1 Summary of logical equivalences

Law	Form 1	Form 2
Commutative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double negative	$\sim (\sim p) \equiv p$	
Idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$
Universal bound	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negation of $\mathbf{t}$ and $\mathbf{c}$	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$
Implication	$p \rightarrow q \equiv \sim p \vee q$	$\sim (p \rightarrow q) \equiv p \wedge \sim q$
Variant absorption law	$p \wedge (\sim \vee q) \equiv p \wedge q$	$p \vee (\sim p \wedge q) \equiv p \vee q$

Note associative law only applies to expressions with the same logical connectives
Note works for $\sim p$ and $\sim q$

2 Rules of inference

Rule of inference is a form of argument that is valid and *modus ponens* and *modus tollens* are examples of rules of inference

1. **Modus Ponens**

   $p \rightarrow q$

   p

   $\therefore q$

   Note type of *syllogism* and also known as *method of affirming*

2. **Modus Tollens**

   $p \rightarrow q$

   $\sim q$

   $\therefore \sim p$

Note uses *proof by contradiction* and also known as *method of denying*

3. **Generalization** (since at least one variable is true, the $\vee$ must be true)

   a. p

   $\therefore p \vee q$

   b. q

   $\therefore p \vee q$

4. **Specialization** (implies that both p *and* q are true)

   a. $p \wedge q$

   $\therefore p$

   b. $p \wedge q$

   $\therefore q$

5. **Conjunction**

   p

   q

   $\therefore p \wedge q$

6. **Elimination** (in an $\vee$ statement, at least one variable must be true, so we are determining which is true)

   a. $p \vee q$

   $\sim q$

   $\therefore p$

   b. $p \vee q$

   $\sim p$

   $\therefore q$

7. **Transitivity**

   $p \to q$

   $q \to r$

   $\therefore p \to r$

8. **Proof by Division into Cases**

   $p \vee q$

   $p \to r$

   $q \to r$

   $\therefore r$

9. **Contradiction Rule**

   $\sim p \to \mathbf{c}$

   $\therefore p$

3 Rules of inference for quantified statements

1. Universal instantiation
   $\forall x \in D, P(x)$
   $\therefore P(a)$ if $a \in D$
2. Universal generalization
   $P(a)$ for every $a \in D$
   $\therefore x \in D, P(x)$
3. Existential instantiation
   $\exists x \in D, P(x)$
   $\therefore P(a)$ for some $a \in D$
4. Existential generalization
   $P(a)$ for some $a \in D$
   $\therefore x \in D, P(x)$

4 Properties of sets

4.1 Subset relations

- **Inclusion of intersection:** $A \cap B \subseteq A$ or $A \cap B \subseteq B$
- **Inclusion of union:** $A \subseteq A \cup B$ or $B \subseteq A \cup B$
- **Transitive property of subsets:** $A \subseteq B \wedge B \subseteq C \rightarrow A \subseteq C$

(L5 S39)

4.2 Procedural versions of set definitions

Let X and Y be subsets of a universal set U and suppose $a, b \in U$

- $a \in X \cup Y \Leftrightarrow a \in X \vee a \in Y$
- $a \in X \cap Y \Leftrightarrow a \in X \wedge a \in Y$
- $a \in X \setminus Y \Leftrightarrow a \in X \wedge a \notin Y$
- $a \in \overline{X} \Leftrightarrow a \notin X$
- $(a, b) \in X \times Y \Leftrightarrow a \in X \wedge b \in Y$

4.3 Set identities

Law	**Form 1**	**Form 2**
Commutative	$A \cap B = B \cap A$	$A \cup B = B \cup A$
Associative	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Identity	$A \cap U = A$	$A \cup \emptyset = A$
Complement	$A \cup \overline{A} = U$	$A \cap \overline{A} = \emptyset$
Double complement	$\overline{\overline{A}} = A$	
Idempotent	$A \cap A = A$	$A \cup A = A$
Universal bound	$A \cup U = U$	$A \cap \emptyset = \emptyset$
De Morgan's	$\overline{A \cap B} = \overline{A} \cup \overline{B}$	$\overline{A \cup B} = \overline{A} \cap \overline{B}$
Absorption	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Complements of U and $\emptyset$	$\overline{U} = \emptyset$	$\overline{\emptyset} = U$
Set difference	$A \setminus B = A \cap \overline{B}$	