付録

1　最適性条件を用いたラグランジュ関数の変形

$$L(\boldsymbol{w},\gamma,\boldsymbol{\xi},\boldsymbol{\alpha},\boldsymbol{\beta}) = \frac{1}{2}\|\boldsymbol{w}\| + C\sum_{i=1}^{n}\xi_i - \textstyle\sum_{i=1}^{n}\alpha_i\{(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x_i}+\gamma)y_i - 1 + \xi_i\} - \sum_{i=1}^{n}\beta_i\xi_i \tag{1}$$

について、

$$\frac{\partial L}{\partial \boldsymbol{w}} = \begin{pmatrix} \frac{\partial L}{\partial w_1} \\ \vdots \\ \frac{\partial L}{\partial w_n} \end{pmatrix} = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} - \begin{pmatrix} \sum_{i=1}^{n}\alpha_i x_i^{(1)} y_i \\ \vdots \\ \sum_{i=1}^{n}\alpha_i x_i^{(n)} y_i \end{pmatrix} = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} - \begin{pmatrix} \sum_{i=1}^{n}\alpha_i y_i x_i^{(1)} \\ \vdots \\ \sum_{i=1}^{n}\alpha_i y_i x_i^{(n)} \end{pmatrix} = \boldsymbol{w} - \sum_{i=1}^{n}\alpha_i y_i \boldsymbol{x_i} = \boldsymbol{0} \tag{2}$$

$$\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{n}\alpha_i y_i = 0 \tag{3}$$

$$\frac{\partial L}{\partial \boldsymbol{\xi}} = \begin{pmatrix} \boldsymbol{C} \\ \vdots \\ \boldsymbol{C} \end{pmatrix} - \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} - \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} = \boldsymbol{0} \tag{4}$$

2　ラグランジュ関数における w,ξ,γ の消去

$$\boldsymbol{w} = \sum_{i=1}^{n}\alpha_i y_i \boldsymbol{x_i},\ \boldsymbol{\alpha}+\boldsymbol{\beta} = \boldsymbol{C}$$

により、

$L\boldsymbol{w},\gamma,\xi,\alpha,\beta =$
$\frac{1}{2}(\sum_{i=1}^{n}\alpha_i y_i \boldsymbol{x_i})^{\mathrm{T}}\ (\sum_{j=1}^{n}\alpha_j y_j \boldsymbol{x_j}) + \sum_{i=1}^{n}(\alpha_i+\beta_i)\xi\ - \sum_{i=1}^{n}\alpha((\sum_{j=1}^{n}\alpha_j y_j \boldsymbol{x_j})^{\mathrm{T}}\ \boldsymbol{x_i}+\gamma)y_i - 1 + \xi_i - \sum_{i=1}^{n}\beta_i\xi_i$
$= \ \displaystyle\frac{1}{2}(\sum_{i,j=1}^{n}\alpha_i y_i \alpha_j y j \boldsymbol{x_i}^{\mathrm{T}}\ \boldsymbol{x_j}) + \sum_{i=1}^{n}(\alpha_i+\beta_i)\xi_i - \sum_{i=1}^{n}\alpha_i\ \sum_{j=1}^{n}\alpha_j y_j \boldsymbol{x_j}^{\mathrm{T}}\boldsymbol{x_i} y_i + \gamma\alpha_i y_i\ - \alpha_i + \alpha_i\xi_i - \sum_{i=1}^{n}\beta_i\xi_i$
$= \ \displaystyle\frac{1}{2}(\sum_{i,j=1}^{n}\alpha_i y_i \alpha_j y j \boldsymbol{x_i}^{\mathrm{T}}\ \boldsymbol{x_j}) + \sum_{i=1}^{n}(\alpha_i+\beta_i)\xi_i - \sum_{i,j=1}^{n}\alpha_i\ \alpha_j y_j \boldsymbol{x_i}^{\mathrm{T}}\boldsymbol{x_j} y_i + \gamma\alpha_i y_i\ - \alpha_i + \alpha_i\xi_i - \sum_{i=1}^{n}\beta_i\xi_i$
$=\sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum i,j=1^n\alpha_i\alpha_j\ y_i y_j \boldsymbol{x_i}^{\mathrm{T}}\boldsymbol{x_j}$ (5)