

Final Coalgebra Properties

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module Final-CoAlgebra-Properties where

open import Final-CoAlgebraP
open import CoAlgebraP
open import FunctorP

module _ (fe : Fun-Ext) (UA : Univalence) func (fc' : Final-CoAlgebra {U = U} func)
where
  open Functor func
  open Final-CoAlgebra func fc'
  open CoAlgebra func
  open CoAlgebra2 func

  f-co : CoAlgebra func
  f-co = Fn { fc } , Fm (fc ↴)

  inv : co-morphism f-co fc
  inv = uni f-co .pr1

  open Morphism f-co fc

  morph : co-morphism fc fc
  morph = (inv →) ∘ (fc ↴) , λ x → (Fm-comp (inv →) (fc ↴) ((fc ↴) x))-1 • (inv comm) ((fc ↴) x)

  morph-id : co-morphism fc fc
  morph-id = (λ x → x) , (λ x → Fm-id ((fc ↴) x))

  inv◦Qf=id : (inv →) ∘ (fc ↴) = id
  inv◦Qf=id = l2-1 • l3 where
    l1 = uni fc
    c = l1 .pr1
    l2 : c .pr1 = morph .pr1
    l2 = ap pr1 (l1 .pr2 morph)

    l3 : c .pr1 = morph-id .pr1
    l3 = ap pr1 (l1 .pr2 morph-id)

  Qf◦inv=id : (fc ↴) ∘ (inv →) = (λ x → x)
  Qf◦inv=id = (dfunext fe λ x → (inv comm) x-1 • (Fm-comp (inv →) (fc ↴) x) • ((ap (λ z → Fm z x) inv◦Qf=id) • Fm-id x))

  QE=FQE : { fc } = Fn { fc }
  QE=FQE = eqtoid (UA _) { fc } (Fn { fc }) (qinveq (fc ↴) ((inv →) , (λ x → ap (λ f → f x) inv◦Qf=id) , (λ x → ap (λ f → f x) Qf◦inv=id)))
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