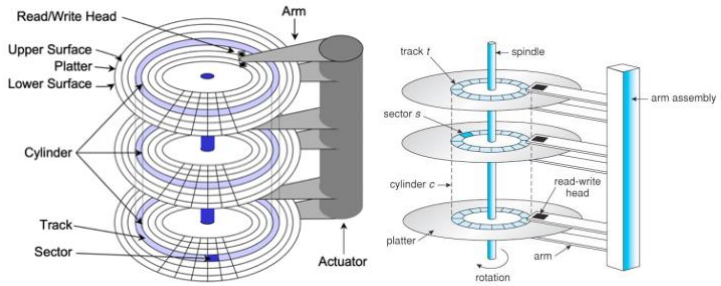
Disk access time

Command processing time: interpreting access command by disk controller



Write order: write to fill a cylinder first before moving to the next cylinder

Read order: all blocks read in cylinder before next cylinder; all tracks on one side read first before the next side; all blocks read as contiguous sectors

Blocks: sequence of one or more contiguous sectors

Seek time: moving arms to position disk head on track

- Seek per cylinder (if sequentially stored)
- Average: 5 – 6 milliseconds

Rotational delay: waiting for block to rotate under head

- Incurred once if sequentially stored/read
- Depends on rotation speed (rotations per minute)
- Average: time for ½ revolution

Transfer time: moving data to/from disk surface

- N = number of requested sectors on same track
- N * (time for one revolution / # of sectors per track)
- Applicable per track/sector/block
- Average: 100 – 200 microseconds

Access time: seek time + rotational delay + transfer time

Response time: queuing delay + access time

Storage manager components

- Data stored & retrieved in units called disk blocks/pages

Files & access methods layer (file layer)	Deals with organization and retrieval of data
Buffer manager	Controls reading/writing of disk pages
Disk space manager	Keeps track of pages used by file layer

Buffer manager

- Check if requested page in frame, if so, increment pin count and return address of f
  - Otherwise, use replacement policy to find victim page, increment count, replace page, and write to disk if victim page was dirty
- Coordinates with transaction manager for data correctness and recoverability

Buffer pool: main memory allocated for DBMS

- Partitioned into block-sized pages: frames
- Clients:
  - Request for disk page to be fetched into buffer pool
  - Release a disk page in buffer pool

Frame information:

- Pin count: number of clients using page (default 0)
- Dirty flag: if page is dirty (modified, not on disk) (default false)

Pinning: incrementing pin count of request page in frame

Unpinning: decrementing pin count

- Should update dirty flag to true if page is dirty

Page replacement: only if pin count is 0

Buffer replacement policies

Others: random, first in first out (FIFO), most recently used (MRU)

LRU: uses a queue of pointers to frames with pin count = 0

- Sequential flooding: evicting all the time, same effect as having 1 buffer page

Clock: variant of LRU

- Current variable: points to a buffer frame
- Each frame has a referenced bit which is turned on when pin count is 0 (i.e. during unpin)
- Replace page with referenced bit off and pin count is 0
- Reference bit turned off once visited

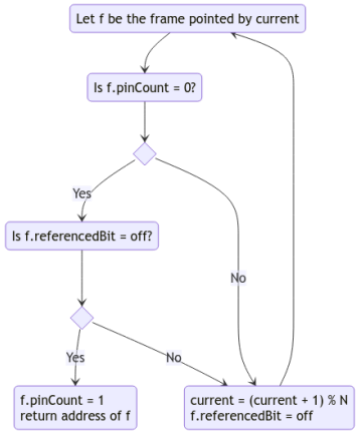
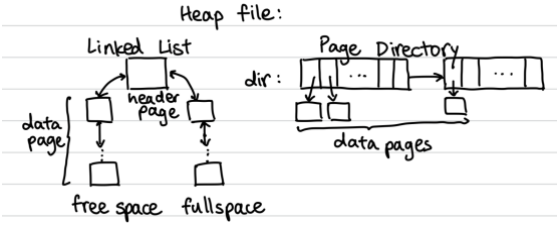


Figure 1: Clock replacement policy

Files

- Each relation is a file of records
- Each record has a unique record identifier (RID/TID)
- Each relation has a file, each file has many pages, each page has many records

File organization: heap file (unordered), sorted file (ordered by search key), hashed file (located in blocks via hash function)



Page formats

- Determines how records are organized within a page
- RID = (page id, slot number)

Fixed length records: may result in internal fragmentation, packed and unpacked organization (use bit map)

- **Packed organization:** costly delete operation matters, shift and move to fill in a gap

Variable length records: slotted page organization

- Divide single data pages into data space and free space
- Contains data and slot directory
- Slot directory points to entries in the data space, storing the byte offset of each entry and left address
- Slot directory: <entries> <# of slots> <free space start ptr>
  - Filled from both directions
- Space managed using compaction (to remove fragmentation)

Record formats

- Determines how fields are organized within a record

Fixed length records: fields are stored consecutively

Variable length records: delimited by special symbol or stored with a prefix array of offsets (indicates the offset to beginning of a field)

Index

- Data structure to speed up retrieval of data records based on search key

Search key: sequence of *k* data attributes, *k >= 1,* composite (k>1)

Unique index: search key is a candidate key

Storage: stored as a file (separate from original relation file) with records in an index referred to as data entries

- Simple index: index + pointer to record in relation in a linked list
  - Stores more records per page since record size smaller

Types:

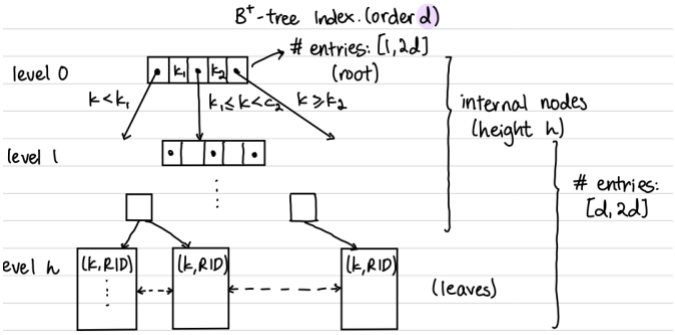
1. Tree-based index: based on sorting of search key value
2. Hash-based index: access data entries with hashing function

Considerations: search performance, storage overhead, update performance

- Types of search: equality (k = v), range (v1 <= k <= v2)
  - Hash-based index cannot support range search

B+-tree index

- Every node is a data page/disc block
- Dynamic index structure so data updates update index



Leaf nodes: store data entries; doubly linked list
Internal nodes: store index entries (p0, k1, p1, k2, p2, …, pn)

- K1 < K2 < … < Kn
- *pi:* disk page address (root node of index subtree *Ti*)
- Index entry: (ki, pi), ki is separator between of p(i-1) and pi
  - p0 is not index entry

Properties: height balanced

- Order of index tree, *d*
  - Each non-root node: *m* entries, *m* in [d, 2d]
  - Root node: *n* entries, *n* in [1, 2d]

Include columns: including column data with leaf entries
Minimum leaf nodes: at level i, 2(d+1)^(i-1) (all lower bound)
Maximum leaf nodes: at level i, (2d+1)^i (all upper bound)
Calculating order: 2d(key size) + (2d+1)(index size)
Duplicate handling: use format 3 or external linked list or use composite key for index instead
Data entry format: what each leaf data entry stores

1. Format 1: k* is actual data record
2. Format 2: k* = (k, rid), rid is record identifier
3. Format 3: k* = (k, rid-list), rid-list is list of record identifiers

Overflow: node contains 2d entries and a new entry is inserted

- Resolution: attempt re-distributing (only leaf) first, before splitting (both leaf and internal)

Redistribution: distributing entries from the overflowed node to a non-full adjacent sibling node (leaf only); try right first

- If sibling has < 2d elements, keep 2d entries in original node and move the first (right) or last (left) entry to sibling and update separator node
- For internal nodes, perform rotation (like underflow) but moving separator node into sibling internal node while pushing overflow to parent (become new separator node)

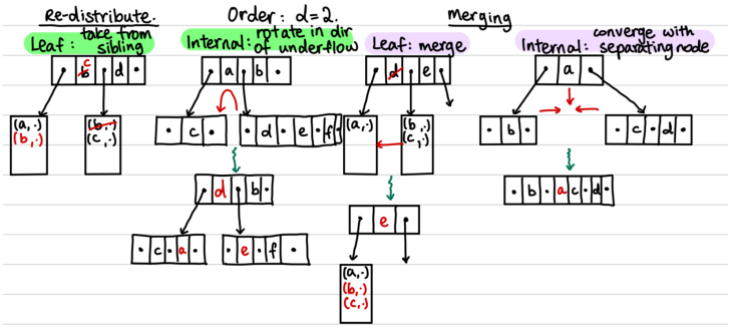
Splitting: split leaf node by distributing d+1 entries to new leaf node

- Push new leaf node as index entry
- If internal (parent) overflow, push middle key (d+1) to parent
  - The overflowed nodes both do not contain the middle key
- If root node overflows, create new root

Underflowed node: existing entry deleted from node with d entries

- Resolution: try re-distributing first (right first; leaf & internal), before merging (only leaf; target must be at most d + 1 entries)

Delete operations:



Inserting into empty B+-tree: start with single leaf/root node, on overflow, create new root and split
Bulk loading:

- Efficient construction, leaf pages allocated sequentially
- Only advantageous during loading, all disk info is separate

1. Sort data entries by search key
2. Load leaf pages with sorted entries (group into page size)
3. Initialize B+-tree with empty root
4. For each leaf page, insert its index entry into the rightmost parent-of-leaf level page
   - If parent overflows, move splitting key to new parent and continue

Clustered vs unclustered index

- Order of its data entries is the same as or 'close to' the order of data records (in original file)
- Format 1 index is always clustered index
- Requires explicit cluster command & future entries may not be clustered
- At most 1 clustered index per relation
- Clustered: incurs less I/O as matching data entries are contiguous (so retrieving page j for data contains all other related records)
- Unclustered: incurs more I/O to retrieve pages that are not contiguous

Dense vs sparse index

- There is an index record for every search key value in data
- Unclustered => dense
- Format 1 is always sparse, format 2/3 is always dense
- Format 1 treats only internal nodes as index records
- Format 2 treats leaf nodes as index records

Linear hashing

- Dynamic hashing that grows hash file linearly by systematic splitting of buckets
- Overflow pages needed, overflowed bucket may not be split

Overflow: if all pages in Bi (primary + overflow) are full
Split: add bucket Bj (split image of Bi) and redistribute entries in Bi between Bi and Bj

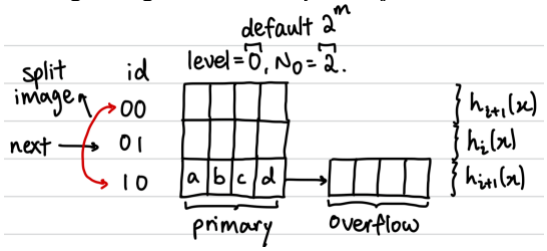
- Triggered when bucket overflows
- Split image of Bj is B(Ni + j) where Ni new buckets added
- After splitting, e remains iff last (m + i + 1) bit of h(k) is 0

1. Redistribute entries of B_next into B_(next+Ni) using h_(i+1)
2. Next += 1
3. If (next = Ni), level += 1; next = 0

Round of splits: file size doubles

- Round i (level), using 2 hash functions, h_i and h_i+1
- $h_i(k) = h(k)\%N_i$ (typically taking the last (m + i) bits of h(k))

File size at beginning of round i: $N_i = 2^i N_0 = 2^i \times 2^m = 2^{i+m}$



Insertion: if there is space in bucket (given by *(m + i)* bits), add to bucket, otherwise, split the bucket at *next* and redistribute between *next* and its split image (using *(m + i + 1)* bits)

- Given that the split bucket may not be the bucket to insert, create overflow page if necessary
- If split bucket is target bucket, use *(m + i + 1)* bits to decide if it stays or goes to split image

$$GetBucketNum(k) = \begin{cases} h_i(k) \text{ if } h_i(k) \geq next \\ h_{i+1}(k) \end{cases}$$

Deletion: locate bucket and delete entry

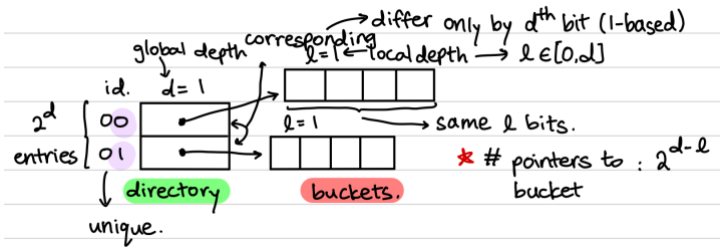
- If last bucket becomes empty, remove it
  - If next > 0, decrement by 1
  - If (next = 0) & (level > 0), update next to point to last bucket in previous round and decrement level by 1 (undoing a round)

Performance: one disk I/O unless overflow pages exists

- On average: 1.2 disk I/O for uniformly distributed data
- Worst case: I/O cost linear in the number of data entries
- Poor space utilization with skewed data distribution (one bucket storing everything)

Extendible hashing

- Adds a bucket (split image) when bucket overflows
- Does not require as many overflow pages except when number of collisions exceed page capacity
- Directory entries are transitive so if inserting into bucket j and directory entry of j points to bucket k, then insert into bucket k first



Insertion: if there is space in bucket (given by *d* bits), insert entry into bucket, otherwise, increase local depth of target bucket, create split image with same local depth and re-distribute across original and split using *l* bits

- If local depth > global depth (after update), then double directory and re-distribute all entries by the local depth bits (*l* bits)
- Must maintain the invariant that the number of pointers to a given bucket is 2^{d-l}

Deletion: locate Bi containing e and delete e

- Merge Bi and Bj if all entries fit in 1 bucket (use the original)
  - l -= 1
- If all corresponding entries point to same bucket, halve directory (all l < d)
  - d -= 1

Performance: at most 2 disk I/Os for equality selection

- At most 1 if directory first in main memory

Last split bucket: same local depth, last l – 1 bits same, l bit is 0 and 1, and total entries in both buckets more than page size

External merge sort

- Given file of *N* pages with *B* buffer pages (B >= 3)
- Sorted runs are temporary tables

1. Creation of sorted runs (pass 0): read and sort B pages in memory
   - Creates $N_0 = \lceil \frac{N}{B} \rceil$ sorted runs with each run being at most B pages
2. Subsequent passes: merge sorted runs using B-1 pages for input and 1 page for output; (B-1)-way merge

Analysis: number of I/O is 2 * N read/write pages * # of passes

$$2N(\lceil \log_{B-1} N_0 \rceil + 1)$$

Blocked I/O: reading/writing in units of buffer blocks of *b* pages

- Trade-off between maximizing the merge factor with reducing I/O cost
- Allocate *b* for output (no closed formula for *b*)
  - Use remaining for blocked input: $\lfloor \frac{B-b}{b} \rfloor$ with at most that many sorted runs each pass
- **Analysis:** *N* pages in file, *B* buffer pages, *b* block size
  - $N_0 = \lceil \frac{N}{B} \rceil$
  - $F = \lfloor \frac{B}{b} \rfloor + 1$ number of runs that can be merged per pass
  - $\lceil \log_F N_0 \rceil + 1$ total passes
- Given *j* input buffers each *m* size and *k* output pages and *s* seek time + rotational delay (typically m = k)

$$\lceil \log_j \lceil N/B \rceil \rceil \times N/m \times (s+k)$$

Achieving k merge passes: given N pages, buffer pages needed is $N_0 = \lceil N/B \rceil \geq B - 1 \Rightarrow B^2 - B - N \leq 0$

Sorting using B+-tree

Format 1: scan leaf pages of B+ tree and return

Format 2/3: scan leaf pages and for each leaf page visited, retrieve data records using RIDs

Selection $\sigma_p(R)$

- Select rows from relation R that satisfy predicate p

Access path

- Way of accessing data records/entries
  - Table scan, index scan, index intersection

Selectivity: number of index & data pages retrieved to access data records/entries

- Most selective => smallest selectivity => retrieve fewest pages

Covering index

- Covering index I for query Q if all attributes referenced in Q are part of the key/include column(s) of I
  - Includes the SELECT, WHERE, GROUP BY, etc
- Q can be evaluated without RID lookups => index-only plans

Index scan

- Find the leftmost element that satisfies the given query and traverse the leaf pages till the predicate is no longer true
- Works with either covering index or format 1 index

Index scan + RID lookups

- Similar to index scan but for each element, perform RID lookups (incurring additional I/O cost)
- Works if non-covering index or format 2/3 index

Index intersection

- Used when there is AND predicate
- Perform index scans for each predicate and find the intersection of all results
- Does not need to be same type of index

CNF predicate

Term: form R.A op c or R.Ai op R.Aj

Conjunct: one or more terms connected via OR

Disjunctive conjunct: conjunct that contains OR

Conjunctive Normal Form (CNF) predicate: consist of one or more conjunct with AND

Matching predicate: B+-tree

- Given index I = (K1, K2, …, Kn) and non-disjunctive CNF predicate p (i.e. cannot contain OR conditions)
- I matches p iff p is in form:

$$(K_1 = c_1) \wedge \ldots \wedge (K_i \; op_i \; c_i), i \in [1, n]$$

  - Follows index attribute list order, cannot skip attributes
  - Zero or more equality predicates (must follow attribute list)
  - At most 1 non-equality predicate (must be the last attribute of attribute list present, but can be multiple non-equality checks on the same attribute)
- If I matches p, use index scan since matching data entries will be in contiguous order

Matching predicate: Hash index

- I matches p iff p is in form:

$$(K_1 = c_1) \wedge (K_2 = c_2) \wedge \ldots \wedge (K_n = c_n)$$

- Does not allow range based predicates

Primary conjuncts

- Subset of conjuncts in p that I matches
- Can re-arrange to find the subsets that match

Covered conjuncts

- All attributes in C in p in the key or include column(s) of I
  - Do not have to match
- Primary conjuncts are a subset of covered conjuncts

Evaluating non-disjunctive conjuncts

1. Table scan: read straight from heap file
2. Hash index scan: use hash to read for information
3. B+ index scan: use B+ tree to read for information
4. Index intersection: combine results from index scans

- For OR predicates, must use covering index or index intersection, else use table scan

Cost of B+-tree index evaluation

- p' = primary conjuncts of p, pc = covered conjuncts of p

1. Navigate internal nodes to locate first leaf: height of tree

$$\text{cost}_{\text{internal}} = \begin{cases} \lceil \log_F (\lceil \frac{||R||}{b_d} \rceil) \rceil & \text{if I is a format-1 index} \\ \lceil \log_F (\lceil \frac{||R||}{b_i} \rceil) \rceil & \text{otherwise} \end{cases}$$

2. Scan leaf pages to access all qualifying data entries: number of pages that contain qualifying data entries

$$\text{cost}_{\text{leaf}} = \begin{cases} \lceil \frac{||\sigma_{p'}(R)||}{b_d} \rceil & \text{if I is a format-1 index} \\ \lceil \frac{||\sigma_{p'}(R)||}{b_i} \rceil & \text{otherwise} \end{cases}$$

3. Retrieve qualified data records via RID lookups: number of records

$$\text{cost}_{\text{RID}} = \begin{cases} 0 & \text{if I is a covering format-1 index,} \\ ||\sigma_{p_c}(R)|| & \text{otherwise} \end{cases}$$

- Cost of RID lookups can be reduced with clustered data records (sorted data) since don't need repeated I/O cost: $\min\left\{\left|\left|\sigma_{p_c}(R)\right|\right|, |R|\right\}$
- If no primary conjunct, perform full leaf scan

Cost of hash index evaluation

- May be more due to thrashing/collisions

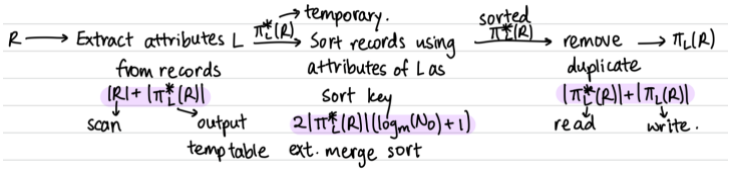
- Format 1: at least $\left\lceil \frac{||\sigma_{p'}(R)||}{b_d} \right\rceil$ (number of buckets)
- Format 2: at least $\left\lceil \frac{||\sigma_{p'}(R)||}{b_i} \right\rceil$ and cost to retrieve data records:
  - Cost is 0 if I is a covering index, else $||\sigma_{p'}(R)||$
- Range-based queries default to table scan
- Maximum I/O with overflow is 1 + # of overflow pages

Projection

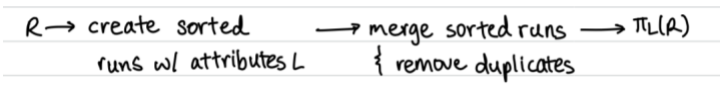
- Project columns given by list L from relation R
- $\pi(R)$: removes duplicates
- $\pi^*(R)$: preserves duplicates (default of DBMS)

Projection using sorting

- Results in sorted output
- Good if there are many duplicates or distribution of hashed values is non-uniform (increases chance to overflow)

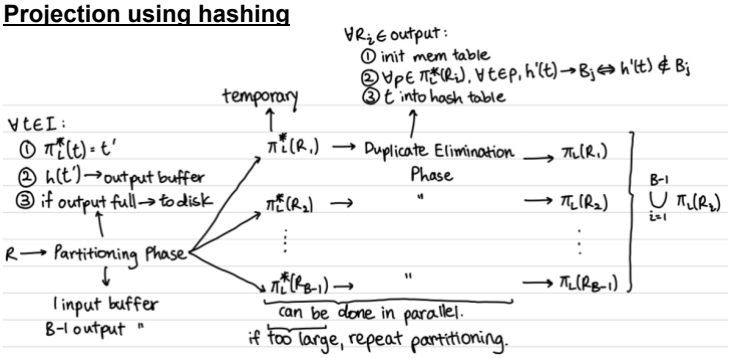


Optimization: break up step 2 and merge with 1 and 3 respectively



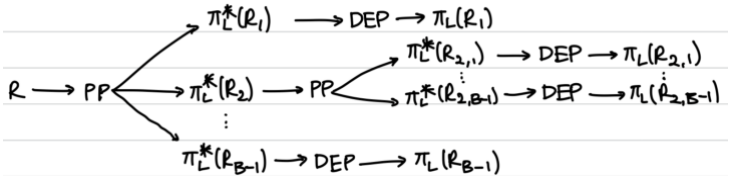
- If $B > \sqrt{|\pi^*_L(R)|}$, sort based performs similar to hash-based
  - $N_0 = \left\lceil \frac{|R|}{B} \right\rceil \approx \sqrt{|\pi^*_L(R)|}$ initial sorted runs
  - $\log_{B-1} N_0 \approx 1$ merge passes

Projection using hashing



Partition overflow: output hash tables too large than available memory buffer

- Recursively apply partitioning to overflowed partition



Analysis: effective if B is large relative to |R|

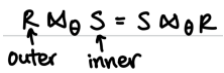
- B size: assume h distributes tuples in R uniformly
  - Approximately, B > $\sqrt{f \times |\pi^*_L(R)|}$ (f is fudge factor)
- Assume no partition overflow: cost of partitioning + cost of duplicate elimination

$$|R| + |\pi^*_L(R)| + |\pi^*_L(R)|$$

Projection using indexes

- Replace table scan with index scan iff there is an index with search key containing all projected attributes
- If index is ordered (like B+-tree) with projected attributes as prefix (no sorting needed)
  - Scan data entries in order
  - Compare adjacent data entries for duplicates
- If not all attributes included, create k partitions of distinct keys, sort individually, and merge (lower I/O cost overall)

Join



Considerations:

1. Type of join predicate: equality, inequality
2. Size of join operands
3. Available buffer space
4. Available access methods

Join algorithms

- Cost analysis ignores write & assumes R as outer relation
- Outer relation should always have less records
- Most optimal join: do it in-memory (|R| + |S|)

Type	Description/Cost
Tuple-based	For each tuple r in R: For each tuple s in S: If (r matches s): output (r, s) to result
	|R| + ||R|| x |S|
Page-based	For each page Pr of R: For each page Ps of S: For each tuple of Pr: For each tuple of Ps: If (r matches s): Output (r, s) to result Brings page and uses it fully to provide more efficient I/O
	|R| + |R| x |S|, min 3 pages

Block nested	While (scan of R is not done):
	Read next (B - 2) pages of R into buffer
	For each page Ps of S:
	Read Ps into inner buffer
	For each tuple r of R in outer buffer:
	For each tuple s in Ps:
	If (r matches s):
	Output (r, s) to result
	
	Allocate 1 page for inner, 1 page for output, remaining (B - 2) for outer

	$\|R\| + (\left\lceil \frac{\|R\|}{B-2} \right\rceil \times \|S\|)$
Index nested	For each tuple r in R: Use r to probe S's index to find matching tuples • There exists an index on the join attributes of inner relation • Cost analysis involves uniform distribution assumption (every tuple in outer loop has the same number of matches in inner relation) $\left\lceil \frac{\|\|S\|\|}{\|\|\pi_{B_j}(S)\|\|} \right\rceil$ • Cost analysis also assumes format 1 B+ tree index for S.A
	|R| + ||R|| x J (cost to find matching entries) $J = \log_F \left\lceil \frac{\|\|S\|\|}{b_d} \right\rceil + \left\lceil \frac{\|\|S\|\|}{b_d \|\|\pi_{B_j}(S)\|\|} \right\rceil$ • J is the height of the tree + search for leaf nodes

Notation

r	Relational algebra expression
||r||	Number of tuples in output of r
|r|	Number of pages in output of r
b_d	Number of data records that fit on page
b_i	Number of data entries* that fit on page
F	Average fanout of B+-tree index (i.e. # of pointers to child nodes)
h	Height of B+-tree index
B	Number of available buffer pages

* Data entries are (k, RID), data records are the full record from a relation

Cost analysis notes

- If format 2, remember to include cost of RID lookups
- If unclustered, remember to duplicate cost of page lookup
- If clustered, cost of RID lookups can be simplified (to 1 per page)
- Think in terms of cost to read/write
- Blocked I/O -> reading/writing in blocks, 1 time I/O vs N time