

# Coalgebras

```
CoAlgebra : (func : Functor  $\mathcal{U}$ )  $\rightarrow \mathcal{U} + \cdot$ 
CoAlgebra func =  $\Sigma A : \_ , (A \rightarrow F.Fn A)$  where
  private
    module F = Functor func

module CoAlgebra func (co : CoAlgebra { $\mathcal{U}$ } func) where

  open Functor func

   $\langle \_ \rangle : \mathcal{U} + \cdot$ 
   $\langle \_ \rangle = co.pr_1$ 

   $\_ \downarrow : \langle \_ \rangle \rightarrow Fn \langle \_ \rangle$ 
   $\_ \downarrow = co.pr_2$ 

module CoAlgebra2 func (a b : CoAlgebra { $\mathcal{U}$ } func) where
  open Functor func

  open CoAlgebra func

  co-morphism :  $\mathcal{U} + \cdot$ 
  co-morphism =  $\Sigma f : (\langle a \rangle \rightarrow \langle b \rangle) , Fm f \circ (a \downarrow) \sim (b \downarrow) \circ f$ 

  module Morphism (m : co-morphism) where
     $\_ \rightarrow : \langle a \rangle \rightarrow \langle b \rangle$ 
     $\_ \rightarrow = m.pr_1$ 

     $\_comm : Fm \_ \circ (a \downarrow) \sim (b \downarrow) \circ \_$ 
     $\_comm = m.pr_2$ 
```