

## 1 Introduction, Definitions & Overview

### Reliability

- ... is a characteristic of an item, expressed by the probability that the item performs its required function under given conditions during a stated time interval, i.e.  $(0, t]$
- Item = entity for investigation, i.e. component, assembly, equipment, subsystem, system
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## 2 Probability Theory and Reliability Analysis

### Definitions:

- Experiment  $\epsilon$
- Sample space  $\Omega$
- Event  $E$

An event  $E$  is a subset of the sample space  $\Omega$  and the experiment  $\epsilon$  yields a set of possible outcomes ( $= E$ ) of the experiment

Certain Events follow Boolean Logic, an event  $E$  can occur or not occur, meaning an Indicator Variable  $X_E$  is 0 when  $E$  does not occur and 1 if  $E$  occurs

Uncertain Events follow can either be true or false, with each a probability associated to it. Event  $E$  in sample space  $\Omega$  is triggered with a probability that the outcome has happened or not

### Classical Probability

- The experiment  $\epsilon$  has  $N$  possible, elementary, mutually exclusive and equally probable outcomes  $A_1, A_2, \dots, A_N \in \Omega$
- The event  $E = A_1 \cup A_2 \cup \dots \cup A_M, M \leq N$
- The probability of event  $E$  is defined as  $p(E) = M/N$

### Kolmogorov Axioms

1.  $0 \leq P(E) \leq 1$
2.  $P(\Omega) = 1, P(\emptyset) = 0$
3. Mutually exclusive events:  $P(\cup_i E_i) = \sum p(E_i)$
4. Non-mutually exclusive events:  
 $P(A \cup B) = P_A + P_B - P(A \cap B)$
5. Conditional probability:  $P(A|B) = P(A \cap B)/P(B)$
6. Theorem of total probability: Given an event  $A$  in  $\Omega$  where the space is consisting of exclusive and exhaustive events  $\cup_j E_j = \Omega$ :  $P(A) = \sum_i (P(A|E_i)P(E_i))$

### Random Variables

- **CDF**: Is a non-decreasing function and returns the probability (state) from random variable  $X$  from 0 to a given point  $A$ :  $F_X(X = A) = P(0 < X \leq A)$
- **pdf**: Probability of per unit  $x$  (continuous)
- **pmf**: Histogram, it assigns the probability to discrete values  $x$

### Summary

- Distribution Percentile  $x_\alpha$ :  
–  $F_X(x_\alpha) = \alpha/100 = \int_{-\infty}^{x_\alpha} f_X(x)dx$
- Median:  
–  $F_X(x_{50}) = 0.5$
- Mean:  
–  $\mu_X = E[X] = \langle X \rangle = \sum_i x_i p_i$  (discrete)  
–  $= \int_{-\infty}^{\infty} x f_X(x)dx$  (continuous)
- Variance:  
–  $\sigma_X^2 = \sum (x_i - \mu_X)^2 p_i$  (discrete)  
–  $= \int_{-\infty}^{\infty} (x - \mu_X)^2 f_X(x)dx$  (continuous)

### Hazard Function