Nonlinear Hamiltonian analysis of the novel Poincaré gauge theories

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I. INTRODUCTION

Break quadratic action down using irreducible projection operators (IPOs) with respect to SO(1, 3) – six for curvature and three for torsion. We write this as

$$
\begin{aligned}
L_{\mathrm{T}} = {}& m_{\mathrm{p}}{}^2 \hat{\beta}_I \mathcal{T}^i_{jk} \, {}^I\mathcal{P}_i{}^{jk}{}_l{}^{nm} \mathcal{T}^l_{nm} \\
& + \hat{\alpha}_I \mathcal{R}^{ij}{}_{kl} \, {}^I\mathcal{P}_{ij}{}^{kl}{}_{nm}{}^{op} \mathcal{R}^{nm}{}_{op} + L_{\mathrm{m}}
\end{aligned}
\tag{1}
$$

Some definitions follow from this, for the parallel momenta

$$
J^{-1}\hat{\pi}_i{}^{\overline{k}} = 4m_{\mathrm{p}}{}^2 \hat{\beta}_I \, {}^I\mathcal{P}_i{}^{\perp\overline{k}}{}_n{}^{mo} \mathcal{T}^n_{mo} ,
\tag{2a}
$$

$$
J^{-1}\hat{\pi}_{ij}{}^{\overline{k}} = 4\hat{\alpha}_I \, {}^I\mathcal{P}_{ij}{}^{\perp\overline{k}}{}_{mo}{}^{pq} \mathcal{R}^{mo}{}_{pq}.
\tag{2b}
$$

Next we project out O(3) irreps using n_i to break Lorentz symmetry (not really broken because of lapse and shift). Next we evaluate Poisson brackets between these. The following very useful identities for this process in ADM are not given in the literature:

$$
\begin{aligned}
\frac{\partial n_k}{\partial b^i{}_\mu} = -n_i h_{\overline{k}}{}^\mu , \quad & \frac{\partial h_j{}^\nu}{\partial b^i{}_\mu} = -h_i{}^\nu h_j{}^\nu , \\
\frac{\partial b}{\partial b^i{}_\nu} = b h_i{}^\nu , \quad & \frac{\partial J}{\partial b^k{}_\nu} = J h_{\overline{k}}{}^\nu
\end{aligned}
\tag{3}
$$

Template for the Poisson matrix of PICs:

	φ	$\hat{\varphi}_{\overline{kl}}$	$\varphi_{\perp\overline{k}}$	$\widetilde{\varphi}_{\overline{kl}}$	$\varphi_\perp$	${}^{\mathrm{P}}\varphi$	$\hat{\varphi}_{\perp\overline{kl}}$	$\overrightarrow{\varphi}_{\overline{k}}$	$\widetilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
φ	.	.	.	.	.	.	.	.	.	.	
$\hat{\varphi}_{\overline{kl}}$	.	.	.	.	.	.	.	.	.	.	
$\varphi_{\perp\overline{k}}$	.	.	.	.	.	.	.	.	.	.	
$\widetilde{\varphi}_{\overline{kl}}$	.	.	.	.	.	.	.	.	.	.	
$\varphi_\perp$	.	.	.	.	.	.	.	.	.	.	
${}^{\mathrm{P}}\varphi$	.	.	.	.	.	.	.	.	.	.	
$\hat{\varphi}_{\perp\overline{kl}}$	.	.	.	.	.	.	.	.	.	.	
$\overrightarrow{\varphi}_{\overline{k}}$	.	.	.	.	.	.	.	.	.	.	
$\widetilde{\varphi}_{\perp\overline{kl}}$	.	.	.	.	.	.	.	.	.	.	
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	.	.	.	.	.	.	.	.	.	.	

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	φ	$\hat{\varphi}_{\overline{kl}}$	$\varphi_{\perp\overline{k}}$	$\widetilde{\varphi}_{\overline{kl}}$	$\varphi_\perp$	${}^{\mathrm{P}}\varphi$	$\hat{\varphi}_{\perp\overline{kl}}$	$\overrightarrow{\varphi}_{\overline{k}}$	$\widetilde{\varphi}_{\perp\overline{kl}}$	${}^{\mathrm{T}}\varphi_{\overline{klo}}$	
φ	.	.	.	.	.	.	.	.	.	.	1
$\hat{\varphi}_{\overline{kl}}$	.	.	.	.	.	.	.	.	.	.	1
$\varphi_{\perp\overline{k}}$	.	.	.	.	.	.	.	.	.	.	1
$\widetilde{\varphi}_{\overline{kl}}$	.	.	.	.	.	.	.	.	.	.	1
$\varphi_\perp$	.	.	.	.	.	.	.	.	.	.	1
${}^{\mathrm{P}}\varphi$	.	.	.	.	.	.	.	.	.	.	1
$\hat{\varphi}_{\perp\overline{kl}}$	.	.	.	.	.	.	.	.	.	.	1
$\overrightarrow{\varphi}_{\overline{k}}$	.	.	.	.	.	.	.	.	.	.	1
$\widetilde{\varphi}_{\perp\overline{kl}}$	.	.	.	.	.	.	.	.	.	.	1
${}^{\mathrm{T}}\varphi_{\overline{klo}}$	.	.	.	.	.	.	.	.	.	.	1
	1	1	1	1	1	1	1	1	1	1	

II. KLEIN-GORDON THEORIES

From general considerations, the novel
Case 20:

$$
\begin{array}{c|ccccc|}
& \varphi_\perp & \hat{\varphi}_{\perp\overline{kl}} & \overrightarrow{\varphi}_{\overline{k}} & \widetilde{\varphi}_{\perp\overline{kl}} & {}^{\mathrm{T}}\varphi_{\overline{klo}} \\
\hline
\varphi_\perp & \cdot & \cdot & \cdot & \cdot & \cdot \\
\hat{\varphi}_{\perp\overline{kl}} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\overrightarrow{\varphi}_{\overline{k}} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\widetilde{\varphi}_{\perp\overline{kl}} & \cdot & \cdot & \cdot & \cdot & \cdot \\
{}^{\mathrm{T}}\varphi_{\overline{klo}} & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\tag{4}
$$

Case 24:

$$
\begin{array}{c|cccc|}
& \widetilde{\varphi}_{\overline{kl}} & \varphi_\perp & \widetilde{\varphi}_{\perp\overline{kl}} & {}^{\mathrm{T}}\varphi_{\overline{klo}} \\
\hline
\widetilde{\varphi}_{\overline{kl}} & \pi & \cdot & \pi & \pi \\
\varphi_\perp & \cdot & \cdot & \cdot & \cdot \\
\widetilde{\varphi}_{\perp\overline{kl}} & \pi & \cdot & \cdot & \cdot \\
{}^{\mathrm{T}}\varphi_{\overline{klo}} & \pi & \cdot & \cdot & \cdot \\
\end{array}
\tag{5}
$$

Case 25:

$$
\begin{array}{c|cccccc|}
& \widetilde{\varphi}_{\overline{kl}} & \varphi_\perp & \hat{\varphi}_{\perp\overline{kl}} & \overrightarrow{\varphi}_{\overline{k}} & \widetilde{\varphi}_{\perp\overline{kl}} & {}^{\mathrm{T}}\varphi_{\overline{klo}} \\
\hline
\widetilde{\varphi}_{\overline{kl}} & \pi & \cdot & \cdot & \cdot & \cdot & \pi \\
\varphi_\perp & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\hat{\varphi}_{\perp\overline{kl}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\overrightarrow{\varphi}_{\overline{k}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\widetilde{\varphi}_{\perp\overline{kl}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
{}^{\mathrm{T}}\varphi_{\overline{klo}} & \pi & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\tag{6}
$$

Case 26:

	φ	$\varphi_{\perp k}$	$\widetilde{\varphi}_{\overline{kl}}$	$\varphi_\perp$	$\hat{\varphi}_{\perp\overline{kl}}$	$\vec{\varphi}_{\overline{k}}$	$\widetilde{\varphi}_{\perp\overline{kl}}$	$^T\varphi_{\overline{klo}}$
φ	·	·	·	·	·	·	·	·
$\varphi_{\perp\overline{k}}$	·	π	·	·	π	·	·	·
$\widetilde{\varphi}_{\overline{kl}}$	·	·	π	·	·	·	·	π
$\varphi_\perp$	·	·	·	·	·	·	·	·
$\hat{\varphi}_{\perp\overline{kl}}$	·	π	·	·	·	·	·	·
$\vec{\varphi}_{\overline{k}}$	·	·	·	·	·	·	·	·
$\widetilde{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	·	·	·
$^T\varphi_{\overline{klo}}$	·	·	π	·	·	·	·	·

(7)

Case 28:

	φ	$\varphi_{\perp\overline{k}}$	$\widetilde{\varphi}_{\overline{kl}}$	$\varphi_\perp$	$\widetilde{\varphi}_{\perp\overline{kl}}$	$^T\varphi_{\overline{klo}}$
φ	·	·	·	·	·	·
$\varphi_{\perp\overline{k}}$	·	π	·	π	π	π
$\widetilde{\varphi}_{\overline{kl}}$	·	·	π	·	π	π
$\varphi_\perp$	·	π	·	·	·	·
$\widetilde{\varphi}_{\perp\overline{kl}}$	·	π	π	·	·	·
$^T\varphi_{\overline{klo}}$	·	π	π	·	·	·

(8)

Case 32:

	φ	$\varphi_\perp$	$\hat{\varphi}_{\perp\overline{kl}}$	$\vec{\varphi}_{\overline{k}}$	$\widetilde{\varphi}_{\perp\overline{kl}}$	$^T\varphi_{\overline{klo}}$
φ	·	·	·	·	·	·
$\varphi_\perp$	·	·	·	·	·	·
$\hat{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	·
$\vec{\varphi}_{\overline{k}}$	·	·	·	·	·	·
$\widetilde{\varphi}_{\perp\overline{kl}}$	·	·	·	·	·	·
$^T\varphi_{\overline{klo}}$	·	·	·	·	·	·

(9)

test Case:

III.   WITH WITTEN'S GRAVITON

Case 3:

	φ	$\hat{\varphi}_{\overline{kl}}$	$\varphi_\perp$	$\widetilde{\varphi}_{\perp\overline{kl}}$	$^T\varphi_{\overline{klo}}$
φ	·	$\partial b + A$	·	·	·
$\hat{\varphi}_{\overline{kl}}$	$\partial b + A$	·	π	π	π
$\varphi_\perp$	·	π	·	·	·
$\widetilde{\varphi}_{\perp\overline{kl}}$	·	π	·	·	·
$^T\varphi_{\overline{klo}}$	·	π	·	·	·

(10)

Case 17:

	φ	$\hat{\varphi}_{\overline{kl}}$	$\varphi_\perp$	$^P\varphi$	$\widetilde{\varphi}_{\perp\overline{kl}}$	$^T\varphi_{\overline{klo}}$
φ	·	$\partial b + A$	·	·	·	·
$\hat{\varphi}_{\overline{kl}}$	$\partial b + A$	·	π	π	π	π
$\varphi_\perp$	·	π	·	·	·	·
$^P\varphi$	·	π	·	·	·	·
$\widetilde{\varphi}_{\perp\overline{kl}}$	·	π	·	·	·	·
$^T\varphi_{\overline{klo}}$	·	π	·	·	·	·

(11)