The Rate and Lightcurve of Exploding PBHs

Stefano Profumo

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1 Rate of PBH explosions today

We intend to calculate the rate of "exploding" PBHs (to use Hawking's original term [3]) if there exists a subdominant dark matter component made up of PBHs with a lifetime comparable to the age of the universe, i.e. a mass of $M_* \simeq 5 \times 10^{14}$ grams [2]. Evaporation constraints for that mass (today) are $f_{\rm PBH} \simeq 10^{-4}(10^{16}/(5 \times 10^{14}))^-3 \simeq 10^{-8}$. Assuming a local DM density of $\rho_{\rm DM} \sim$0.4 GeV cm^{-3}, the number density of exploding PBHs is

$$n_* \simeq \frac{\rho_{\rm DM} f_{\rm PBH}}{M_*} \simeq 4 \times 10^8 \ {\rm pc}^{-3}. \tag{1}$$

Let's consider a mass function $\psi(M) \propto M \frac{dn_{\rm PBH}}{dM}$ normalized as customary to $f_{\rm PBH}$, i.e.

$$\int dM \psi(M) = f_{\rm PBH}. \tag{2}$$

Notice that the lower limit of integration is set always to M_*, as lighter black holes have already evaporated away.

The rate of PBH "explosions" per unit volume is then given per unit PBH as

$$\dot{n}_{\rm PBH} = \frac{dn_{\rm PBH}}{dM}\left(-\frac{dM}{dt}\right) = \rho_{\rm DM}\frac{\psi(M)}{M}\frac{\alpha(M)}{M^2}, \tag{3}$$

where $\alpha(M)$ describes the evaporation rate at mass M, and

$$\alpha(M) \simeq 4 \times 10^{-4} M_*^2 \ {\rm g/sec}. \tag{4}$$

Consider a lognormal mass function

$$\psi(M) = \frac{\exp\left(-\frac{\log^2(M/M_*)}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma} \tag{5}$$

The PBH explosion rate per unit volume is then

$$\dot{n}_{\rm PBH}(M_*) = \Gamma_{\rm PBH} n_* \simeq \frac{0.0085 \ {\rm pc}^{-3} \ {\rm yr}^{-1}}{\sigma} \tag{6}$$

Current limits are at the level of 3400 pc^{-3} yr^{-1} [**?**], and thus at present for this particular mass function they constrain $\sigma \lesssim 10^{-6}$.

For a power-law mass function [2]

$$\psi(M) \propto M^{\gamma-1} \tag{7}$$

the normalization of $\psi(M)$ requires, for $\gamma < 0$

$$\psi(M) \propto (-\gamma)\frac{M^{\gamma-1}}{M_*^{\gamma}} \tag{8}$$

The PBH explosion rate per unit volume becomes, similar to above,

$$\dot{n}_{\rm PBH}(M_*) = (-\gamma)(0.011\ \mathrm{pc}^{-3}\ \mathrm{yr}^{-1}) \tag{9}$$

Finally, for a critical collapse function,

$$\psi(M) \propto M^{2.85}\exp(-(M/M_f)^{2.85}), \tag{10}$$

upon proper normalization, we get the following results as a function of the ratio of the cutoff mass M_f to M_*

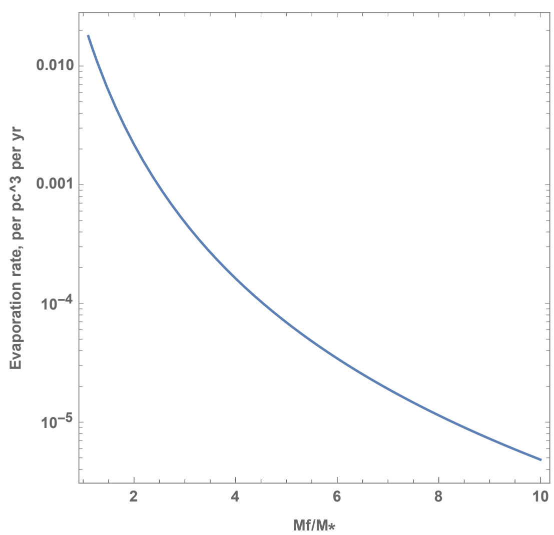

Figure 1: Caption

2 Lightcurve for evaporation with additional degrees of freedom

Suppose at time t_D the evaporation rate $\alpha \to \alpha + \alpha_D$, because of "dark" degrees of freedom not contributing to photon emission; for simplicity assume both $alpha$, $alpha_D$ constant in the range of mass/time of interest; the evolution of mass with time reads:

$$M^3(t) = M_0^3 - \alpha t \ (t < t_D); \tag{11}$$

$$M^3_{t>t_D}(t) = M_0^3 - \alpha t_D - \alpha_D (t - t_D) = M(t)^3 - (\alpha - \alpha_D)(t - t_D). \tag{12}$$

As a result, with the additional degrees of freedom, the time t' at which $M_{t>t_D}(t) = M(t')$ is

$$t' = t + \frac{\Delta\alpha}{\alpha}(t - t_D), \quad \Delta\alpha = \alpha_D - \alpha > 0. \tag{13}$$

Thus, given the lightcurve $\phi_\gamma(t)$ for "standard" evaporation, with the additional degrees of freedom, at $t > t_D$

$$\phi_\gamma(t) \to \phi_\gamma\left(t + \frac{\Delta\alpha}{\alpha}(t - t_D)\right).$$

Defining τ the time to evaporation, i.e. $\tau = t_\dagger - t$, and $\tau_D = t_\dagger - t_D$, one has the following equivalent parameterization for the modified lightcirve:

$$\phi^D_\gamma(\tau) = \phi_\gamma\left(\tau\left(1 + \frac{\Delta\alpha}{\alpha}\right)\right), \ [\tau < \tau_D]; \tag{14}$$

$$\phi^D_\gamma(\tau) = \phi_\gamma\left(\tau + \tau_D\frac{\Delta\alpha}{\alpha}\right), \ [\tau \geq \tau_D]. \tag{15}$$

Below is an example for $\Delta\alpha/\alpha = 5,\ 50$ and $\tau_D = 1000, 100$ s (the blue curve is the standard evaporation, for $E_\gamma = 200$ GeV) and one for .

In order to ascertain from data both τ_D and $\Delta\alpha/\alpha$ it will be most convenient to consider the ratio $\phi^D_\gamma(\tau)/\phi_\gamma(\tau)$, shown in the figure as example: One can compute the location of the ratio being close to 1, and the location and depth of the absolute minimum of the ratio.

In particular, the location of the minimum of the ratio is a direct proxy for τ_D, while the depth of the minimum of the ratio, i.e. $\phi^D_\gamma(\tau_D)/\phi_\gamma(\tau_D)$ correlates with $\Delta\alpha/\alpha$: assuming (as it is largely the case, especially at low energy) that $\phi_\gamma(\tau) \sim \tau^{-\beta}$, with $\beta \sim 1/3$, we have

$$\frac{\phi^D_\gamma(\tau_D)}{\phi_\gamma(\tau_D)} \simeq \frac{\tau_D^{-\beta}\left(1 + \frac{\Delta\alpha}{\alpha}\right)^{-\beta}}{\tau_D^{-\beta}} = \left(1 + \frac{\Delta\alpha}{\alpha}\right)^{-\beta}. \tag{16}$$

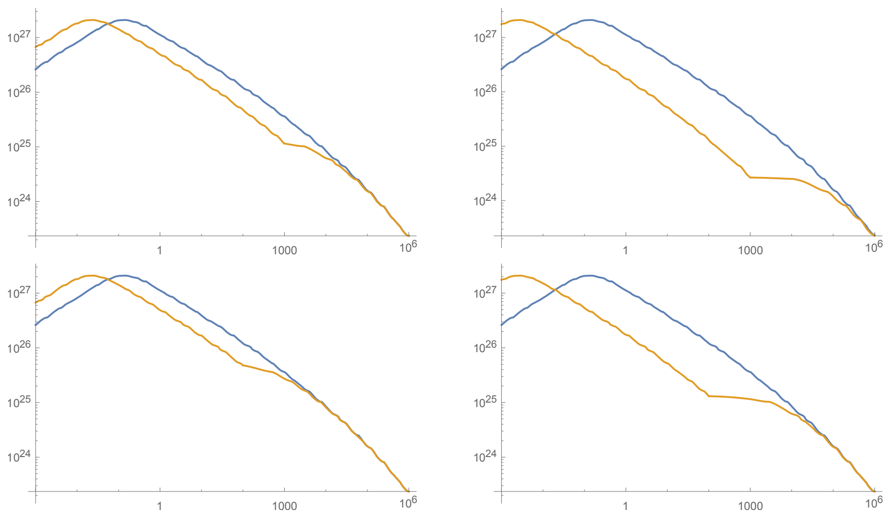

Figure 2: $\Delta\alpha/\alpha = 5$ (left) and 50 (right); $\tau_D = 1000$ s (top) and 100 s (bottom).

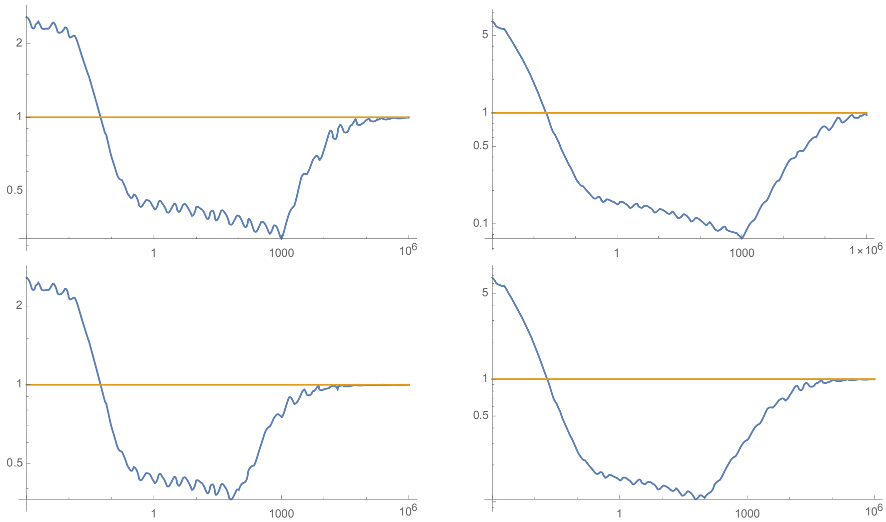

Figure 3: $\phi_\gamma^D(\tau)/\phi_\gamma(\tau)$ for $\Delta\alpha/\alpha = 5$ (left) and 50 (right); $\tau_D = 1000$ s (top) and 100 s (bottom).

Notice that $\beta \simeq 1/3$ is a generic prediction in the limit $E_\gamma \ll T$: in that limit,

$$\phi_\gamma(E_\gamma \ll T) \sim \frac{1}{e^{E_\gamma/T}-1} \sim T$$

and since

$$M(\tau) \simeq \left(M_0^3 - \alpha\left(t_\dagger - \tau\right)\right)^{1/3} = \left(M_0^3 - \alpha\left(\frac{M_0^3}{\alpha} - \tau\right)\right)^{1/3} = (\alpha\tau)^{1/3},$$

we get that $\phi_\gamma \sim T \sim 1/M(\tau) \sim \tau^{-1/3}$.

Notice that in BlackHawk there is a weird non-physical break to the $\phi_\gamma \sim \tau^{-1/3}$ behavior around $t \sim 0.1$ sec, or a mass $M \sim 5 \times 10^8$ grams, corresponding to a $T \sim 20$ TeV; I believe this break is simply due to a lack of correct hadronization for $T \gg m_W/\alpha_W \simeq 10$ TeV. I believe the correction should incorporate final state radiation of massive gauge bosons, as discussed in [1]. and in the HDMSpectra code[1]

References

[1] C. W. Bauer, N. L. Rodd, and B. R. Webber. Dark matter spectra from the electroweak to the Planck scale. *JHEP*, 06:121, 2021.

[2] B. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama. Constraints on primordial black holes. *Rept. Prog. Phys.*, 84(11):116902, 2021.

[3] S. W. Hawking. Black hole explosions. *Nature*, 248:30–31, 1974.

[1] github.com/nickrodd/HDMSpectra