One-Zone Model for the Accretion Disk Evolution in Collapsars

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Abstract

This note discusses the formation of accretion disk around a black hole during the core-collapse of a rapidly rotating massive star. The input of the model are the stellar density and rotational angular frequency profiles and the output is the full accretion history of the black hole.

Let the density and angular frequency profile of the star be $\rho(r)$ and $\Omega(r)$. We assume that the star rotates much below the Keplerian rate so its shape is close to spherical. The profile for the enclosed mass is

$$M(r) = \int_0^r \rho 4\pi r'^2 \mathrm{d}r'. \tag{1}$$

The moment of inertial of a spherical shell at radius r is $\Delta I = (2/3)\Delta M r^2$, so the profile for the enclosed angular momentum (AM) is given by

$$J(r) = \int_0^r \frac{2}{3}\Omega r'^2 \rho 4\pi r'^2 \mathrm{d}r'. \tag{2}$$

Now suppose the star suddenly loses internal pressure and starts collapsing from inside out. Each fluid element undergoes nearly radial free-fall, and the time it takes to reach the center (or radii much smaller than the initial radius r) is given by the integral

$$t_{\mathrm{ff}}(r) = \int_0^r v_r^{-1}(r')\mathrm{d}r' \approx \frac{\pi}{2^{1/5}}\sqrt{\frac{r^3}{GM(r)}}, \tag{3}$$

where the radial velocity has been approximated by the free-fall speed $0.5 v_r^2(r') \approx GM(r)(1/r' - 1/r)$. Thus, the mass and AM fallback rates are given by

$$\dot{M}_{\mathrm{fb}} = \frac{\mathrm{d}M(r)}{\mathrm{d}t_{\mathrm{ff}}(r)}, \quad \dot{J}_{\mathrm{fb}} = \frac{\mathrm{d}J(r)}{\mathrm{d}t_{\mathrm{ff}}(r)}. \tag{4}$$

These are the rates at which mass and AM are added to either the black hole (BH) or the accretion disk at a given time $t = t_{\mathrm{ff}}(r)$. Realistically, a fluid element only starts free-fall after the information of the loss of pressure has reached to its position, which takes a sound-crossing time. Here we ignore this sound travel effect, which will only give order unity change to our fallback rate.

Let us assume that the innermost regions directly collapse to form a BH (there might be a transient neutron star phase that is not included here). Subsequently, the shells at larger and larger radii will fall towards the BH. If all the enclosed mass below radius r has fallen into the BH's horizon, then the BH mass and AM are $M_{\mathrm{bh}} = M(r)$ and $J_{\mathrm{bh}} = J(r)$, so the dimensionless spin is given by

$$a = \frac{cJ_{\mathrm{bh}}}{GM_{\mathrm{bh}}^2}, \tag{5}$$

and its current ISCO radius is (for a prograde orbit, Bardeen et al. 1972)

$$\frac{R_{\rm isco}}{R_{\rm g}} = 3 + z_2 - \sqrt{(3 - z_1)(3 + z_1 + 2z_2)},\ R_{\rm g} = \frac{GM_{\rm bh}}{c^2},$$
$$z_1 = 1 + (1 - a^2)^{1/3}\left[(1 + a)^{1/3} + (1 - a)^{1/3}\right],\ z_2 = \sqrt{3a^2 + z_1^2}. \tag{6}$$

The specific AM at the ISCO is

$$\frac{j_{\rm isco}}{R_{\rm g}c} = \frac{x^2 + 2ax^{1/2} + a^2}{x^{3/4}\sqrt{x^{3/2} - 3x^{1/2} - 2a}},\ \ x = R_{\rm isco}/R_{\rm g}. \tag{7}$$

Note that the apparent singularity at $a = 1$ does not exist, because in this case $j_{\rm isco}/R_{\rm g}c = 2/\sqrt{3}$.

For a sufficiently rapidly rotating star, there will be a critical radius $r_{\rm c}$ where the local specific AM exceeds that of the ISCO for the BH formed by the enclosed mass within this radius, i.e.

$$\frac{2}{3}\Omega(r_{\rm c})r_{\rm c}^2 = j_{\rm isco}(M_{\rm bh} = M(r_{\rm c}), J_{\rm bh} = J(r_{\rm c})). \tag{8}$$

For each stellar model, we first solve the above equation for $r_{\rm c}$ and determine the disk formation time $t_{\rm disk} = t_{\rm ff}(r_{\rm c})$ — after this time, the infalling material has enough AM to circularize beyond the ISCO. The exact disk formation time may be slightly different from our value of $t_{\rm disk}$ for a few reasons: (1) materials falling from closer to the polar axis may directly plunge into the BH whereas it is easier for equatorial materials to circularize; (2) for a nearly zero energy particle, the marginally bound (parabolic) orbit that barely avoids direct capture has slightly larger specific angular momentum than $j_{\rm isco}$; (3) gas may undergo shock interactions before hitting the centrifugal barrier. However, these will only mildly change our disk formation time.

At $t > t_{\rm disk}$, the newly infalling material is first incorporated into a rotating disk which then accretes onto the BH. Based on the works of Popham et al. (1999); Narayan et al. (2001); Di Matteo et al. (2002); Kohri et al. (2005); Chen & Beloborodov (2007), the disk may be sufficiently dense and hot to trigger URCA reactions and undergo neutrino cooling. In the case where the disk is strongly cooled due to neutrino emission, then we expect that most of the mass will be transported to smaller radii by viscosity (Shakura & Sunyaev 1973); whereas when the disk is not neutrino-cooled (in the advection-dominated accretion flow, or ADAF regime, Narayan & Yi 1994), then we expect most of the mass to be driven away from the system by disk wind (Blandford & Begelman 1999). A crude one-zone model for the time evolution of the accretion disk is provided by Kumar et al. (2008), who included the combined actions of mass infall, viscous accretion, and disk wind. The model is summarized as follows.

Let us start from the initial disk mass $M_{\rm d} = M_{\rm d,0}$ and AM $J_{\rm d} = J_{\rm d,0}$ (the initial conditions are unimportant as long as the initial mass is much smaller than the total circularizing fallback mass). Assuming the disk to be rotating at the Keplerian frequency, we obtain the characteristic disk radius $R_{\rm d}$ (where most of the mass and AM is located) by equating the specific AM $J_{\rm d}/M_{\rm d}$ to the Keperian value $\sqrt{GM_{\rm d}R_{\rm d}}$, and hence

$$R_{\rm d} = \frac{(J_{\rm d}/M_{\rm d})^2}{GM_{\rm bh}}. \tag{9}$$

The time evolution of these two quantities are given by

$$\dot{M}_{\rm d} = \dot{M}_{\rm fb} - \dot{M}_{\rm acc}, \quad \dot{J}_{\rm d} = \dot{J}_{\rm fb} - \dot{J}_{\rm acc}, \tag{10}$$

where the decrease in disk mass due to viscous accretion is approximated by

$$\dot{M}_{\rm acc} = \frac{M_{\rm d}}{t_{\rm vis}}, \quad t_{\rm vis} \simeq \frac{2}{\alpha \Omega_{\rm K}}, \tag{11}$$

and the AM loss from the disk are due to BH accretion and disk wind

$$\dot{J}_{\rm acc} = \dot{M}_{\rm bh} j_{\rm isco} + f_{\rm AM} \dot{M}_{\rm acc} (J_{\rm d}/M_{\rm d}). \tag{12}$$

The dimensionless viscous parameter α is likely in the range between 0.01 and 0.1. The Keplerian frequency at the disk radius $R_{\rm d}$ is defined as $\Omega_{\rm K} = \sqrt{GM_{\rm bh}/R_{\rm d}^3}$.

An important assumption in the above model is that the mechanical feedback from the already accreted material does not impede the fallback of the outer shells of the star. On the energetic ground, the binding energy of the outer envelope of the star is of the order 10^{50} erg. As we show later, the mechanical energy of the disk wind ($\sim 10^{52}$ erg) is much higher than the binding energy, so it is in principle possible that the accretion of only a very small fraction of the infalling mass drives the rest of the envelope unbound. However, the disk wind (and jet) power is concentrated near the rotational axis of the star and the interactions with the gas in the polar regions produce a cocoon (highly pressurized shocked gas) which expands laterally away from the rotational axis. In the case of a successful cocoon breakout, most of the disk wind/jet energy after the breakout will escape nearly freely at small polar angles and the gas closer to the equatorial plane is likely unaffected by the feedback. Based on the above argument, we multiply the mass and AM fallback rates $\dot{M}_{\rm fb}$ and $\dot{J}_{\rm fb}$ at $t > t_{\rm disk}$ by a factor of $\cos\theta_{\rm min}$ and $1.5(\cos\theta_{\rm min} - \cos^3\theta_{\rm min}/3)$ respectively, where $\theta_{\rm min} = 30^{\rm o}$ is the minimum polar angle below which the infall is impeded by the feedback.

In the following, we estimate the mass gaining rate of the BH $\dot{M}_{\rm bh}$ and the coefficient $f_{\rm AM}$ which describes the AM loss rate of the disk wind.

Based on 1D (height-integrated) studies of steady-state accretion disks (e.g., Narayan et al. 2001; Kohri et al. 2005; Chen & Beloborodov 2007), the disk at radius r is neutrino-cooled if the local accretion rate is sufficiently high

$$\dot{M}(r) \gtrsim 10^{-2.5}\, M_\odot\, {\rm yr}^{-1}\, \frac{r}{2R_{\rm g}}. \tag{13}$$

The normalization in the above equation depends on the viscous parameter α, BH mass and spin (see Chen & Beloborodov 2007; Siegel et al. 2019), but we keep it fixed for simplicity (this assumption will affect our final accretion energy by a factor of a few). If the accretion rate at the outer disk radius $R_{\rm d}$ is below the critical value, then wind mass loss leads to a decrease in accretion rate at smaller radii, which is typically taken to be in a power-law form (Blandford & Begelman 1999)

$$\dot{M} = \dot{M}_{\rm acc} (r/R_{\rm d})^s, \tag{14}$$

where $s \in (0, 1)$ is a free parameter of the model and it is likely somewhere between 0.3 and 0.8 (Yuan & Narayan 2014). Therefore, we can define a transitional radius (and limit it to be in between $R_{\rm isco}$ and $R_{\rm g}$)

$$R_{\rm t} = \max\left[R_{\rm isco}, \min\left[R_{\rm d}, \left(\frac{2R_{\rm g}}{R_{\rm d}^s}\dot{M}_{\rm acc}10^{2.5}\,M_\odot\,{\rm yr}^{-1}\right)^{\frac{1}{1-s}}\right]\right], \tag{15}$$

and then the mass-gaining rate of the BH is given by

$$\dot{M}_{\rm bh} = \dot{M}_{\rm acc}(R_{\rm t}/R_{\rm d})^s. \tag{16}$$

The AM carried away by the disk wind can be obtained under the assumption that the local wind mass loss from each radius r, $\mathrm{d}\dot{M}(r)/\mathrm{d}r$, carries the Keplerian AM $\sqrt{GM_{\rm bh}r}$ at that radius, and this gives

$$f_{\rm AM} = \frac{2s}{2s+1}\left[1 - (R_{\rm t}/R_{\rm d})^{(2s+1)/2}\right]. \tag{17}$$

Now the full set of equations are closed and we can integrate eq. (10) to obtain the full accretion history of a collapsar. At a given time, the total accretion luminosity (including both neutrinoes and wind) is given by

$$L_{\rm acc} \simeq \eta_{\rm NT}\dot{M}_{\rm bh}c^2, \tag{18}$$

where the Novikov-Thorne efficiency $\eta_{\rm NT}(a) = 1 - \sqrt{1 - 2R_{\rm g}/3R_{\rm isco}}$ is given by the difference between the energy of a particle at infinity and that at the ISCO. When the disk near the ISCO is neutrino-cooled ($R_{\rm t} > R_{\rm isco}$), $L_{\rm acc}$ is dominated by the neutrino luminosity; whereas when the disk near the ISCO is advection-dominated, $L_{\rm acc}$ gives the wind power. It is likely that in the case where the disk near the ISCO is in the ADAF regime, an ultra relativistic jet is launched by the Blandford-Znajek mechanism (it is unclear if a neutrino-cooled, geometrically thin disk can launched jets). Under the assumption that each wind fluid element lifted from radius r carries positive specific energy of $GM_{\rm bh}/2r$, we estimate the wind luminosity to be

$$L_{\rm w} \simeq \frac{s}{2(1-s)}\frac{GM_{\rm bh}}{R_{\rm d}}\dot{M}_{\rm acc}\left[(R_{\rm d}/R_{\rm t})^{1-s} - 1\right]. \tag{19}$$

The results for a rapidly spinning MESA model is shown in Fig. 1. For this model, the total energy generated by accretion is $E_{\rm acc} = 1.27 \times 10^{53}$ erg, a fraction of which is released when the disk near the ISCO is in the advection-dominated regime $E_{\rm acc,ADAF} = 2.2 \times 10^{52}$ erg (and neutrinos carried away an energy of $E_{\rm acc} - E_{\rm acc,ADAF}$). If the jet efficiency is $\eta_{\rm j}$, then the total jet energy is $E_{\rm j} = \eta_{\rm j}E_{\rm acc}$ (note that one might argue that only $E_{\rm acc,ADAF}$ is relevant here). The total wind energy is $E_{\rm w} = 4.3 \times 10^{52}$ erg, which is capable of driving a strong explosion of the star. The wind is enriched in ^{56}Ni (and perhaps a small amount of r-process elements as well Siegel et al. 2019), which can power the type-Ic supernova emission.

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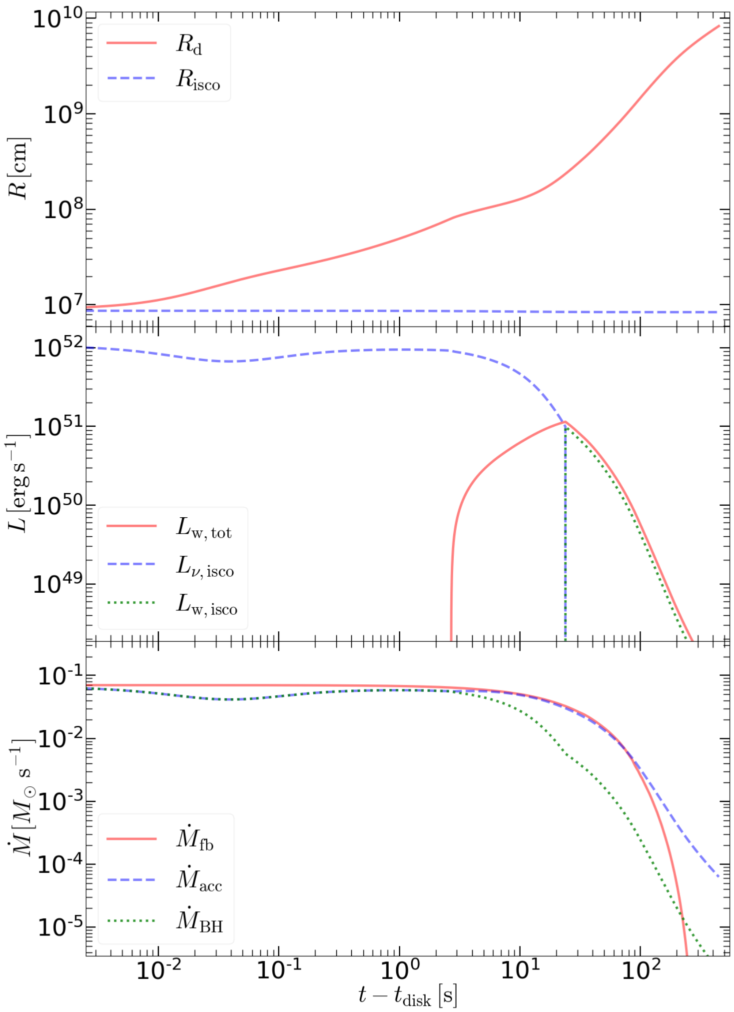

Fig. 1.— Accretion history for "profile12". For this case, the disk formation time is $t_{\rm disk} = 36.9\,$s. The disk undergoes two state transitions: (1) at $t - t_{\rm disk} \approx 2\,$s, the outer disk at $R_{\rm d}$ transitions from the neutrino-cooled regime to ADAF (but the inner disk is still neutrino-cooled); (2) at $t - t_{\rm disk} \approx 23\,$s, the inner disk near the ISCO transitions from the neutrino-cooled regime to ADAF (and after this time the entire disk is in the ADAF regime). In the middle panel, we see that the accretion power is dominated by neutrino luminosity near the ISCO $L_{\nu,\rm isco}$ at $t - t_{\rm disk} < 23\,$s and afterwards, URCA processes shut down and the accretion power is carried by the disk wind launched near the ISCO $L_{\rm w,isco}$. In the time range $2\,{\rm s} < t - t_{\rm disk} < 23\,$s, there is strong disk wind from radii $R_{\rm t} < r < R_{\rm d}$.

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