

lsbi: linear simulation based inference

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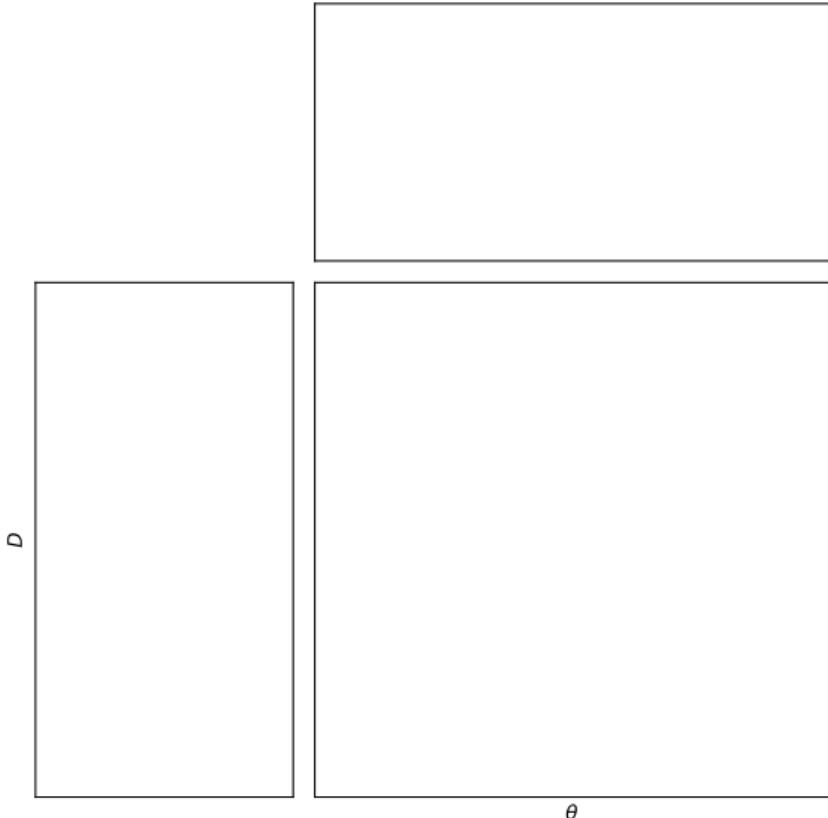


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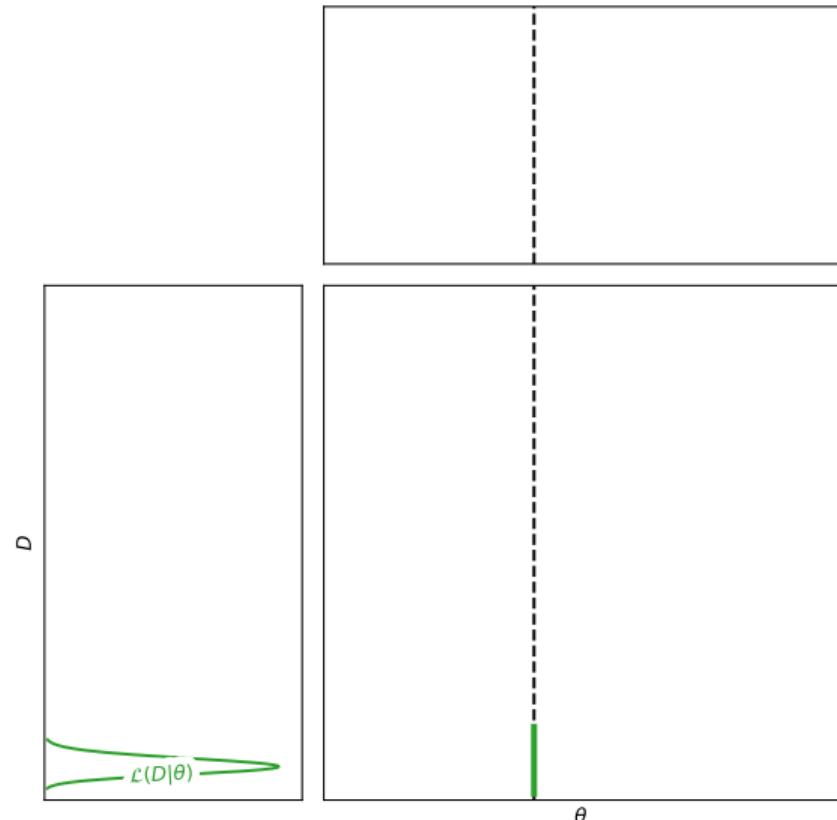
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- ▶ What do you do if you don't know $\mathcal{L}(D|\theta)$?
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- ▶ Task of SBI is take joint \mathcal{J} samples and
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- ▶ Present state of the art achieves this using
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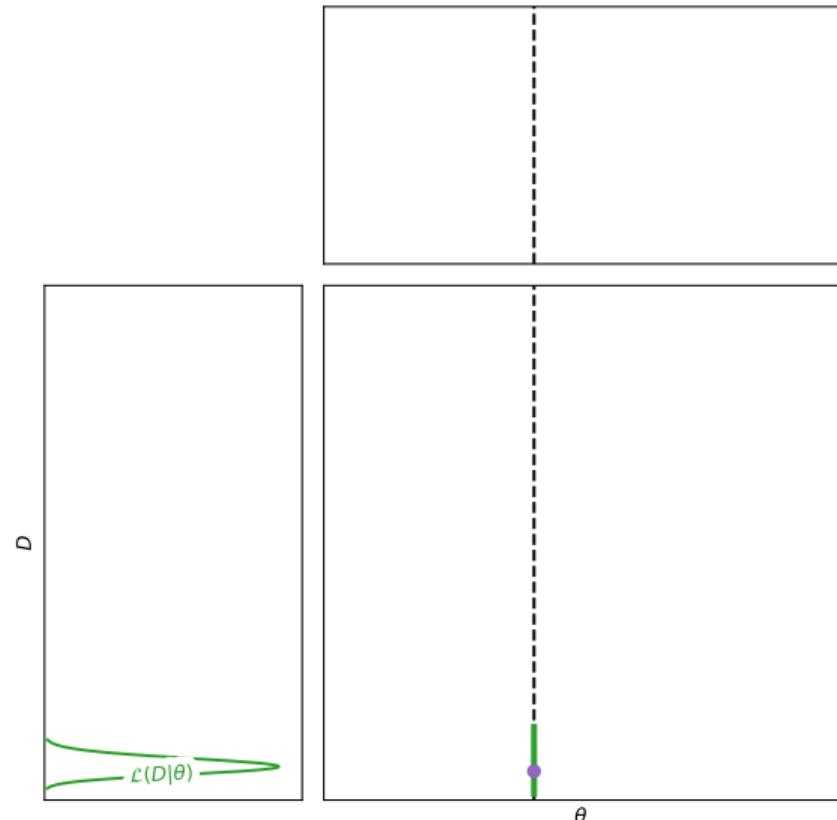
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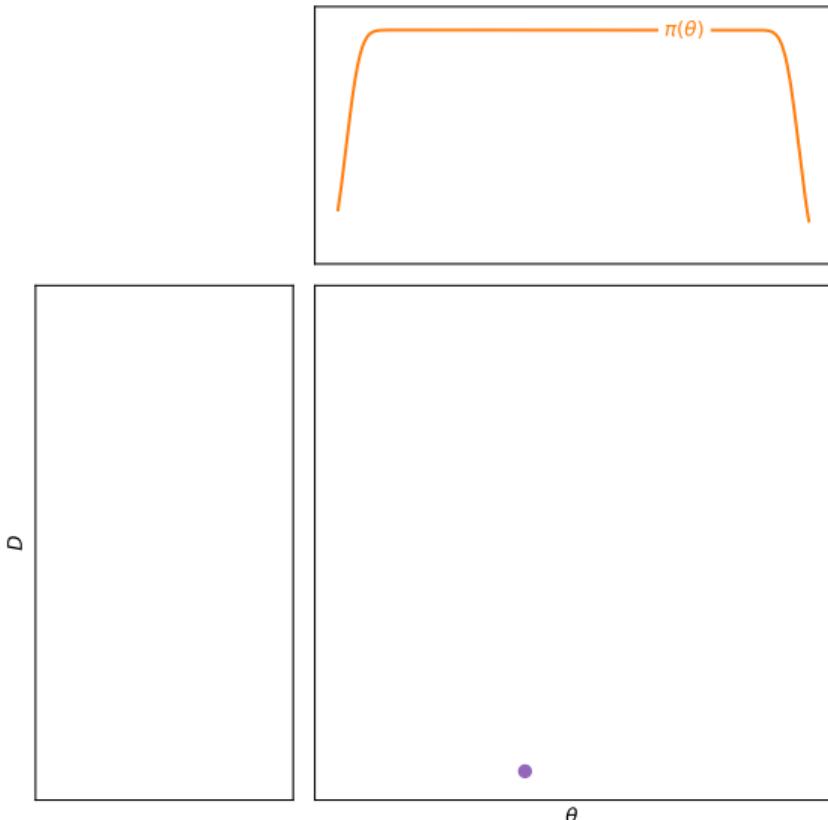
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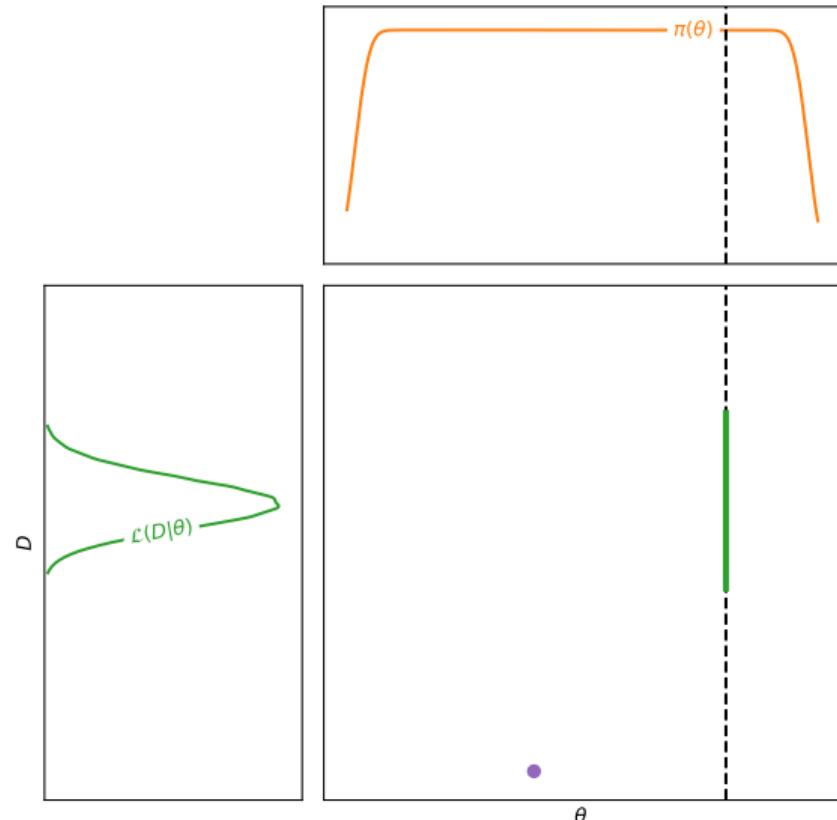
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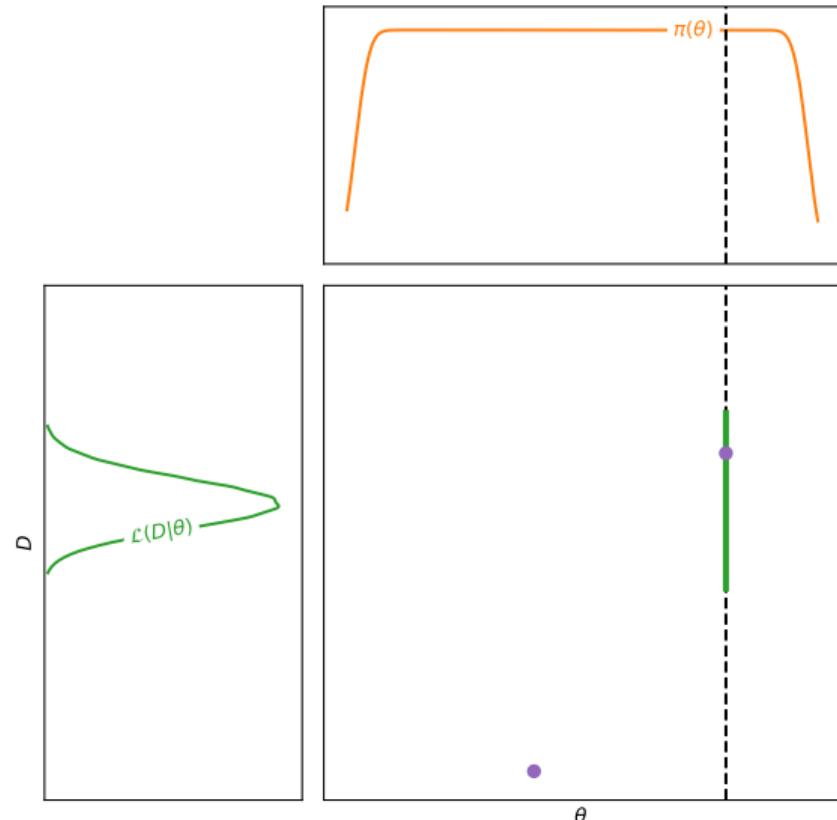
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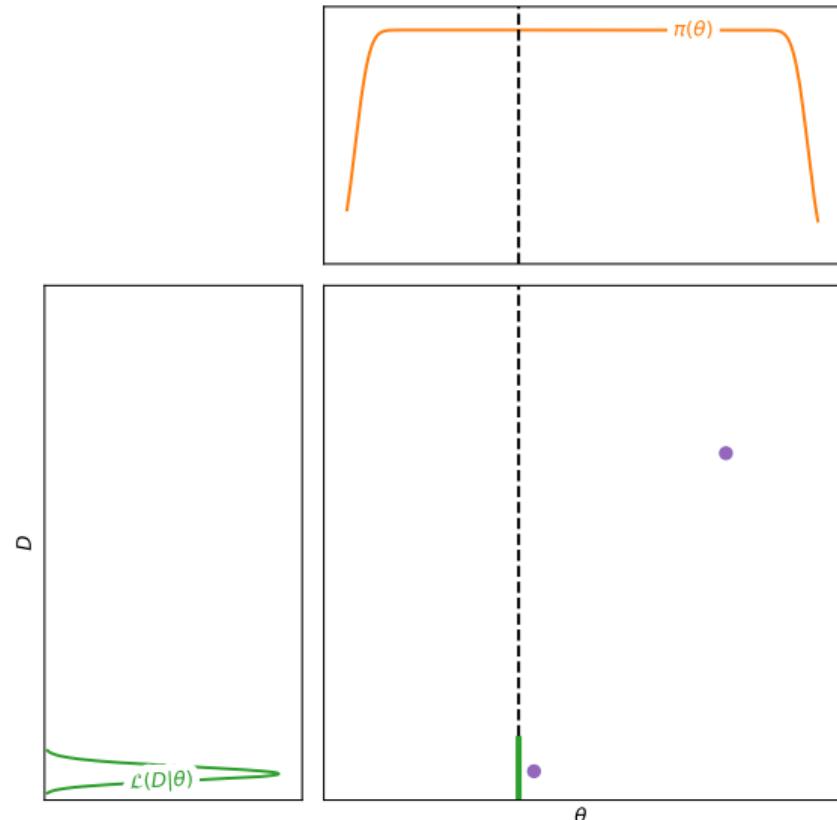
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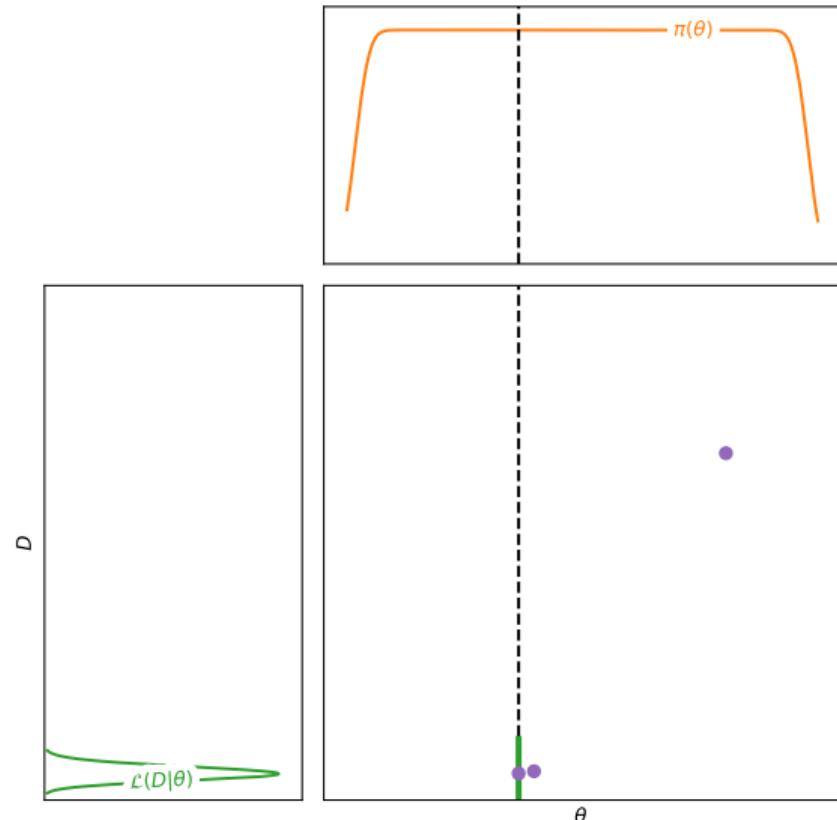
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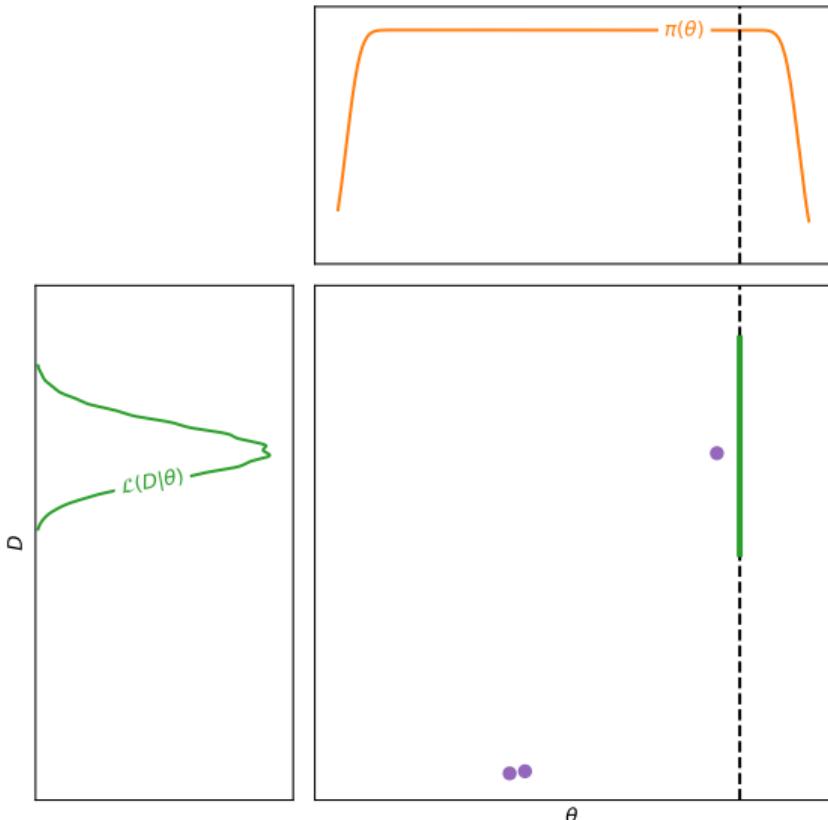
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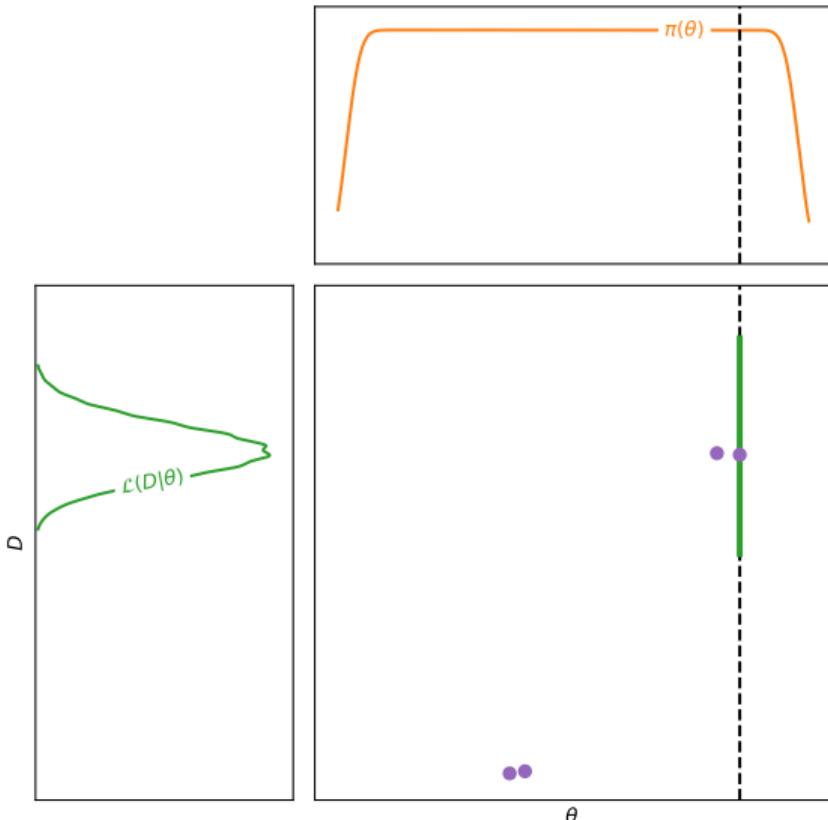
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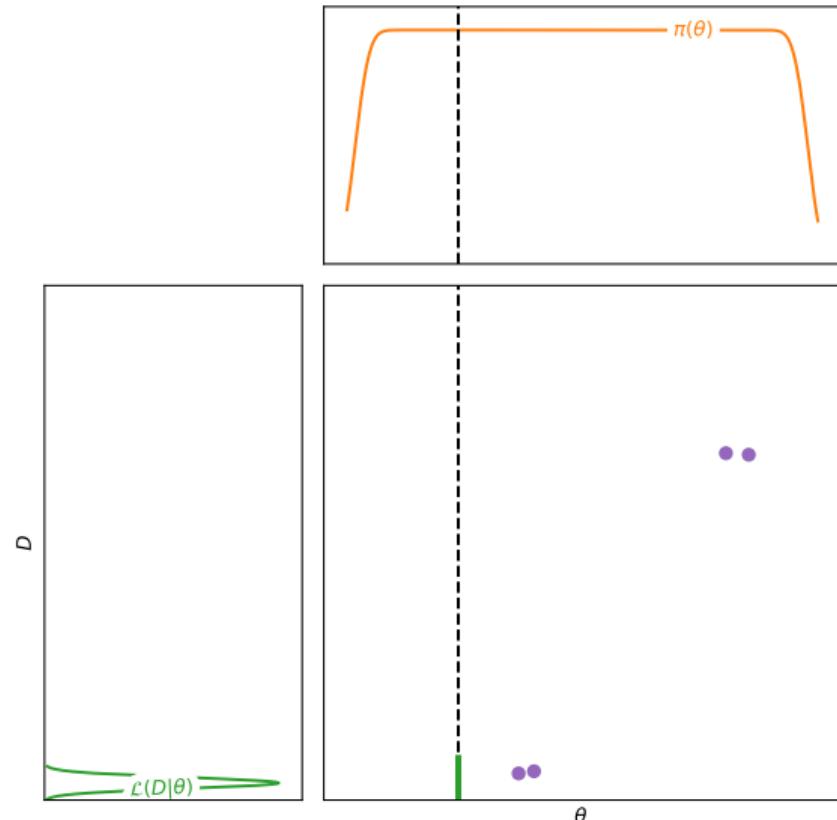
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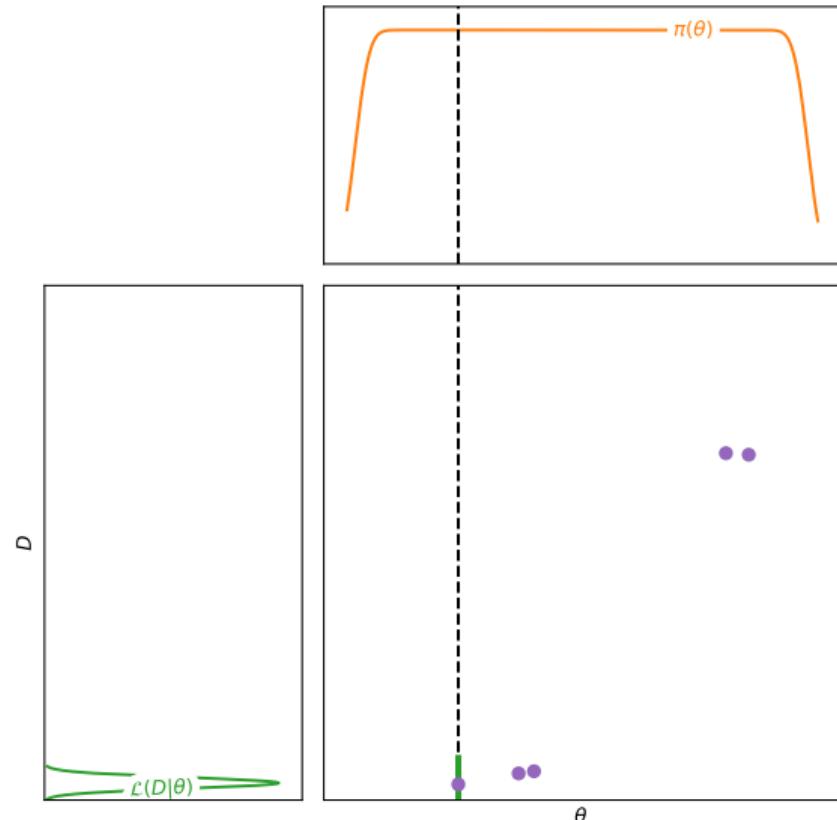
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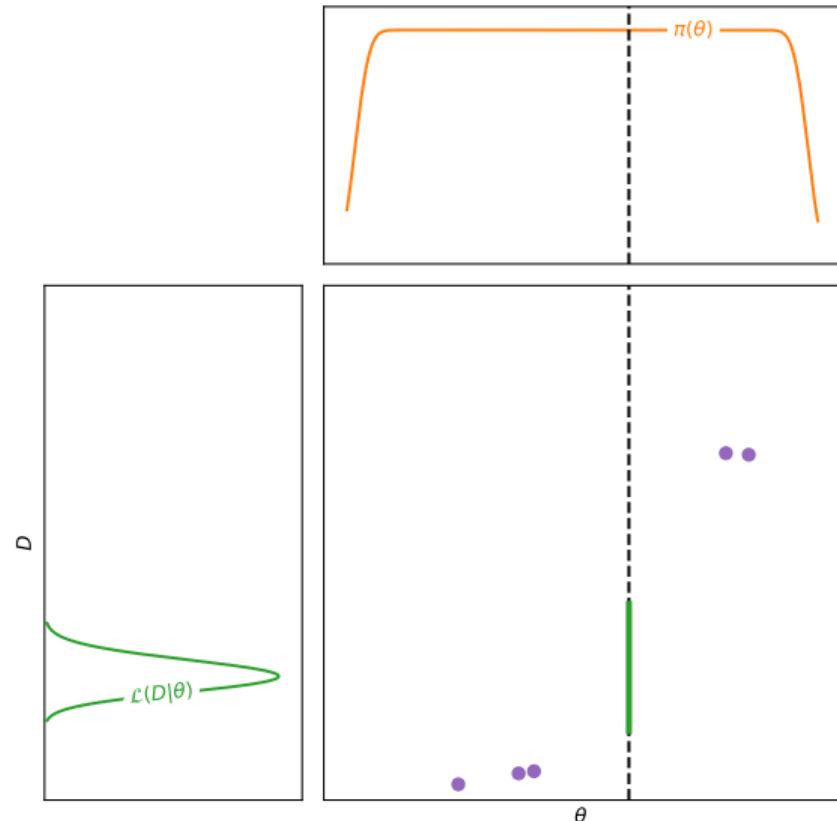
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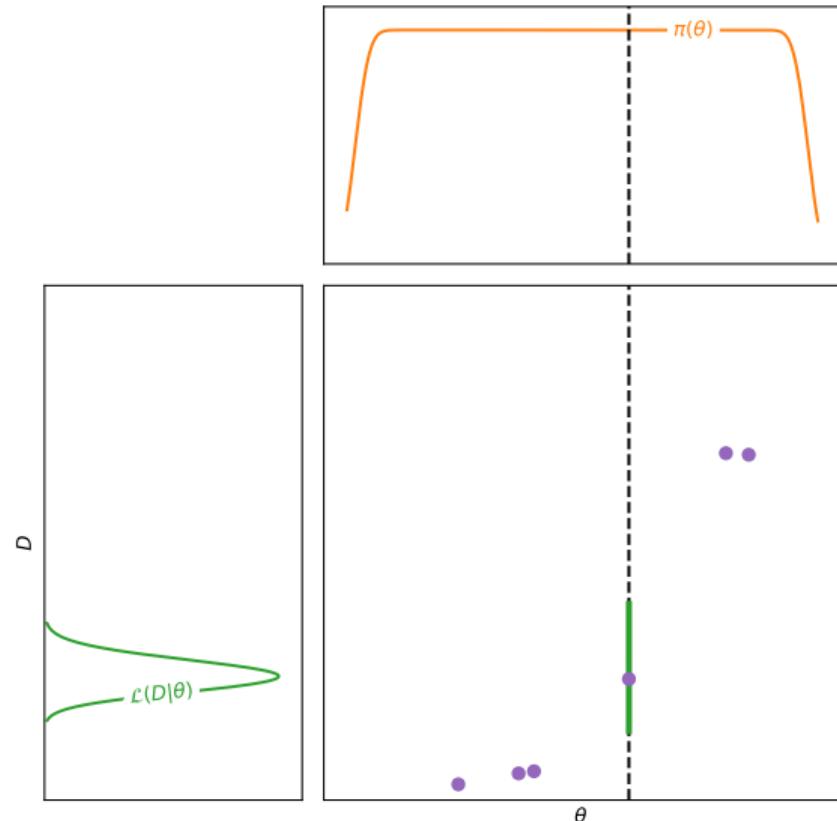
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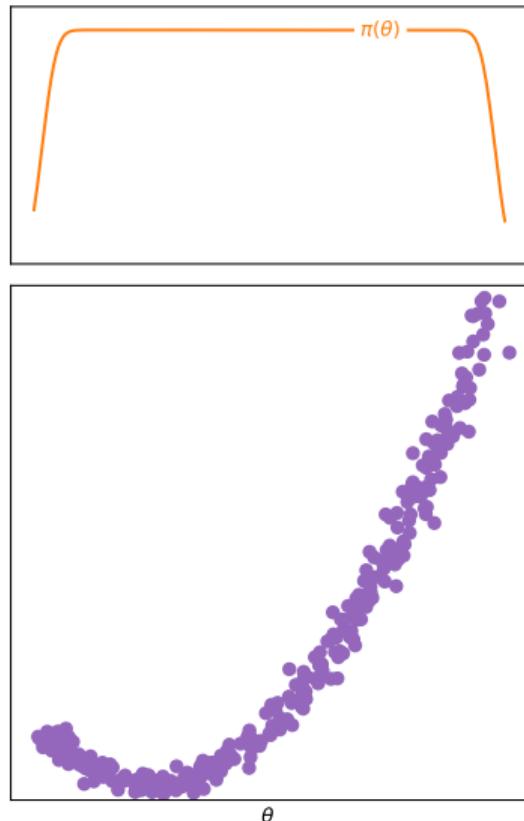
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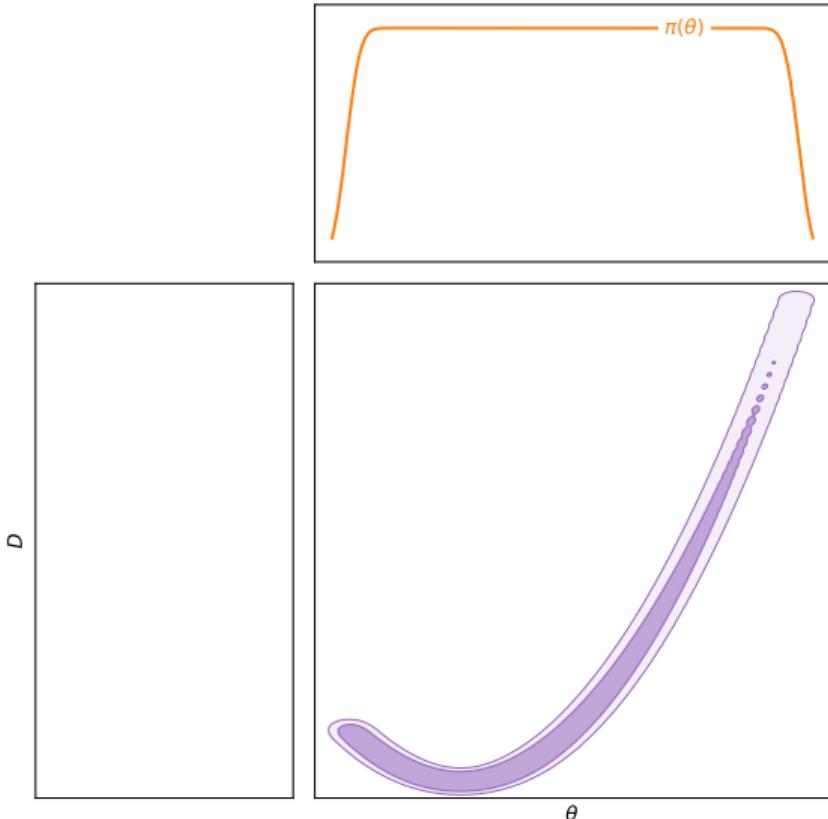
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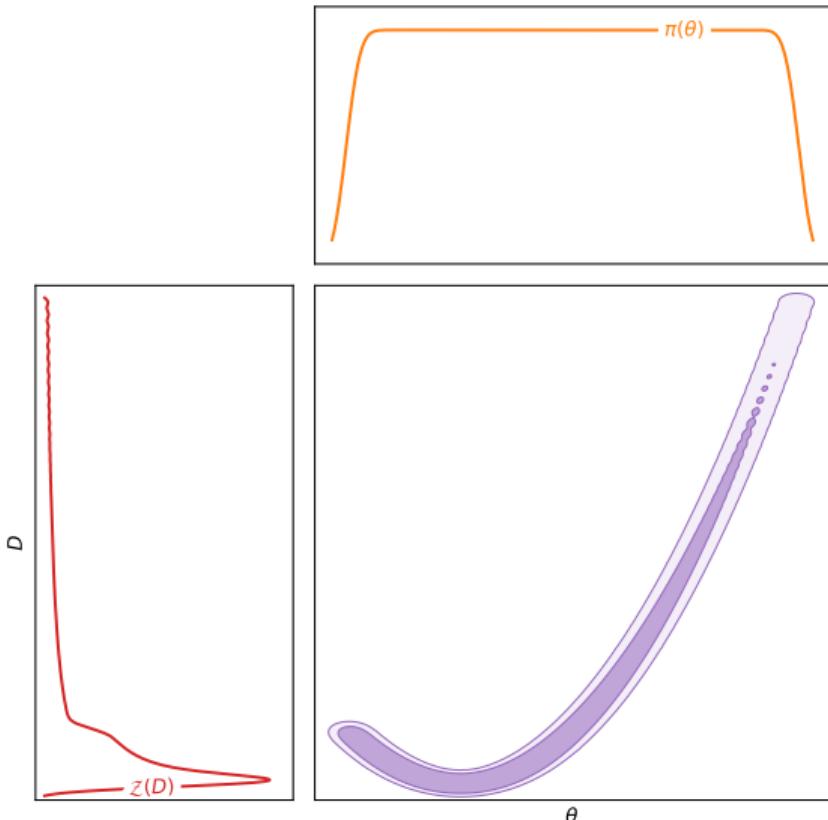
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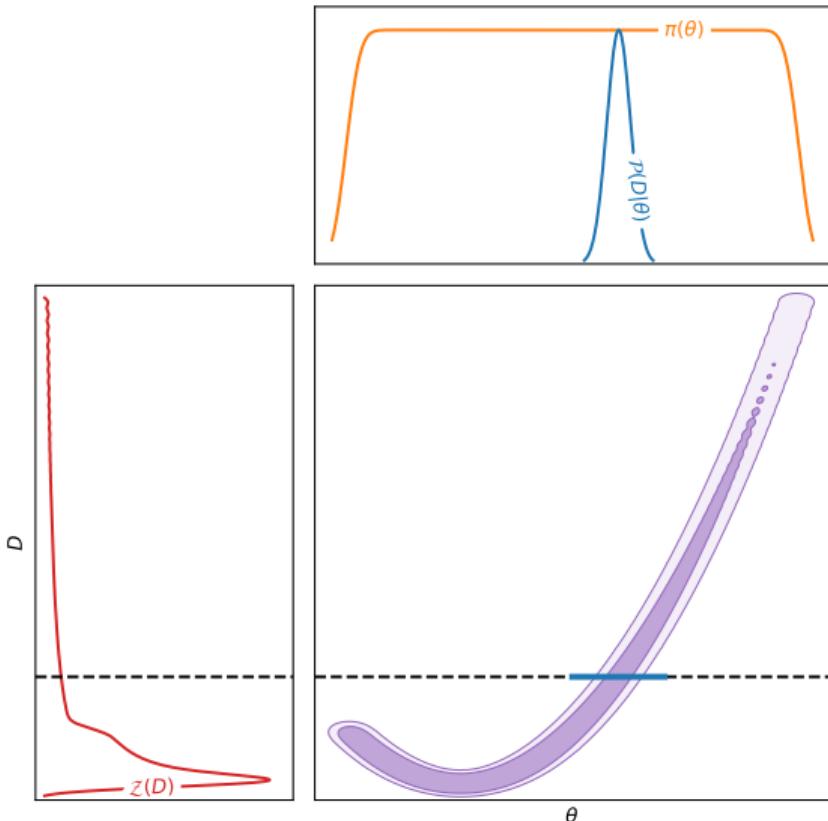
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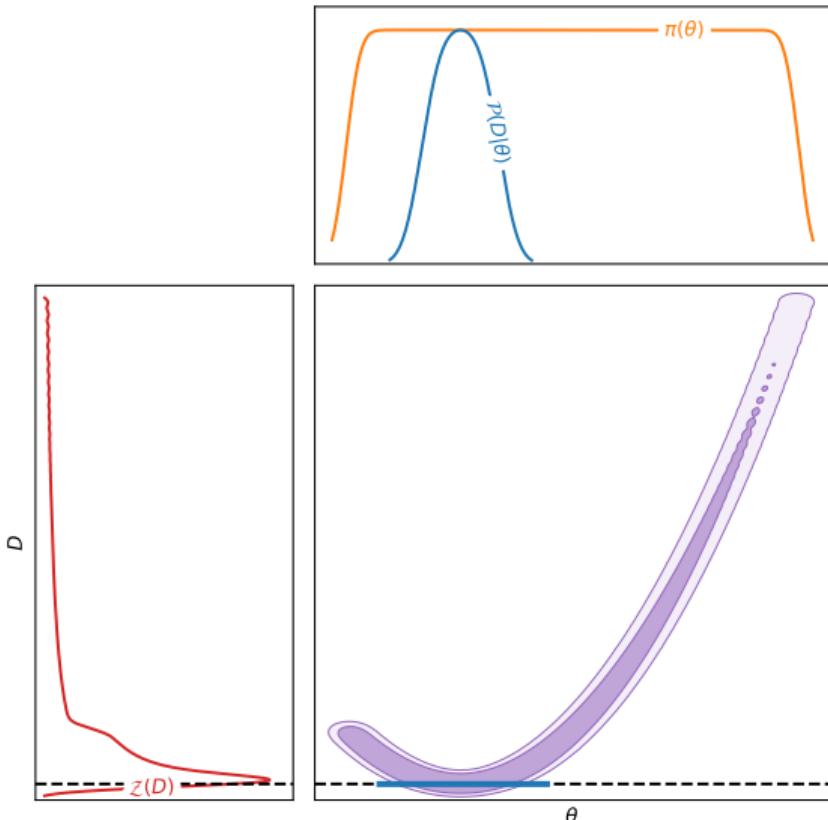
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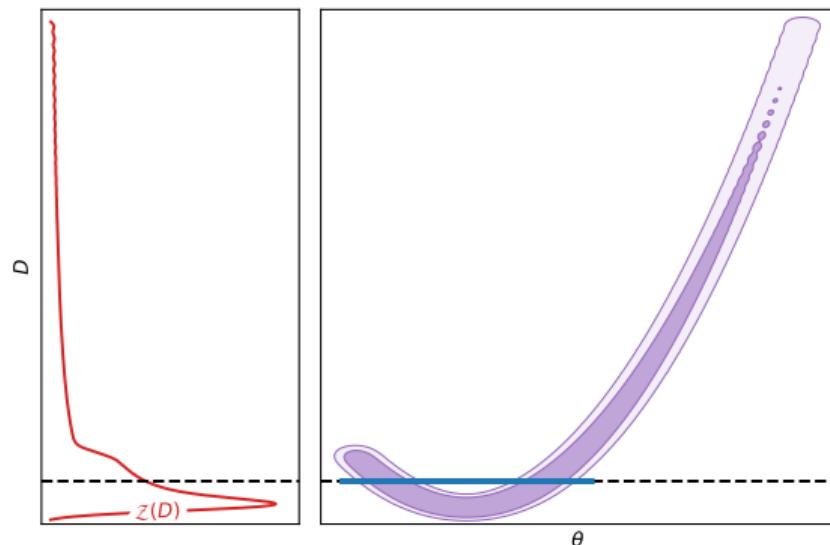
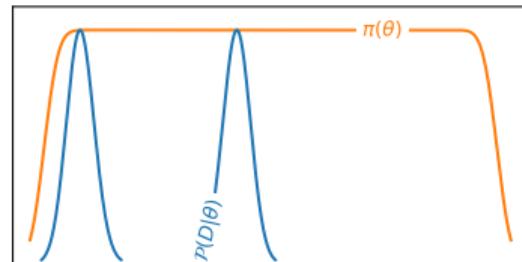
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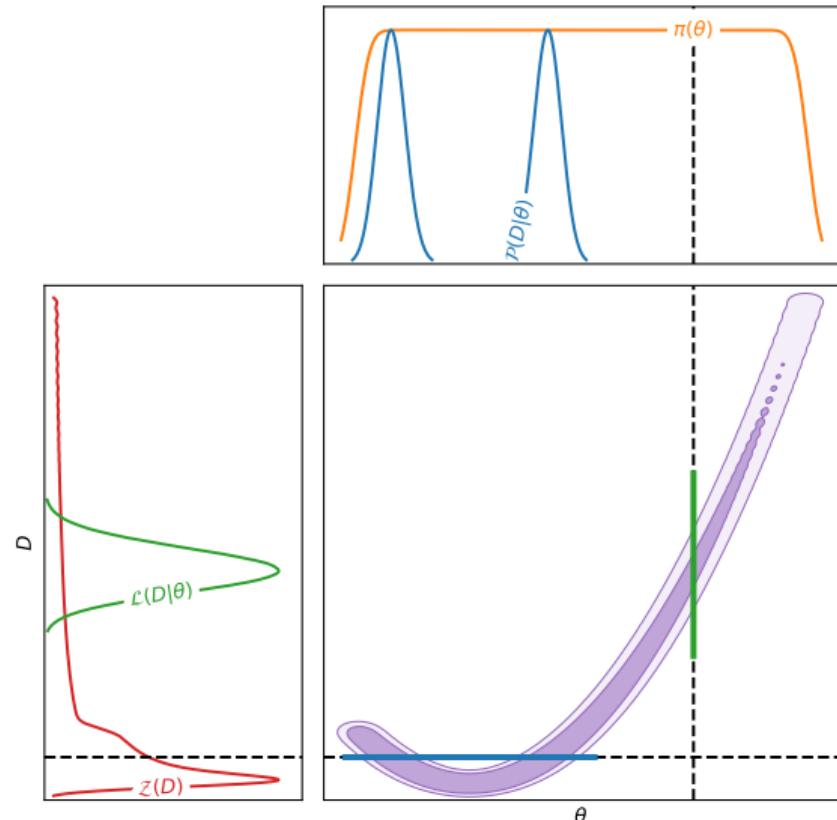
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Why linear SBI?

If neural networks are all that, why should we consider the regressive step of going back to linear versions?

- ▶ It is **pedagogically** helpful
 - ▶ separates general principles of SBI from the details of neural networks
 - ▶ (particularly for ML skeptics)
- ▶ It is **practically** useful
 - ▶ for producing expressive examples with known ground truths
- ▶ It is **pragmatically** useful
 - ▶ competitive with neural approaches in terms of accuracy
 - ▶ faster and more interpretable

Linear Simulation Based Inference

Mathematical setup and Bayesian Inference

- ▶ Linear generative model (m, M, C)

$$D \sim \mathcal{N}(m + M\theta, C)$$

where:

θ : n dimensional parameters

D : d dimensional data

M : $d \times n$ transfer matrix

m : d -dimensional shift

C : $d \times d$ data covariance

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$$\pi : \begin{cases} m | M, C, \sim \mathcal{N}(D_0 - M\theta_0, \frac{1}{\lambda_0} C), \\ M | C, \sim \mathcal{MN}(M_0, C, \Omega_0^{-1}), \\ C \sim \mathcal{W}_{\nu_0}^{-1}(\Psi_0) \end{cases}$$

Linear Simulation Based Inference

Mathematical setup and Bayesian Inference

Toby Lovick

mathematical derivations



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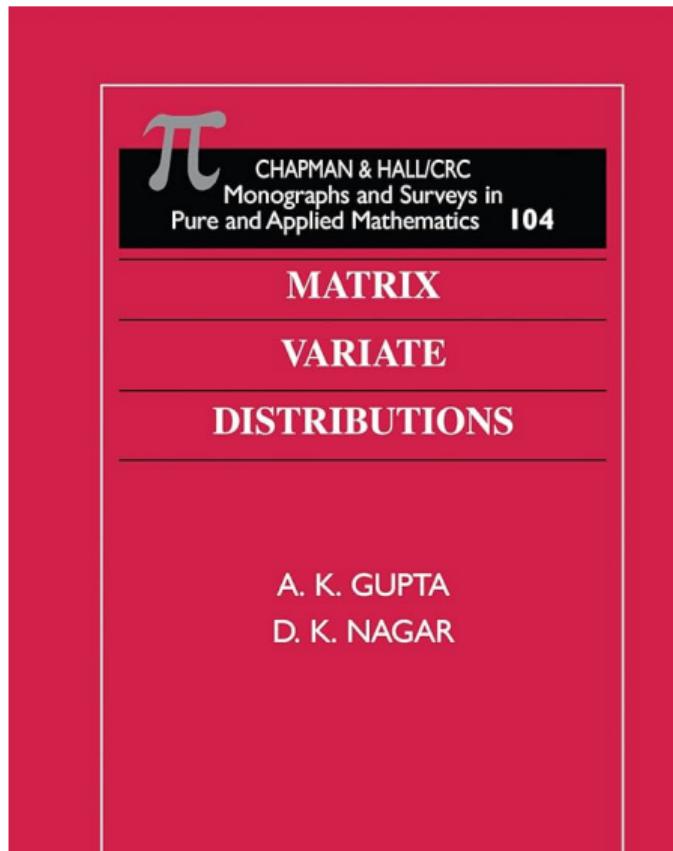
LSBI vs Neural SBI: Explainability

Neural SBI challenges:

- ▶ Black-box approximations
- ▶ Hyperparameter tuning required
- ▶ Saturation at $-5 < \log r < 5$
- ▶ Training instability
- ▶ Difficult convergence diagnostics

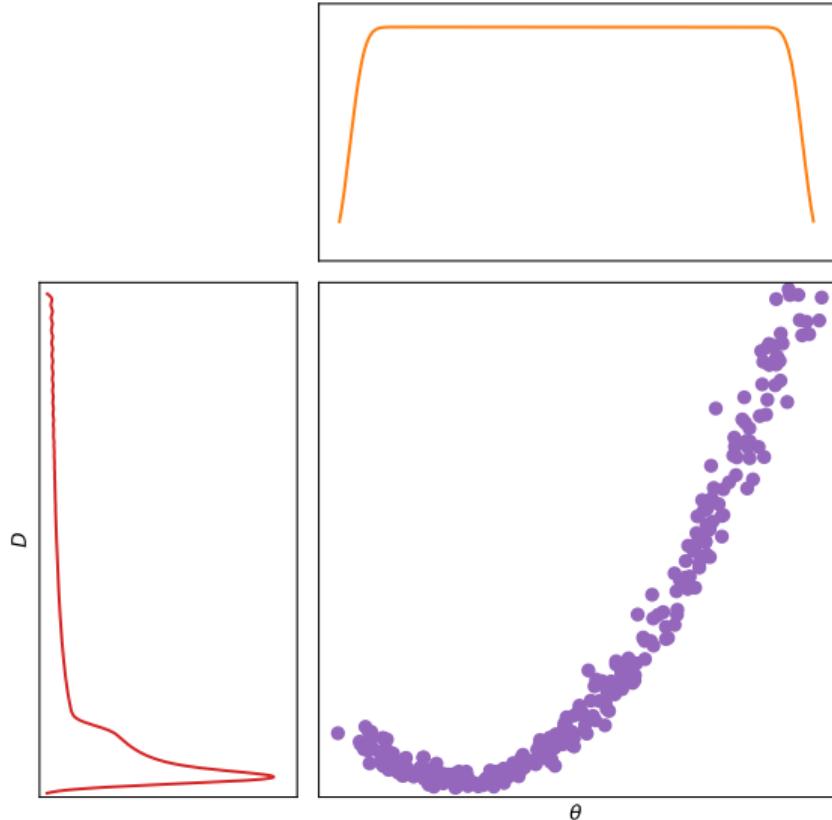
LSBI advantages:

- ▶ Transparent mathematical foundation
- ▶ Self-tuning (no hyperparameters)



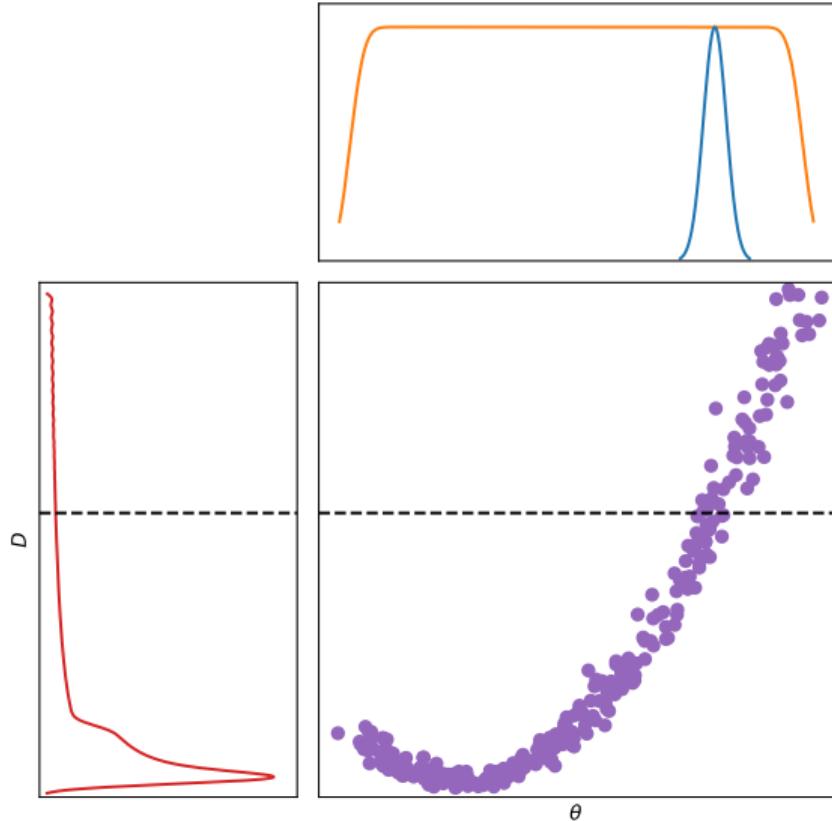
Sequential LSBI

- ▶ Same model as before
- ▶ Mark the observed data D_{obs}
- ▶ Fit a model using lsbi
- ▶ Evaluate the posterior (cheap as linear)
- ▶ Now use this posterior to pick $\{\theta_i\}$
- ▶ Generate $\{D_i\}$ from original simulator
- ▶ Fit lsbi to these
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- ▶ Iterate until convergence



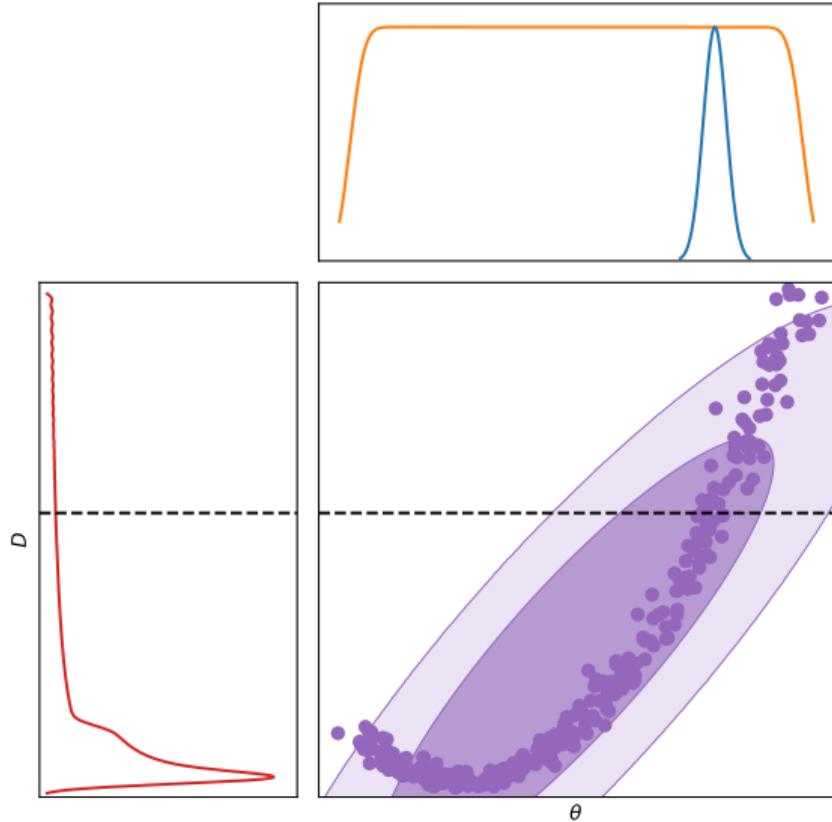
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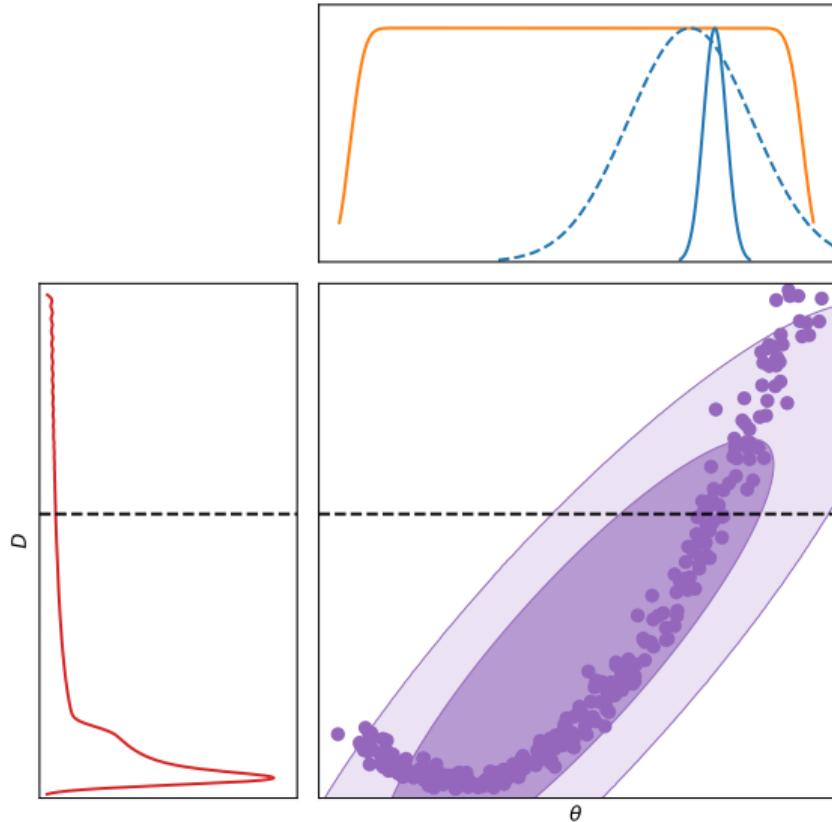
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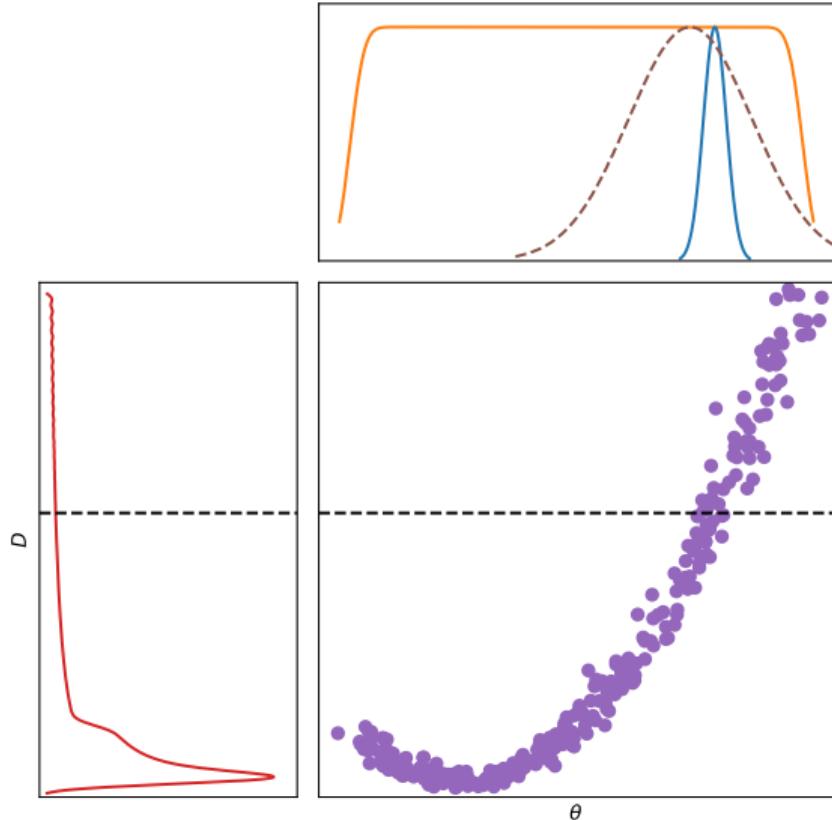
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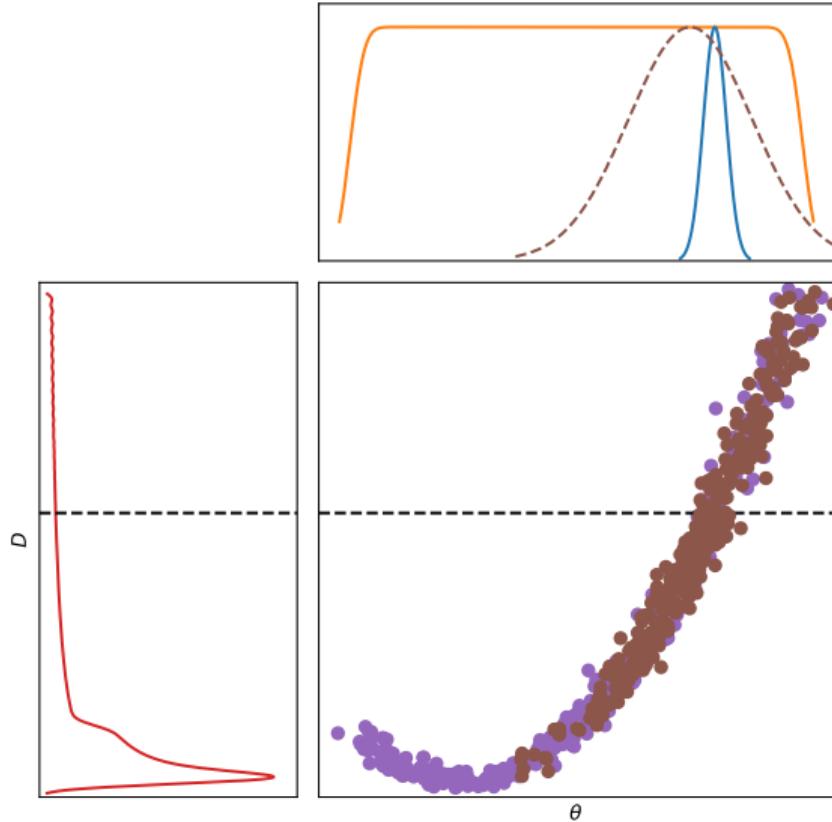
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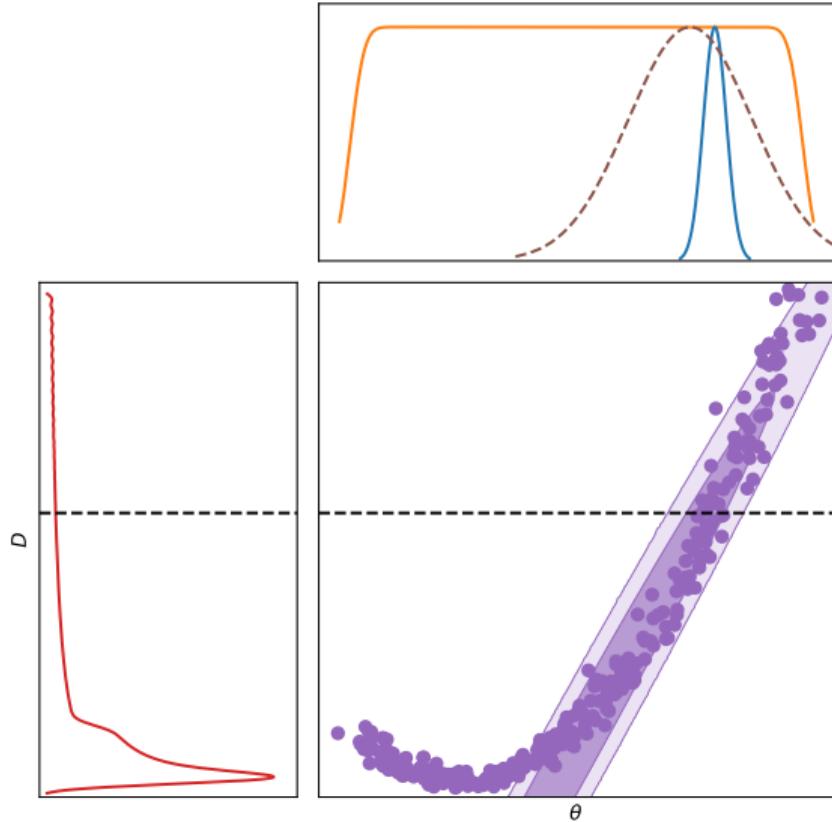
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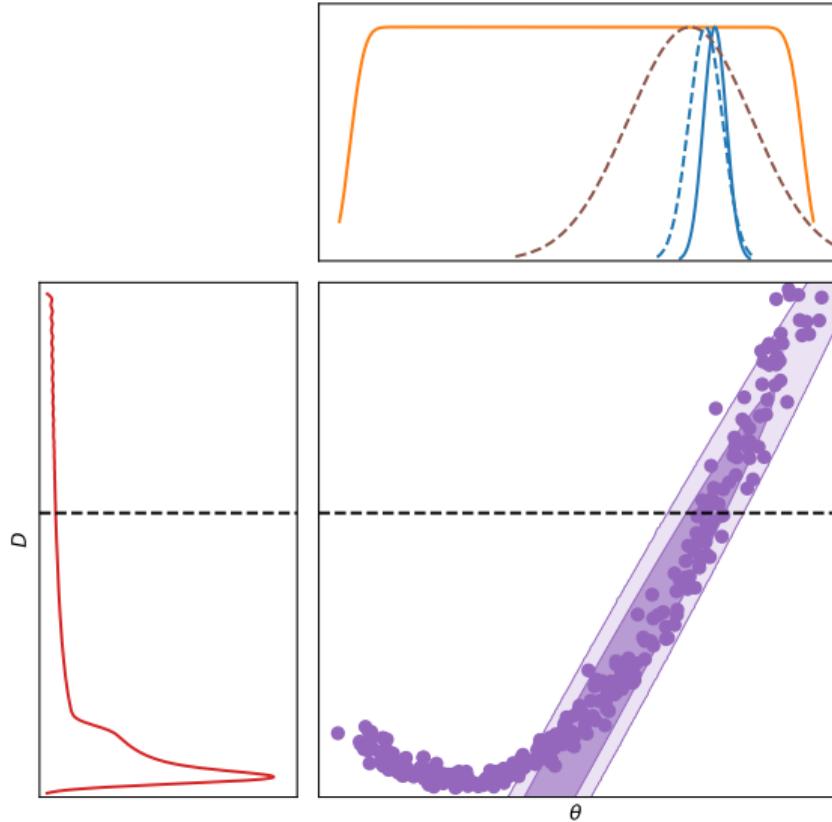
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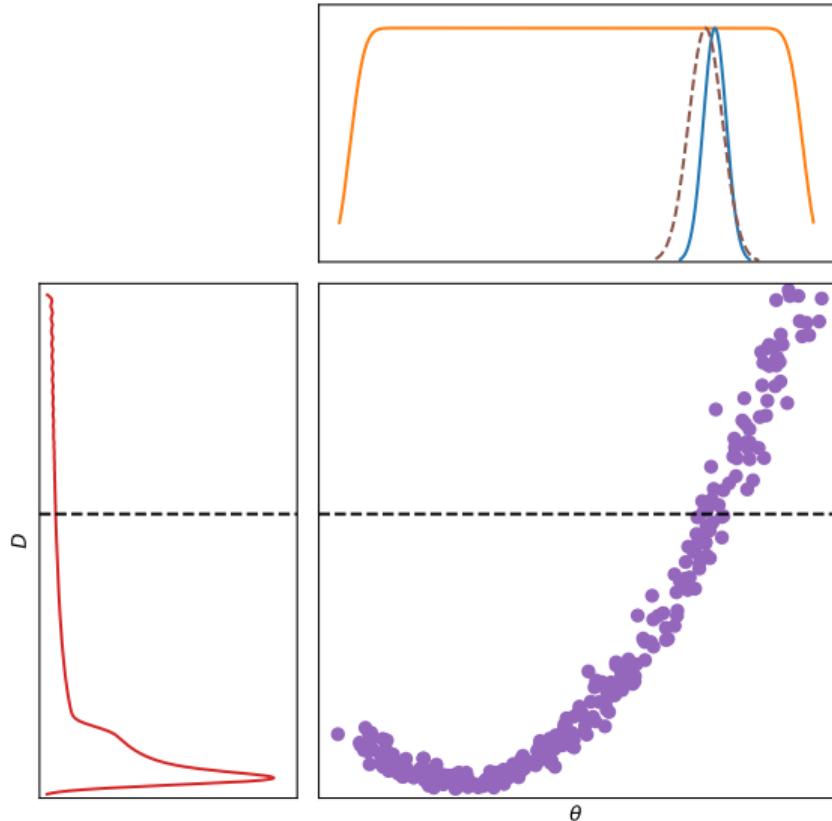
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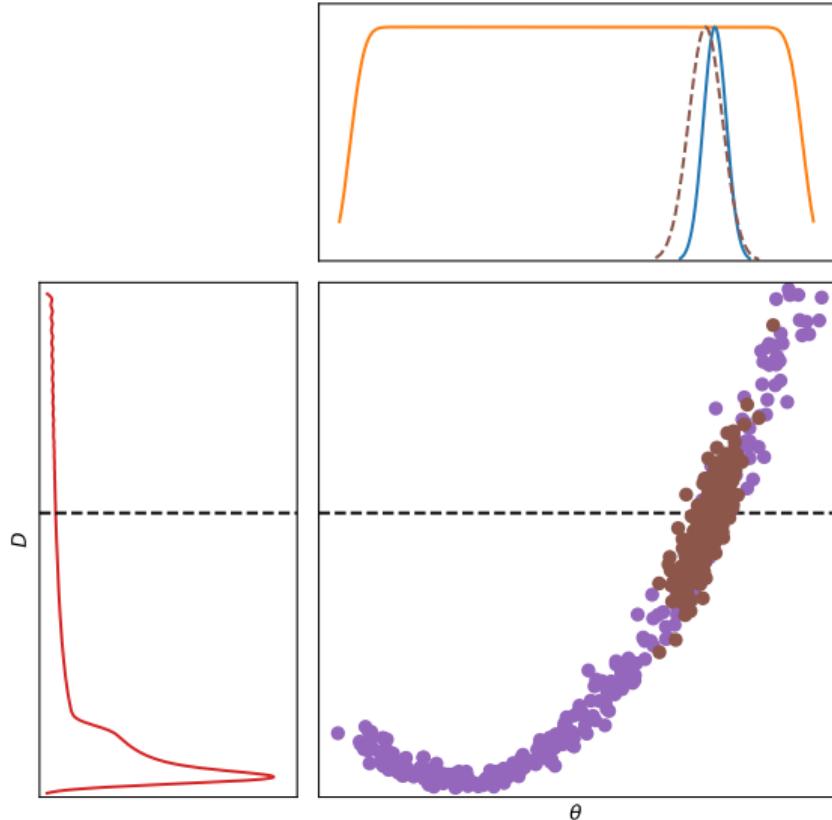
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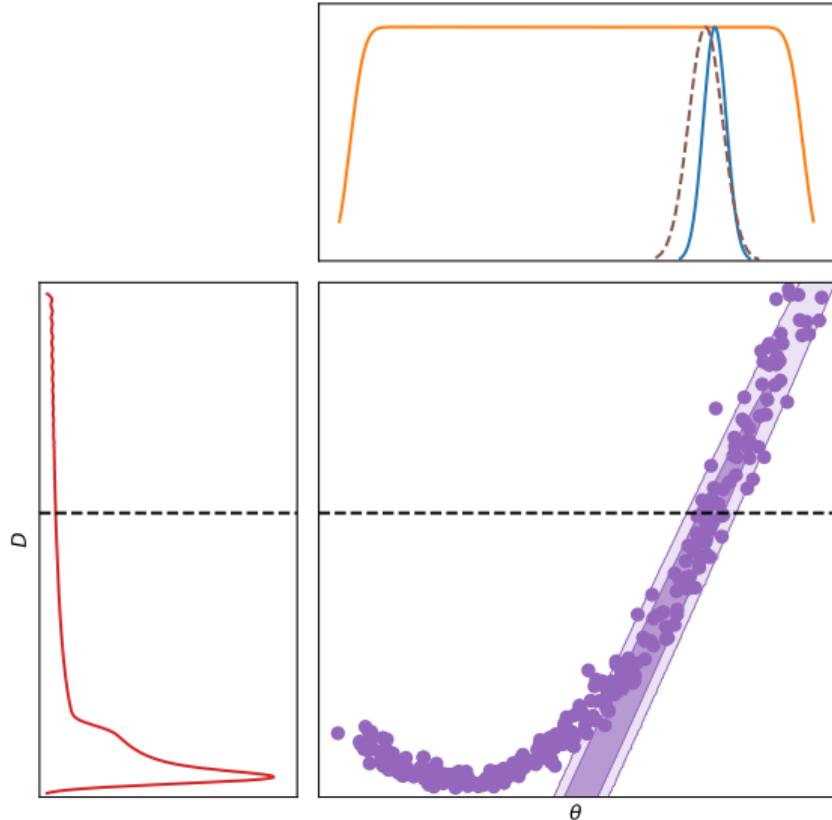
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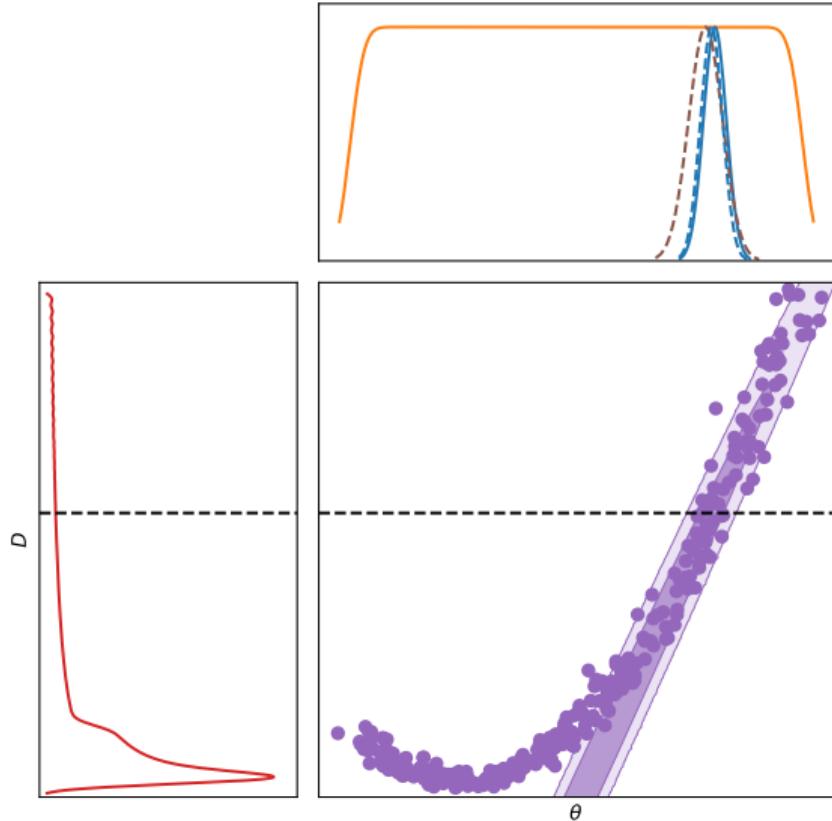
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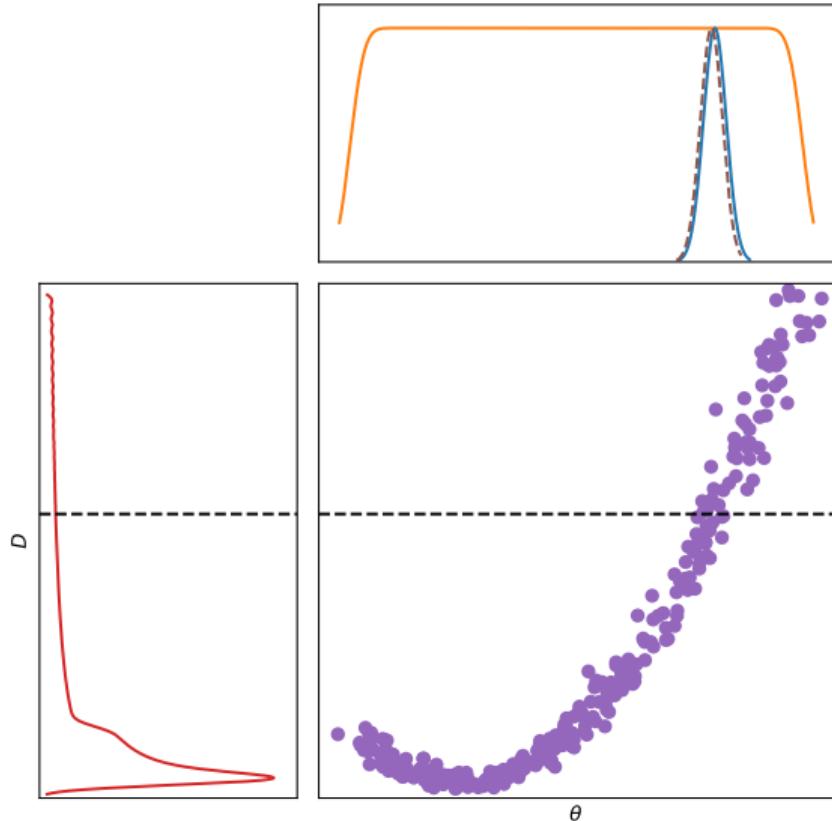
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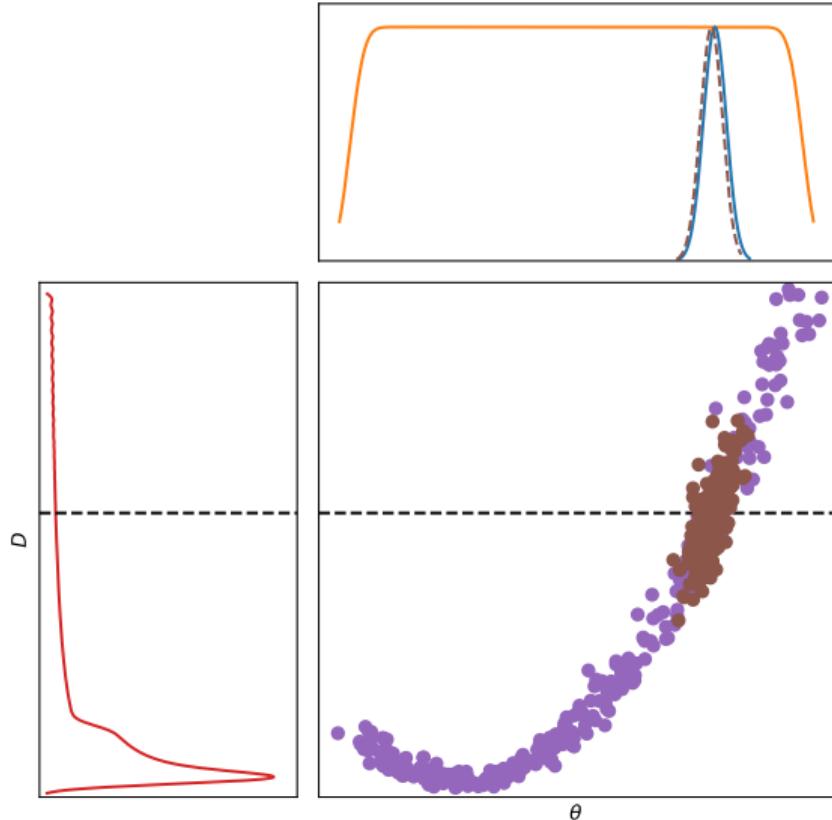
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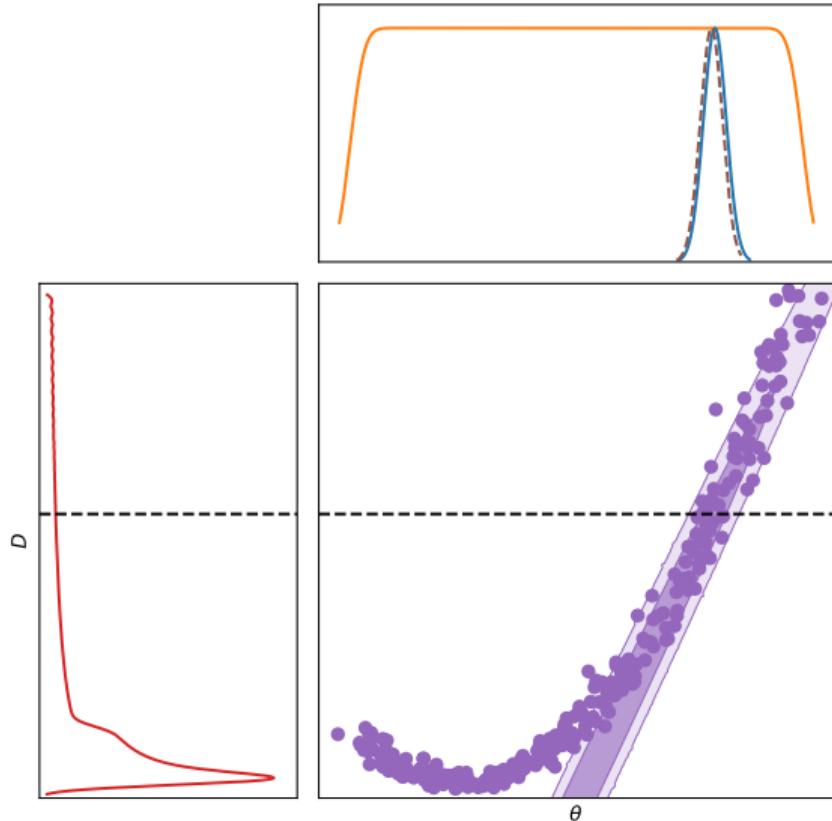
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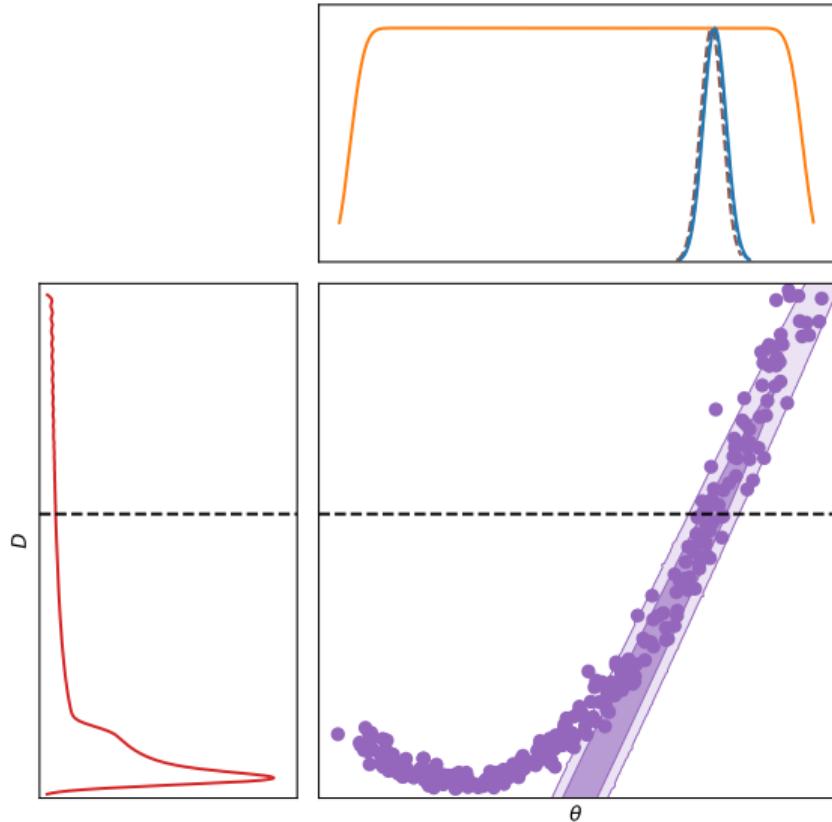
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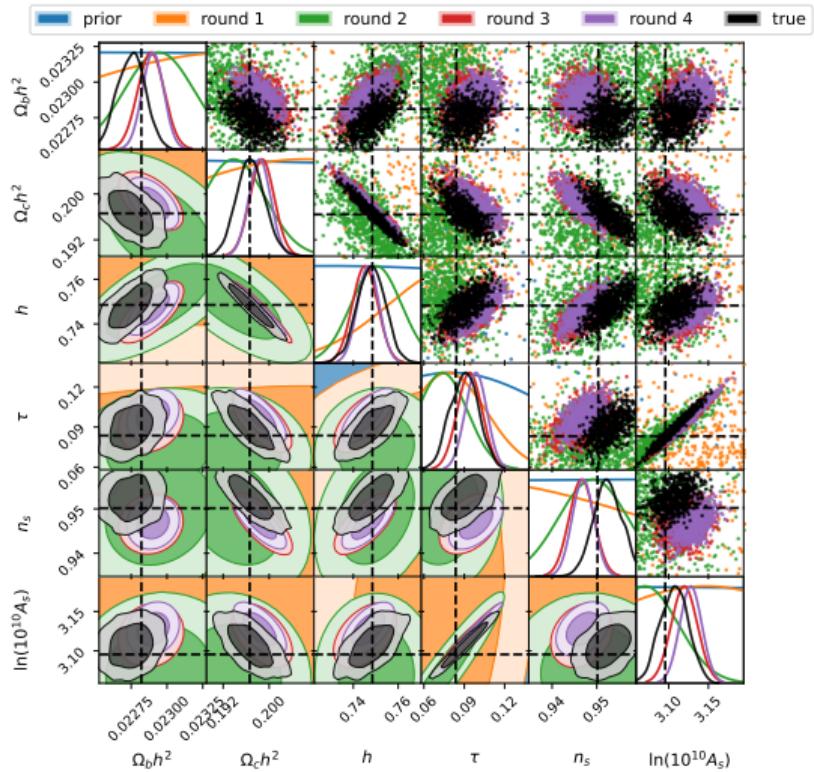
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Cosmology Example: Λ CDM from CMB

- ▶ Apply LSBI to cosmic microwave background
- ▶ Cosmic-variance limited, temperature-only, full sky
- ▶ $n = 6$ parameters, $d = 2500$ data dimensions
- ▶ Sequential improvement around observed data
- ▶ Competitive with traditional methods



lsbi: Code & Implementation

- ▶ lsbi is a pip-installable python package
- ▶ Extends `scipy.stats.multivariate_normal`
 - ▶ Vectorised distributions with broadcastable arrays
 - ▶ `.marginalise(...)` and `.condition(...)` methods
 - ▶ Built-in plotting functionality
- ▶ Implements `LinearModel` class with interpretable methods:
 - ▶ `.prior()`, `.likelihood(theta)`, `.posterior(D)`
 - ▶ `.evidence()` - all return distributions
- ▶ Also implements `MixtureModel` for nonlinear effects
- ▶ [2501.03921] and github.com/handley-lab/lsbi

Results & Recent Work: [2501.03921]

Performance, scaling, and key contributions

Key advantages:

- ▶ Analytical solutions (no iterative training)
- ▶ Transparent mathematical operations
- ▶ Competitive with neural methods
- ▶ Faster than neural training, no GPU requirements
- ▶ Self-tuning (no hyperparameter search)
- ▶ Handles $n \approx 6$, $d \approx 2500$ problems

Key contributions ([2501.03921]):

- ▶ Mathematical foundation using matrix-variate distributions
- ▶ Sequential refinement algorithm for nonlinear problems

Results demonstrate:

- ▶ Competitive accuracy on standard benchmarks
- ▶ Significant computational speedup
- ▶ Enhanced interpretability
- ▶ Pedagogical value for understanding SBI principles

Available at:

- ▶ [\[2501.03921\]](#)
- ▶ github.com/handley-lab/lsbi

Master's student project led to

willhandley.co.uk/talks

<wh260@cam.ac.uk>

Conclusions



github.com/handley-lab/group

- ▶ **lsbi** provides explainable alternative to neural SBI methods
- ▶ Separates SBI principles from ML implementation details
- ▶ Competitive accuracy with interpretable mathematical foundation
- ▶ Available: [[2501.03921](#)] and github.com/handley-lab/lsbi
- ▶ Future: realistic simulations, model comparison, importance sampling