Determinant Of A Matrix

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Matrix Determinant

A square matrix A,

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,n} \end{bmatrix} \tag{1}$$

Definition

Be A a matrix $n \times n$ and $n \geqslant 2$. The determinant of A is scalar give by $|A| = det(A)$,

$$det(A) = \sum_{k=1}^{n} a_{i,j} C_{i,j}$$

where $C_{i,j}$ is cofactor,

$$C_{i,j} = (-1)^{i+j} det(A_{i,j})$$

Example

- Compute the determinant of matrix A showed bellow,

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 5 & 7 \\ 12 & 6 & 1 \end{bmatrix}$$

Resolution Choice expansion for row 1,

$$det(A) = \sum_{k=1}^{3} a_{1,k}(-1)^{1+k} det\left(A_{1,k}\right) =$$

$$= 1\ (\text{-}1)^{1+1} det\left(\begin{bmatrix} 5 & 7 \\ 6 & 1 \end{bmatrix}\right) + 1(-1)^{1+2} det\left(\begin{bmatrix} 2 & 7 \\ 12 & 1 \end{bmatrix}\right) +$$

$$3(-1)^{1+3} det\left(\begin{bmatrix} 2 & 5 \\ 12 & 6 \end{bmatrix}\right) =$$

$$= 1 \cdot(5 \cdot 1 - 7 \cdot 6) + (-1)(2 \cdot 1 - 7 \cdot 12) + 3 \cdot 1 \cdot (2 \cdot 6 - 5 \cdot 12) =$$

$$= \text{-}37 + 82 \text{ - } 144 = \boxed{-99}$$