

# Nonlinear Hamiltonian analysis of the novel Poincaré gauge theories

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## I. INTRODUCTION

Break quadratic action down using irreducible projection operators (IPOs) with respect to  $SO(1,3)$  – six for curvature and three for torsion. We write this as

$$L_T = m_P^2 \hat{\beta}_I \mathcal{T}_{jk}^i \mathcal{P}_{i\ l}^{jk\ nm} \mathcal{T}_{nm}^l + \hat{\alpha}_I \mathcal{R}_{kl}^{ij} \mathcal{P}_{ij\ nm}^{kl\ op} \mathcal{R}_{op}^{nm} + L_m \quad (1)$$

Some definitions follow from this, for the parallel momenta

$$J^{-1} \hat{\pi}_i^{\bar{k}} = 4m_P^2 \hat{\beta}_I \mathcal{P}_{i\ n}^{j\ \bar{k}\ mo} \mathcal{T}_{mo}^n, \quad (2a)$$

$$J^{-1} \hat{\pi}_{ij}^{\bar{k}} = 4\hat{\alpha}_I \mathcal{P}_{ij\ mo}^{i\ \bar{k}\ pq} \mathcal{R}_{pq}^{mo}. \quad (2b)$$

Next we project out  $O(3)$  irreps using  $n_i$  to break Lorentz symmetry (not really broken because of lapse and shift) – this process is very long. The computer cannot canonicalise the results so they have to be done by hand. This projection gives us the primary if-constraints. Next we evaluate Poisson brackets between these – this requires field derivatives so must also be done by hand. Some very useful identities for this process in ADM are not given by Blagojević, Yo and Nester etc., so we write them out

$$\begin{aligned} \frac{\partial n_k}{\partial b_{\mu}^i} &= -n_i h_{\bar{k}}^{\mu}, & \frac{\partial h_j^{\nu}}{\partial b_{\mu}^i} &= -h_i^{\nu} h_j^{\nu}, \\ \frac{\partial b}{\partial b_{\nu}^i} &= b h_i^{\nu}, & \frac{\partial J}{\partial b_{\nu}^k} &= J h_{\bar{k}}^{\nu} \end{aligned} \quad (3)$$

Next the PIC matrix

$$\begin{array}{c} \varphi \quad \hat{\varphi}_{\bar{k}l} \quad \varphi_{\perp\bar{k}} \quad \tilde{\varphi}_{\perp\bar{k}l} \quad \varphi_{\perp} \quad {}^P\varphi \quad \hat{\varphi}_{\perp\bar{k}l} \quad \vec{\varphi}_{\bar{k}} \quad \tilde{\varphi}_{\perp\bar{k}l} \quad {}^T\varphi_{\bar{k}lo} \\ \begin{array}{c} \varphi \\ \hat{\varphi}_{\bar{k}l} \\ \varphi_{\perp\bar{k}} \\ \tilde{\varphi}_{\perp\bar{k}l} \\ \varphi_{\perp} \\ {}^P\varphi \\ \hat{\varphi}_{\perp\bar{k}l} \\ \vec{\varphi}_{\bar{k}} \\ \tilde{\varphi}_{\perp\bar{k}l} \\ {}^T\varphi_{\bar{k}lo} \end{array} \end{array} \left| \begin{array}{cccccccccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array} \right|$$

$$\begin{array}{c} \varphi \quad \hat{\varphi}_{\bar{k}l} \quad \varphi_{\perp} \quad {}^P\varphi \quad \tilde{\varphi}_{\perp\bar{k}l} \quad {}^T\varphi_{\bar{k}lo} \\ \begin{array}{c} \varphi \\ \hat{\varphi}_{\bar{k}l} \\ \varphi_{\perp} \\ {}^P\varphi \\ \tilde{\varphi}_{\perp\bar{k}l} \\ {}^T\varphi_{\bar{k}lo} \end{array} \end{array} \left| \begin{array}{cccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \pi & \pi & \pi & \pi \\ \cdot & \pi & \cdot & \cdot & \cdot & \cdot \\ \cdot & \pi & \cdot & \cdot & \cdot & \cdot \\ \cdot & \pi & \cdot & \cdot & \cdot & \cdot \\ \cdot & \pi & \cdot & \cdot & \cdot & \cdot \end{array} \right| \quad (4)$$

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