Hamiltonian analysis of novel Poincaré gauge theories: promising cases

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I. INTRODUCTION

Break quadratic action down using irreducible projection operators (IPOs) with respect to $\mathrm{SO}(1,3)$ – six for curvature and three for torsion. We write this as

$$\begin{aligned} L_{\mathrm{T}} = {} & {m_{\mathrm{p}}}^2 \hat{\beta}_I \mathcal{T}^{i}_{\ jk} \, {}^{I}\mathcal{P}_{i\ \ l}^{\ jk\ \ nm} \mathcal{T}^{l}_{\ nm} \\ & + \hat{\alpha}_I \mathcal{R}^{ij}_{\ \ kl} \, {}^{I}\mathcal{P}_{ij\ \ nm}^{\ \ kl\ \ \ op} \mathcal{R}^{nm}_{\ \ \ op} + L_{\mathrm{m}} \end{aligned} \tag{1}$$

Some definitions follow from this, for the parallel momenta

$$J^{-1} \hat{\pi}_i^{\ \bar{k}} = 4 \hat{\beta}_I n_{[j} \delta^{\bar{k}}_{l]} \, {}^{I}\mathcal{P}_{i\ \ n}^{\ jl\ \ mo} \mathcal{T}^{n}_{\ mo} \,, \tag{2a}$$

$$J^{-1} \hat{\pi}_{ij}^{\ \ \bar{k}} = 4 \hat{\alpha}_I n_{[l} \delta^{\bar{k}}_{n]} \, {}^{I}\mathcal{P}_{ij\ \ mo}^{\ \ ln\ \ \ pq} \mathcal{R}^{mo}_{\ \ \ pq}. \tag{2b}$$

Next we project out $\mathrm{O}(3)$ irreps using n_i to break Lorentz symmetry (not really broken because of lapse and shift) – this process is very long. The computer cannot canonicalise the results so they have to be done by hand. This projection gives us the primary if-constraints. Next we evaluate Poisson brackets between these – this requires field derivatives so must also be done by hand. Some very useful identities for this process in ADM are not given by Blagojević, Yo and Nester etc., so we write them out

$$\begin{aligned} & \frac{\partial n_j}{\partial {b^i}_\mu} = -n_i {h_{\bar{j}}}^\mu, \quad \frac{\partial {h_j}^\nu}{\partial {b^i}_\mu} = -{h_i}^\nu {h_j}^\nu, \quad \frac{\partial b}{\partial {b^i}_\nu} = b {h_i}^\nu, \\ & \frac{\partial J^{-1} {b^k}_\alpha \delta^\alpha_\nu}{\partial {b^i}_\mu} = J^{-1} \delta^k_i \delta^\mu_\alpha \delta^\alpha_\nu - J^{-1} {b^k}_\alpha \delta^\alpha_\nu {h_{\bar{i}}}^\mu. \end{aligned} \tag{3}$$

Note the last identity is made more annoying by the restriction to the 3-space (i.e. you can't just replace the contractions).

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