MATH 484 QUIZ 08 DUALITY & RATES OF CHANGE

Name____________________

Find max $z(x,y) = 17x + 11y - 9$
Subject to

$$
\begin{aligned}
g_1(x1,x2) &= 2x1 + x2 \le 32 = b_1 \\
g_2(x1,x2) &= x1 + x2 \le 20 = b_2 \\
g_3(x1,x2) &= x1 + 2x2 \le 38 = b_3 \\
&x >= 0
\end{aligned}
$$

1. Graph the feasible space

2. Find the limits of each of b1, b2, b3, within which the original dual solution remains valid. That is, find the max and min of b1 when b2=20 & b3=38, the max and min of b2 when b1=32 & b3=38, and the max and min of b3 when b1=32 & b2=20.

3. a) Put the LP in row tableau, solve using the simplex algorithm, and extract the basic solution of the LP and its dual. That is, get the values of x* and z*=max z, and y* and w* = min w.

   b) Put the LP in column tableau, solve using the exact same simplex algorithm. That is, imagine the row LP, which is the dual, is actually the LP you are solving, and follow the simplex algorithm as you would on any other row tableau. Extract the basic solution of both the row LP (aka the dual) and the column LP (aka the primal). Your solutions (a) & (b) must agree.

4. Formulate z*[b1, b2] as z[*[b1, b2]] and differentiate to verify that dz*/bi equals y*[[i]] for all i using y* obtained from part 3.

5. Re-solve the tableau from 3(a) or 3(b) … you're choice …. for each given combination of dbi given by the vectors db = (db1, db2, db3): (I defined db more explicitly here than in HW12)

   a) db = (0 2 0)

   b) db = (3 0 0)

   c) db = (3 2 0)

   d) db = (3 -2 0)

For each solution (a) - (d), use boolean comparisons (==) in Mathematica to verify the string of equalities verified in HW12, namely,

dz* = del(z).dx* =(y1* del(g1) + y2* del(g2) + y3* del(g3)).dx*
= y1* del(g1).dx* + y2* del(g2).dx* + y3* del(g3).dx* = y1*db1+y2*db2+y3*db3

You can copy the code from HW 12 and all you need to use it is to redefine symbols for
del(z), del(gi) for i= 1,…,3, which are constant,
dbi for i = 1,…,3, which are inputs to each case a,b,c,d,
yi* for i = 1,…,3, from (3), as these db are within the ranges found in (2) so y* is the same for these db,
dz* and dx* from the primal basic solution of each optimal tableau and these do change with each db.