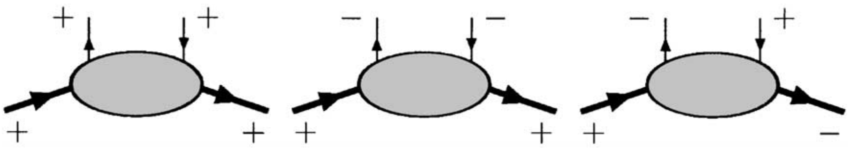

Fig. 9. The three quark–nucleon helicity amplitudes.

Time-reversal invariance,

$$\mathscr{A}_{\Lambda\lambda,\Lambda'\lambda'} = \mathscr{A}_{\Lambda'\lambda',\Lambda\lambda}, \tag{4.5.5}$$

adds no further constraints. Hence, we are left with three independent amplitudes (see Fig. 9)

$$\mathscr{A}_{++,++}, \quad \mathscr{A}_{+-,+-}, \quad \mathscr{A}_{+-,-+}. \tag{4.5.6}$$

Two of the amplitudes in (4.5.6), $\mathscr{A}_{++,++}$ and $\mathscr{A}_{+-,+-}$, are diagonal in the helicity basis (the quark does not flip its helicity: $\lambda = \lambda'$), the third, $\mathscr{A}_{+-,-+}$, is off-diagonal (helicity flip: $\lambda = -\lambda'$). Using the optical theorem we can relate these quark–nucleon helicity amplitudes to the three leading-twist quark distribution functions, according to the scheme

$$f(x) = f_+(x) + f_-(x) \sim \mathrm{Im}(\mathscr{A}_{++,++} + \mathscr{A}_{+-,+-}), \tag{4.5.7a}$$

$$\Delta f(x) = f_+(x) - f_-(x) \sim \mathrm{Im}(\mathscr{A}_{++,++} - \mathscr{A}_{+-,+-}), \tag{4.5.7b}$$

$$\Delta_T f(x) = f_\uparrow(x) - f_\downarrow(x) \sim \mathrm{Im}\, \mathscr{A}_{+-,-+}. \tag{4.5.7c}$$

In a transversity basis (with $\uparrow$ directed along y)

$$| \uparrow \rangle = \frac{1}{\sqrt{2}}[|+\rangle + \mathrm{i}|-\rangle],$$

$$| \downarrow \rangle = \frac{1}{\sqrt{2}}[|+\rangle - \mathrm{i}|-\rangle], \tag{4.5.8}$$

the transverse polarisation distributions $\Delta_T f$ is related to a diagonal amplitude

$$\Delta_T f(x) = f_\uparrow(x) - f_\downarrow(x) \sim \mathrm{Im}(\mathscr{A}_{\uparrow\uparrow,\uparrow\uparrow} - \mathscr{A}_{\uparrow\downarrow,\uparrow\downarrow}). \tag{4.5.9}$$

Reasoning in terms of parton–nucleon forward helicity amplitudes, it is easy to understand why there is no such thing as leading-twist transverse polarisation of gluons. A hypothetical $\Delta_T g$ would imply an helicity flip gluon–nucleon amplitude, which cannot exist owing to helicity conservation. In fact, gluons have helicity ± 1 but the nucleon cannot undergo an helicity change $\Delta\Lambda = \pm 2$. Targets with higher spin may have an helicity-flip gluon distribution.

If transverse momenta of quarks are not neglected, the situation becomes more complicated and the number of independent helicity amplitudes increases. These amplitudes combine to form six $\boldsymbol{k}_\perp$-dependent distribution functions (three of which reduce to $f(x)$, $\Delta f(x)$ and $\Delta_T f(x)$ when integrated over $\boldsymbol{k}_\perp$).