

## 1 General Considerations

$$\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + u_i \frac{\partial\rho}{\partial x_i} \neq 0 \quad (1)$$

- Wave propagation
- Convective flows with buoyancy
- Flows with variable temperature, friction, sources of heat
- High speed flows with Mach numbers  $Ma \geq 1$

Compressible flows can still be described through the continuum model and conservation laws. The assumption is also that the thermodynamic state of the fluid is in a local equilibrium.

### Assumptions

- Length scale of flows large compared to molecular scales (mean free path  $\lambda$ )
- Length scale of flows small compared to the geometric scales (length  $L$ )
- Time scale  $\tau_F$  of the flow long compared to the molecular process (relaxation) time constants  $\tau_R$

### Description of the “Continuum” Flow State

- Three components of flow velocity  $\underline{u}(\underline{x}, t)$
- The fluid density  $\rho(\underline{x}, t)$
- The fluid pressure  $p(\underline{x}, t)$
- The energy  $e(\underline{x}, t)$

The required equations are the conservation laws for mass, momentum and energy together with suitable thermodynamic equations of state. With corresponding initial and boundary conditions, the evolution can then be computed.

## 2 Thermodynamic Relations

### State Variables

- Density:  $\rho = \rho(p, T)$
- Pressure:  $p = p(\rho, T)$
- Temperature:  $T = T(\rho, p)$
- Internal energy:  $e = e(\rho, T)$  [ $e$ ] = J/kg
- Enthalpy:  $h = h(p, T)$
- Entropy:  $s = s(\rho, T)$

### Van der Waals Gas

$$(p + a\rho^2) \left( \frac{1}{\rho} - b \right) = RT \quad (2)$$

### Incompressible Fluid

$$\rho = \text{const.} \neq \rho(p, T) \quad (3)$$

## 3 Conservation Laws for Continuum Flows

### Mass Conservation

#### Material Volume

$$\frac{Dm}{Dt} = \frac{D}{Dt} \int_V \rho dV = 0 \quad (4)$$

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho}{\partial x_i} (\rho u_i) = 0 \quad (5)$$

#### Eulerian Volume

$$\int_V \frac{\partial\rho}{\partial t} dV + \int_S \rho(\vec{u} \cdot \vec{n}) dS = 0 \quad (6)$$

$$\frac{D\rho}{Dt} = -\rho \frac{\partial u_i}{\partial x_i} \quad (7)$$

### Momentum Conservation

### Energy Conservation

### Dissipation Function

### Entropy Equation

## 4 Simplification Strategies

## 5 Conservation Laws for Stream Tubes

## 6 Steady one-dimensional Flow without Friction and Heat

## 7 Unsteady one-dimensional Flows

## 8 Two-dimensional steady supersonic Flow

## 9 Method Characteristics for planar homentropic supersonic Flows

## 10 Homentropic Flow around slender Wings

## 11 Homentropic Flow around axisymmetric slender Bodies

## 12 Similarity Relations