Proofs

1. The product of any two odd integers is an odd integer **(tutorial 1 question 9)**
2. Let n be an integer. Then n^2 is odd if and only if n is odd **(tutorial 1 question 10)**
3. Rational numbers are closed under addition **(tutorial 2 question 3b)**
4. $\forall x \in \mathbb{R}\,((x^2 > x) \rightarrow (x < 0) \vee (x > 1))$ **(tutorial 2 question 7)**
5. If n is a product of two positive integers a and b, then $a \leq \sqrt{n}\ or\ b \leq \sqrt{n}$ **(tutorial 2 question 10)**
6. Let $A = \{2n + 1 : n \in \mathbb{Z}\}$ and $B = \{2n - 5 : n \in \mathbb{Z}\}$. $A = B$ **(tutorial 3 question 4)**
7. $A \cap (B \setminus C) = (A \cap B) \setminus C$ **(tutorial 3 question 5)**
8. $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$ **(tutorial 3 question 6)**
9. $A \oplus B = (A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$ **(tutorial 3 question 7)**
10. Let A and B be set. $A \subseteq B$ if and only if $A \cup B = B$ **(tutorial 3 question 8)**
11. Given that $\bigcup_{i=1}^{k} B_i \subseteq \bigcap_{j=1}^{l} C_j$, then $B_i \subseteq C_j$ for any $i \in \{1, 2, \cdots, k\}$ and any $j \in \{1, 2, \cdots, l\}$ **(tutorial 4 question 12)**
12. The following are logically equivalent **(tutorial 4 question 2)**
    a. R is symmetric, i.e. $\forall x, y \in A\,(x\,R\,y \Rightarrow y\,R\,x)$
    b. $\forall x, y \in A\,(x\,R\,y \Leftrightarrow y\,R\,x)$
    c. $R = R^{-1}$
13. Given that $S = \{(m, n) \in \mathbb{Z}^2 : m^3 + n^3\ is\ even\}$, then **(tutorial 4 question 5)**
    a. $S^{-1} = S$
    b. $S \circ S = S$
    c. $S \circ S^{-1} = S$
14. Let A, B, C, D be sets and $R \subseteq A \times B$, $S \subseteq B \times C$, $T \subseteq C \times D$, then $T \circ (S \circ R) = (T \circ S) \circ R$ **(tutorial 4 question 6)**

15. If $x \in S \in \tau$, then $[x] \in S$ **(tutorial 4 question 9a)**
16. $A/{\sim} = \tau$ **(tutorial 4 question 9b)**
17. The binary relation $\subseteq$ $on\ P(A)$ is a partial order **(tutorial 5 question 3)**
18. Asymmetry is $\forall x, y \in A\ (x\ R\ y \Rightarrow\ !\,(y\ R\ x))$ **(tutorial 5 question 6)**
19. Given a set A and a total order $\preccurlyeq$ on A, all minimal elements are smallest **(tutorial 5 question 7)**
20. a, b are compatible iff there exists $c \in A$ such that $a \preccurlyeq c$ and $b \preccurlyeq c$ **(tutorial 5 question 8)**
21. In all partially ordered sets, any two comparable elements are compatible **(tutorial 5 question 10a)**
22. In all partially ordered sets, any two compatible elements are NOT compatible **(tutorial 5 question 10b)**
23. Variant absorption law **(assignment 1 question 1)**
    a. $p \wedge ({\sim} p \vee q) \equiv p \wedge q$
    b. $p \vee ({\sim} p \wedge q) \equiv p \vee q$

Quick tips

1. For logical equivalence statements, if the same law is used more than once, can condense into one step and explain how many times the step is applied
2. Do not create loaded predicates, keep each predicate as atomic as possible
3. Work backwards when proving logical equivalence if necessary
4. Don’t need to quote theorems involving the sum of even and odd integers