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Nonlinear Hamiltonian analysis of the novel Poincaré gauge theories

W.E.V. Barker,[1,2,*] A.N. Lasenby,[1,2,†] M.P. Hobson,[1,‡] and W.J. Handley[1,2,§]

[1] *Astrophysics Group, Cavendish Laboratory, JJ Thomson Avenue, Cambridge CB3 0HE, UK*
[2] *Kavli Institute for Cosmology, Madingley Road, Cambridge CB3 0HA, UK*

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I. INTRODUCTION

Break quadratic action down using irreducible projection operators (IPOs) with respect to $\mathrm{SO}(1,3)$ – six for curvature and three for torsion. We write this as

$$\begin{aligned} L_{\mathrm{T}} = {} & {m_{\mathrm{p}}}^2 \hat{\beta}_I \mathcal{T}^i{}_{jk} \, {}^I\mathcal{P}_i{}^{jk}{}_l{}^{nm} \mathcal{T}^l{}_{nm} \\ & + \hat{\alpha}_I \mathcal{R}^{ij}{}_{kl} \, {}^I\mathcal{P}_{ij}{}^{kl}{}_{nm}{}^{op} \mathcal{R}^{nm}{}_{op} + L_{\mathrm{m}} \end{aligned} \tag{1}$$

Some definitions follow from this, for the parallel momenta

$$J^{-1}\hat{\pi}_i{}^{\overline{k}} = 4{m_{\mathrm{p}}}^2 \hat{\beta}_I \, {}^I\mathcal{P}_i{}^{\perp\overline{k}}{}_n{}^{mo} \mathcal{T}^n{}_{mo} \, , \tag{2a}$$

$$J^{-1}\hat{\pi}_{ij}{}^{\overline{k}} = 4\hat{\alpha}_I \, {}^I\mathcal{P}_{ij}{}^{\perp\overline{k}}{}_{mo}{}^{pq} \mathcal{R}^{mo}{}_{pq}. \tag{2b}$$

Next we project out $\mathrm{O}(3)$ irreps using n_i to break Lorentz symmetry (not really broken because of lapse and shift). Next we evaluate Poisson brackets between these. The following very useful identities for this process in ADM are not given in the literature:

$$\begin{aligned} \frac{\partial n_k}{\partial b^i{}_\mu} = -n_i h_{\overline{k}}{}^\mu, \quad \frac{\partial h_j{}^\nu}{\partial b^i{}_\mu} = -h_i{}^\nu h_j{}^\nu, \\ \frac{\partial b}{\partial b^i{}_\nu} = b h_i{}^\nu, \quad \frac{\partial J}{\partial b^k{}_\nu} = J h_{\overline{k}}{}^\nu \end{aligned} \tag{3}$$

Template for the Poisson matrix of PICs:

$$\begin{array}{c|cc|c} & \hat{\varphi}_{\overline{kl}} & {}^{\mathrm{T}}\varphi_{\overline{klo}} & \\ \hline \hat{\varphi}_{\overline{kl}} & \cdot & \cdot & 1 \\ {}^{\mathrm{T}}\varphi_{\overline{klo}} & \cdot & \cdot & 1 \\ \hline & 1 & 1 & \end{array} \tag{4}$$

$$\begin{array}{c|cccc|c} & \tilde{\varphi}_{\overline{kl}} & \varphi_{\perp} & \tilde{\varphi}_{\perp\overline{kl}} & {}^{\mathrm{T}}\varphi_{\overline{klo}} & \\ \hline \tilde{\varphi}_{\overline{kl}} & \pi & A & b & \partial & 5 \\ \varphi_{\perp} & A & \pi & \pi & \partial b & 1 \\ \tilde{\varphi}_{\perp\overline{kl}} & b & \pi & b & \cdot & 5 \\ {}^{\mathrm{T}}\varphi_{\overline{klo}} & \partial & \partial b & \cdot & \pi & 5 \\ \hline & 5 & 1 & 5 & 5 & \end{array} \tag{5}$$

II. KLEIN-GORDON THEORIES

From general considerations, the novel

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* wb263@cam.ac.uk
† a.n.lasenby@mrao.cam.ac.uk
‡ mph@mrao.cam.ac.uk
§ wh260@cam.ac.uk