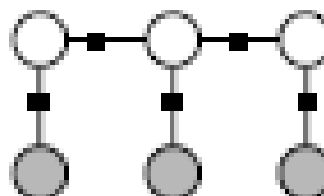
CONDITIONAL



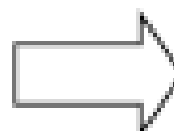
Logistic Regression



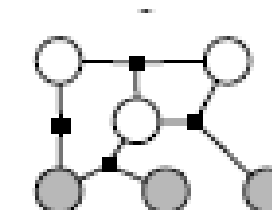
SEQUENCE



Linear-chain CRFs



**GENERAL
GRAPHS**



General CRFs

Linear-chain CRF

$$f_j(y_{t-1}, y_t, x, t) = \begin{cases} 1 & (y_{t-1} = IN) \wedge (y_t = NNP) \wedge (x_t = \textit{Sept}) \\ 0 & \textit{otherwise} \end{cases}$$

$$F_j(y, x) = \sum_{t=1}^T f_j(y_{t-1}, y_t, x, t)$$

$$\begin{aligned} P(y|x) &= \frac{1}{Z(x)} \exp(\sum_j \lambda_j F_j(y, x)) \\ &= \frac{1}{Z(x)} \exp(\sum_j (\lambda_j \sum_{t=1}^T f_j(y_t, y_{t-1}, x, t))) \\ &= \frac{1}{Z(x)} \exp(\sum_j \sum_{t=1}^T (\lambda_j f_j(y_t, y_{t-1}, x, t))) \\ &= \frac{1}{Z(x)} \exp(\sum_{t=1}^T \sum_j (\lambda_j f_j(y_t, y_{t-1}, x, t))) \\ &= \frac{1}{Z(x)} \prod_{t=1}^T \exp(\sum_j (\lambda_j f_j(y_t, y_{t-1}, x, t))) \\ &= \frac{1}{Z(x)} \prod_{t=1}^T \phi_t(y_t, y_{t-1}, x) \end{aligned}$$

Training and decoding

$$P(y|x) = \frac{1}{Z(x)} \prod_{t=1}^T \phi_t(y_t, y_{t-1}, x)$$

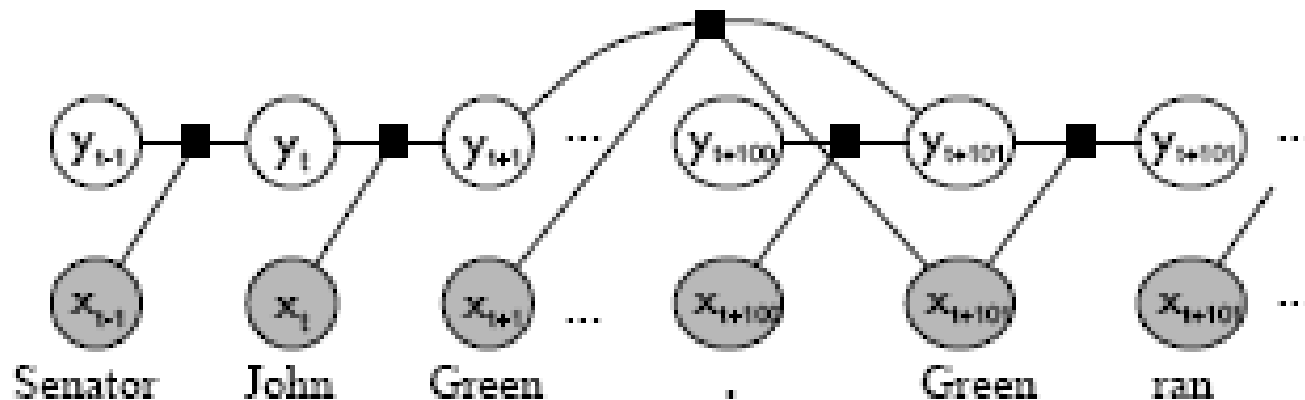
$$\phi_t(y_t, y_{t-1}, x) = \exp(\sum_j (\lambda_j f_j(y_t, y_{t-1}, x, t)))$$

- **Training:** estimate λ_j
 - similar to the one used for MaxEnt
 - Ex: L-BFGS
- **Decoding:** find the best sequence y
 - similar to the one used for HMM
 - Viterbi algorithm

Skip-chain CRF

Motivation

- Sometimes, we need to handle long-distance dependency, which is not allowed by linear-chain CRF
- An example: NE detection
 - “Senator John Green ... Green ran ...”



Linear-chain CRF:

$$P(y|x) = \frac{1}{Z(x)} \prod_{t=1}^T \phi_t(y_t, y_{t-1}, x)$$

$$\phi_t(y_t, y_{t-1}, x) = \exp(\sum_k (\lambda_k f_k(y_t, y_{t-1}, x, t)))$$

Skip-chain CRF:

$$P(y|x) = \frac{1}{Z(x)} \prod_{t=1}^T \phi_t(y_t, y_{t-1}, x) \prod_{(u,v) \in D} \phi_{uv}(y_u, y_v, x)$$

$$\phi_t(y_t, y_{t-1}, x) = \exp(\sum_k (\lambda_k f_k(y_t, y_{t-1}, x, t)))$$

$$\phi_{uv}(y_u, y_v, x) = \exp(\sum_k (\lambda_{2k} f_{2k}(y_u, y_v, x, u, v)))$$

Summary

- Graphical models:
 - Bayesian network (BN)
 - Markov random field (MRF)
- CRF is a variant of MRF:
 - Linear-chain CRF: HMM + MaxEnt
 - Skip-chain CRF: can handle long-distance dependency
 - General CRF
- Pros and cons of CRF:
 - Pros: higher accuracy than HMM and MaxEnt
 - Cons: training and inference can be very slow