Analysis of Dynamics on Scale-free Network

The most obvious way to analyze complex objects and systems is to separate it into elements which are used to investigate and understand the way the whole object or system functions. However, today there is a need to analyze the macroscopic properties of an object.

There are a lot of complex systems that can be analyzed by its topology, for example: cancer patients do not have a separate cancer gene, human consciousness cannot be reduced to the behavior of the only one neuron, economic crisis starts from somewhere inside of a tremendous system of debts in worlds financial system, the solution of traffic jams problem requires investigation of the whole transport system of the city.

Last years showed that the majority of social, technical, biological and transport systems have similar network topology and can be successfully simulated using scale free networks. Due to high structure complexity of these networks the better method (comparing with the traditional one) to analyze them is to look at the macro-signals of these networks.

The scale free networks are the networks which node distribution over its degree (the number of links the node have) obeys a power law. So that p – is the probability of the node to have a degree d:

$$p = cd^{-k}$$

k – is a constant which is calculated empirically and is own for each network

c – is a normalizing factor (to have an area under the curve equal to 1)

The most significant qualities of the scale free networks are:

1. They are quite a rarefied
2. The have quite a small diameter of the whole network

The aim of this research is to simulate the scale free network and investigate its behavior in the case of Kuramoto oscillators placed into its nodes.

The dynamics of each node in the network is defined by the system of differential equations:

$$\forall i \in V: \dot{\varphi}_i = \omega_i + \lambda \sum_{j \in V} w_{ij} \sin(\varphi_j - \varphi_i)$$

with initial conditions: $\varphi_i(0) \in [0; 2\pi]$

V – set of all nodes from the considered network;

φ_i – the phase value of node i;

$\omega_i \in [1; 10]$ – the own frequency of node i;

w_{ij} – the number of links from node i to node j;

$\lambda = const$ – the bond strength, it shows how much the neighbors of node i affect it.

The observation of the network is based on 2 macro-signals:

1. The sum signal of the whole network: $X(t) = A \sum_{i \in V} \cos(\varphi_i(t))$

   Where A – a normalizing factor

2. The coherency rate of nodes in the network: $r(t) = \frac{1}{n}\sum_{j \in V} e^{i\varphi_j(t)}$

The following methods were used to make a number of computer experiments:

- The ‘Static modification’ method (also known as ‘LCD-diagram’ method) was used to build scale free network.
- The Fourth-Order Runge-Kutta method was used to solve the system of differential equations

The following stages has been done:

1. The software for building scale free network and calculating dynamics on its nodes has been developed.
2. The software for observation of macro-signals and the changes of phases in nodes of scale free network has been developed.

- A number of experiments have been done. The dependencies of macro-signals from time, link strength and numerical method grid step have been received.

The following IT tools has been used:

- The Microsoft Visual Studio Community 2015 environment has been used to develop all applications in this research work
- The C++11 has been used to create console applications for
    1) building scale free networks
    2) computing the dynamics on the nodes of the scale free network
- The C# with Windows Forms Application (without any third-party frameworks) has been used to create applications for visualizing
    1) the dependencies of macro properties of network from the value of simulating time

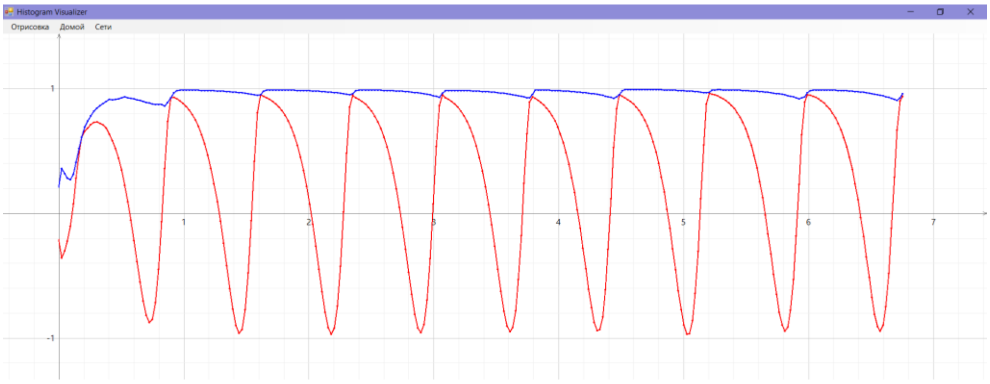

Figure 1 The dependency of ‘sum signal’ (red) and ‘coherency rate’ (blue) on simulating time for one of the random networks

    2) the dynamics of each node of scale free network at every value of simulating time

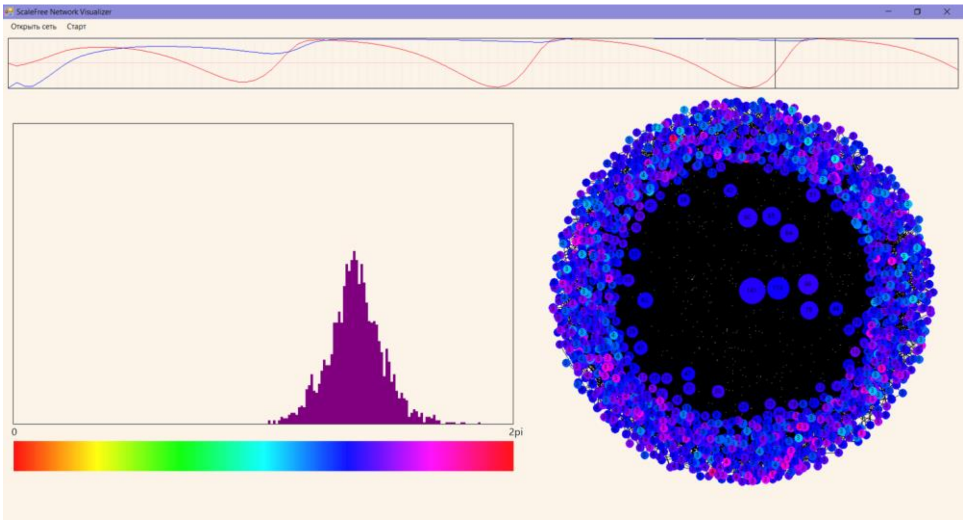

Figure 2 Dynamics visualization (at the 'consistent mode' moment) for the scale free network consisting of 2000 nodes

A number of experiments demonstrated the following facts:

- The behavior of the scale free networks for the whole simulating time can be separated into 2 dynamic modes:
    1. "Transient mode" – arises in the network at the very beginning and corresponds to the network state when each node is in different phase without any visual dependencies between them
    2. "Consistent mode" – follows after the transient mode and corresponds to the network state when every node is synchronized with each other and their phase values are close to some nonstationary value in the network.
- All possible states of the network sooner or later leads to the consistent mode which networks continue to stay in up to the very last moment of the simulation.
- The 'bond strength' rate in the differential equations effects on the duration of the transient mode and on the general node coherence in the network. This fact can be viewed on the behavior of coherency degree macro signal.
- Both 'sum signal' and 'coherency degree' macro properties show the periodicity in the law of changing the sum phase of the network and the correlation between them.
- In the consistent mode the distribution of nodes over their phases has the form which is close to the form of normal distribution.

The perspective research may be extended by complication the simulation part of dynamics on the links (in addition to dynamics on the nodes) of the network and by adding a wavelet transform of the sum signal into the list of observing macroscopic properties of the scale free network.