Cataloging the Motion of Co-orbitals of the Galilean Satellites. B.R. Scott and B.G. Bills
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Introduction: Small bodies in the solar system commonly move along orbits which are in 1:1 mean motion resonances with larger bodies. The best known case is the large population of asteroids ("Greeks" and "Trojans") co-orbital with Jupiter [**1**]. Other planets, including Earth, Mars, Venus and Neptune have smaller populations [**2,3,4,5**], while Saturn appears to have none. On the other hand, two of Saturn's satellites (Tethys and Dione) each have at least two co-orbital companions [**6,7**].

This prompts us to ask: do any of Jupiter's Galilean satellites have co-orbitals? When informally posing this question to colleagues, a common response has been that the Laplace resonance would quickly destabilize any such co-orbitals. The asymmetry between Jupiter and Saturn, in this regard, is often attributed to the effect of the near 5:2 mean motion resonance (the "Great Inequality") between them [**8**]. However, both Tethys and Dione are in low order resonances [**9**]. This raises the additional question: how do other orbital resonances influence a 1:1 mean motion resonance?

In general, co-orbitals will only exist if some process, such as in situ formation or capture, supplies them to the Lagrange points and the local environment leads to long term stability. In the latter respect, the Galilean satellite system is an attractive object of study. The system consists of four large satellites, with the inner three in the 4:2:1 Laplace resonance, and the outer two in a 7:3 resonance (the "De Haerdtl inequality"). [**10, 11**].

We begin our study of the stability of Galilean satellites co-orbitals by numerically integrating the equations of motion and analyzing the Fourier amplitude spectrum of the resulting motion.

Numerical Integration: In our analysis, the motions of the Galilean satellites are specified by the model E5 of Lieske, truncated to include the dominant terms [**12, 13**]. This model includes the oblate figure of Jupiter, mutual perturbations between pairs of satellites, and perturbations from Saturn and the Sun.

The initial positions and velocities of co-orbital test particles are specified by a rotation of the state vector of the Galilean satellite with which it shares an orbit, on a reference date, through a given angle. That rotation angle is the single parameter we use to explore the 6 dimensional space of possible initial conditions.

Integrations are carried out for 100,000 days, which is several hundred times the longest forcing period. Uncertainties in the integration are checked by integrating back to the initial time from the final time. Because our model does not include any dissipation, a perfect integrator would return the body exactly to its initial state. In our case, the errors are consistently less than the uncertainty in the specified satellite positions.

Catalog of Motion: In order to examine the dynamics of the co-orbitals, we compute the time-varying angular separation between the co-orbital and its parent. We then compute the corresponding Fourier amplitude spectrum, in order to identify the frequencies at which the angular separation oscillates. These frequencies are then compared to the known forcing frequencies.

Fundamental Frequencies. A linearized stability analysis of motion about the L4 or L5 Lagrange points, of the circular restricted three body problem, predicts oscillations in angular separation at two main frequencies. The short period term is close to the orbital period of the parent, and the long period term depends on the mass ratio between the parent and the central body. [**14**]. In the 3-body problem, with large angular separation, harmonics of these two fundamental terms emerge. In the six body problem that we consider here, these same frequencies appear. . Additional frequencies emerge at the Solar and Saturn forcing frequencies, through their perturbations to the motions of the Galilean satellites.

Harmonics and Beats. Additional major peaks primarily occur at integer multiples and linear combinations of the linearized periods and additional forcing terms. These terms grow in prominence with increasing departure from the linearized case, and for the inner partners in the Laplace resonance (e.g. Harmonics and Beats are more important for co-orbitals of Io and Europa than Ganymede and Callisto).

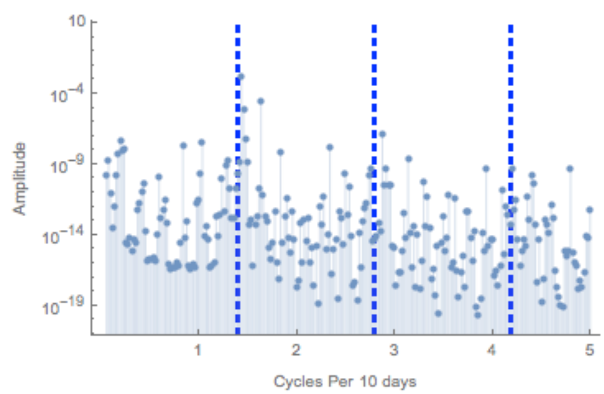

Figure 1 Amplitude Spectrum for the Angular Separation of a Co-orbital of Ganymede at a rotation of the Ganymede state vector through an angle π/6. The dashed lines are placed at harmonics of the long period term in the linearized approximation.

Stability. In some previous work on stability in the three body problem, the infinitesimal mass is asserted to be unstable if its motion crosses the axis linking the two finite masses [**15**]. Elsewhere, we have shown that a class of stable orbits exists, which can cross the x-axis without adverse influence [**16**], and that this particular criterion is not meaningful. Instead, we argue that stability is better understood in terms of the response of the system to external perturbations.

Co-orbitals in the rotating frame exhibit the behavior expected from the circular restricted three body problem. The minimum amplitude "tadpole" is found at the approximate L4/L5 location in the rotating frame. Transitions to horseshoe orbits occur near the approximate L3 location, while transitions to unstable orbits occur when the initial state vector is obtained by a small rotations from the parent body. For rotations to points near L4/L5 and in the range of rotations between those points and L3, the co-orbital exhibits stable oscillations in the rotating frame on long timescales.

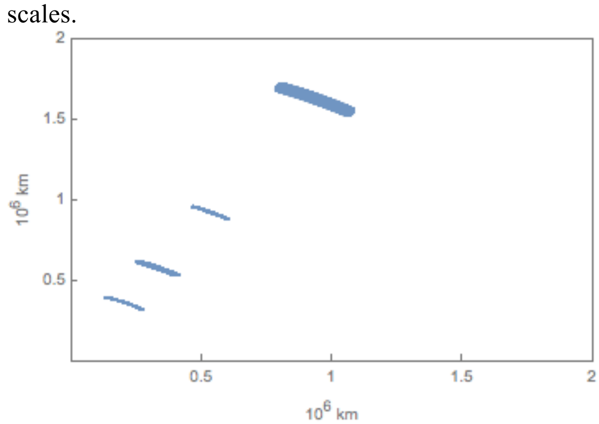

Figure 2 Positions in the rotating frame for coorbitals initially placed at the nominal L4 point in the rotating frame. These represent amplitudes near the minimum case.

Preliminary Conclusions: If placed there today, a body co-orbital with any of the four Galilean satellites would exhibit stable long and short period oscillations predicted by the restricted three body problem on long timescales. Resonant interactions do not appear to enhance the amplitude of oscillation leading to instability.

Searches for these objects are complicated by scattered light from Jupiter and the Galilean satellites, and to our knowledge, no dedicated search has been conducted. If these bodies do not exist, the Laplace resonance is unlikely to be the explanation. The orbital environment may have been hostile to co-orbitals in the past, or no suitable reservoir population may have existed to transfer material to the Lagrange points.

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