1 Introduction, Definitions & Overview

Reliability

- ... is a characteristic of an item, expressed by the probability that the item performs its required function under given conditions during a stated time interval, i.e. $(0, t]$
- Item = entity for investigation, i.e. component, assembly, equipment, subsystem, system
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2 Probability Theory and Reliability Analysis

Definitions:

- Experiment ϵ
- Sample space Ω
- Event E

An event E is a subset of the sample space Ω and the experiment ε yields a set of possible outcoms ($= E$) of the experiment

Certain Events follow Boolean Logic, an event E can occur or not occur, meaning an Indicator Variable X_E is 0 when E does not occur and 1 if E occurs

Uncertain Events follow can either be true or false, with each a probability associated to it. Event E in sample space Ω is triggered with a probability that the outcome has happened or not

Classical Probability

- The experiment ϵ has N possible, elementary, mutually exclusive and equally probable outcomes $A_1, A_2, ..., A_N \in \Omega$
- The event $E = A_1 \cup A_2 \cup ... \cup A_M$, $M \leq N$
- The probability of event E is defined as $p(E) = M/N$

Kolmogorov Axioms

1. $0 \leq P(E) \leq 1$
2. $P(\Omega) = 1$, $P(\emptyset) = 0$
3. Mutually exclusive events: $P(\cup_i E_i) = \sum p(E_i)$
4. Non-mutually exlusive events: $P(A \cup B) = P_A + P_B - P(A \cap B)$
5. Conditional probability: $P(A|B) = P(A \cap B)/P(B)$
6. Theorem of total probability: Given an event A in Ω where the space is consisting of exclusive and exhaustive events $\cup_j E_j = \Omega$: $P(A) = \Sigma_i(P(A|E_i)P(E_i))$

Random Variables

- **CDF**: Is a non-decreasing function and returns the probabilty (state) from random variable X from 0 to a given point A: $F_X(X = A) = P(0 < X \leq A)$
- **pdf**: Probability of per unit x (continuous)
- **pmf**: Histogram, it assignes the probability to discrete values x

Summary

- Distribution Percentile x_α:
  - $F_X(x_\alpha) = \alpha/100 = \int_{-\infty}^{x_\alpha} f_X(x)dx$
- Median:
  - $F_X(x_50) = 0.5$
- Mean:
  - $\mu_X = E[X] =< X >= \sum_i x_i p_i$ (discrete)
  - $= \int_{-\infty}^{\infty} x f_X(x)dx$ (continuous)
- Variance:
  - $\sigma_X^2 = \sum(x_i - \mu_X)^2 p_i$ (discrete)
  - $= \int_{-\infty}^{\infty}(x - \mu_X)^2 f_X(x)dx$ (continuous)

Hazard Function (Failure Rate)

For risk and reliability analyses, we can use models whereas the time to failure of a component T can be expressed through a CDF $F_T(t)$ and a pdf $f_T(t)$. The complementary, cumulative function is

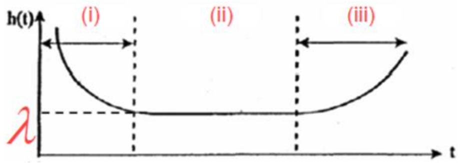
$$R(t) = 1 - F_T(t) = P(T \geq t) \tag{1}$$

which is decribed as the **Reliability or Survival Function** of the component T at time t and gives the probability of it surviving up to time t without failures.

In order to monitor the failure evolution, given the component has survived up to time t in a time interval dt, one can define a so called **Hazard Function or Failure Rate** $h_T(t)$.

$$h_T(t)dt = P(t < T \leq t{+}dt|T > t) = \frac{P(t < T \leq t + t + dt)}{P(T > t)} = \frac{f_T(t)dt}{R(t)} \tag{2}$$

The hazard function is depending on time, and is often discribed through the bathtub curve. The failure rate at the beginning is higher (infant mortality, burn in) and decreses after a certain time. The failure rate becomes constant λ and increases at the end through ageing.



Through the definition of $R(t)$ and integrating the hazard function, we receive:

$$F_T(t) = 1 - e^{-\int_0^t h_T(\tilde{t})d\tilde{t}} \tag{3}$$

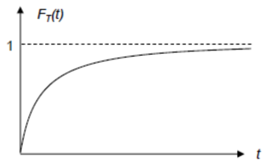
$$R(t) = e^{-\int_0^t h_T(\tilde{t})d\tilde{t}} \tag{4}$$

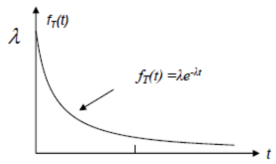
If our hazard function is in its constant phase (constant hazard rate), the failure evolution follows the **Exponential Distribution**:

$$h_T(t)\lambda, t > 0 \tag{5}$$

$$F_T(t) = P(T \leq t) = 1 - e^{-\lambda t} \tag{6}$$

$$R(t) = \begin{cases} f_T(t) = \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases} \tag{7}$$





The mean time to failure (MTTF) can be found through the expectation value

$$E[T] = \frac{1}{\lambda} = MTTF \tag{8}$$

$$Var[T] = \frac{1}{\lambda^2} \tag{9}$$