1 Differentiation

Limit existence: $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = \lim_{x\to c} f(x)$
Continuity: Applicable to endpoints, (1) $f(c)$ exists, (2) $\lim_{x\to c} f(x)$ exists, and (3) $\lim_{x\to c} f(x) = f(c)$
Rational functions:

$$\lim_{x\to\pm\infty} \frac{P(x)}{Q(x)} = \lim_{x\to\pm\infty} \frac{\overbrace{Ax^\alpha}^{\text{leading term}} + \cdots}{\underbrace{Bx^\beta}_{\text{leading term}} + \cdots} = \begin{cases} 0, \alpha < \beta \\ \frac{A}{B}, \alpha = \beta \\ \infty / -\infty, \alpha > \beta \end{cases}$$

Common limits: If $\lim_{x\to c} g(x) = 0$, then

- $\lim_{n\to\infty} \frac{\ln n}{n} = 0$
- $\lim_{n\to\infty} x^{\frac{1}{n}} = 1 (x > 0)$
- $\lim_{n\to\infty} (1 + \frac{x}{n})^n = e^x (\text{any x})$
- $\lim_{n\to\infty} \sqrt[n]{n} = 1$
- $\lim_{n\to\infty} x^n = 0 (|x| < 1)$
- $\lim_{n\to\infty} \frac{x^n}{n!} = 0 (\text{any x})$
- $\lim_{x\to c} \frac{\sin g(x)}{g(x)} = 1$
- $\lim_{x\to c} \frac{1 - \cos g(x)}{g(x)} = 0$
- $\lim_{x\to c} \frac{\tan g(x)}{g(x)} = 1$
- $\lim_{x\to\infty} \frac{1}{x^n} = \lim_{x\to-\infty} \frac{1}{x^n} = 0$

Derivative of inverse function: Let f be bijective and differentiable on an open interval I

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Parametric equations:

$$x = f(t) \text{ and } y = g(t), \therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

Common differentiation identities

Function	Derivative	Function	Derivative
x^n	nx^{n-1}	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\sin x$	$\cos x$	$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$	$\sec^{-1} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert > 1$
$\sec x$	$\sec x \tan x$	$\csc^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert > 1$
$\csc x$	$-\csc x \cot x$	$\cot^{-1} x$	$-\frac{1}{1+x^2}$
$\cot x$	$-\csc^2 x$	$\ln x$	$\frac{1}{x}$
e^x	e^x		

Evaluating $\csc^{-1} x, \sec^{-1} x, \cot^{-1} x$: $\sin^{-1} \frac{1}{x}, \cos^{-1} \frac{1}{x}, \tan^{-1} \frac{1}{x}$
Point of inflection: $f''(c) = 0$ if $(c, f(c))$ is a point of inflection and $f''(c)$ exists
Maximum and minimum values:

- **Maximum:** $f(x) \leq f(c) \forall x \in D$ (global) or $f(x) \leq f(c \forall x)$ in open interval
- **Minimum:** $f(x) \geq f(c) \forall x \in D$ (global) or $f(x) \geq f(c \forall x)$ in open interval

Critical point: of f is a) not an endpoint, and b) either $f'(c) = 0$ or $f'(c)$ does not exist
First derivative test: Let f be differentiable on an open interval containing a critical point c except possibly at c and f is continuous at c. If f' goes from...

- $-ve \to +ve \Rightarrow$ minimum at $f(c)$
- $+ve \to -ve \Rightarrow$ maximum at $f(c)$
- Unchanged $\Rightarrow$ no extreme

Second derivative test: Let f be a twice differentiable function defined in an open interval containing c. If $f'(c) = 0$ and $f''(c)$ is...

- < 0, $f(c)$ is a local maximum
- > 0, $f(c)$ is a local minimum
- $= 0$ is inconclusive

L'Hôpital's Rule:
If $\lim_{x\to c} \frac{f(x)}{g(x)}$ is $\frac{\infty}{\infty}$ or $\frac{0}{0}$, then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

2 Integrals

Partial fractions
Only possible if $deg(P(x)) < deg(Q(x))$. If $deg(P(x)) \geq deg(Q(x))$, then perform polynomial division first.

$$ax + b \longrightarrow \frac{A}{ax+b}$$

$$(ax+b)^k \longrightarrow \frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_k}{(ax+b)^k}$$

$$ax^2 + bx + c \longrightarrow \frac{Ax+B}{ax^2+bx+c}$$

$$(ax^2+bx+c)^k \longrightarrow \frac{A_1 x + B}{ax^2+bx+c} + \cdots + \frac{A_k x + B}{(ax^2+bx+c)^k}$$

Integration by substitution:

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Integration by parts:

$$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$$

Priority of functions:

Differentiation	Integration
Logarithmic	Exponential
Inverse trigonometric	Trigonometric
Algebraic (polynomials)	
Trigonometric	

Riemann sums: Approximating the area under function f from a to b by dividing each section into rectangles of width $\frac{1}{n}$

$$\int_a^b f(x)dx = \lim_{n\to\infty} \left\{ \sum_{k=1}^{n} \left(\frac{b-a}{n}\right) f\left(a + k\left(\frac{b-a}{n}\right)\right) \right\}$$

Fundamental Theorem of Calculus:

$$\frac{d}{dx} \int_a^{g(x)} f(t)dt = f(g(x))g'(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F(x)$ is the antiderivative of $f(x)$
Type 1 Improper Integral: If limit does not exist, improper integral diverges, else it converges
If $f(x)$ is continuous on given range,

$$\int_a^\infty f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx$$

Type 2 Improper Integral:

1. $\int_a^b f(x)dx = \lim_{c\to a^+} \int_c^b f(x)dx$ if $f(x)$ is continuous on $(a, b]$ and discontinuous at a

2. $\int_a^b f(x)dx = \lim_{c\to b^-}\int_a^c f(x)dx$ if $f(x)$ is continuous on $[a,b)$ and discontinuous at b
3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ if $f(x)$ is discontinuous at c with $a<c<b$

Common integration identities:

Function	Antiderivative
$\int (ax+b)^n dx$	$\frac{(ax+b)^{n+1}}{(n+1)a} + C, (n \neq 1)$
$\int \frac{1}{ax+b}dx$	$\frac{1}{a}\ln\lvert ax+b\rvert + C$
$\int e^{ax+b}dx$	$\frac{1}{a}e^{ax+b}+C$
$\int \sin(ax+b)dx$	$-\frac{1}{a}\cos(ax+b)+C$
$\int \cos(ax+b)dx$	$\frac{1}{a}\sin(ax+b)+C$
$\int \tan(ax+b)dx$	$\frac{1}{a}\ln\lvert\sec(ax+b)\rvert+C$
$\int \sec(ax+b)dx$	$\frac{1}{a}\ln\lvert\sec(ax+b)+\tan(ax+b)\rvert+C$
$\int \csc(ax+b)dx$	$-\frac{1}{a}\ln\lvert\csc(ax+b)+\cot(ax+b)\rvert+C$
$\int \cot(ax+b)dx$	$-\frac{1}{a}\ln\lvert\csc(ax+b)\rvert+C$
$\int \sec^2(ax+b)dx$	$\frac{1}{a}\tan(ax+b)+C$
$\int \csc^2(ax+b)dx$	$-\frac{1}{a}\cot(ax+b)+C$
$\int \sec(ax+b)\tan(ax+b)dx$	$\frac{1}{a}\sec(ax+b)+C$
$\int \csc(ax+b)\cot(ax+b)dx$	$-\frac{1}{a}\csc(ax+b)+C$
$\int \frac{1}{a^2+(x+b)^2}dx$	$\frac{1}{a}\tan^{-1}(\frac{x+b}{a})+C$
$\int \frac{1}{\sqrt{a^2-(x+b)^2}}dx$	$\sin^{-1}(\frac{x+b}{a})+C$
$\int \frac{-1}{\sqrt{a^2-(x+b)^2}}dx$	$\cos^{-1}(\frac{x+b}{a})+C$
$\int \frac{1}{a^2-(x+b)^2}dx$	$\frac{1}{2a}\ln\left\lvert\frac{x+b+a}{x+b-a}\right\rvert+C$
$\int \frac{1}{(x+b)^2-a^2}dx$	$\frac{1}{2a}\ln\left\lvert\frac{x+b-a}{x+b+a}\right\rvert+C$
$\int \frac{1}{\sqrt{(x+b)^2\pm a^2}}dx$	$\ln\left\lvert(x+b)+\sqrt{(x+b)^2\pm a^2}\right\rvert+C$
$\int \sqrt{a^2-x^2}dx$	$\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}(\frac{x}{a})+C$
$\int \sqrt{x^2-a^2}dx$	$\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left\lvert x+\sqrt{x^2-a^2}\right\rvert+C$

Area between curves:
Regardless of whether $f(x)\geq g(x)$ or vice versa

$$A = \int_a^b \lvert f(x)-g(x)\rvert\, dx$$

If lower bound by x-axis, then $g(x)$ is a constant function of 0
Note: split into smaller integrals if necessary
Volume of solid of revolution: Assuming revolution about x-axis or y-axis only
Disk method:

$$V = \pi\int_a^b [f(x)]^2dx - \pi\int_a^b [g(x)]^2dx$$

where $f(x)$ and $g(x)$ is the radius of the outer and inner disk and $f(x)\geq g(x)$ (applicable for functions of y)
Cylindrical shell method:

$$V = 2\pi\int_a^b x\,\lvert f(x)-g(x)\rvert\, dx$$

where x is the radius of the shell, $f(x)-g(x)$ is the height of the shell, and $f(x)\geq g(x)$ (applicable for functions of y)
Arc length of curve:

$$L = \int_a^b \sqrt{1+[f'(x)]^2}dx$$

applicable to $x=f(y)$ as well

3 Sequences and series

$$\sum_{n=1}^{\infty} a_n$$

(Infinite) series is convergent is the sequence of partial sums S_n is convergent, otherwise, it is divergent
Types of series:

1. Geometric series
$$\sum_{n=1}^{\infty} ar^{n-1}, (a\neq 0)$$
Convergent to $\frac{a}{1-r}$ when $\lvert r\rvert<1$, otherwise, divergent
2. Harmonic series
$$\sum_{n=1}^{\infty}\frac{1}{n}$$
is always divergent
3. p-series
$$\sum_{n=1}^{\infty}\frac{1}{n^p}$$
is convergent iff $p>1$
4. Alternating series
$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1-b_2+b_3-b_4+\cdots$$
b_n is positive and used in alternating series test
5. Power series
$$\sum_{n=0}^{\infty}c_n(x-a)^n = c_0+c_1(x-a)+c_2(x-a)^2+c_3(x-a)^3+\cdots$$
centered at a. Is the representation of the final function after converging, e.g. $\sum_{n=0}^{\infty}x^n$ is the power series representation of the function $\frac{1}{1-x}$ about $x=0$
   (a) Converges at $x=a$ only
   (b) Converges for all x
   (c) Converges if $\lvert x-a\rvert<R$ (radius of convergence) and diverges if $\lvert x-a\rvert>R$ ($\lvert x-a\rvert=R$ needs to be checked separately)
6. Taylor and Maclaurin series (if f has a power series representation at $x=a$, then it can be represented as a Taylor series of f at $x=a$
$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$
If $a=0$, then it is a Maclaurin series of f
$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$$
compute $f^{(n)}(a)$ first and then solve for a given a before finding the rest of the Taylor series

Radius of convergence: Given a power series and $\lim_{n\to\infty}\left\lvert\frac{c_{n+1}}{c_n}\right\rvert = L$ or $\lim_{n\to\infty}\sqrt[n]{\lvert c_n\rvert}=L$, then $R=\frac{1}{L}$
If $R>0$, then

$$f(x)=\sum_{n=0}^{\infty}c_n(x-a)^n$$

$$f'(x)=\sum_{n=1}^{\infty}nc_n(x-a)^{n-1}, \lvert x-a\rvert<R$$

$$\int f(x)dx = \sum_{n=1}^{\infty}c_n\frac{(x-a)^{n+1}}{n+1}, \lvert x-a\rvert<R$$

Tests

1. **Test for divergence:** If $\lim_{n\to\infty}a_n$ does not exist or not equals 0, then the series $\sum_{n=1}^{\infty}a_n$ is divergent

2. **Integral Test:** If a_n is a sequence of positive terms and $f(n)$ is continuous, positive, decreasing function for all $n \geq 1$, then if $\int_1^\infty f(x)dx$ is convergent, so is $\sum_{n=1}^{\infty} a_n$, otherwise, divergent

3. **Comparison Test:** Given $a_n \leq b_n \forall n$, then if $\sum_{n=1}^{\infty} b_n$ converges, so does $\sum_{n=1}^{\infty} a_n$. If $\sum_{n=1}^{\infty} a_n$ diverges, so does $\sum_{n=1}^{\infty} b_n$

4. **Ratio Test:** $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$, then

5. **Root Test:** $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$, then

   (a) $0 \leq L < 1$: $\sum_{n=1}^{\infty} a_n$ is absolutely convergent ($\sum_{n=1}^{\infty} |a_n|$)

   (b) $L > 1$: $\sum_{n=1}^{\infty} a_n$ is divergent

   (c) $L = 1$: inconclusive

6. **Alternative Series Test:** If b_n is a sequence of positive numbers such that (1) b_n is decreasing (using derivative) and (2) $\lim_{n\to\infty} b_n = 0$, then $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ is convergent

7. **Absolute Convergence:** If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges (converse is not always true, i.e. converge conditionally)

4 Vectors

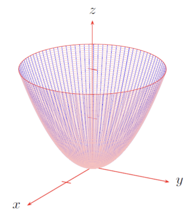
Distance between points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$: $|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$
Equation of sphere: Center (h, k, l): $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$
Vector from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$: $\overrightarrow{AB} = \vec{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1\rangle$
Length of $\vec{u} = \langle u_1, u_2, u_3\rangle$: $||\vec{u}|| = \sqrt{u_1^2 + u_2^2 + u_3^2}$
Unit vector: $\vec{u} = \frac{\vec{a}}{||\vec{a}||}$
Dot product: $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
Angle between vectors: $\vec{a} \cdot \vec{b} = ||\vec{a}||||\vec{b}|| \cos\theta$
Orthogonal vectors: $\vec{a} \cdot \vec{b} = 0$
Vector projection of $\vec{b}$ onto $\vec{a}$: $\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a}$
Shortest distance from point $P(x_0, y_0, z_0)$ to the plane $ax+by+cz = d$: $\frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2+b^2+c^2}}$
Cross product:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

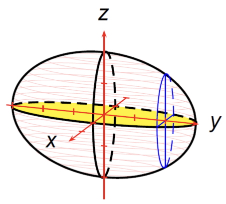
Remember the minus in the middle
$\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$ and $\vec{b}$
Angle between vectors using cross product: $||\vec{a} \times \vec{b}|| = ||\vec{a}||||\vec{b}|| \sin\theta$
Area of parallelogram forms by $\vec{a}$ and $\vec{b}$: $||\vec{a} \times \vec{b}||$, area of triangle bound is half of that
Distance from point to line: $||\vec{a}|| \sin\theta = \frac{||\vec{a} \times \vec{b}||}{||\vec{b}||}$ where $\vec{b}$ is the projection of $\vec{a}$ onto $\vec{c}$
Equation of line given point already on the line ($\vec{r_0} = \langle x_0, y_0, z_0\rangle$) and parallel vector ($\vec{v} = \langle a, b, c\rangle$): $\vec{r} = \vec{r_0} + t\vec{v}$. In parametric form: $\langle x_0 + at, y_0 + bt, z_0 + ct\rangle$
Skew lines: Non-parallel and non-intersecting lines (show non-parallel if both lines are not multiples of each other (using the parallel vector from the line) and show non-intersecting by equating the coordinates and showing inconsistency)
Equation of plane: $\vec{n} \cdot (\vec{r} - \vec{r_0}) = 0$ given that $\vec{n}$ is orthogonal to the plane (using cross product of two chosen vectors) and $\vec{r_0}$ is the starting point used
Linear equation of plane: $ax + by + cz + d = 0$ where $d = -(ax_0 + by_0 + cz_0)$
Non-parallel planes intersect at a line and the angle between these planes is the acute angle between their normal vectors.

5 Functions of several variables

Derivative of vector-valued function ($\vec{r}(t) = \langle f(t), g(t), h(t)\rangle$): $\vec{r}'(t) = \langle f'(t), g'(t), h'(t)\rangle$
Arc length in space given $\vec{r}(t) = \langle f(t), g(t), h(t)\rangle$: $\int_a^b ||\vec{r}'(t)||$
Cylinder: there exists a plane P such that all planes parallel to P intersect the surface in the same curve ($x^2 + y^2 = a^2$ in 3-d is a cylinder)
Quadric surface: graph of second-degree equation with three variables i.e. includes x^2, y^2, z^2
Elliptic paraboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$



Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



Partial derivatives (with respect to a given variable): differentiate while keeping all other variables constant - $\frac{\partial f}{\partial x} = f_x$
Higher order partial derivatives: f_{xx} or f_{xy}
Clairaut's Theorem (used to check if f exists based on f_x and f_y): $f_{xy} = f_{yx}$
Equation of tangent plane with normal vector $\langle f_x(a, b), f_y(a, b), -1\rangle$: $f_x(a, b)(x-a) + f_y(a, b)(y-b) - (z - f(a, b)) = 0$
Chain rule: $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial f}\frac{\partial f}{\partial x} + \frac{\partial w}{\partial g}\frac{\partial g}{\partial y} + \cdots$ (draw tree diagram of relations and pick all branches that have intended variable)
Implicit differentiation (z is a function of x and y):

$$\frac{\partial z}{\partial x} = -\frac{F_x(x, y, z)}{F_z(x, y, z)}$$

Increments of Δz: $\Delta z \approx dz = f_x(a, b)dx + f_y(a, b)dy = f_x(a, b)\Delta x + f_y(a, b)\Delta y$
Directional derivative in direction of unit vector ($\vec{u}$): $D_u f(x, y, z) = \nabla f \cdot \vec{u}$ where ∇f is the gradient vector: $\langle f_x, f_y, f_z\rangle$ at (x, y, z)
Tangent plane to level surface: $\nabla F(x_0, y_0, z_0) \cdot \langle x - x_0, y - y_0, z - z_0\rangle = 0$
Change in given direction: $D_u f(x, y, z) * ds$ (directional derivative times the increment)
Maximum rate of increase/decrease: in the direction of the gradient vector ∇f ($-\nabla f$ for decrease)
Second derivative test (on critical points $f_{xx} = f_{yy} = 0$): $D = D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$

1. $D > 0, f_{xx}(a, b) > 0 \Rightarrow f(a, b)$ is local minimum

2. $D > 0, f_{xx}(a, b) < 0 \Rightarrow f(a, b)$ is local maximum

3. $D < 0 \Rightarrow f(a, b)$ is saddle point

4. $D = 0 \Rightarrow$ is inconclusive

6 Double Integrals

Find the area under the graph ($R = \{(x, y)|c \leq x \leq d, a \leq y \leq b\}$) and integrate over it

$$\iint_R f(x,y)dA = \int_c^d \int_a^b f(x,y)dydx$$

order of integration does not matter (Fubini's Theorem)

$$\iint_R f(x,y)dA = \left(\int_a^b g(x)dx\right)\left(\int_c^d h(y)dy\right)$$

Type 1 region: $R = \{(x, y)|a \leq x \leq b, h(x) \leq y \leq g(x)\}$

$$\int_a^b \int_{h(x)}^{g(x)} f(x,y)dydx$$

Type 2 region: $R = \{(x, y)|h(y) \leq x \leq g(y), c \leq y \leq d\}$

$$\int_c^d \int_{h(y)}^{g(y)} f(x,y)dxdy$$

$$\iint_D f(x,y)dA = \iint_{D_1} f(x,y)dA + \cdots + \iint_{D_n} f(x,y)dA$$

Area of plane region:

$$\iint_D 1dA$$

Convert to polar coordinates (if region is circular): $r^2 = x^2 + y^2, x = r\cos\theta, y = r\sin\theta$ (r is the radius and θ is the quadrants occupied)

$$R = \{(r, \theta)|a \leq r \leq b, \alpha \leq \theta \leq \beta\}$$

$$\iint_R f(x,y)dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)rdrd\theta$$

Surface area:

$$\iint_D dS = \iint_D \sqrt{f_x^2 + f_y^2 + 1}dA$$

7 Ordinary Differential Equations

Separable first order ODE:

$$\frac{dy}{dx} = f(x)g(y)$$

$$\int \frac{1}{g(y)}dy = \int f(x)dx + C$$

Proportional rate of change:

$$\frac{dy}{dx} = ky$$

Half-life formula: $y = Ae^{-kt}$

Reduction to separable form: substitute common variables with new variables, differentiate new variable and substitute back into original differential equation to solve

Linear first order ODE:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

create integrating factor $I(x) = e^{\int P(x)dx}$ and use for

$$\frac{d}{dx}(y \cdot I(x)) = Q(x) \cdot I(x)$$

Bernoulli Equation:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

find $u = y^{1-n}$ and substitute into original equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x)$$

solve new linear first order ODE

Standard Malthus's model:

$$N(t) = \hat{N}e^{kt}$$

Logistic Malthus' model:

$$N(t) = \frac{N_\infty}{1 + \left(\frac{N_\infty}{\hat{N}} - 1\right)e^{-Bt}}$$

where $\hat{N}$ is the initial population and N_∞ is the carrying capacity (max)

8 Basics

Solving quadratic inequalities

Find the roots, select x between roots, apply x to quadratic inequality to check if applicable. If applicable, include root in solution

Solving absolute inequalities

Applicable to $< and >$
$|f(x)| \leq g(x) \Leftrightarrow -g(x) \leq f(x) \leq g(x)$
$|f(x)| \geq g(x) \Leftrightarrow f(x) \leq -g(x) \vee f(x) \geq g(x)$

Solving rational inequalities

Bring all terms with x to one side and solve accordingly

Change of base formulae

$$\log_a x = \frac{\ln x}{\ln a}, a > 0 \text{ and } a \neq 1$$

9 Appendix

Trigonometric identities:

- $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
- $\sin 2x = 2\sin x \cos x$
- $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$
- $\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$
- $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sec^2 x - 1 = \tan^2 x$
- $\csc^2 x - 1 = \cot^2 x$
- $\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$
- $\cos A \sin B = \frac{1}{2}(\sin(A+B) - \sin(A-B))$
- $\cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$
- $\sin A \sin B = -\frac{1}{2}(\cos(A+B) - \cos(A-B))$
- $\sqrt{a^2 - (x+b)^2} \to x + b = a\sin\theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
- $\sqrt{a^2 + (x+b)^2} \to x + b = a\tan\theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
- $\sqrt{(x+b)^2 - a^2} \to x + b = a\sec\theta, 0 \leq \theta \leq \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$

Common Maclaurin series:

- $f(x) = f(0) + xf'(0) + \dfrac{x^2}{2!}f''(0) + \cdots + \dfrac{x^n}{n!}f^{(n)}(0) + \cdots$
- $(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \cdots + \dfrac{n(n-1)\cdots n(n-r+1)}{r!}x^r + \cdots$
- $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots + \dfrac{x^r}{r!} + \cdots$
- $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \cdots + \dfrac{(-1)^r x^{2r+1}}{(2r+1)!} + \cdots$
- $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \cdots + \dfrac{(-1)^r x^{2r}}{(2r)!} + \cdots$
- $\ln 1 + x = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \cdots + \dfrac{(-1)^{r+1}x^r}{r} + \cdots$

Others:

- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- Ellipse: $x = a\cos t + x_0; y = b\sin t + y_0$
- Circle: $x = r\cos t + x_0; y = r\sin t + y_0$
- Hyperbola: $x = a\sec t + x_0; y = b\tan t + y_0$