

①

1. rot  $-45^\circ$  in  $z$
2. rot  $-30^\circ$  in  $x$
3. trans  $(-15, -15, 12)$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & -15 \\ 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(30) & -\sin(30) & 0 \\ 0 & \sin(30) & \cos(30) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(-45) & -\sin(-45) & 0 & 0 \\ \sin(-45) & \cos(-45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3)                      (2)                      (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

②

1. XYZ scaling  $(2, 1, 3)$  in  $x, y$  and  $z$  direction

or: double torus' size in  $x$ -axis  
 triple torus' size in  $z$ -axis  
 keep torus' size in  $y$ -axis

2. rotation about the  $y$ -axis by  $-45^\circ$

3. XYZ Translation of  $(5, -4, 2)$

or moving torus in 5 units in  $x$ -axis  
 $-4$  units in  $y$ -axis  
 2 units in  $z$ -axis

Transformation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(-45) & 0 & \sin(-45) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-45) & 0 & \cos(-45) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

(3)                      (2)                      (1)

③

1. rotation about X-axis by  $-60^\circ$
2. XYZ scale to  $(1.5, 3, 2)$
3. XYZ translation of  $(-7, 0, 2)$

④ ①. 
$$\begin{pmatrix} \cos(30^\circ) & -\sin(30^\circ) & 0 & 0 \\ \sin(30^\circ) & \cos(30^\circ) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
 rotation about Z-axis by  $30^\circ$

②. 
$$\begin{pmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
 XYZ scale to  $(0.5, 1.5, 2)$

③. 
$$\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
 XYZ translation of  $(4, 5, -1)$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{③}} \cdot \underbrace{\begin{pmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{②}} \cdot \underbrace{\begin{pmatrix} \cos(30^\circ) & -\sin(30^\circ) & 0 & 0 \\ \sin(30^\circ) & \cos(30^\circ) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{①}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$