

1 Introduction, Definitions & Overview

Reliability

- ... is a characteristic of an item, expressed by the probability that the item performs its required function under given conditions during a stated time interval, i.e. $(0, t]$
- Item = entity for investigation, i.e. component, assembly, equipment, subsystem, system
- from a **qualitative** point of view, reliability is defined as the ability of an item to **remain functional**
- from a **quantitative** point of view, reliability is defined as the probability that **no operational interruptions** will occur during a stated time interval $R(t)$

In comparison to Availability, the Reliability analysis includes the fact that no item is allowed to fail. This means, the function performed cannot be interrupted (redundant system however can be repaired). Availability incurs that failures can happen on the item level.

Risk

2 Probability Theory and Reliability Analysis

Definitions:

- Experiment ϵ
- Sample space Ω
- Event E

An event E is a subset of the sample space Ω and the experiment ϵ yields a set of possible outcomes ($= E$) of the experiment

Certain Events follow Boolean Logic, an event E can occur or not occur, meaning an Indicator Variable X_E is 0 when E does not occur and 1 if E occurs

Uncertain Events follow can either be true or false, with each a probability associated to it. Event E in sample space Ω is triggered with a probability that the outcome has happened or not

Classical Probability

- The experiment ϵ has N possible, elementary, mutually exclusive and equally probable outcomes $A_1, A_2, \dots, A_N \in \Omega$
- The event $E = A_1 \cup A_2 \cup \dots \cup A_M, M \leq N$
- The probability of event E is defined as $p(E) = M/N$

Kolmogorov Axioms

1. $0 \leq P(E) \leq 1$
2. $P(\Omega) = 1, P(\emptyset) = 0$
3. Mutually exclusive events: $P(\cup_i E_i) = \sum P(E_i)$
4. Non-mutually exclusive events:
 $P(A \cup B) = P_A + P_B - P(A \cap B)$
5. Conditional probability: $P(A|B) = P(A \cap B)/P(B)$
6. Theorem of total probability: Given an event A in Ω where the space is consisting of exclusive and exhaustive events $\cup_j E_j = \Omega$: $P(A) = \sum_i (P(A|E_i)P(E_i))$

Random Variables

- **CDF**: Is a non-decreasing function and returns the probability (state) from random variable X from 0 to a given point A : $F_X(X = A) = P(0 < X \leq A)$
- **pdf**: Probability of per unit x (continuous)
- **pmf**: Histogram, it assigns the probability to discrete values x

Summary

- Distribution Percentile x_α :
– $F_X(x_\alpha) = \alpha/100 = \int_{-\infty}^{x_\alpha} f_X(x)dx$
- Median:
– $F_X(x_{50}) = 0.5$

Mean:

- $\mu_X = E[X] = \langle X \rangle = \sum_i x_i p_i$ (discrete)
- $= \int_{-\infty}^{\infty} x f_X(x) dx$ (continuous)

Variance:

- $\sigma_X^2 = \sum (x_i - \mu_X)^2 p_i$ (discrete)
- $= \int_{-\infty}^{\infty} (x - \mu_X)^2 f_X(x) dx$ (continuous)

Hazard Function (Failure Rate)

For risk and reliability analyses, we can use models whereas the time to failure of a component T can be expressed through a CDF $F_T(t)$ and a pdf $f_T(t)$. The complementary, cumulative function is

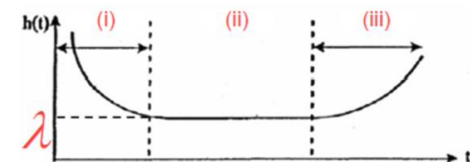
$$R(t) = 1 - F_T(t) = P(T \geq t) \quad (1)$$

which is described as the **Reliability or Survival Function** of the component T at time t and gives the probability of it surviving up to time t without failures.

In order to monitor the failure evolution, given the component has survived up to time t in a time interval dt , one can define a so called **Hazard Function or Failure Rate** $h_T(t)$.

$$h_T(t)dt = P(t < T \leq t+dt | T > t) = \frac{P(t < T \leq t+dt)}{P(T > t)} = \frac{f_T(t)dt}{R(t)} \quad (2)$$

The hazard function is depending on time, and is often described through the bathtub curve. The failure rate at the beginning is higher (infant mortality, burn in) and decreases after a certain time. The failure rate becomes constant λ and increases at the end through ageing.



Through the definition of $R(t)$ and integrating the hazard function, we receive:

$$F_T(t) = 1 - e^{-\int_0^t h_T(\tilde{t}) d\tilde{t}} \quad (3)$$

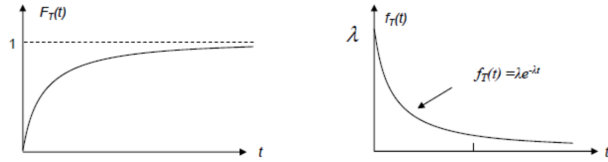
$$R(t) = e^{-\int_0^t h_T(\tilde{t}) d\tilde{t}} \quad (4)$$

If our hazard function is in its constant phase (constant hazard rate), the failure evolution follows the **Exponential Distribution**:

$$h_T(t)\lambda, t > 0 \quad (5)$$

$$F_T(t) = P(T \leq t) = 1 - e^{-\lambda t} \quad (6)$$

$$R(t) = \begin{cases} f_T(t) = \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (7)$$



The mean time to failure (MTTF) can be found through the expectation value

$$E[T] = \frac{1}{\lambda} = MTTF \quad (8)$$

$$Var[T] = \frac{1}{\lambda^2} \quad (9)$$

The failure process is memoryless. Given a component has survived at least until time t_1 , the probability of it failing between time t_1 and t_2 is only depending on the time inbetween and not prior to time t_1

$$P(t_1 < T < t_2) = \frac{P(t_1 < T < t_2)}{P(T > t_1)} = \frac{F_T(t_2) - F_T(t_1)}{1 - F_T(t_1)} = \quad (10)$$

$$\frac{e^{-\lambda t_1} - e^{-\lambda t_2}}{e^{-\lambda t_1}} = 1 - e^{-\lambda(t_2 - t_1)} \quad (11)$$

The influence of an ageing process of the components failure rate shows that is not constant through time and hence can be described through the Weibull distribution.