Parameters

- Aerodynamic Force $\overline{F_A} = \overline{L} + \overline{D}$
- Lift Force $L\ [N]$
- Drag Force $D\ [N]$
- Aerodynamic Moment $M_A\ [Nm]$
- Dynamic Pressure $q = 1/2\rho V^2$ (Bernoulli) $[Pa]$
- Chord Length $c\ [m]$
- Surface Area $S = b \cdot c\ [m^2]$ (Rectangular)
- Wing Span $b\ [m]$
- Lift Coefficient $C_l = L/(1/2\rho V^2 \cdot S)$
- Drag Coefficient $C_d = D/(1/2\rho V^2 \cdot S)$
- Moment Coefficient $C_m = M_A/(1/2\rho V^2 \cdot c \cdot S)$
- Angle of Attack $\alpha\ [rad]$ (positive in clockwise direction)
- Lift curve slope $a = C_l/\alpha \approx tan(angle\, x - axis\, to\, curve)$

Conventions throughout Course

- If L and D absolute → use calculations above
- If L and D per span unit → correct via dividing by b
- Sign conventions: Lift positive, Drag positive in x and y direction
- Moments and angles positive in clockwise direction
- Our system coordinate system is defined by the wing. The angle of attack is defined relative to it
- The variables which describe the airfoil motion are the pitch θ and the plunge h which act at the shear centre of the wing

Mathematical Basics

- A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- Inverse of Matrix (2D): $A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
- The solutions of $Ax = 0$ for a matrix A, x cannot be just the trivial solution if A is not invertible
- $[rad] = \frac{\pi}{180}[deg]$

Steady Aerofoil and Wing Section Aerodynamics

- Aerofoil = 2-D wing section with goal to generate lift force perpendicular to the relative airspeed
- Convention: Lift is up, Drag is in direction of windspeed and Aerodynamic moment in clockwise direction acting on the aerodynamic center. Aerodynamic center is normally at the quarter chord position $c_{m,c/4}$ for syymetric airfoils. $x_{ac} = -m_0/2\pi + 0.25$ with m_0 as a shape constant
- Further assumptions: No viscosity, incompressible fluid, $Ma < 0.2, 0.3$, no vortices, potential flow (Navier-Stokes)
- Another centre is the shear center (elastic axis) from mechanics
- $L = 1/2\rho V^2 ca\alpha$, with a from tables (CFD and Wind Tunnel) $[N/m]$
- $M_A = 1/2\rho V^2 c^2 c_{m0}$ with c_{m0} also from tables $[N]$

Lift curve $C_l(\alpha)$ and drag curve $C_d(\alpha)$

- At small ranges of α, both lift and drag increase with: $C_l \propto \alpha$ and $C_d \propto \alpha^2$
- In aeroelasticity and this course, α will be very small, hence drag will be negligble small

The aerodynamic moment M_A

- The aerodynamic moment is much more important than drag C_d
- M_A varies with α in the small ranges of the angle of attack (very small, p. 7)
- **Important to note:** There exist a point at which the aerodynamic moment does not depend on α. This is the the aerodynamic centre
- The aerodynamic centre is not the same as the centre of pressure, which is defined as the point where the aerodynamic moment is zero given a certain angle of attack α
- Symmetric airfoils at $\alpha = 0$ have no aerodynamic moment at all times ($M_A = 0 = const$). At the aerodynamic centre for symmetric foils results into no moment
- Asymmetric airfoils at $\alpha = 0$ have a non-zero aerodynamic moment at all times (all angles α)

Assessment of C_l/α (Correction of value through Mach Number)

- The linear part of the lift curve is characterised by the slope $a = C_l/\alpha(M) = \frac{C_l/\alpha_{M=0}}{\sqrt{1-M^2}}$
- The Prandtl-Glauert factor is $1/\sqrt{1-M^2}$
- The factor is depending on the Mach number. The slop increases with increasing M (between 0 and 1)
- The dependence on Re is more subtle (p. 8)

Extension to wing aerodynamics (p. 8)

Aerofoil dynamics (2D) refer to the previous topics, however the 3-D case can be also modeled by through a couple examples. A finite wing is less stable and efficient than the airfoil since the tips have vortices on at the wing tips. These “induce” a velocity, which locally reduces the angle of attack. An important parameter is the so called **Aspect Ratio** $AR = b^2/S$. If the wing is assumed to be of surface $S = b \cdot c$, it follows $AR = b/c$.

- The lift curve can become a function of AR if due to the different tips. Approximately, the lift slope a_0 is adjusted via following formula:

- $a = a_0 \frac{AR}{AR+4}$

- The values a and c_{m0} will hence be corrected with a a^* and c^*_{m0}

Strip Theory (p.9)

- If AS is very small (delta wings), the integral of multiple airfoils

- Define multiple airfoils stacked next to each other along the span b

- Example, the wing is an elliptical $f(y) = \sqrt{1 - (\frac{y}{b/2})^2} \cdot \overline{f}_\phi$

- $f(y) = a\alpha c = C_l c$ with $c =$ chord length.

Steady-state (static) Aeroelasticity

Typical Section = 1DOF model

2-D problem with a rigid wing and 2 degrees of freedom (free rotation / pitch and plunge). We can have multiple typical sections. The pitch is modeled via a torsional string and the plunge via a longitudinal one. The idea is later on to model the torsional spring to be a torsional stiffness of a beam (since a real wing is actually a beam with a certain stiffness).

- The torsion acts in a beam section on the shear centre, however in aeroelasticity on the elastic axis

- The goal of engineering is alwayt to move the shear center to the front (comes with risk to thin out the rear longeron and thicken the front longeron)

- In equilibrium, we know that the aerodynamic forces are equal to the spring forces

- $M_t + L \cdot e = \theta k_\theta = (qc^2 c_{m0} + qca\theta e) \cdot b$

- $L =$

Static Instability or Divergence

- If the elastic twist would become

How to solve typical sections

1. Check the number of DOF (= size of problem and unknowns), which defines the matrices later

2. Define angles α (true angle of attack) and θ (elastic twist)

3. Define the measures h (plunge) and e (distance aerodynamic centre and elastic axis)

4. The pitching moment M_t and the lift L can be described on one hand as the response from the springs (α)

5. And on the other hand via the

Ritz Method (p. 18)

... is a energy variational method whereas an equilibrium occurs in correspondence of an extreme of potential energy. A general application is the virtual work. According to the Hamilton's principle and Lagrange equations, we can define a set of equations