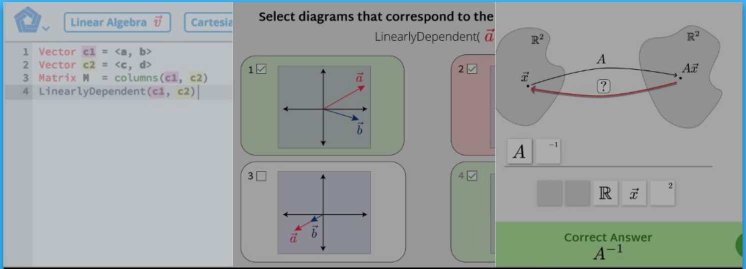
What is Penrose?

Wode "Nimo" Ni

2



Designing Declarative Language Tutorials:
a Guided and Individualized Approach

Anael Kuperwajs Cohen*, Wode Ni, Joshua Sunshine*

15

Defining Visual Mathematical Narratives Declaratively

Max Krieger*, Wode Ni, Joshua Sunshine*

28

What is Penrose?

Wode “Nimo” Ni

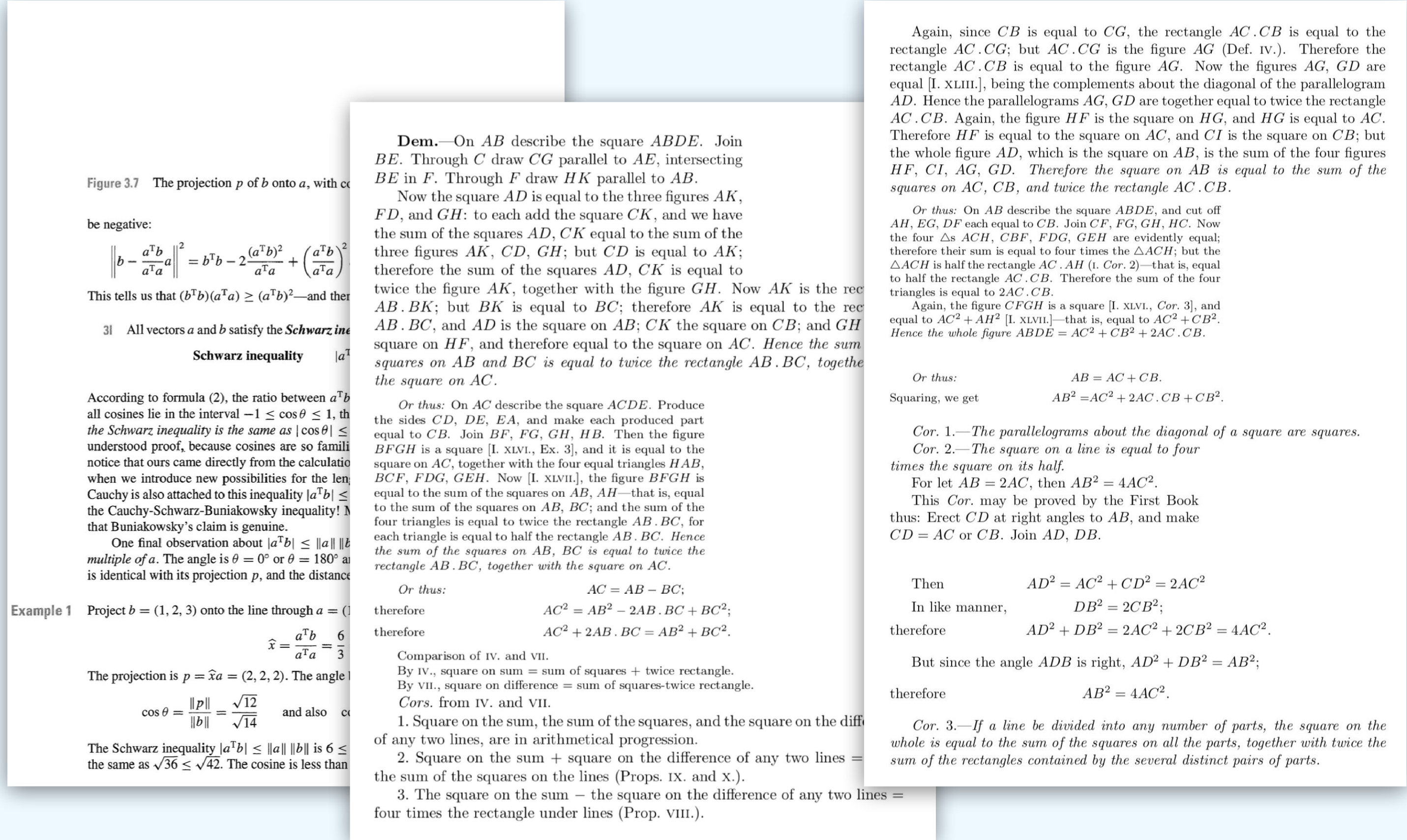


Figure 3.7 The projection p of b onto a, with c

be negative:

$$\left\| b - \frac{a^{\mathrm{T}}b}{a^{\mathrm{T}}a}a \right\|^2 = b^{\mathrm{T}}b - 2\frac{(a^{\mathrm{T}}b)^2}{a^{\mathrm{T}}a} + \left(\frac{a^{\mathrm{T}}b}{a^{\mathrm{T}}a}\right)^2$$

This tells us that $(b^{\mathrm{T}}b)(a^{\mathrm{T}}a) \geq (a^{\mathrm{T}}b)^2$—and ther

3I All vectors a and b satisfy the ***Schwarz ine***

Schwarz inequality $|a^{\mathrm{T}}$

According to formula (2), the ratio between $a^{\mathrm{T}}b$
all cosines lie in the interval $-1 \leq \cos\theta \leq 1$, th
the Schwarz inequality is the same as $|\cos\theta| \leq$
understood proof, because cosines are so famili
notice that ours came directly from the calculatio
when we introduce new possibilities for the len
Cauchy is also attached to this inequality $|a^{\mathrm{T}}b| \leq$
the Cauchy-Schwarz-Buniakowsky inequality! M
that Buniakowsky's claim is genuine.

One final observation about $|a^{\mathrm{T}}b| \leq \|a\|\,\|b$
multiple of a. The angle is $\theta = 0°$ or $\theta = 180°$ a
is identical with its projection p, and the distance

Example 1 Project $b = (1, 2, 3)$ onto the line through $a = ($

$$\hat{x} = \frac{a^{\mathrm{T}}b}{a^{\mathrm{T}}a} = \frac{6}{3}$$

The projection is $p = \hat{x}a = (2, 2, 2)$. The angle

$$\cos\theta = \frac{\|p\|}{\|b\|} = \frac{\sqrt{12}}{\sqrt{14}} \quad \text{and also} \quad \text{c}$$

The Schwarz inequality $|a^{\mathrm{T}}b| \leq \|a\|\,\|b\|$ is $6 \leq$
the same as $\sqrt{36} \leq \sqrt{42}$. The cosine is less than

Dem.—On AB describe the square $ABDE$. Join BE. Through C draw CG parallel to AE, intersecting BE in F. Through F draw HK parallel to AB.

Now the square AD is equal to the three figures AK, FD, and GH: to each add the square CK, and we have the sum of the squares AD, CK equal to the sum of the three figures AK, CD, GH; but CD is equal to AK; therefore the sum of the squares AD, CK is equal to twice the figure AK, together with the figure GH. Now AK is the rec
$AB \,.\, BK$; but BK is equal to BC; therefore AK is equal to the rec
$AB \,.\, BC$, and AD is the square on AB; CK the square on CB; and GH
square on HF, and therefore equal to the square on AC. *Hence the sum*
squares on AB and BC is equal to twice the rectangle $AB \,.\, BC$, togethe
the square on AC.

Or thus: On AC describe the square $ACDE$. Produce the sides CD, DE, EA, and make each produced part equal to CB. Join BF, FG, GH, HB. Then the figure $BFGH$ is a square [I. XLVI., Ex. 3], and it is equal to the square on AC, together with the four equal triangles HAB, BCF, FDG, GEH. Now [I. XLVII.], the figure $BFGH$ is equal to the sum of the squares on AB, AH—that is, equal to the sum of the squares on AB, BC; and the sum of the four triangles is equal to twice the rectangle $AB \,.\, BC$, for each triangle is equal to half the rectangle $AB \,.\, BC$. *Hence the sum of the squares on AB, BC is equal to twice the rectangle $AB \,.\, BC$, together with the square on AC.*

Or thus: $AC = AB - BC$;

therefore $AC^2 = AB^2 - 2AB \,.\, BC + BC^2$;

therefore $AC^2 + 2AB \,.\, BC = AB^2 + BC^2$.

Comparison of IV. and VII.
By IV., square on sum = sum of squares + twice rectangle.
By VII., square on difference = sum of squares-twice rectangle.

Cors. from IV. and VII.

1. Square on the sum, the sum of the squares, and the square on the diff
of any two lines, are in arithmetical progression.
2. Square on the sum + square on the difference of any two lines =
the sum of the squares on the lines (Props. IX. and X.).
3. The square on the sum − the square on the difference of any two lines = four times the rectangle under lines (Prop. VIII.).

Again, since CB is equal to CG, the rectangle $AC \,.\, CB$ is equal to the rectangle $AC \,.\, CG$; but $AC \,.\, CG$ is the figure AG (Def. IV.). Therefore the rectangle $AC \,.\, CB$ is equal to the figure AG. Now the figures AG, GD are equal [I. XLIII.], being the complements about the diagonal of the parallelogram AD. Hence the parallelograms AG, GD are together equal to twice the rectangle $AC \,.\, CB$. Again, the figure HF is the square on HG, and HG is equal to AC. Therefore HF is equal to the square on AC, and CI is the square on CB; but the whole figure AD, which is the square on AB, is the sum of the four figures HF, CI, AG, GD. *Therefore the square on AB is equal to the sum of the squares on AC, CB, and twice the rectangle $AC \,.\, CB$.*

Or thus: On AB describe the square $ABDE$, and cut off AH, EG, DF each equal to CB. Join CF, FG, GH, HC. Now the four $\triangle$s ACH, CBF, FDG, GEH are evidently equal; therefore their sum is equal to four times the $\triangle ACH$; but the $\triangle ACH$ is half the rectangle $AC \,.\, AH$ (I. *Cor.* 2)—that is, equal to half the rectangle $AC \,.\, CB$. Therefore the sum of the four triangles is equal to $2AC \,.\, CB$.

Again, the figure $CFGH$ is a square [I. XLVI., *Cor.* 3], and equal to $AC^2 + AH^2$ [I. XLVII.]—that is, equal to $AC^2 + CB^2$. *Hence the whole figure $ABDE = AC^2 + CB^2 + 2AC \,.\, CB$.*

Or thus: $AB = AC + CB$.

Squaring, we get $AB^2 = AC^2 + 2AC \,.\, CB + CB^2$.

Cor. 1.—*The parallelograms about the diagonal of a square are squares.*

Cor. 2.—*The square on a line is equal to four times the square on its half.*

For let $AB = 2AC$, then $AB^2 = 4AC^2$.

This *Cor.* may be proved by the First Book thus: Erect CD at right angles to AB, and make $CD = AC$ or CB. Join AD, DB.

Then $AD^2 = AC^2 + CD^2 = 2AC^2$

In like manner, $DB^2 = 2CB^2$;

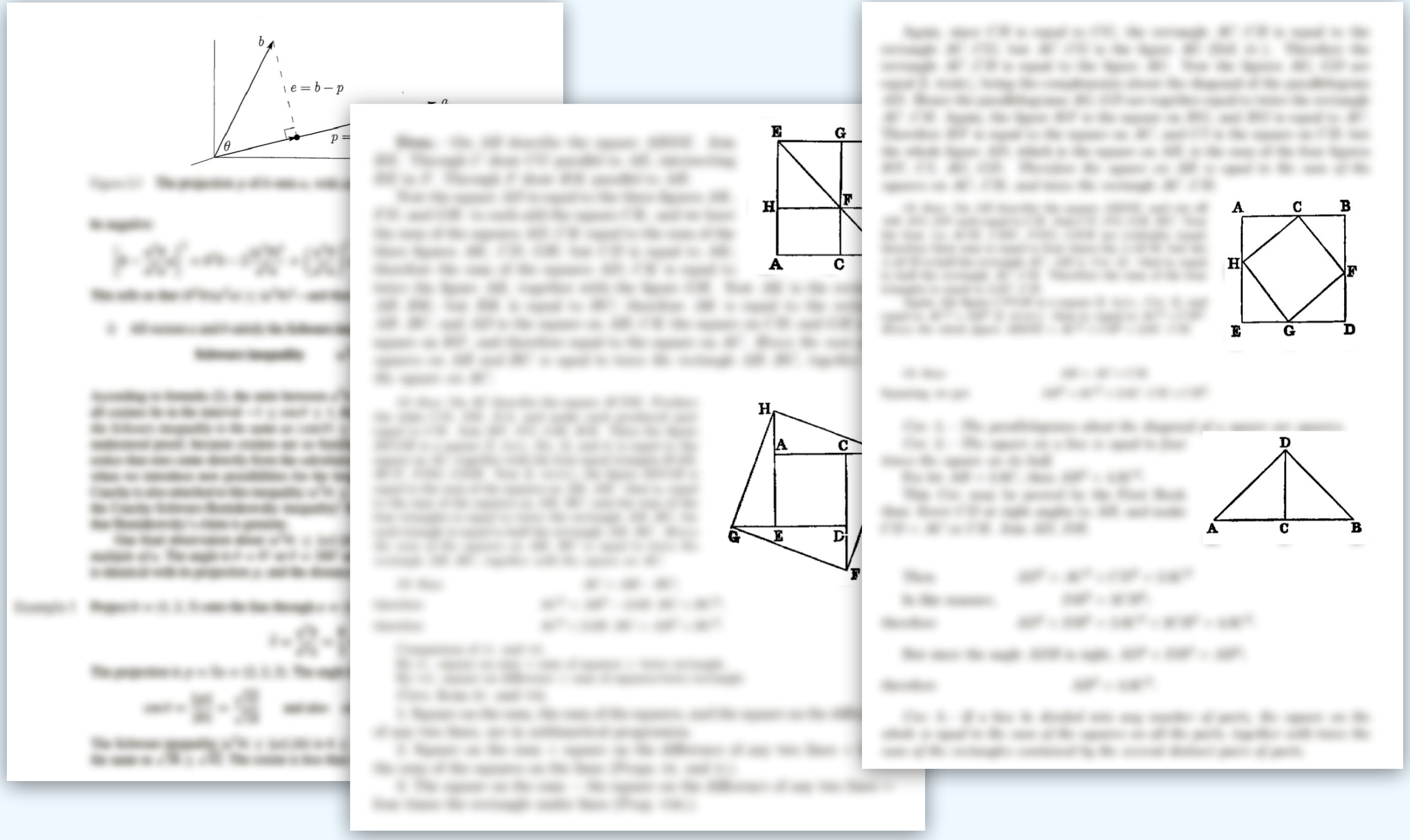
therefore $AD^2 + DB^2 = 2AC^2 + 2CB^2 = 4AC^2$.

But since the angle ADB is right, $AD^2 + DB^2 = AB^2$;

therefore $AB^2 = 4AC^2$.

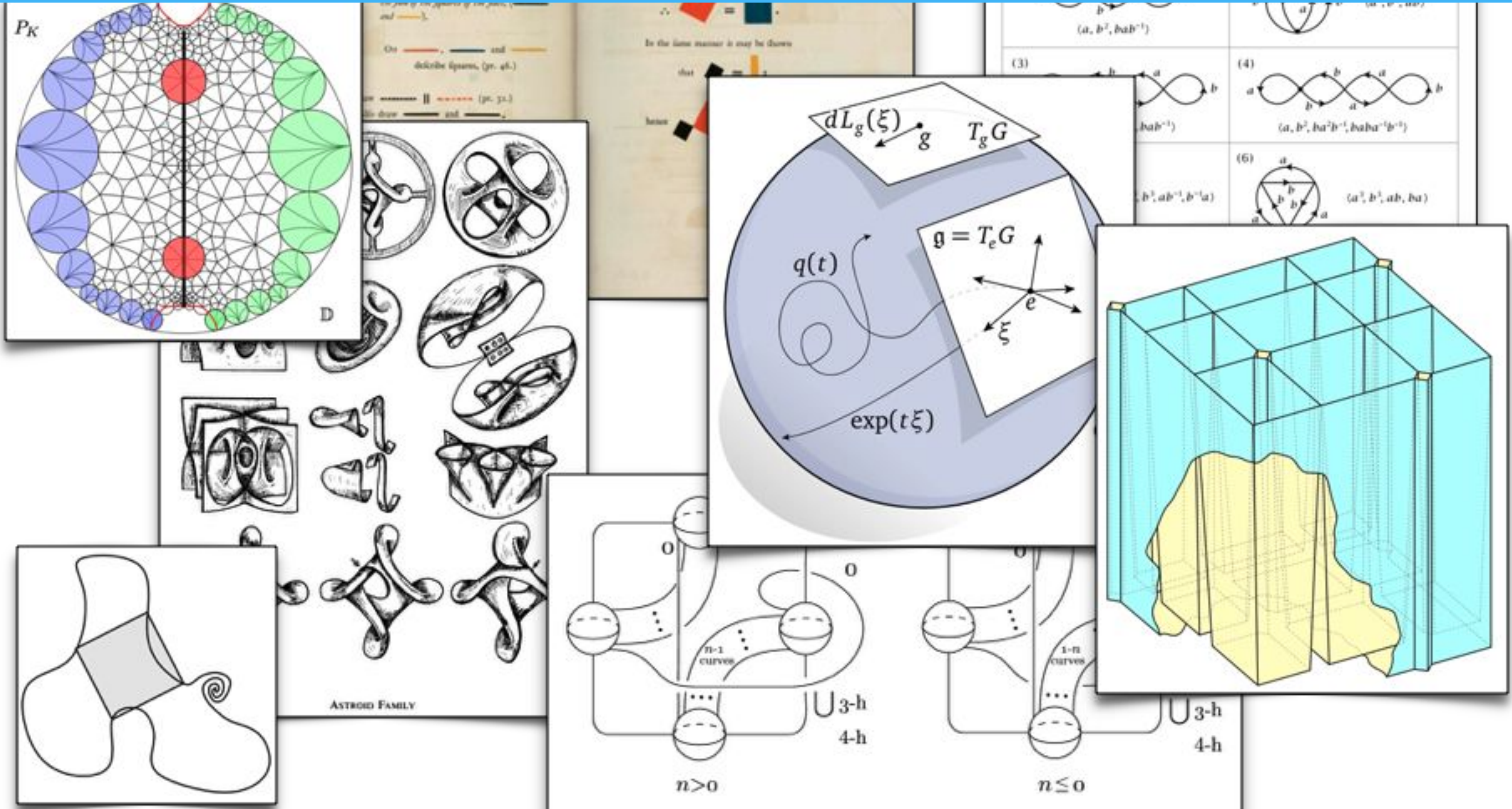
Cor. 3.—*If a line be divided into any number of parts, the square on the whole is equal to the sum of the squares on all the parts, together with twice the sum of the rectangles contained by the several distinct pairs of parts.*

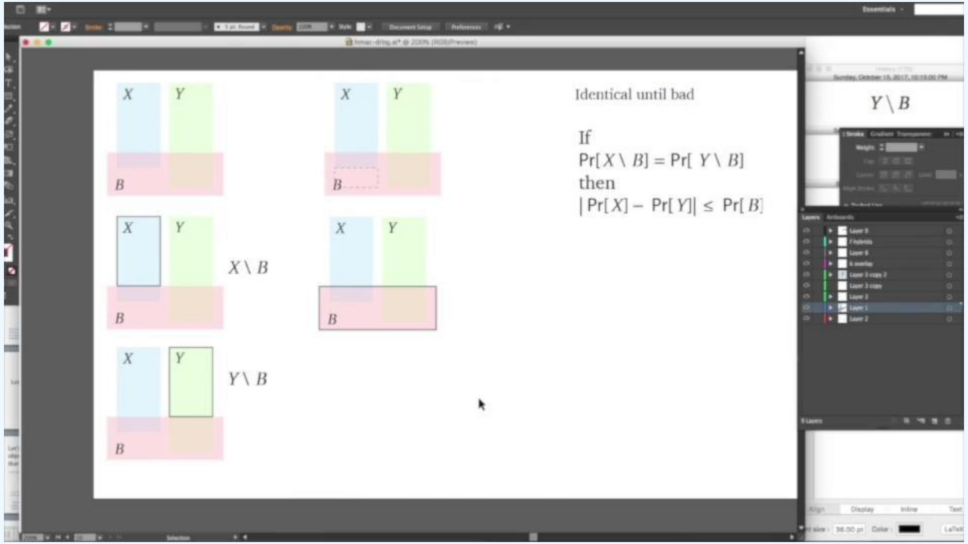
Math notations are great!

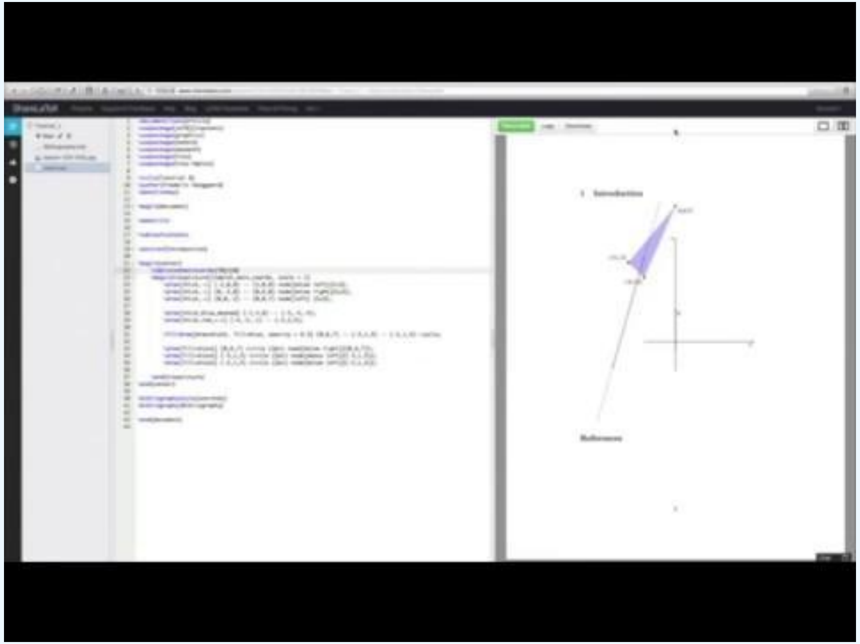


But that’s not the whole story!

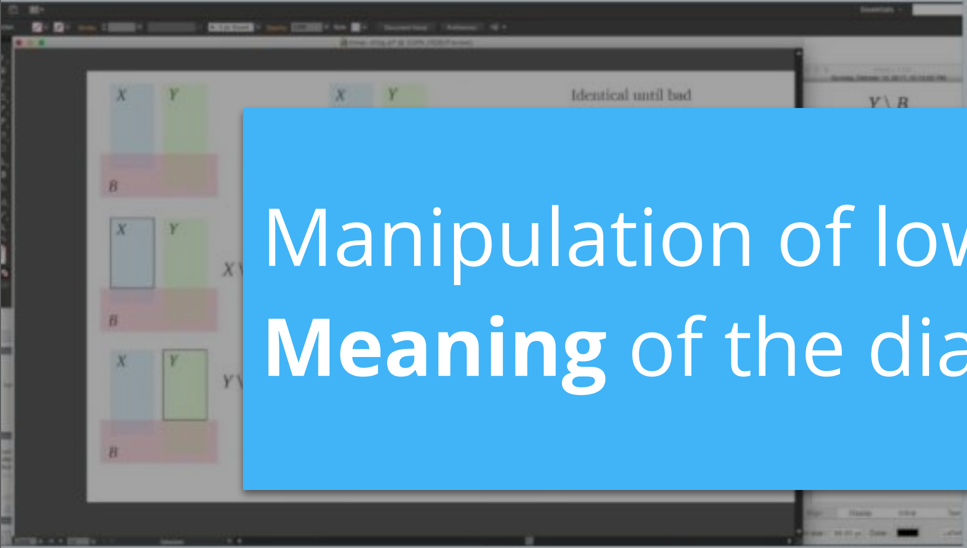
But that's not the whole ~~story~~ picture!

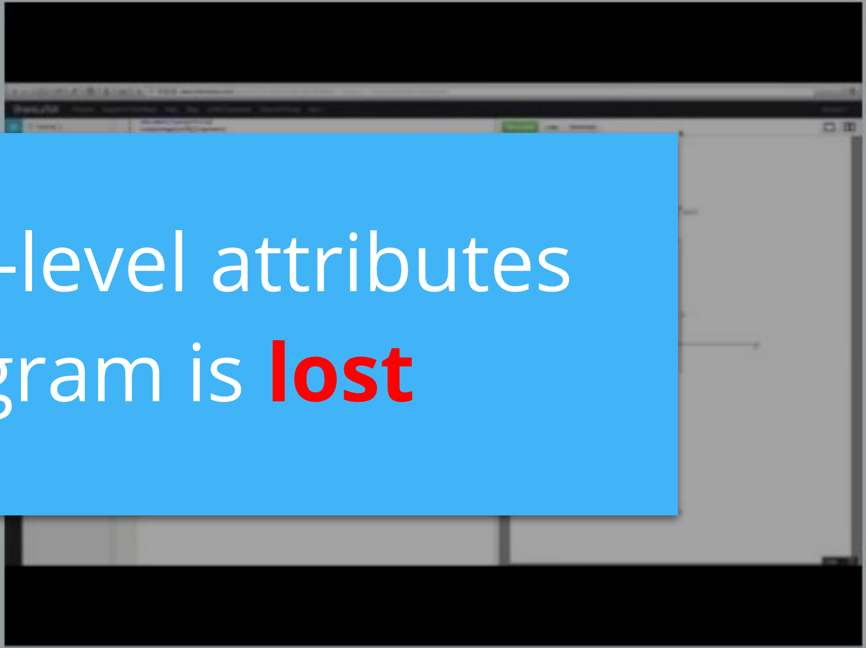






Let’s make a diagram





Manipulation of low-level attributes

Meaning of the diagram is **lost**

Let’s make a diagram (painfully)

Introducing Penrose

Domain

Defines all possible notation in a domain to be visualized

set theory ∩

Real Analysis $\mathbb{R}$

Ray Tracing

Linear Algebra $\vec{v}$

Neural Networks

• • •

Substance

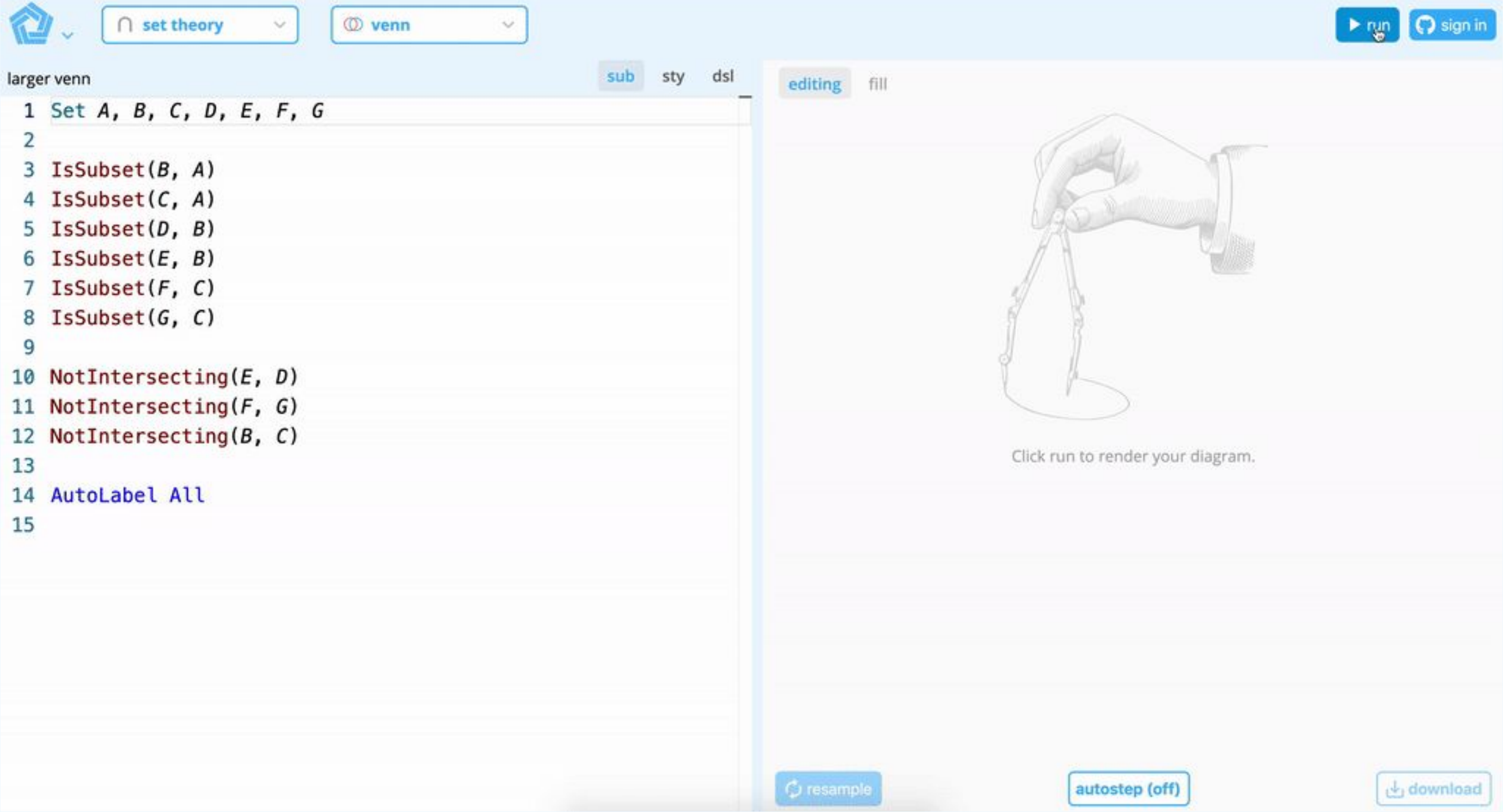
Declare objects and relationships with high-level notation

```
Set A, B
A ∩ B
```

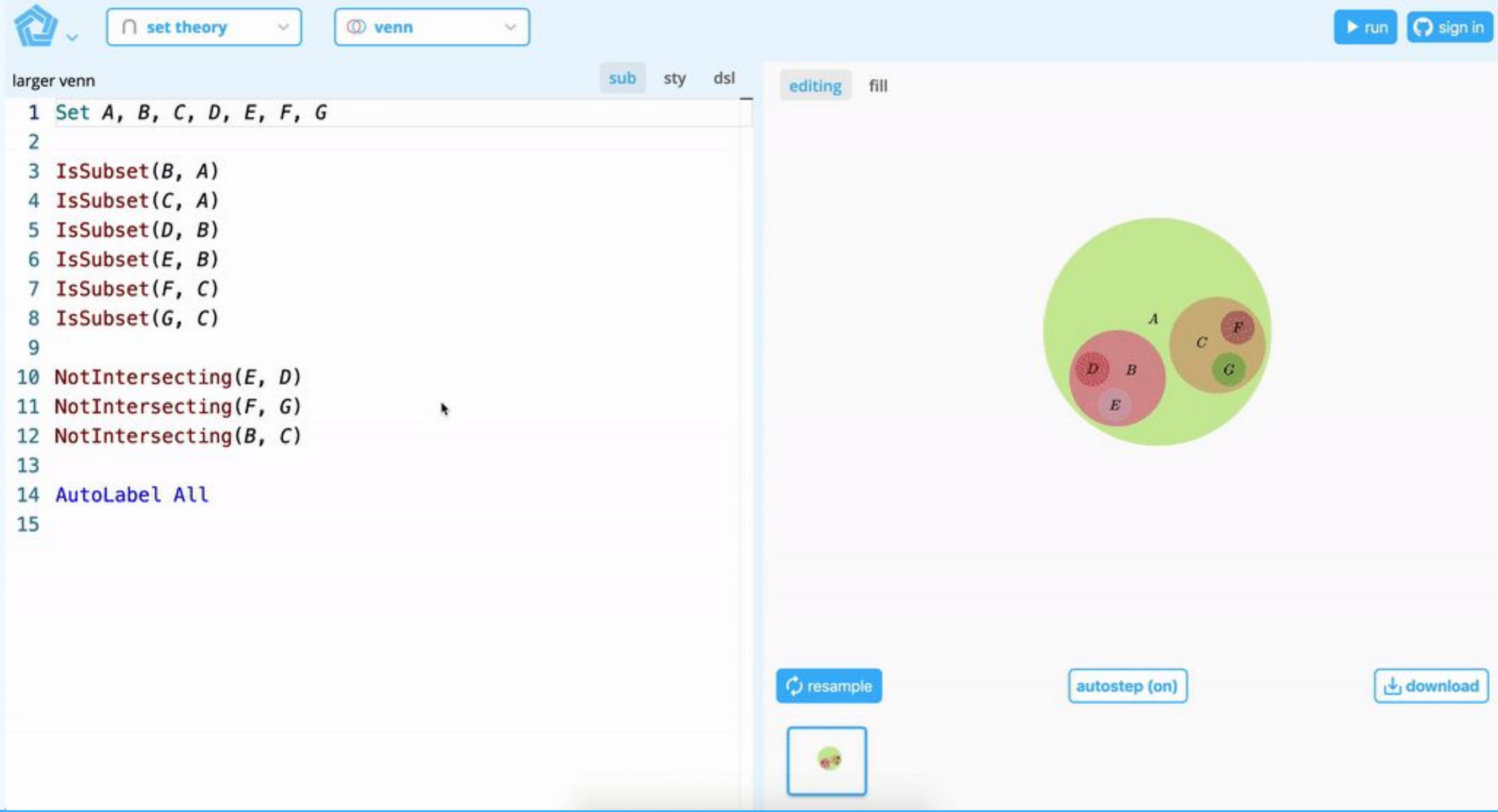
Style

Map the objects and relationships to a concrete visual *representation*

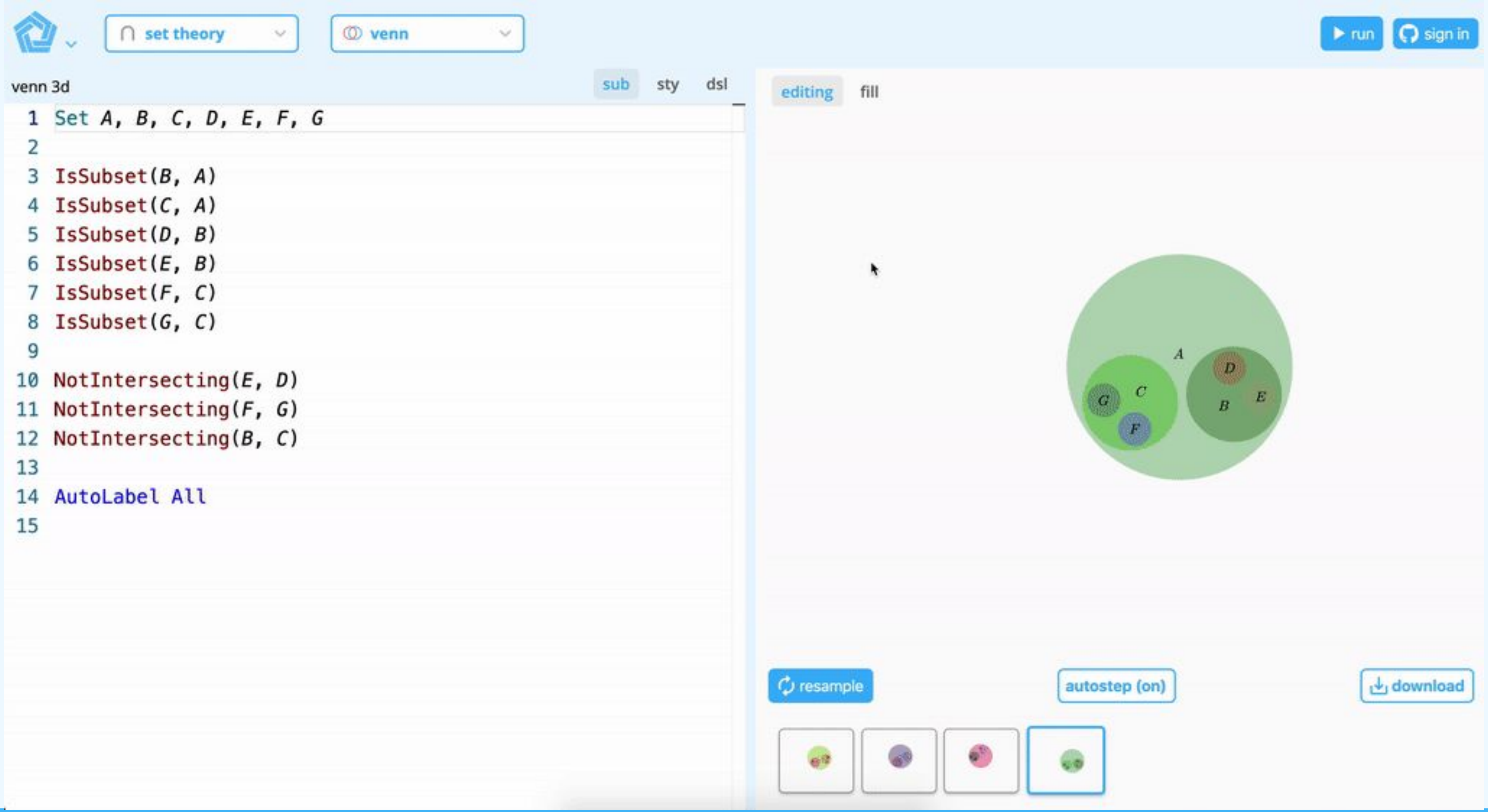
```
Set X { shape = Circle{} }
X ∩ Y {
  ensure X.shape contains Y.shape
}
```



Let’s make a diagram again



Buy 1 get n free



Diagramming with style(s)

Subst ce

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n

Domain

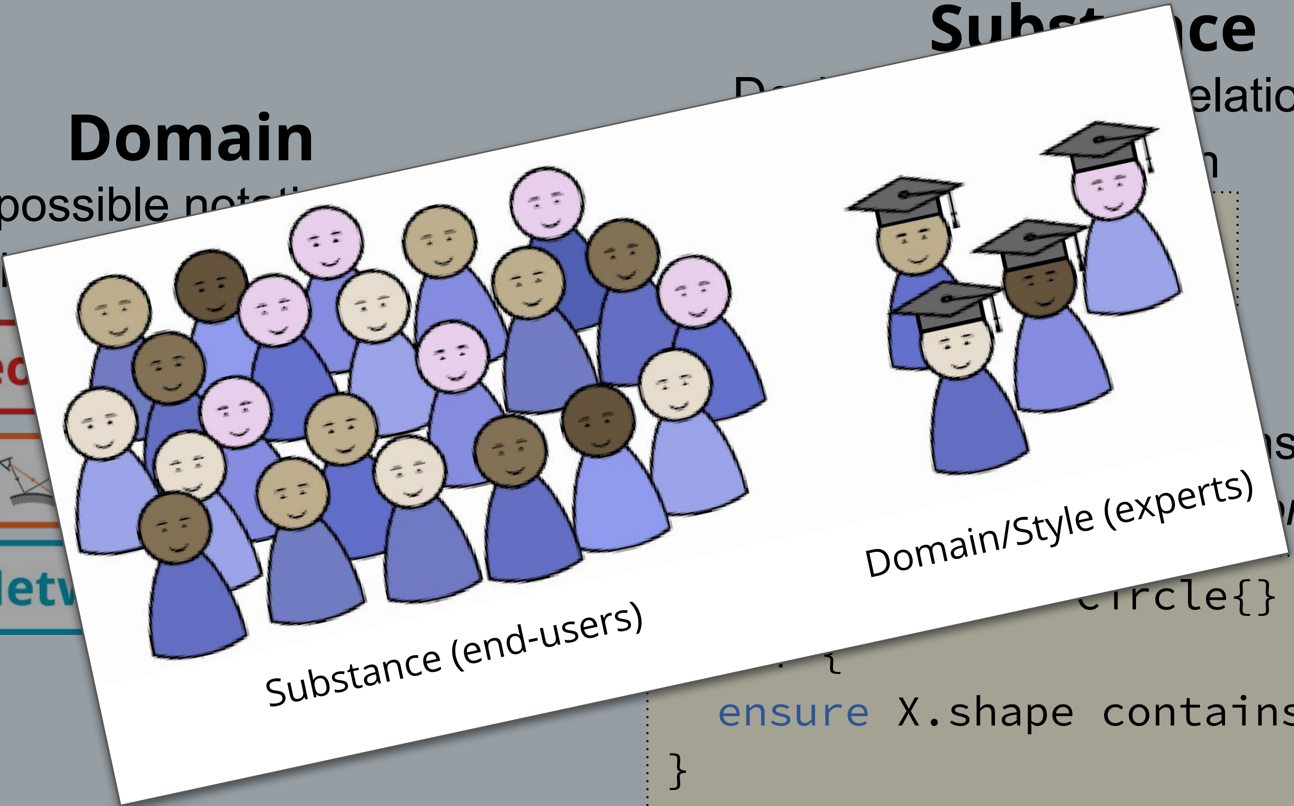
Defines all possible nota

to be visual

set theo

Ray Tracing

Neural Netw



nships to a

on

```
circle{} }
{
  ensure X.shape contains Y.shape
}
```

So many languages!

Domain

Defines all possible notation in a domain to be visualized

set theory ∩

Real Analysis $\mathbb{R}$

Ray Tracing

Linear Algebra $\vec{v}$

Neural Networks

• • •

Substance

Declare objects and relationships with high-level notation

```
Set A, B
A ∩ B
```

Style

Map the objects and relationships to a concrete visual *representation*

```
Set X { shape = Circle{} }
X ∩ Y {
  ensure X.shape contains Y.shape
}
```

What's in the Domain?

Substance

Declare objects and relationships with high-level notation

```
Set A, B
A ∩ B
```

Domain

Defines all possible notation in a domain to be visualized

set theory ∩

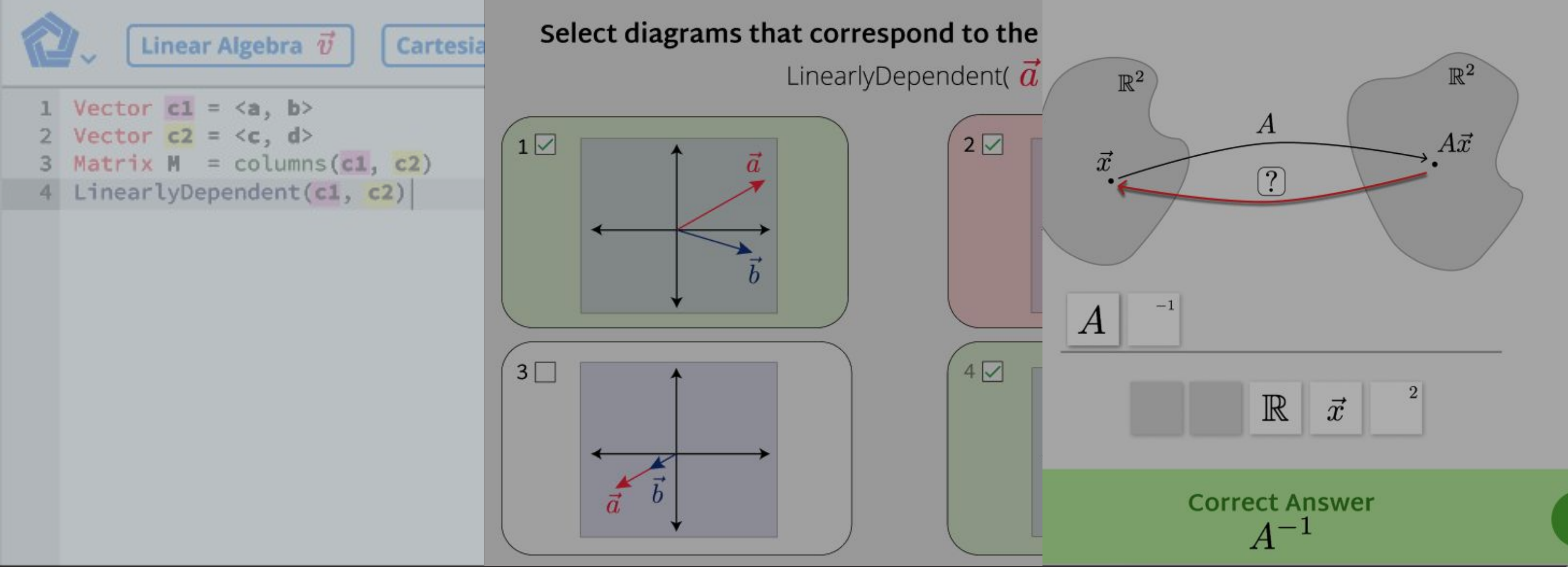
```
type Set
predicate IsSubset : Set s1 * Set s2
notation "A ⊆ B" ~ "IsSubset(A, B)"
```

Style

Map the objects and relationships to a concrete visual *representation*

```
Set X { shape = Circle{} }
X ∩ Y {
  ensure X.shape contains Y.shape
}
```

What's in the Domain?



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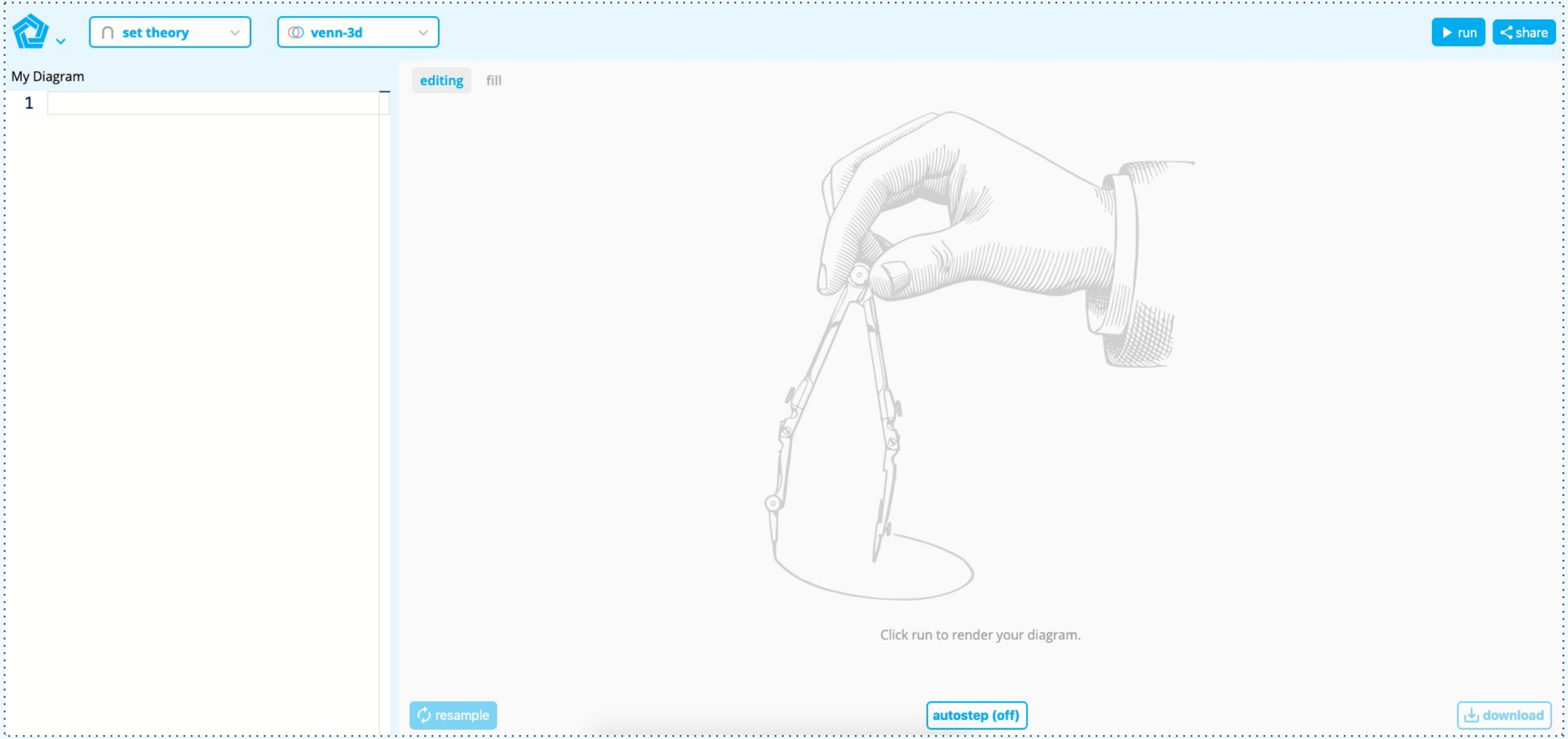

set theory
venn-3d
run
share
My Diagram
1
editing
fill
Click run to render your diagram.
resample
autostep (off)
download

So...what now?

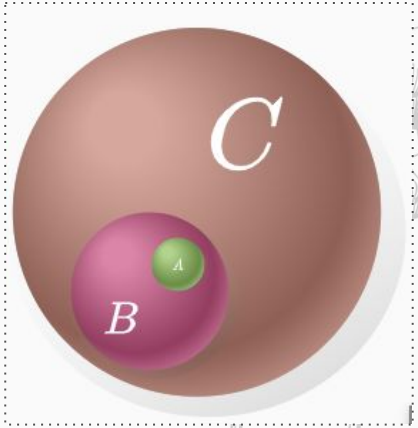
set theory

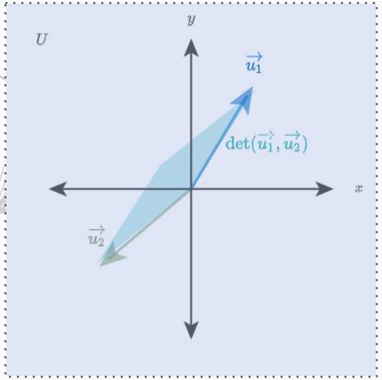
My Diagram

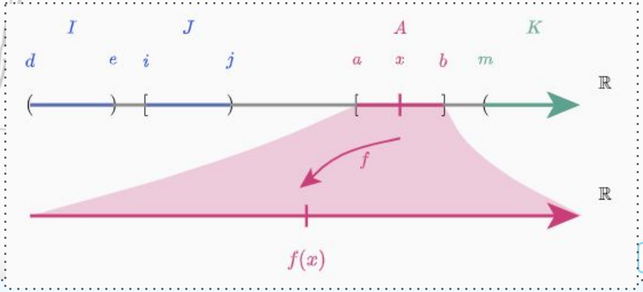
```
Set A, B, C
IsSubset(A, B)
IsSubset(B, C)
```

```
VectorSpace U
Vector u1 ∈ U
Vector u2 ∈ U
Scalar c := det(u1,u2)
```

```
Real a, b
A := [a,b] ⊆ R
Real l ∈ A
f : A -> R
Continuous(f)
Real f_l
f_l := f(l)
Real d, e, i, j
I := (d, e) ⊆ R
J := [i, j) ⊆ R
```







run share resample download

Too many DSLs!

Higher-order Functions

Scala allows the definition of higher-order functions. These are functions that *take other functions as parameters*, or whose *result is a function*. Here is a function `apply` which takes another function `f` and a value `v` and applies function `f` to `v`:

```
def apply(f: Int => String, v: Int) = f(v)
```

Note: methods are automatically coerced to functions if the context requires this.

Here is another example:

```
class Decorator(left: String, right: String) {
  def layout[A](x: A) = left + x.toString() + right
}

object FunTest extends App {
  def apply(f: Int => String, v: Int) = f(v)
  val decorator = new Decorator("[", "]")
  println(apply(decorator.layout, 7))
}
```

ade up of packages.

in package `main`.

he packages with import paths `"fmt"`

kage name is the same as the last
ath. For instance, the `"math/rand"`
s that begin with the statement `package`

t in which these programs are executed
h time you run the example program
he same number.

ber, seed the number generator; see
nstant in the playground, so you will
else as the seed.)

< 1/17 >

packages.go | Imports off | Syntax off

```
package main

import (
	"fmt"
	"math/rand"
)

func main() {
	fmt.Println("My favorite number is",
rand.Intn(10))

```

Reset | Format | Kill

```
./prog.go:11:1: syntax error: unexpected EOF, expecting }

Go build failed.
```

Here's some simple arithmetic.

```
ghci> 2 + 15
17
ghci> 49 * 100
4900
ghci> 1892 - 1472
420
ghci> 5 / 2
2.5
ghci>
```

This is pretty self-explanatory. We can also use several operators on one line and all the usual precedence rules are obeyed. We can use parentheses to make the precedence explicit or to change it.

```
ghci> (50 * 100) - 4999
1
ghci> 50 * 100 - 4999
1
ghci> 50 * (100 - 4999)
-244950
```

Pretty cool, huh? Yeah, I know it's not but bear with me. A little pitfall to watch out for here is negating numbers. If we want to have a negative number, it's always best to surround it with parentheses. Doing `5 * -3` will make GHCI yell at you but doing `5 * (-3)` will work just fine.

Boolean algebra is also pretty straightforward. As you probably know, `&&` means a boolean *and*, `||` means a boolean *or*. `not` negates a `True` or a `False`.

```
ghci> True && False
False
ghci> True && True
True
ghci> False || True
True
ghci> not False
True
ghci> not (True && True)
False
```

Testing for equality is done like so.

```
ghci> 5 == 5
True
ghci> 1 == 0
False
ghci> 5 /= 5
False
ghci> 5 /= 4
True
ghci> "hello" == "hello"
True
```

What about doing `5 + "llama"` or `5 == True`? Well, if we try the first snippet, we get a big scary error message!

```
No instance for (Num [Char])
arising from a use of `+' at <interactive>:1:0-9
Possible fix: add an instance declaration for (Num [Char])
In the expression: 5 + "llama"
```

Building your first TypeScript file

In your editor, type the following JavaScript code in `greeter.ts`:

```
function greeter(person) {
    return "Hello, " + person;
}

let user = "Jane User";

document.body.textContent = greeter(use
r);
```

Compiling your code

We used a `.ts` extension, but this code is just JavaScript. You could have copy/pasted this straight out of an existing JavaScript app.

At the command line, run the TypeScript compiler:

```
tsc greeter.ts
```

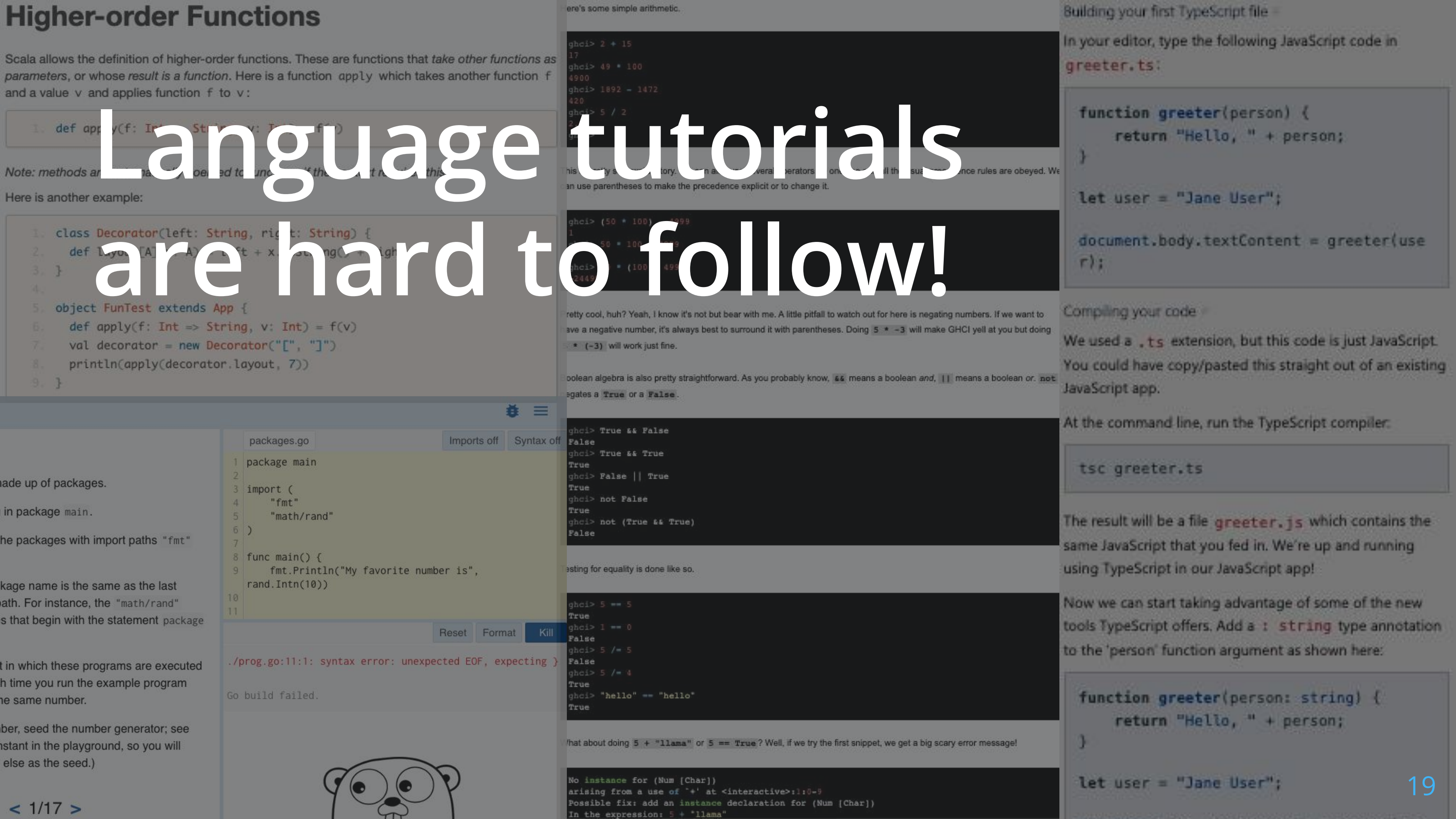
The result will be a file `greeter.js` which contains the same JavaScript that you fed in. We're up and running using TypeScript in our JavaScript app!

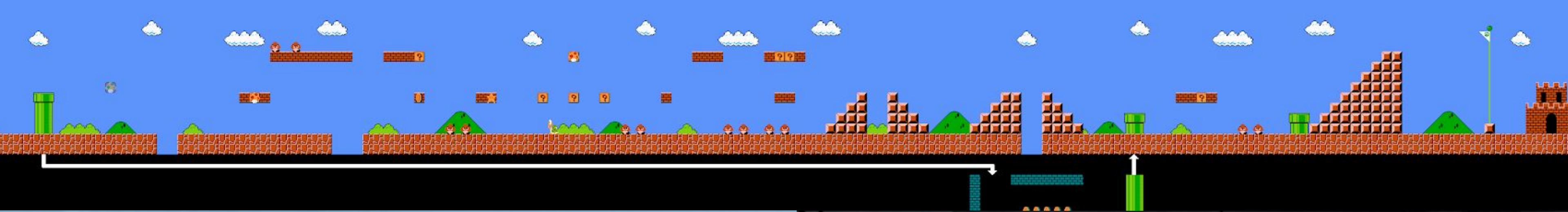
Now we can start taking advantage of some of the new tools TypeScript offers. Add a `: string` type annotation to the 'person' function argument as shown here:

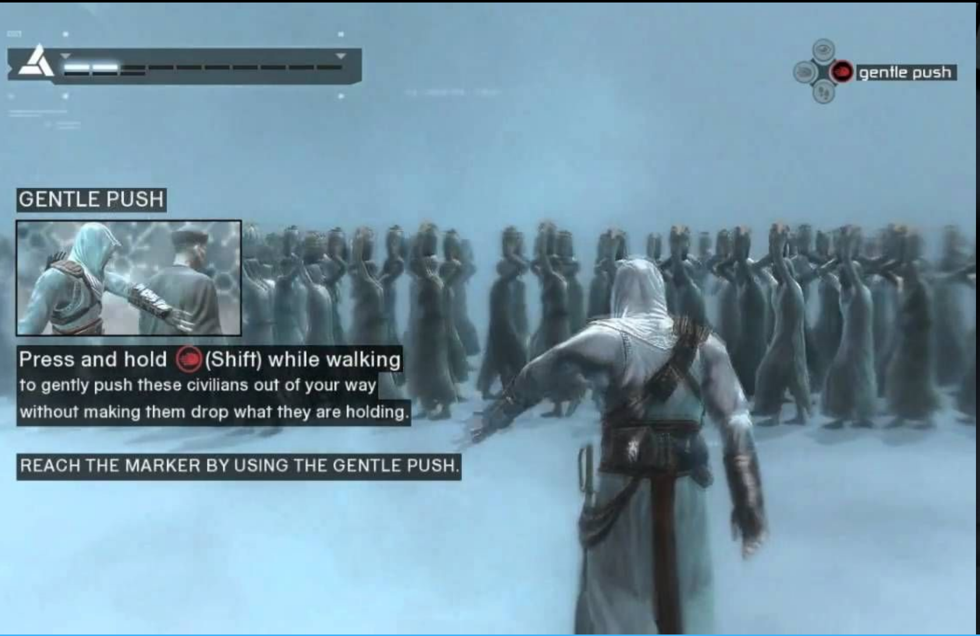
```
function greeter(person: string) {
    return "Hello, " + person;
}

let user = "Jane User";
```

Language tutorials are hard to follow!







Video game tutorials?

Objective

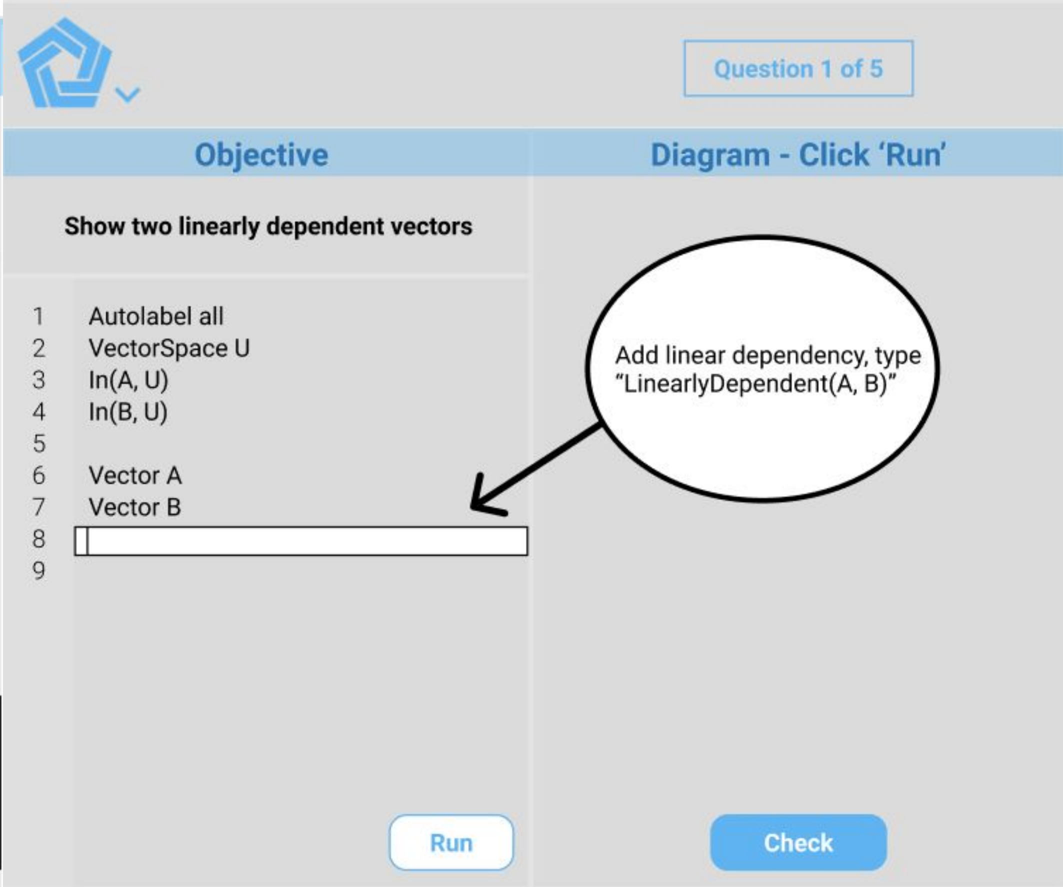
Show two linearly dependent vectors

```
Autolabel All
VectorSpace U
In(A, U)
In(B, U)

Vector A
Vector B
```

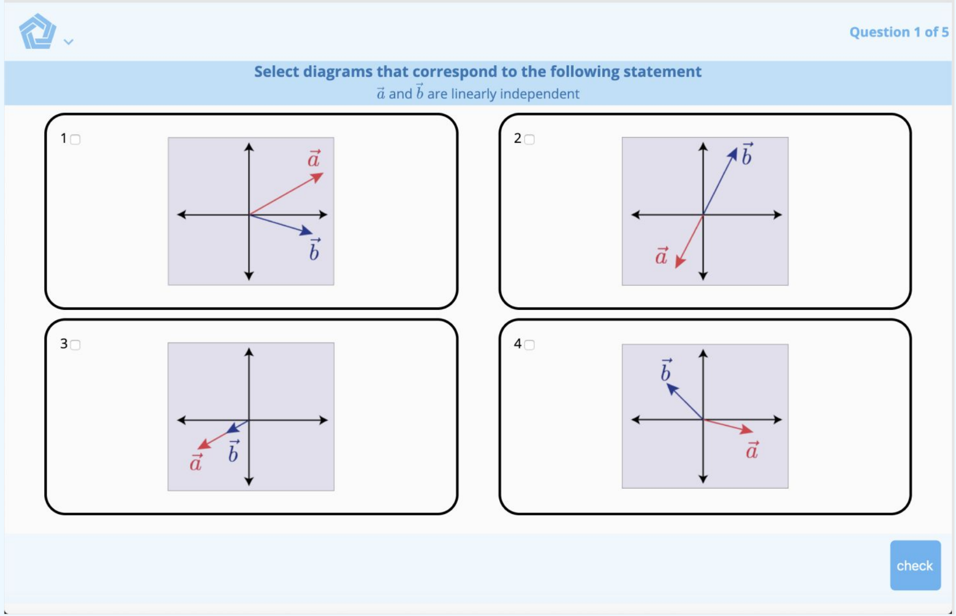
Vector C | LinearlyDependent(A, B)

Scaler s | LinearlyDependent(B, C)

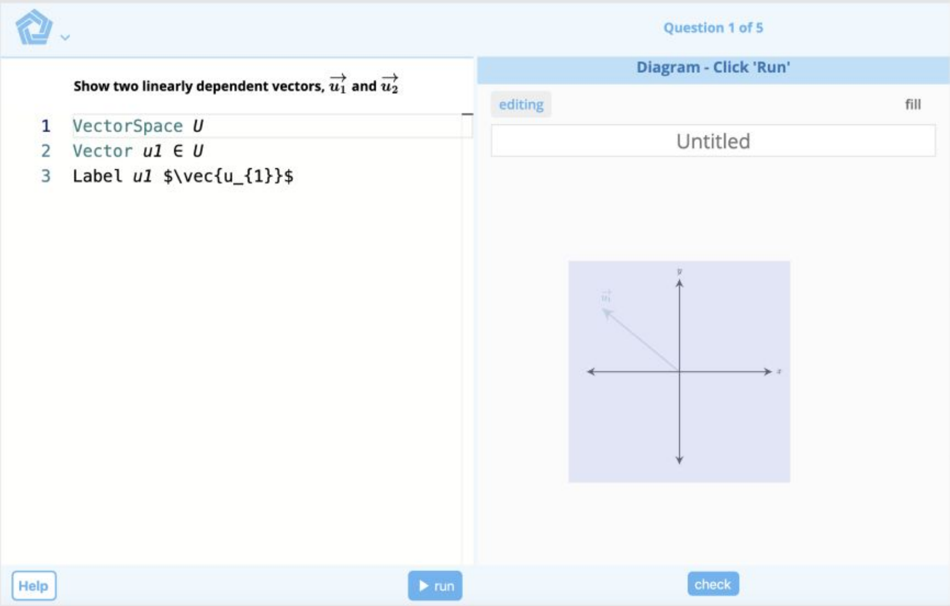


A guided tour of Penrose

Recognition: Recognizing the associations between program outputs and source programs

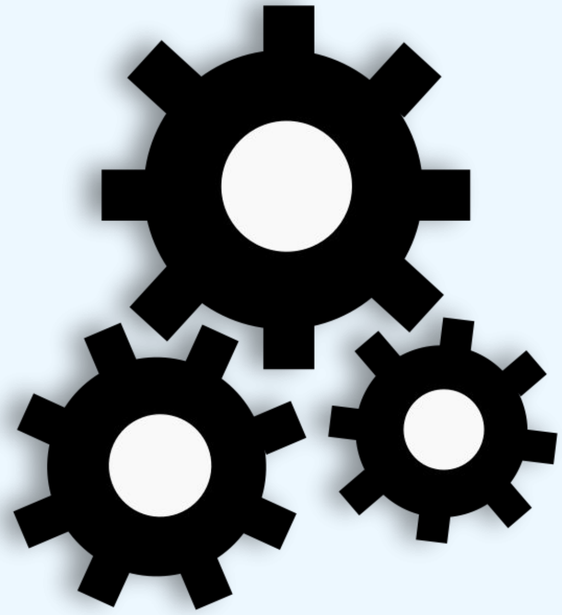


Individualized practice

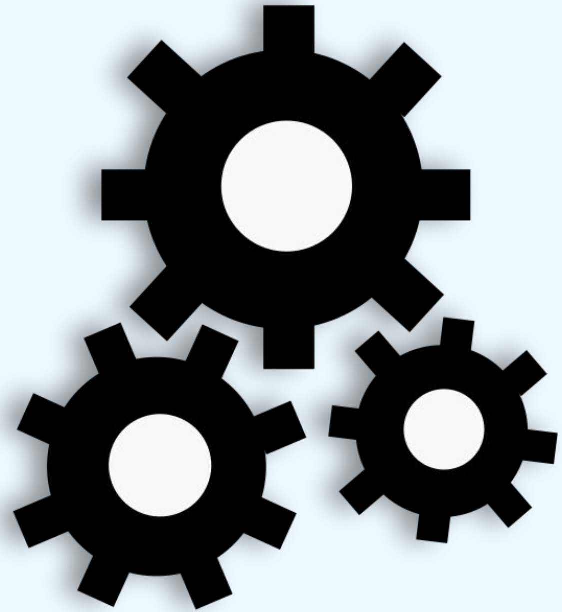


Recall: Recalling language constructs of the DSL

Individualized practice

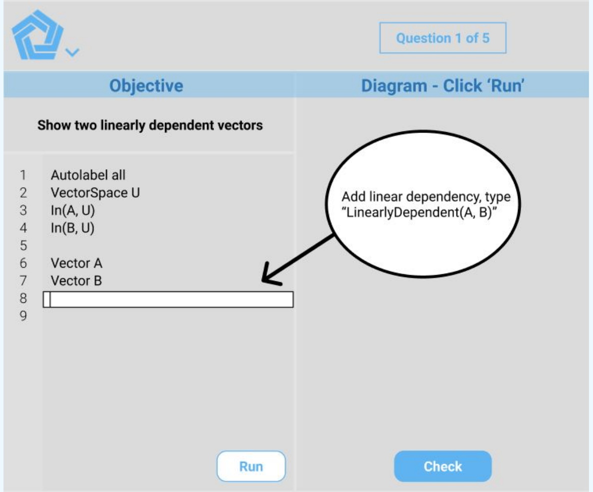


Future Work



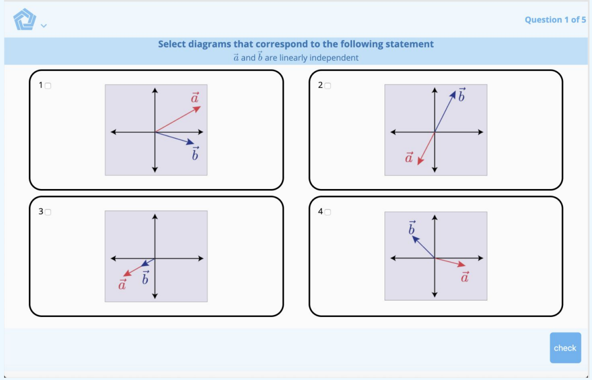
Future Work

Questions?

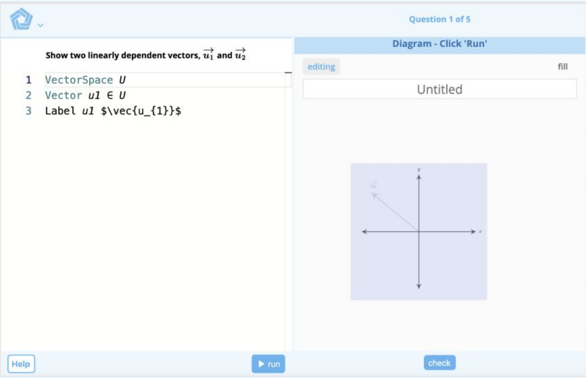


Question 1 of 5
Objective
Diagram - Click 'Run'
Show two linearly dependent vectors
Autolabel all
VectorSpace U
In(A, U)
In(B, U)
Vector A
Vector B
Add linear dependency, type "LinearlyDependent(A, B)"
Run
Check

Question 1 of 5
Objective
Diagram - Click 'Run'
Show two linearly dependent vectors
Autolabel all
VectorSpace U
In(A, U)
In(B, U)
Vector A
Vector B
LinearlyDependent(A, B)
Click 'Run'
Run
Check



Question 1 of 5
Select diagrams that correspond to the following statement
check



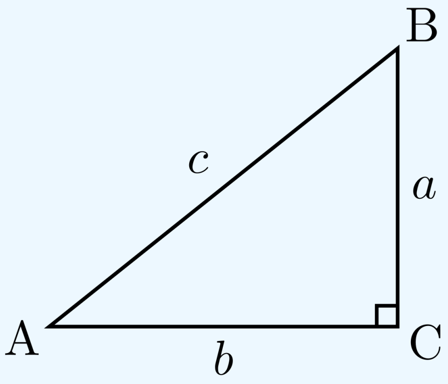
Question 1 of 5
Diagram - Click 'Run'
VectorSpace U
Label u1 $\vec{u_{1}}$
editing
fill
Untitled
Help
run
check

Defining Visual Mathematical Narratives Declaratively

***Max Krieger**, Wode Ni, Joshua Sunshine*

Let's prove

$$a^2 + b^2 = c^2$$



(Pythagorean Theorem)

- Yeah. All right. Well, then... I guess I'll see you around. Or not. OK, Barry. And thank you so much again... for before. Oh, that? That was nothing. Well, not nothing, but... Anyway... This can't possibly work. He's all set to go. We may as well try it. OK, Dave, pull the chute. - Sounds amazing. - It was amazing! It was the scariest, happiest moment of my life. Humans! I can't believe you were with humans! Giant, scary humans! What were they like? Huge and crazy. They talk crazy. They eat crazy giant things. They drive crazy. - Do they try and kill you, like on TV? - Some of them. But some of them don't. - How'd you get back? - Poodle. You did it, and I'm glad. You saw whatever you wanted to see. You had your "experience." Now you can pick out yourjob and be normal. - Well... - Well? Well, I met someone. You did? Was she Bee-ish? - A wasp?! Your parents will kill you! - No, no, no, not a wasp. - Spider? - I'm not attracted to spiders. I know it's the hottest thing, with the eight legs and all. I can't get by that face. So who is she? She's... human. No, no. That's a bee law. You wouldn't break a bee law. - Her name's Vanessa. - Oh, boy. She's so nice. And she's a florist! Oh, no! You're dating a human florist! We're not dating. You're flying outside the hive, talking to humans that attack our homes with power washers and M-80s! One-eighth a stick of dynamite! She saved my life! And she understands me. This is over! Eat this. This is not over! What was that? - They call it a crumb. - It was so stingin' stripey! And that's not what they eat. That's what falls off what they eat! - You know what a Oinnabon is? - No. It's bread and cinnamon and frosting. They heat it up... Sit down! ...really hot! - Listen to me! We are not them! We're us. There's us and there's them! Yes, but who can deny the heart that is yearning? There's no yearning. Stop yearning. Listen to me! You have got to start thinking bee, my friend. Thinking bee! - Thinking bee. - Thinking bee. Thinking bee! Thinking bee! Thinking bee! Thinking bee! There he is. He's in the pool. You know what your problem is, Barry? I gotta start thinking bee? How much longer will this go on? It's been three days! Why aren't you working? I've got a lot of big life decisions to think about. What life? You have no life! You have no job. You're barely a bee! Would it kill you to make a little honey? Barry, come out. Your father's talking to you. Martin, would you talk to him? Barry, I'm talking to you! You coming? Got everything? All set! Go ahead. I'll catch up. Don't be too long. Watch this! Vanessa! - We're still here. - I told you not to yell at him. He doesn't respond to yelling! - Then why yell at me? - Because you don't listen! I'm not listening to this. Sorry, I've gotta go. - Where are you going? - I'm meeting a friend. A girl? Is this why you can't decide? Bye. I just hope she's Bee-ish. They have a huge parade of flowers every year in Pasadena? To be in the Tournament of Roses, that's every florist's dream! Up on a float, surrounded by flowers, crowds cheering. A tournament. Do the roses compete in athletic events? No. All right, I've got one. How come you don't fly everywhere? It's exhausting. Why don't you run everywhere? It's faster. Yeah, OK, I see, I see. All right, your turn. TiVo. You can just freeze live TV? That's insane! You don't have that? We have Hivo, but it's a disease. It's a horrible, horrible disease. Oh, my. Dumb bees! You must want to sting all those jerks. We try not to sting. It's usually fatal for us. So you have to watch your temper. Very carefully. You kick a wall, take a walk, write an angry letter and throw it out. Work through it like any emotion: Anger, jealousy, lust. Oh, my goodness! Are you OK? Yeah. - What is wrong with you?! - It's a bug. He's not bothering anybody. Get out of here, you creep! What was that? A Pic 'N' Save circular? Yeah, it was. How did you know? It felt like about 10 pages. Seventy-five is pretty much our limit. You've really got that down to a science. - I lost a cousin to Italian Vogue. - I'll bet. What in the name of Mighty Hercules is this? How did this get here? Oute Bee, Golden Blossom, Ray Liotta Private Select? - Is he that actor? - I never heard of him. - Why is this here? - For people. We eat it. You don't have enough food of your own? - Well, yes. - How do you get it? - Bees make it. - I know who makes it! And it's hard to make it! There's heating, cooling, stirring. You need a whole Krelman thing! - It's organic. - It's our-ganic! It's just honey, Barry. Just what?! Bees don't know about this! This is stealing! A lot of stealing! You've taken our homes, schools, hospitals! This is all we have! And it's on sale?! I'm getting to the bottom of this. I'm getting to the bottom of all of this! Hey, Hector. - You almost done? - Almost. He is here. I sense it. Well, I guess I'll go home now and just leave this nice honey out, with no one around. You're busted, box boy! I knew I heard something. So you can talk! I can talk. And now you'll start talking! Where you getting the sweet stuff? Who's your supplier? I don't understand. I thought we were friends. The last thing we want to do is upset bees! You're too late! It's ours now! You, sir, have crossed the wrong sword! You, sir, will be lunch for my iguana, Ignacio! Where is the honey coming from? Tell me where! Honey Farms! It comes from Honey Farms! Orazy person! What horrible thing has happened here? These faces, they never knew what hit them. And now they're on the road to nowhere! Just keep still. What? You're not dead? Do I look dead? They will wipe anything that moves. Where you headed? To Honey Farms. I am onto something huge here. I'm going to Alaska. Moose blood, crazy stuff. Blows your head off! I'm going to Tacoma. - And you? - He really is dead. All right. Uh-oh! - What is that?! - Oh, no! - A wiper! Triple blade! - Triple blade? Jump on! It's your only chance, bee! Why does everything have to be so doggone clean?! How much do you people need to see?! Open your eyes! Stick your head out the window! From NPR News in Washington, I'm Oarl Kasell. But don't kill no more bugs! - Bee! - Moose blood guy!! - You hear something? - Like what? Like tiny screaming. Turn off the radio. Whassup, bee boy? Hey, Blood. Just a row of honey jars, as far as the eye could see. Wow! I assume wherever this truck goes is where they're getting it. I mean, that honey's ours. - Bees hang tight. - We're all jammed in. It's a close community. Not us, man. We on our own. Every mosquito on his own. - What if you get in trouble? - You a mosquito, you in trouble. Nobody likes us. They just smack. See a mosquito, smack, smack! At least you're out in the world. You must meet girls. Mosquito girls try to trade up, get with a moth, dragonfly. Mosquito girl don't want no mosquito. You got to be kidding me! Mooseblood's about to leave the building! So long, bee! - Hey, guys! - Mooseblood! I knew I'd catch y'all down here. Did you bring your crazy straw? We throw it in jars, slap a label on it, and it's pretty much pure profit. What is this place? A bee's got a brain the size of a pinhead. They are pinheads! Pinhead. - Oheck out the new smoker. - Oh, sweet. That's the one you want. The Thomas 3000! Smoker? Ninety puffs a minute, semi-automatic. Twice the nicotine, all the tar. A couple breaths of this knocks them right out. They make the honey, and we make the money. "They make the honey, and we make the money"? Oh, my! What's going on? Are you OK? Yeah. It doesn't last too long. Do you know you're in a fake hive with fake walls? Our queen was moved here. We had no choice. This is your queen? That's a man in women's clothes! That's a drag queen! What is this? Oh, no! There's hundreds of them! Bee honey. Our honey is being brazenly stolen on a massive scale! This is worse than anything bears have done! I intend to do something. Oh, Barry, stop. Who told you humans are taking our honey? That's a rumor. Do these look like rumors? That's a conspiracy theory. These are obviously doctored photos. How did you get mixed up in this? He's been talking to humans. - What? - Talking to humans?! He has a human girlfriend. And they make out! Make out? Barry! We do not. - You wish you could. - Whose side are you on? The bees! I dated a cricket once in San Antonio. Those crazy legs kept me up all night. Barry, this is what you want to do with your life? I want to do it for all our lives. Nobody works harder than bees! Dad, I remember you coming home so overworked your hands were still stirring. You couldn't stop. I remember that. What right do they have to our honey? We live on two cups a year. They put it in lip balm for no reason whatsoever! Even if it's true, what can one bee do? Sting them where it really hurts. In the face! The eye! - That would hurt. - No. Up the nose? That's a killer. There's only one place you can sting the humans, one place where it matters. Hive at Five, the hive's only full-hour action news source. No more bee beards! With Bob Bumble at the anchor desk. Weather with Storm Stinger. Sports with Buzz Larvi. And Jeanette Ohung. - Good evening. I'm Bob Bumble. - And I'm Jeanette Ohung. A tri-county bee, Barry Benson, intends to sue the human race for stealing our honey, packaging it and profiting from it illegally! Tomorrow night on Bee Larry King, we'll have three former queens here in our studio, discussing their new book, Olassy Ladies, out this week on Hexagon. Tonight we're talking to Barry Benson. Did you ever think, "I'm a kid from the hive. I can't do this"? Bees have never been afraid to change the world. What about Bee Oolumbus? Bee Gandhi? Bejesus? Where I'm from, we'd never sue humans. We were thinking of stickball or candy stores. How old are you? The bee community is supporting you in this case, which will be the trial of the bee century. You know, they have a Larry King in the human world too. It's a common name. Next week... He looks like you and has a show and suspenders and colored dots... Next week... Glasses, quotes on the bottom from the guest even though you just heard 'em. Bear Week next week! They're scary, hairy and here live. Always leans forward, pointy shoulders, squinty eyes, very Jewish. In tennis, you attack at the point of weakness! It was my grandmother, Ken. She's 81. Honey, her backhand's a joke! I'm not gonna take advantage of that? Quiet, please. Actual work going on here. - Is that that same bee? - Yes, it is! I'm helping him sue the human race. - Hello. - Hello, bee. This is Ken. Yeah, I remember you. Timberland, size ten and a half. Vibram sole, I believe. Why does he talk again? Listen, you better go 'cause we're really busy working. But it's our yogurt night! Bye-bye. Why is yogurt night so difficult?! You poor thing. You two have been at this for hours! Yes, and Adam here has been a huge help. - Frosting... - How many sugars? Just one. I try not to use the competition. So why are you helping me? Bees have good qualities. And it takes my mind off the shop. Instead of flowers, people are giving balloon bouquets now. Those are great, if you're three. And artificial flowers. - Oh, those just get me psychotic! - Yeah, me too. Bent stingers, pointless pollination. Bees must hate those fake things! Nothing worse than a daffodil that's had work done. Maybe this could make up for it a little bit. - This lawsuit's a pretty big deal. - I guess. You sure you want to go through with it? Am I sure? When I'm done with the humans, they won't be able to say, "Honey, I'm home," without paying a royalty! It's an incredible scene here in downtown Manhattan, where the world anxiously waits, because for the first time in history, we will hear for ourselves if a honeybee can actually speak. What have we gotten into here, Barry? It's pretty big, isn't it? I can't believe how many humans don't work during the day. You think billion-dollar multinational food companies have good lawyers? Everybody needs to stay behind the barricade. - What's the matter? - I don't know, I just got a chill. Well, if it isn't the bee team. You boys work on this? All rise! The Honorable Judge Bumbleton presiding. All right. Oase number 4475, Superior Oourt of New York, Barry Bee Benson v. the Honey Industry is now in session. Mr. Montgomery, you're representing the five food companies collectively? A privilege. Mr. Benson... you're representing all the bees of the world? I'm kidding. Yes, Your Honor, we're ready to proceed. Mr. Montgomery, your opening statement, please. Ladies and gentlemen of the jury, my grandmother was a simple woman. Born on a farm, she believed it was man's divine right to benefit from the bounty of nature God put before us. If we lived in the topsy-turvy world Mr. Benson imagines, just think of what would it mean. I would have to negotiate with the silkworm for the elastic in my britches! Talking bee! How do we know this isn't some sort of holographic motion-picture-capture Hollywood wizardry? They could be using laser beams! Robotics! Ventriloquism! Oloning! For all we know, he could be on steroids! Mr. Benson? Ladies and gentlemen, there's no trickery here. I'm just an ordinary bee. Honey's pretty important to me. It's important to all bees. We invented it! We make it. And we protect it with our lives. Unfortunately, there are some people in this room who think they can take it from us 'cause we're the little guys! I'm hoping that, after this is all over, you'll see how, by taking our honey, you not only take everything we have but everything we are! I wish he'd dress like that all the time. So nice! Oall your first witness. So, Mr. Klauss Vanderhayden of Honey Farms, big company you have. I suppose so. I see you also own Honeyburton and Honron! Yes, they provide beekeepers for our farms. Beekeeper. I find that to be a very disturbing term. I don't imagine you employ any bee-free-ers, do you? - No. - I couldn't hear you. - No. - No. Because you don't free bees. You keep bees. Not only that, it seems you thought a bear would be an appropriate image for a jar of honey. They're very lovable creatures. Yogi Bear, Fozzie Bear, Build-A-Bear. You mean like this? Bears kill bees! How'd you like his head crashing through your living room?! Biting into your couch! Spitting out your throw pillows! OK, that's enough. Take him away. So, Mr. Sting, thank you for being here. Your name intrigues me. - Where have I heard it before? - I was with a band called The Police. But you've never been a police officer, have you? No, I haven't. No, you haven't. And so here we have yet another example of bee culture casually stolen by a human for nothing more than a prance-about stage name. Oh, please. Have you ever been stung, Mr. Sting? Because I'm feeling a little stung, Sting. Or should I say... Mr. Gordon M. Sumner! That's not his real name?! You idiots! Mr. Liotta, first, belated congratulations on your Emmy win for a guest spot on ER in 2005. Thank you. Thank you. I see from your resume that you're devilishly handsome with a churning inner turmoil that's ready to blow. I enjoy what I do. Is that a crime? Not yet it isn't. But is this what it's come to for you? Exploiting tiny, helpless bees so you don't have to rehearse your part and learn your lines, sir? Watch it, Benson! I could blow right now! This isn't a goodfella. This is a badfella! Why doesn't someone just step on this creep, and we can all go home?! - Order in this court! - You're all thinking it! Order! Order, I say! - Say it! - Mr. Liotta, please sit down! I think it was awfully nice of that bear to pitch in like that. I think the jury's on our side. Are we doing everything right, legally? I'm a florist. Right. Well, here's to a great team. To a great team! Well, hello. - Ken! - Hello. I didn't think you were coming. No, I was just late. I tried to call, but... the battery. I didn't want all this to go to waste, so I called Barry. Luckily, he was free. Oh, that was lucky. There's a little left. I could heat it up. Yeah, heat it up, sure, whatever. So I hear you're quite a tennis player. I'm not much for the game myself. The ball's a little grabby. That's where I usually sit. Right... there. Ken, Barry was looking at your resume, and he agreed with me that eating with chopsticks isn't really a special skill. You think I don't see what you're doing? I know how hard it is to find the rightjob. We have that in common. Do we? Bees have 100 percent employment, but we do jobs like taking the crud out. That's just what I was thinking about doing. Ken, I let Barry borrow your razor for his fuzz. I hope that was all right. I'm going to drain the old stinger. Yeah, you do that. Look at that. You know, I've just about had it with your little mind games. - What's that? - Italian Vogue. Mamma mia, that's a lot of pages. A lot of ads. Remember what Van said, why is your life more valuable than mine? Funny, I just can't seem to recall that! I think something stinks in here! I love the smell of flowers. How do you like the smell of flames?! Not as much. Water bug! Not taking sides! Ken, I'm wearing a Ohapstick hat! This is pathetic! I've got issues! Well, well, well, a royal flush! - You're bluffing. - Am I? Surf's up, dude! Poo water! That bowl is gnarly. Except for those dirty yellow rings! Kenneth! What are you doing?! You know, I don't even like honey! I don't eat it! We need to talk! He's just a little bee! And he happens to be the nicest bee I've met in a long time! Long time? What are you talking about?! Are there other bugs in your life? No, but there are other things bugging me in life. And you're one of them! Fine! Talking bees, no yogurt night... My nerves are fried from riding on this emotional roller coaster! Goodbye, Ken. And for your information, I prefer sugar-free, artificial sweeteners made by man! I'm sorry about all that. I know it's got an aftertaste! I like it! I always felt there was some kind of barrier between Ken and me. I couldn't overcome it. Oh, well. Are you OK for the trial? I believe Mr. Montgomery is about out of ideas. We would like to call Mr. Barry Benson Bee to the stand. Good idea! You can really see why he's considered one of the best lawyers... Yeah. Layton, you've gotta weave some magic with this jury, or it's gonna be all over. Don't worry. The only thing I have to do to turn this jury around is to remind them of what they don't like about bees. - You got the tweezers? - Are you allergic? Only to losing, son. Only to losing. Mr. Benson Bee, I'll ask you what I think we'd all like to know. What exactly is your relationship to that woman? We're friends. - Good friends? - Yes. How good? Do you live together? Wait a minute... Are you her little... ...bedbug? I've seen a bee documentary or two. From what I understand, doesn't your queen give birth to all the bee children? - Yeah, but... - So those aren't your real parents! - Oh, Barry... - Yes, they are! Hold me back! You're an illegitimate bee, aren't you, Benson? He's denouncing bees! Don't y'all date your cousins? - Objection! - I'm going to pincushion this guy! Adam, don't! It's what he wants! Oh, I'm hit!! Oh, lordy, I am hit! Order! Order! The venom! The venom is coursing through my veins! I have been felled by a winged beast of destruction! You see? You can't treat them like equals! They're striped savages! Stinging's the only thing they know! It's their way! - Adam, stay with me. - I can't feel my legs. What angel of mercy will come forward to suck the poison from my heaving buttocks? I will have order in this court. Order! Order, please! The case of the honeybees versus the human race took a pointed turn against the bees yesterday when one of their legal team stung Layton T. Montgomery. - Hey, buddy. - Hey. - Is there much pain? - Yeah. I... I blew the whole case, didn't I? It doesn't matter. What matters is you're alive. You could have died. I'd be better off dead. Look at

A textual proof?

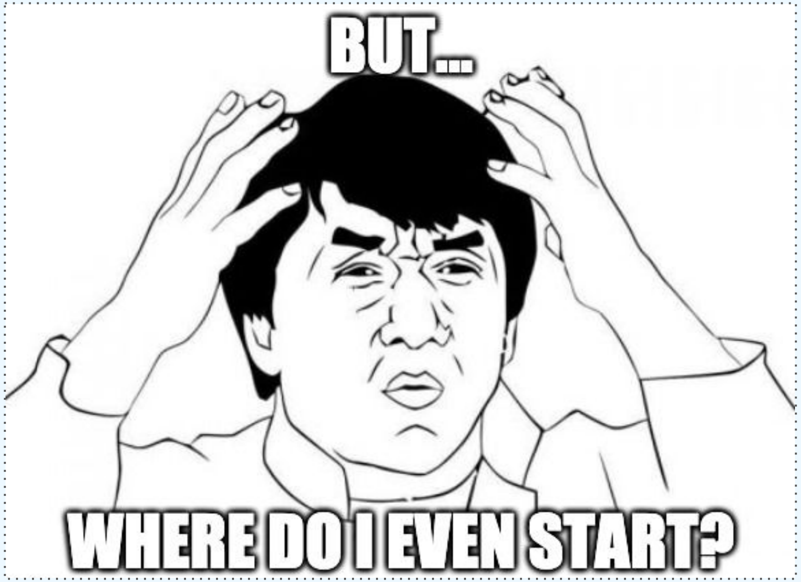
✅ *Decomposability*

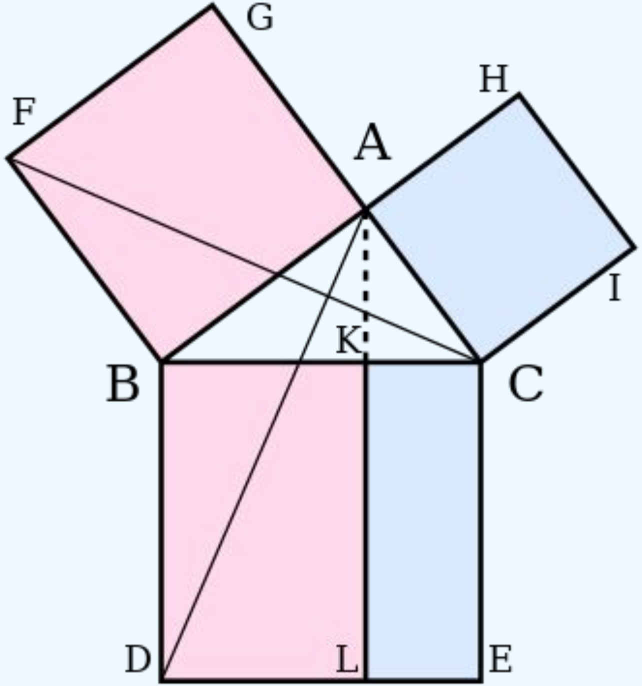
❌ *Structure*

The proof is as follows:

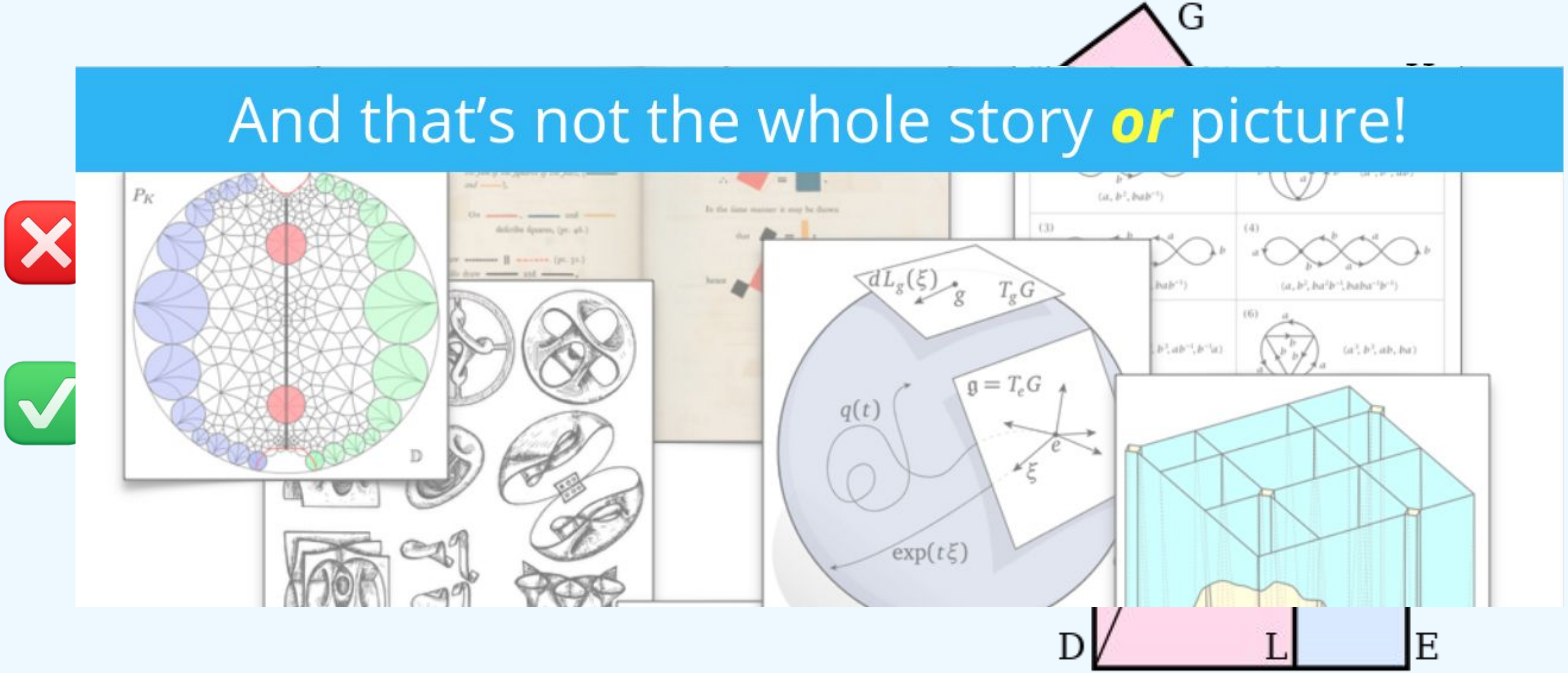
1. Let ACB be a right-angled triangle with right angle CAB.
2. On each of the sides BC, AB, and CA, squares are drawn, CBDE, BAGF, and ACIH, in that order. The construction of squares requires the immediately preceding theorems in Euclid, and depends upon the parallel postulate.[9]
3. From A, draw a line parallel to BD and CE. It will perpendicularly intersect BC and DE at K and L, respectively.
4. Join CF and AD, to form the triangles BCF and BDA.
5. Angles CAB and BAG are both right angles; therefore C, A, and G are collinear. Similarly for B, A, and H.
6. Angles CBD and FBA are both right angles; therefore angle ABD equals angle FBC, since both are the sum of a right angle and angle ABC.
7. Since AB is equal to FB and BD is equal to BC, triangle ABD must be congruent to triangle FBC.
8. Since A-K-L is a straight line, parallel to BD, then rectangle BDLK has twice the area of triangle ABD because they share the base BD and have the same altitude BK, i.e., a line normal to their common base, connecting the parallel lines BD and AL. (lemma 2)
9. Since C is collinear with A and G, square BAGF must be twice in area to triangle FBC.
10. Therefore, rectangle BDLK must have the same area as square BAGF = AB^2.
11. Similarly, it can be shown that rectangle CKLE must have the same area as square ACIH = AC^2.
12. Adding these two results, $AB^2 + AC^2 = BD \times BK + KL \times KC$
13. Since $BD = KL$, $BD \times BK + KL \times KC = BD(BK + KC) = BD \times BC$
14. Therefore, $AB^2 + AC^2 = BC^2$, since CBDE is a square.

A textual proof?

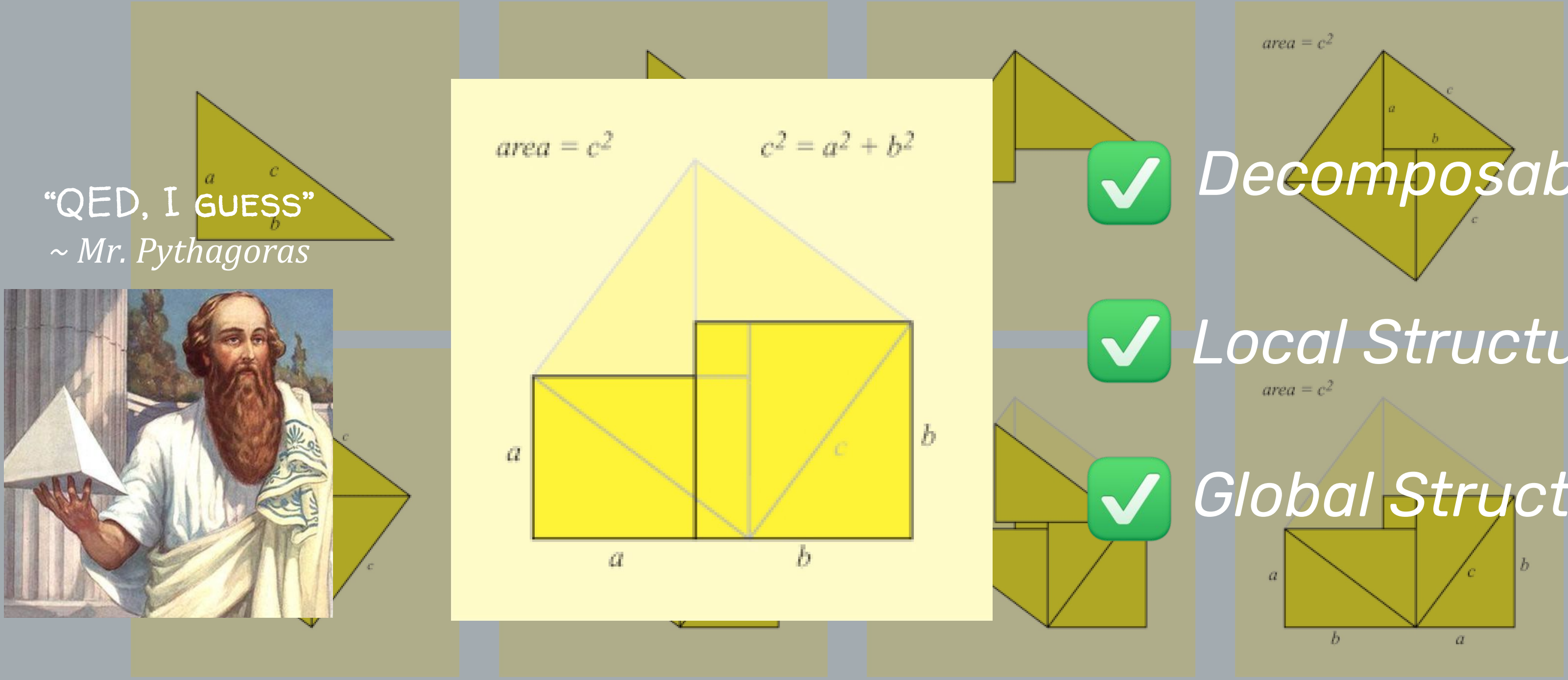




A visual proof?



A visual proof?



"QED, I GUESS"

~ *Mr. Pythagoras*

✅ *Decomposability*

✅ *Local Structure*

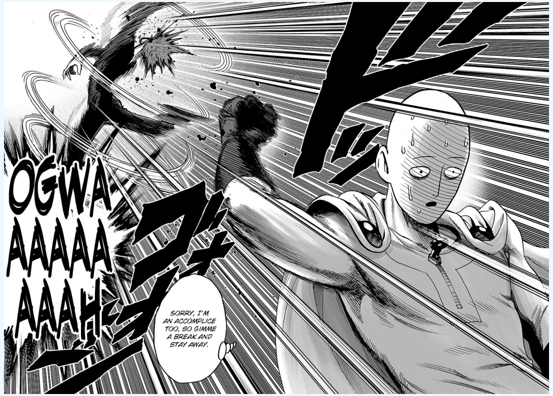
✅ *Global Structure*

A visual narrative

Visual Narrative

Noun. A story told through pictures.

aka Comics, Manga, Cave Paintings…



Visual narratives are hard to make.

We’ve talked about creating diagrams declaratively with *Penrose*

Could we also make diagrammatic ***narratives?*** *

⚠️ *We haven’t implemented any of this yet. ***This is just inquiry!***

What does the language look like?

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Set A, B
A ∩ B
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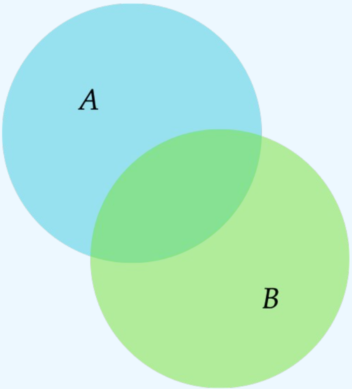


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Set A, B
A ∩ B
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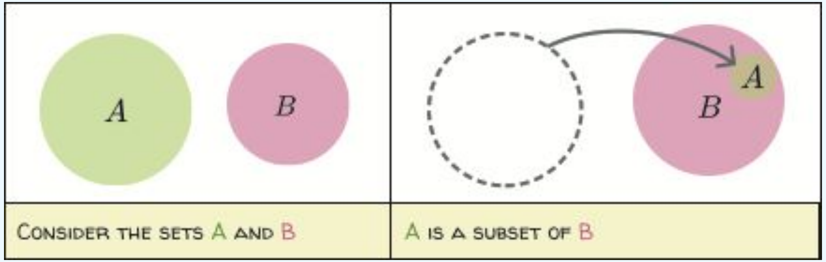
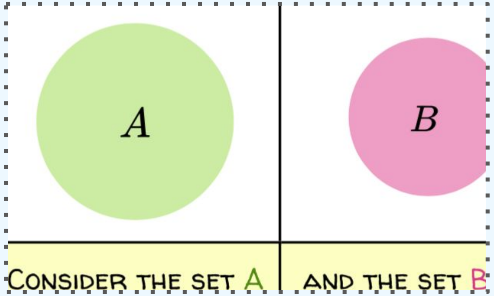
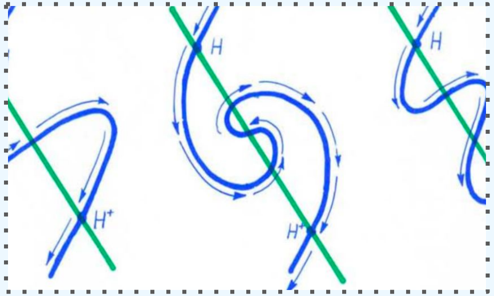
???

penrose inside





Conjunction



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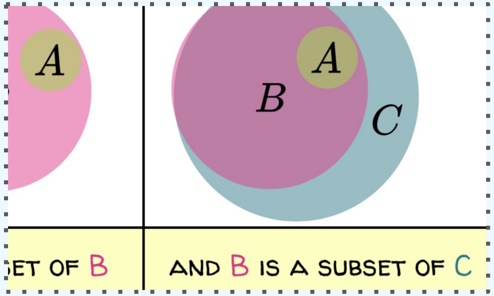

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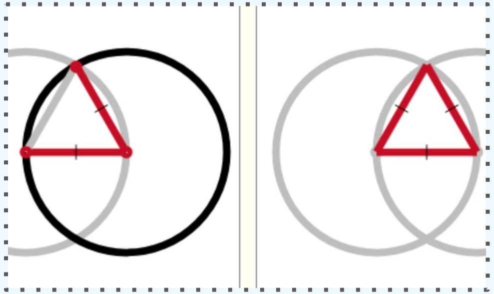
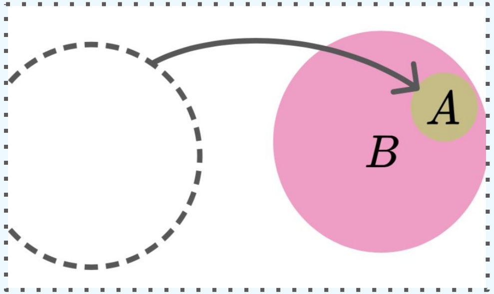
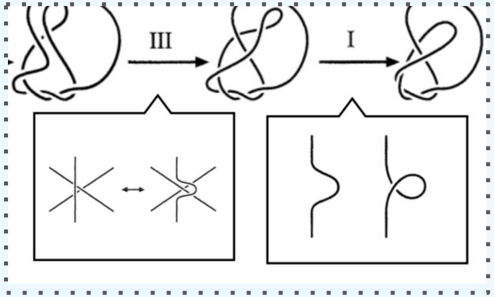


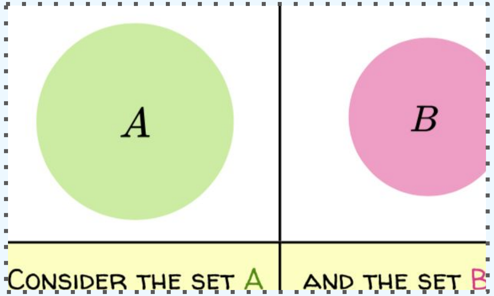
Foci



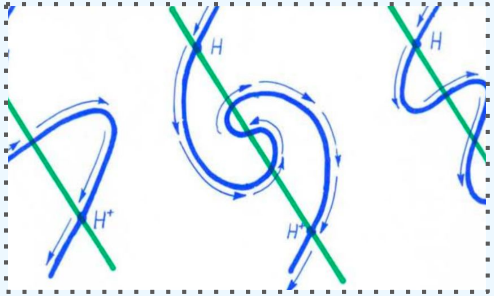
Emanata



Nesting



Conjunction



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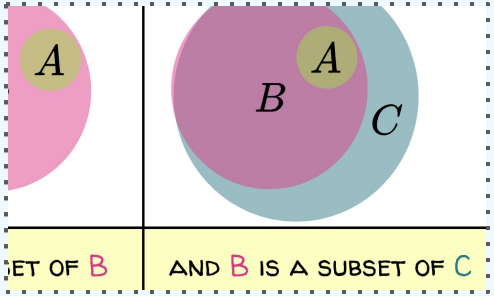
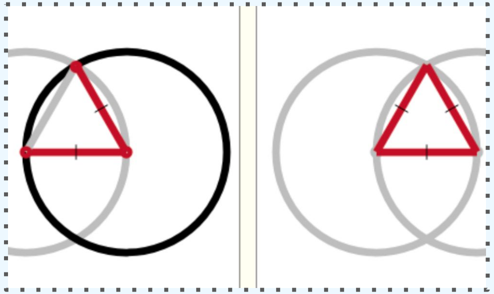
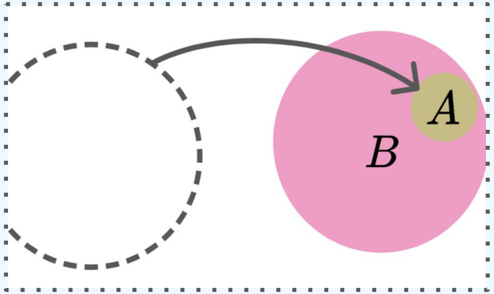


Predicates



Foci



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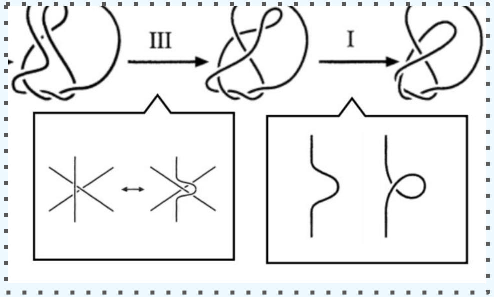

Nesting

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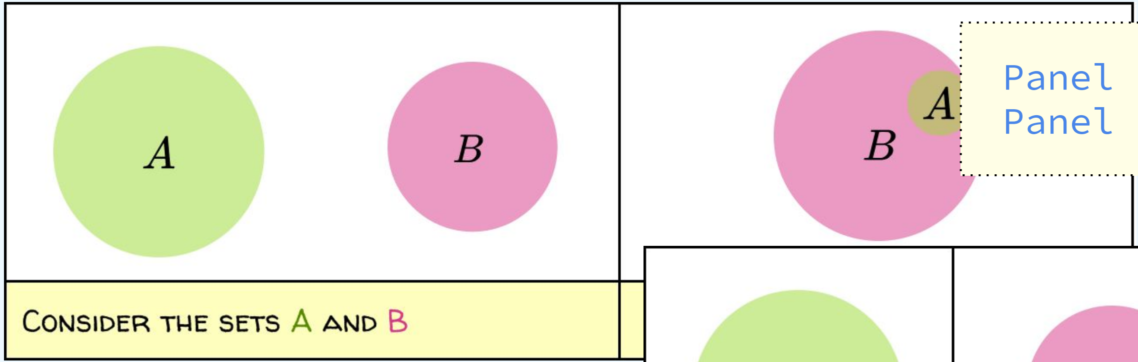
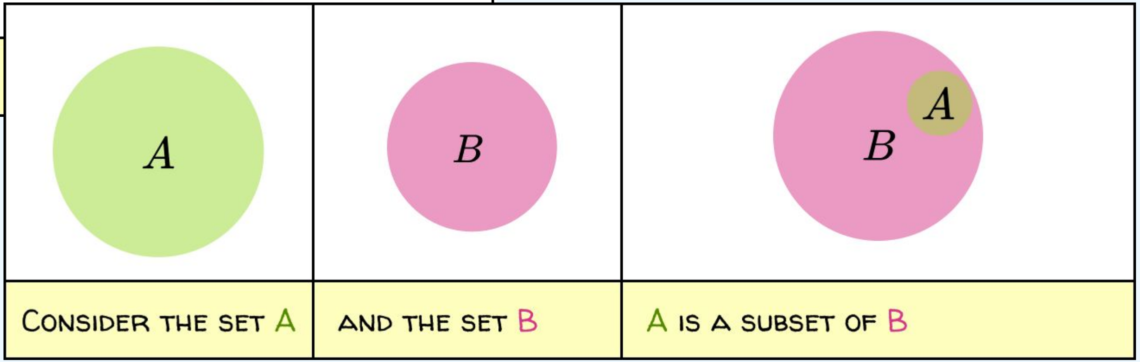
Panel ([A,B],
  "Consider  the sets [A] and [B]")
```

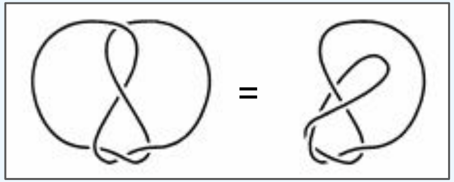



```
Panel ([A], "Consider  the set [A]")
Panel ([B], "and the set [B]")
```



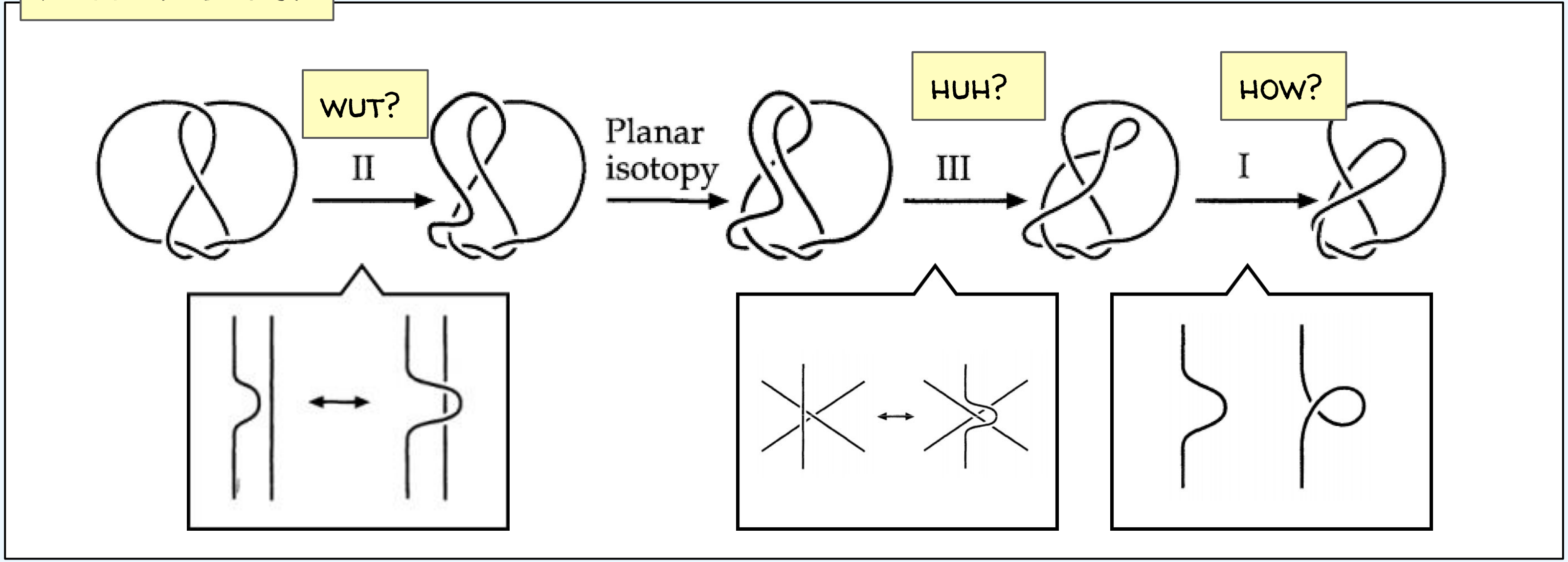
Conjunction

Fig 1.25, The Knot Book

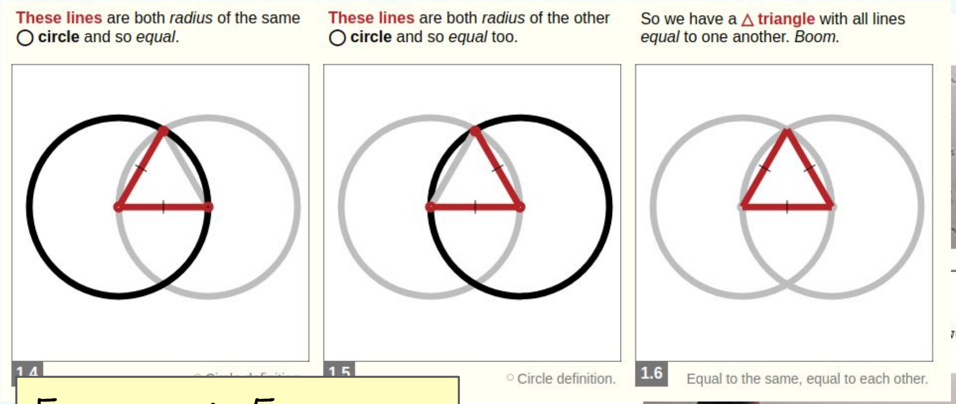



```
NestPanels (p1, [(thm1, t1),
(thm2, t2), (lemma1, t3)])
```

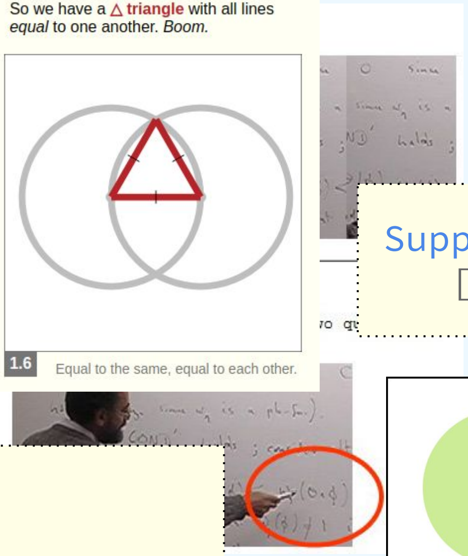
Proof (Fig 1.26)



Nesting

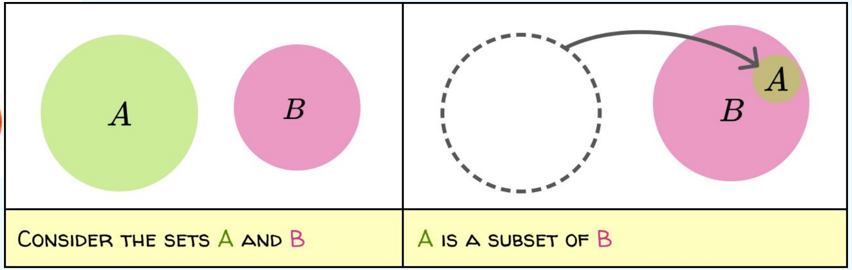


Eli parra's Elements



```
SuppletePanel ({ IsSubset(A,B) },
    [A,B], "[A] is a subset  of [B]")
```

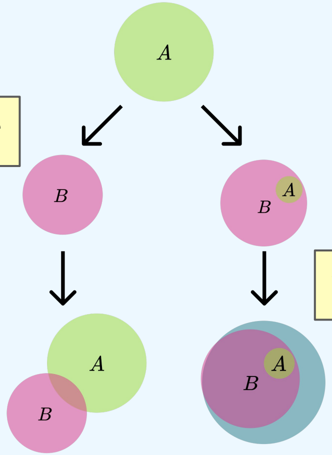
```
Circle C,D
{ Focus(C)
Panel ([C,D]) }
```



Foci & Emanata

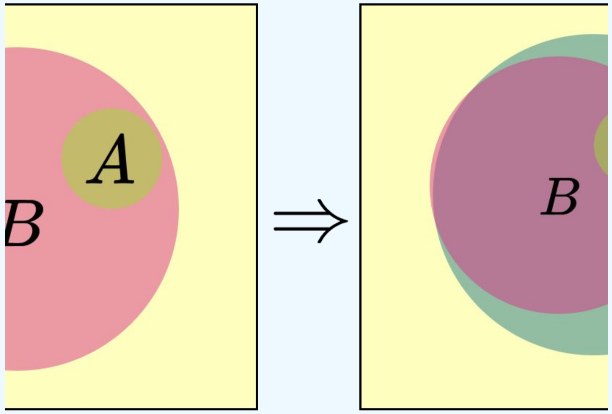
Parallel Counterfactuals

What if?



What if?

Gutter as implication



Future work: sampled layout, interactivity

What emerges?

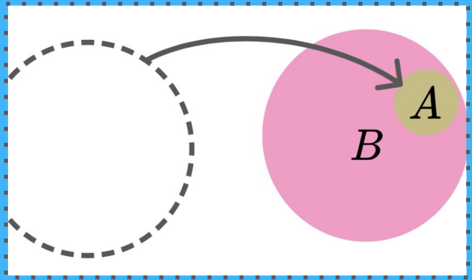
Thank you!

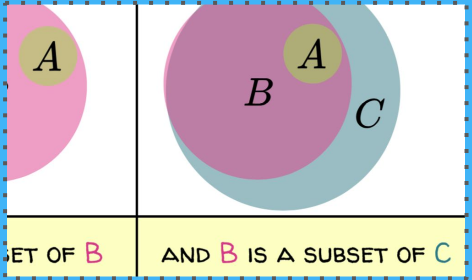
Acknowledgements

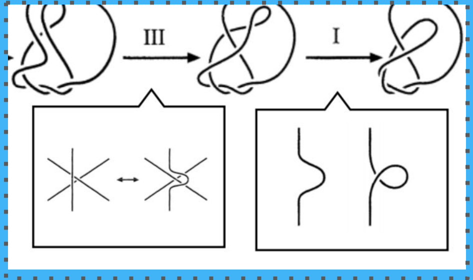
Co-Authors **Nimo Ni** and **Joshua Sunshine**

References via **Katherine Ye**, **Neil Cohn**, **Eli Parra**

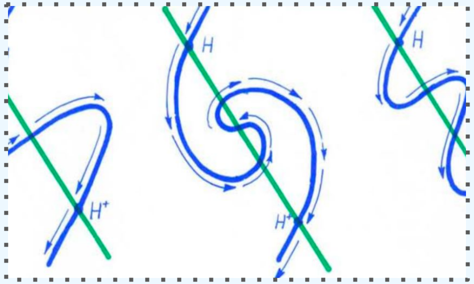
Questions?

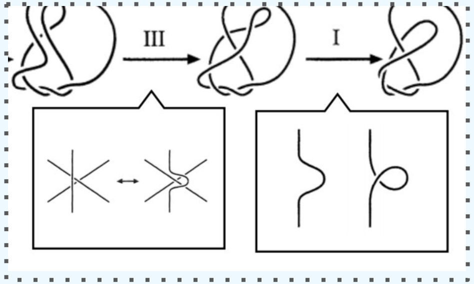


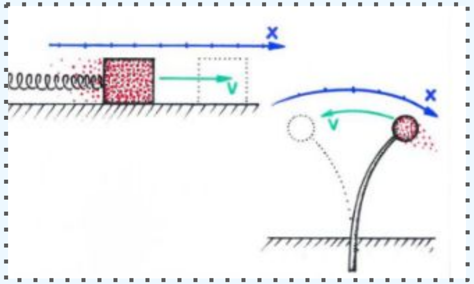


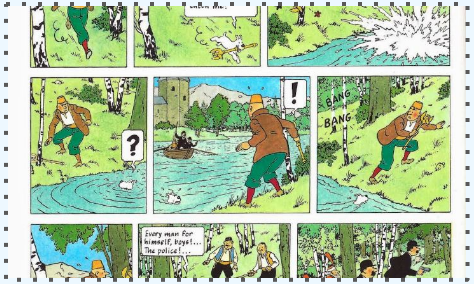


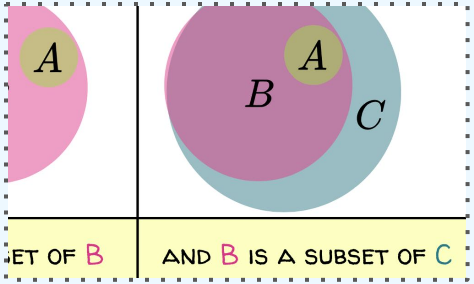

WELL, OKAY.
OKAY, IF THAT'S HOW IT IS...
IF THAT'S HOW YOU WANT TO PLAY IT.
YOU THINK I'M SCARED, HUH?

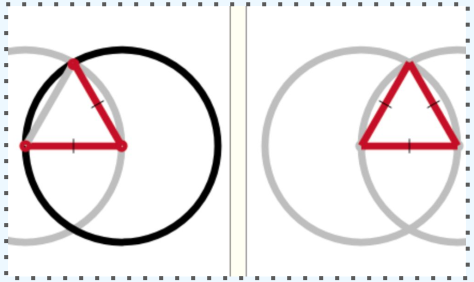

H
H

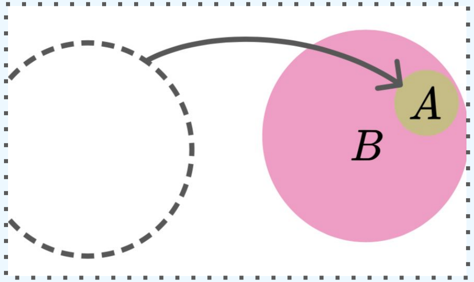

III
I


x
v
x
v


BANG
BANG
Every man for himself, boys!... The police!...


A
A
B
C
ET OF B
AND B IS A SUBSET OF C




A
B


ZAG
GUK
COFF
NO! NO! I CAN'T BE SEEING WHAT I THINK I'M SEEING! THAT'S IMPOSSIBLE!
COUGH! COUGH!
GASP!