

A DISSERTATION

REPRESENTING BIG DATA AS NETWORKS: NEW METHODS AND INSIGHTS

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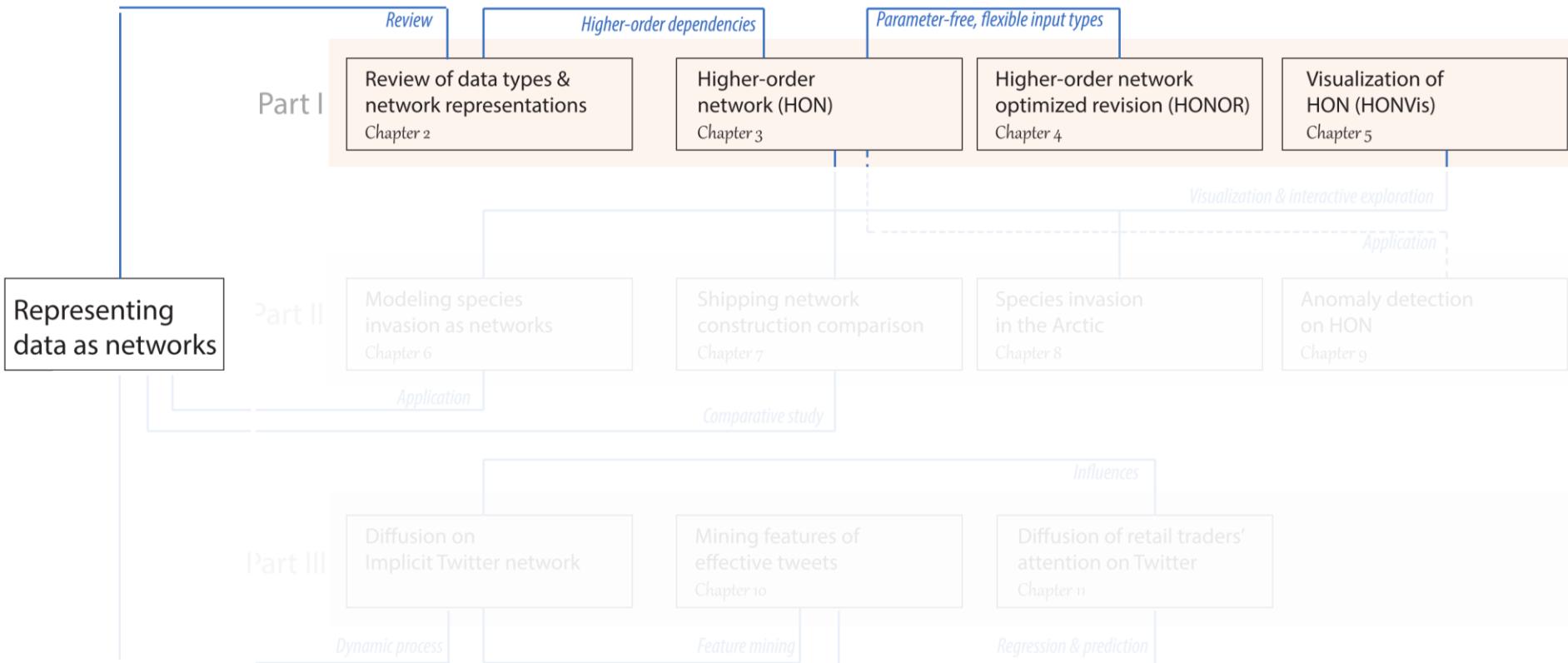
Prof. Zoltan Torontzkai



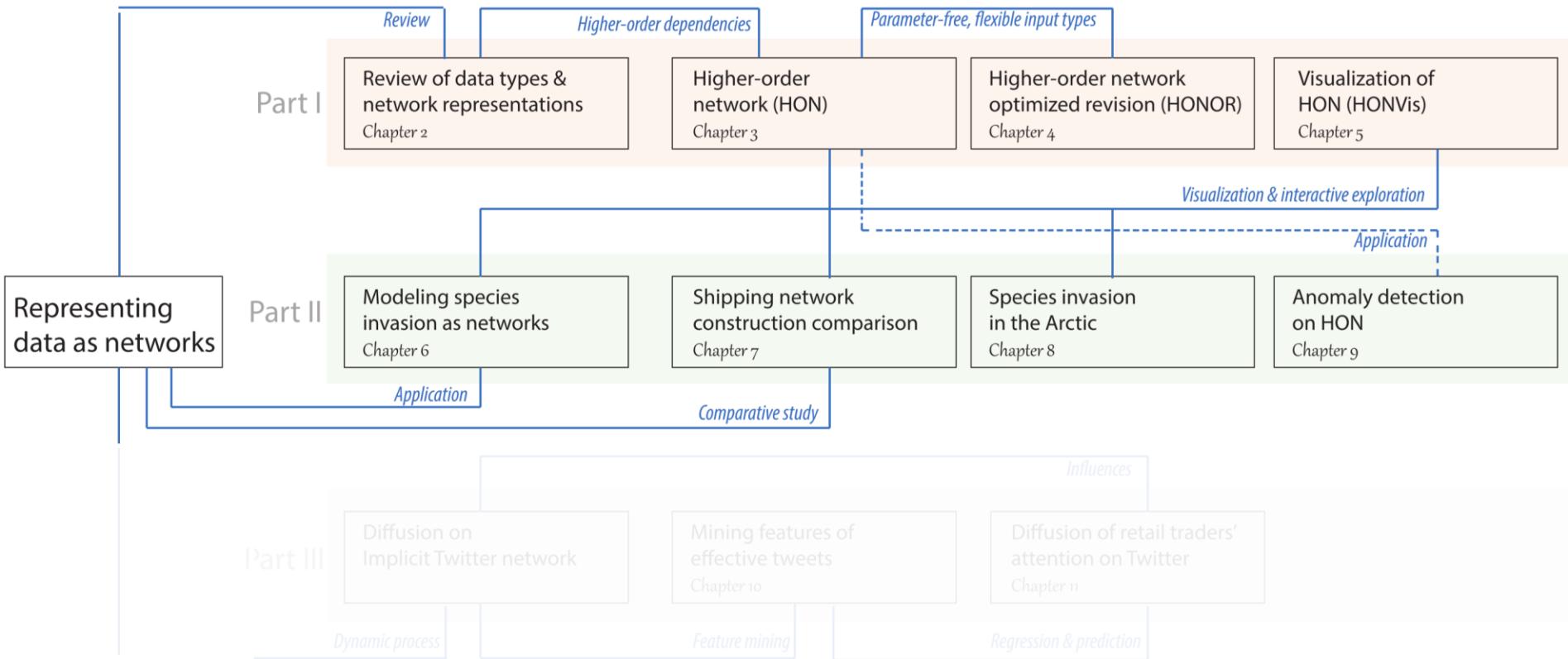
Overview

Representing
data as networks

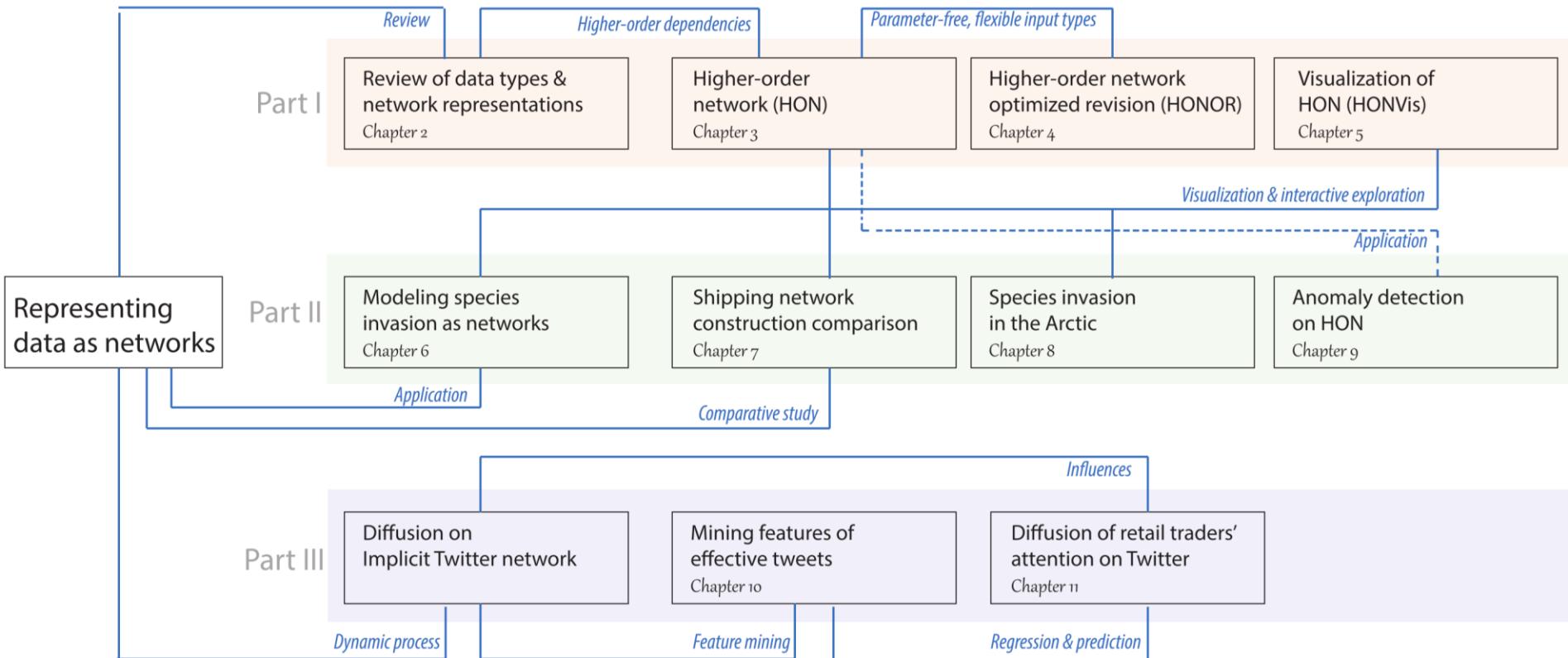
Overview



Overview

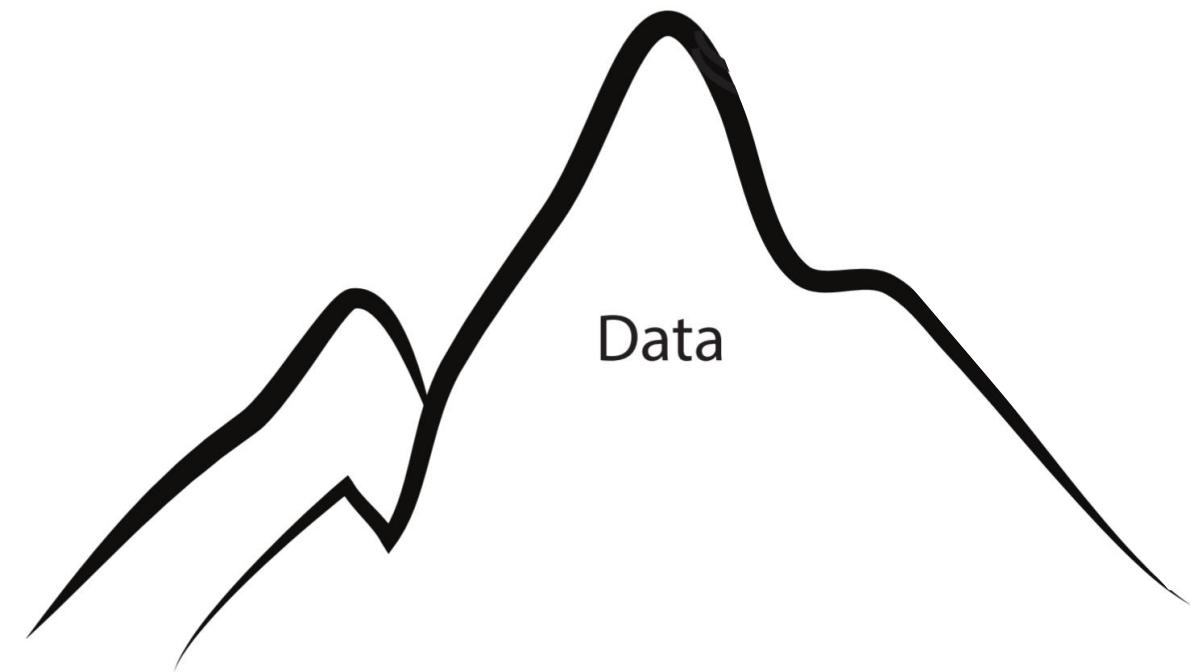


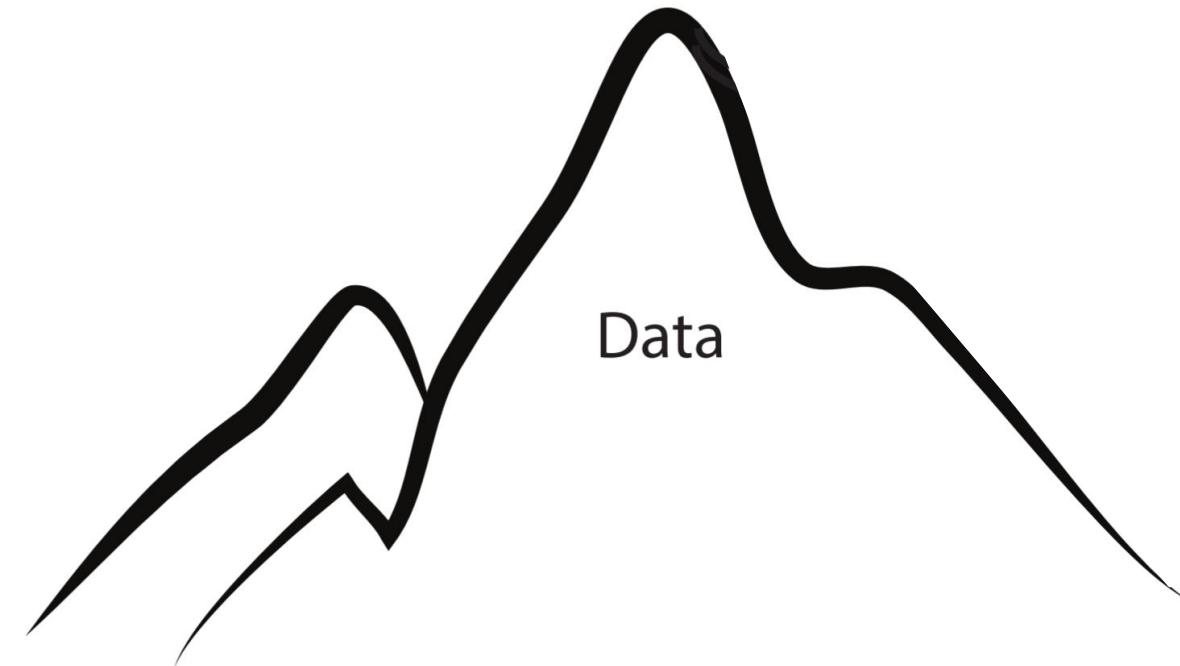
Overview



Part I

Methods to represent data as networks





Pairwise

a - b
a - c
a - d
b - c

Weighted pairwise

a - b : 5
a - c: 3
a - d: 1
b - c: 2

Directed pairwise

a -> b
a -> c
a -> d
b -> c

Temporal pairwise

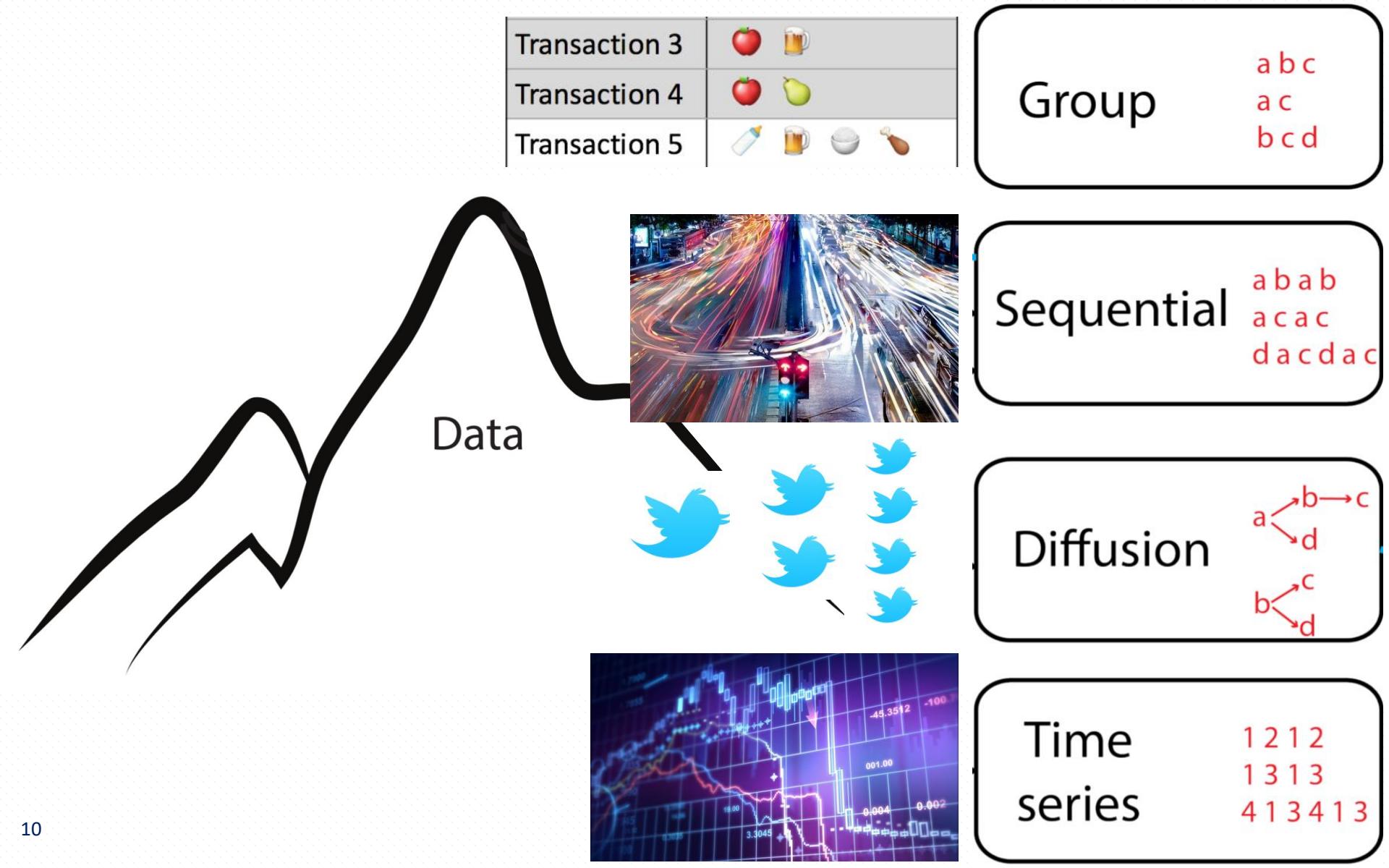
a - b: 1, 3
a - c: 2
a - d: 2, 3
b - c: 1, 4

Matrix

a b c
a 0 3 5
b 3 0 1
c 5 1 0

Tensor

T=1: T=2:
0 3 5 0 1 5
3 0 1 1 0 4
5 1 0 5 4 0



Transaction 3		
Transaction 4		
Transaction 5		

Group

a b c
a c
b c d

Sequential

a b a b
a c a c
d a c d a c

Diffusion

a → b → c
a → d
b → c
b → d

Time
series

1 2 1 2
1 3 1 3
4 1 3 4 1 3

Data

Pairwise

a - b
a - c
a - d
b - c

Weighted pairwise

a - b: 5
a - c: 3
a - d: 1
b - c: 2

Directed pairwise

a -> b
a -> c
a -> d
b -> c

Temporal pairwise

a - b: 1, 3
a - c: 2
a - d: 2, 3
b - c: 1, 4

Matrix

a b c
a 0 3 5
b 3 0 1
c 5 1 0

Group

a b c
a c
b c d

Sequential

a b a b
a c a c
d a c d a c

Diffusion

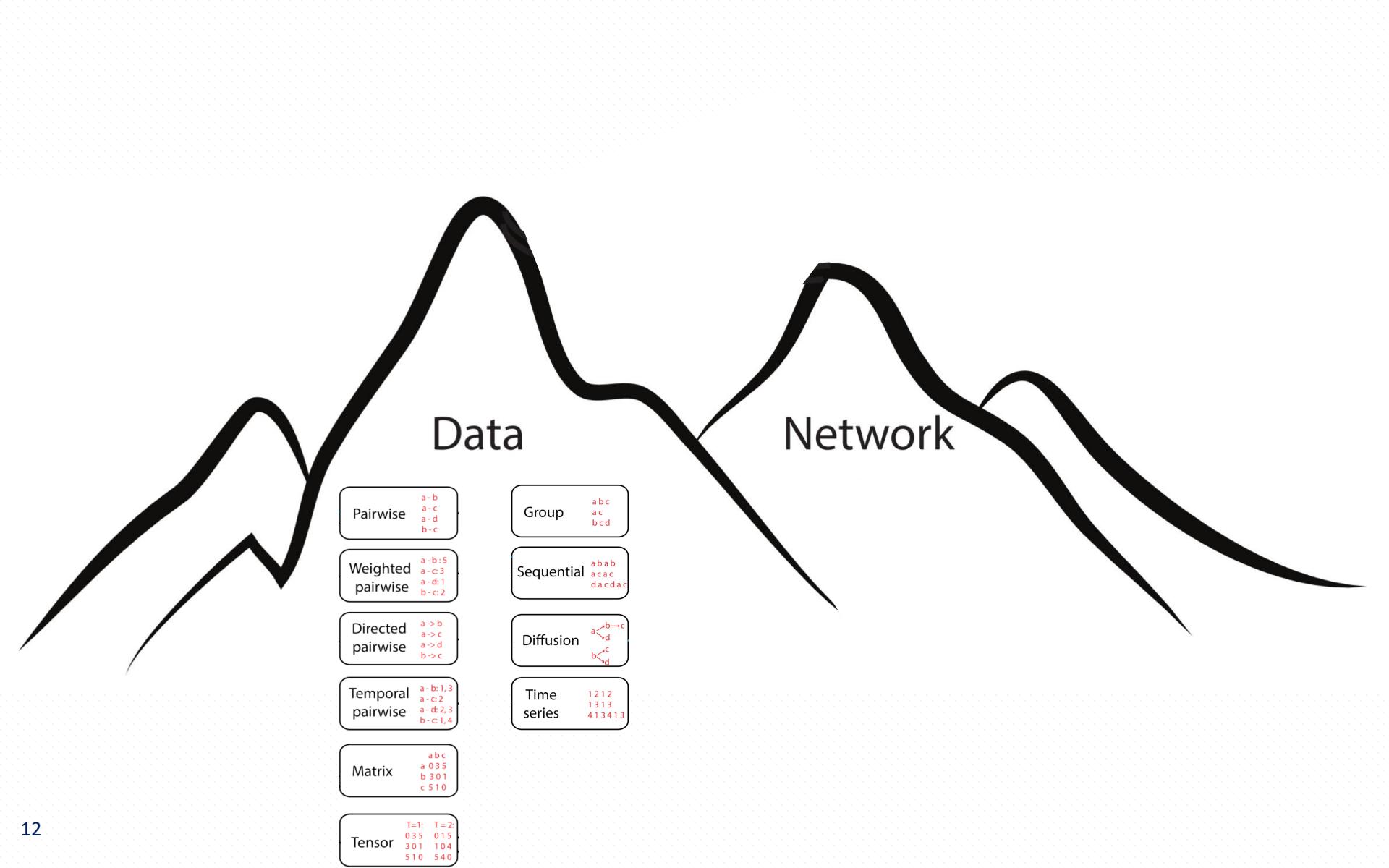
a -> b -> c
a -> d -> c
b -> c -> d

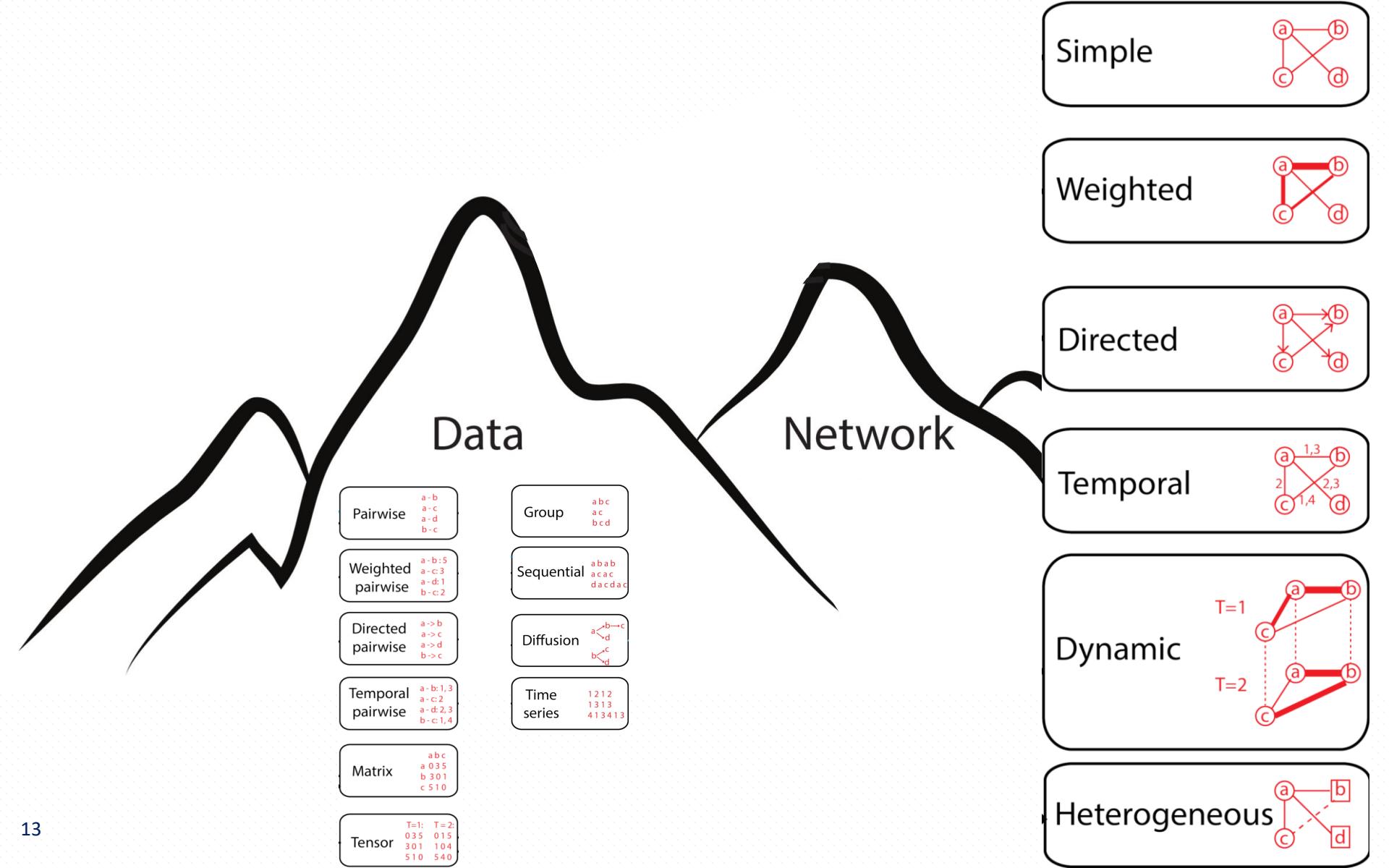
Time series

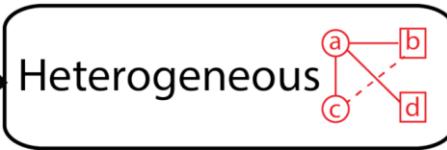
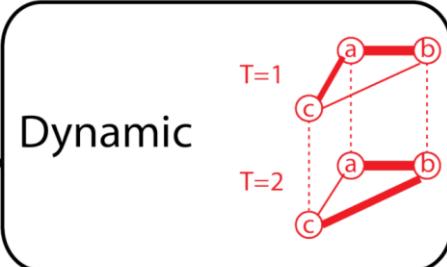
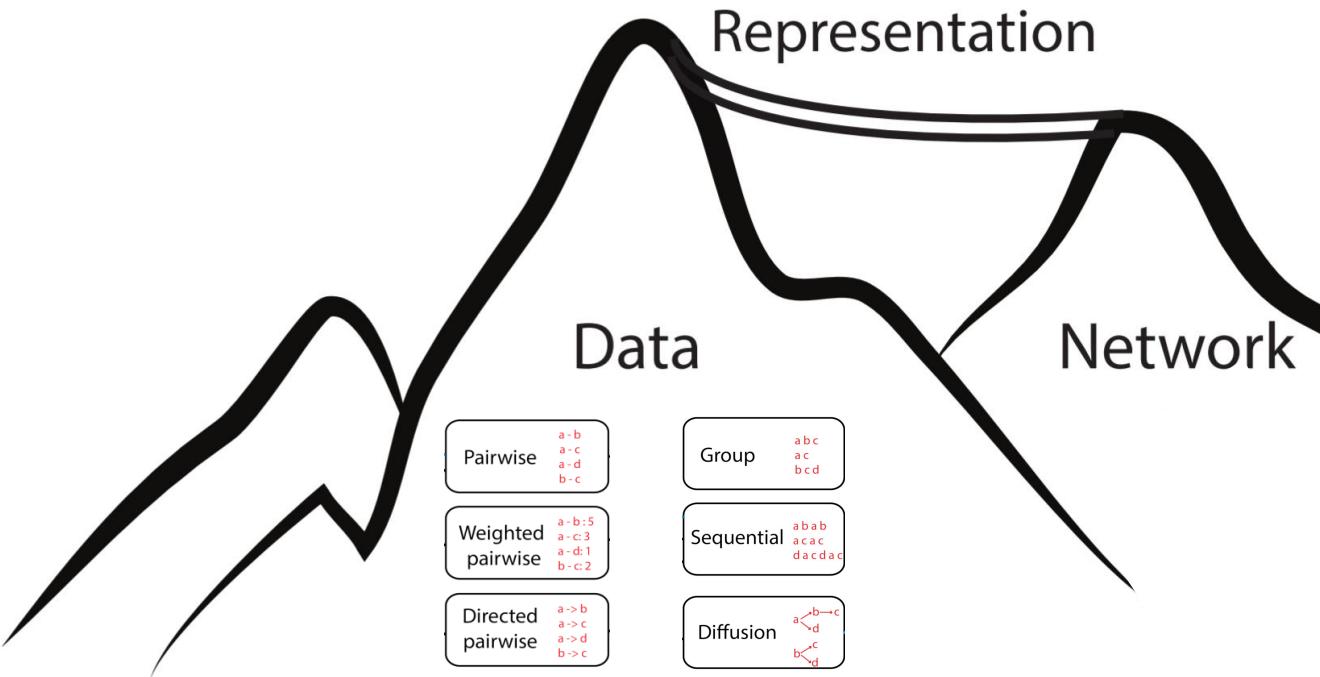
1 2 1 2
1 3 1 3
4 1 3 4 1 3

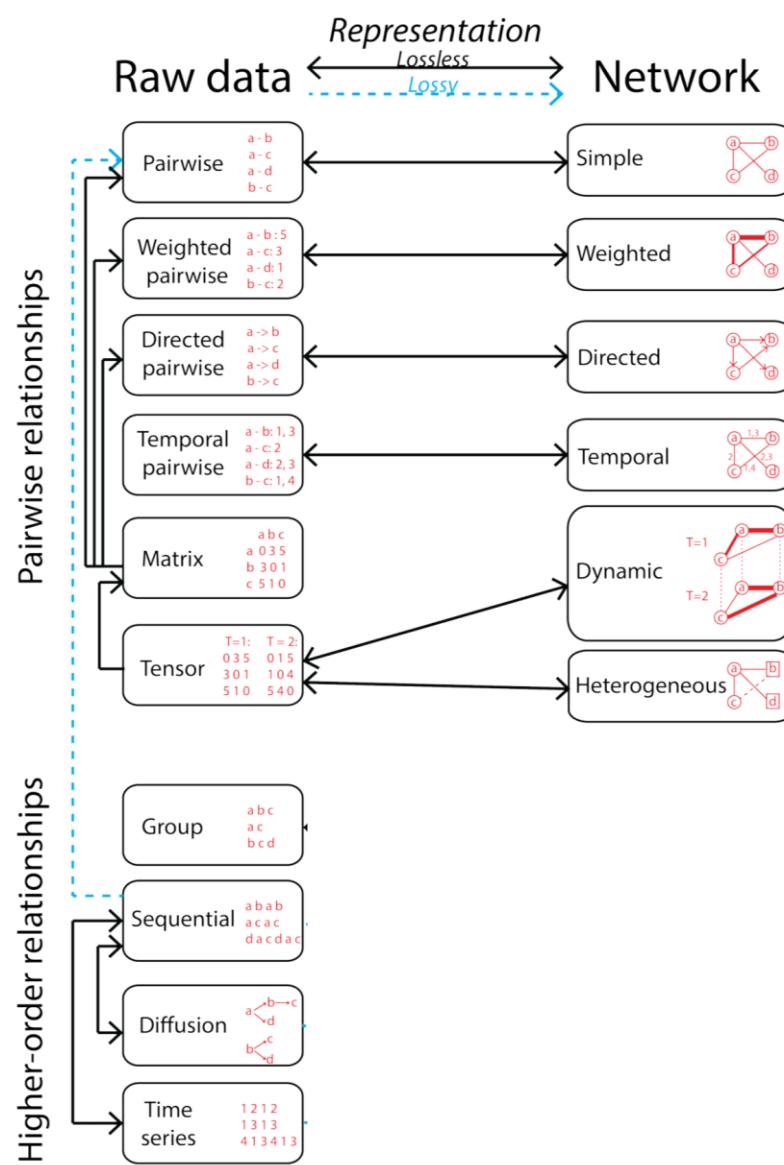
Tensor

T=1: 0 3 5
0 1 5
3 0 1 10 4
5 1 0 5 4 0



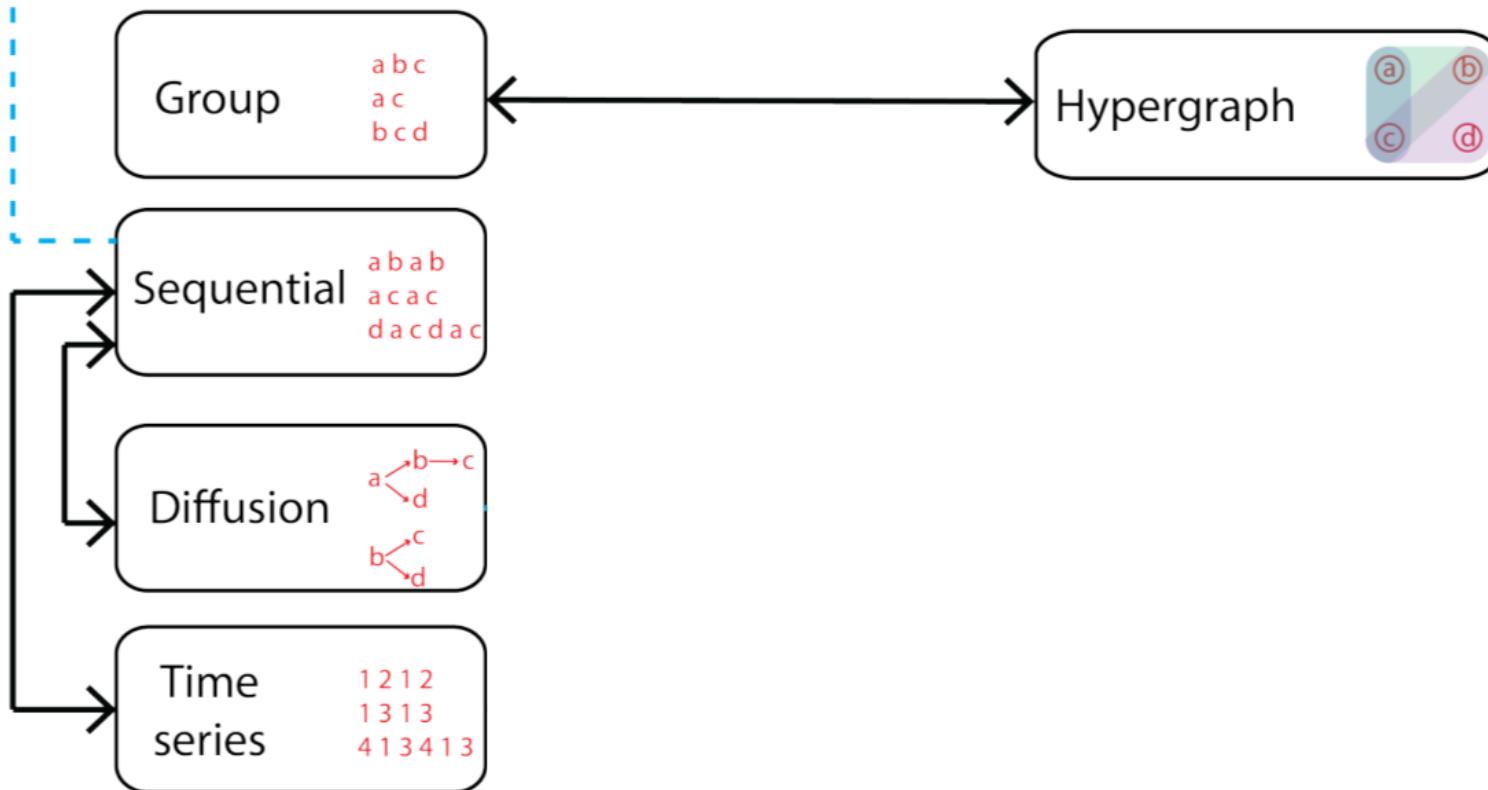




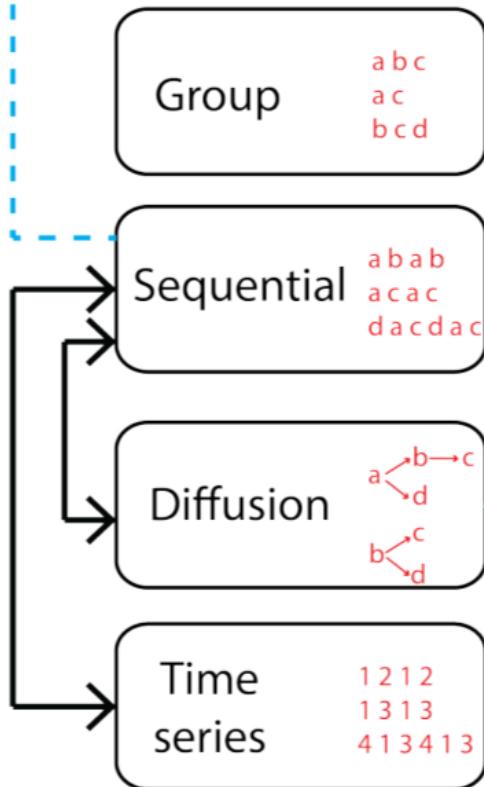


Higher-order relationships

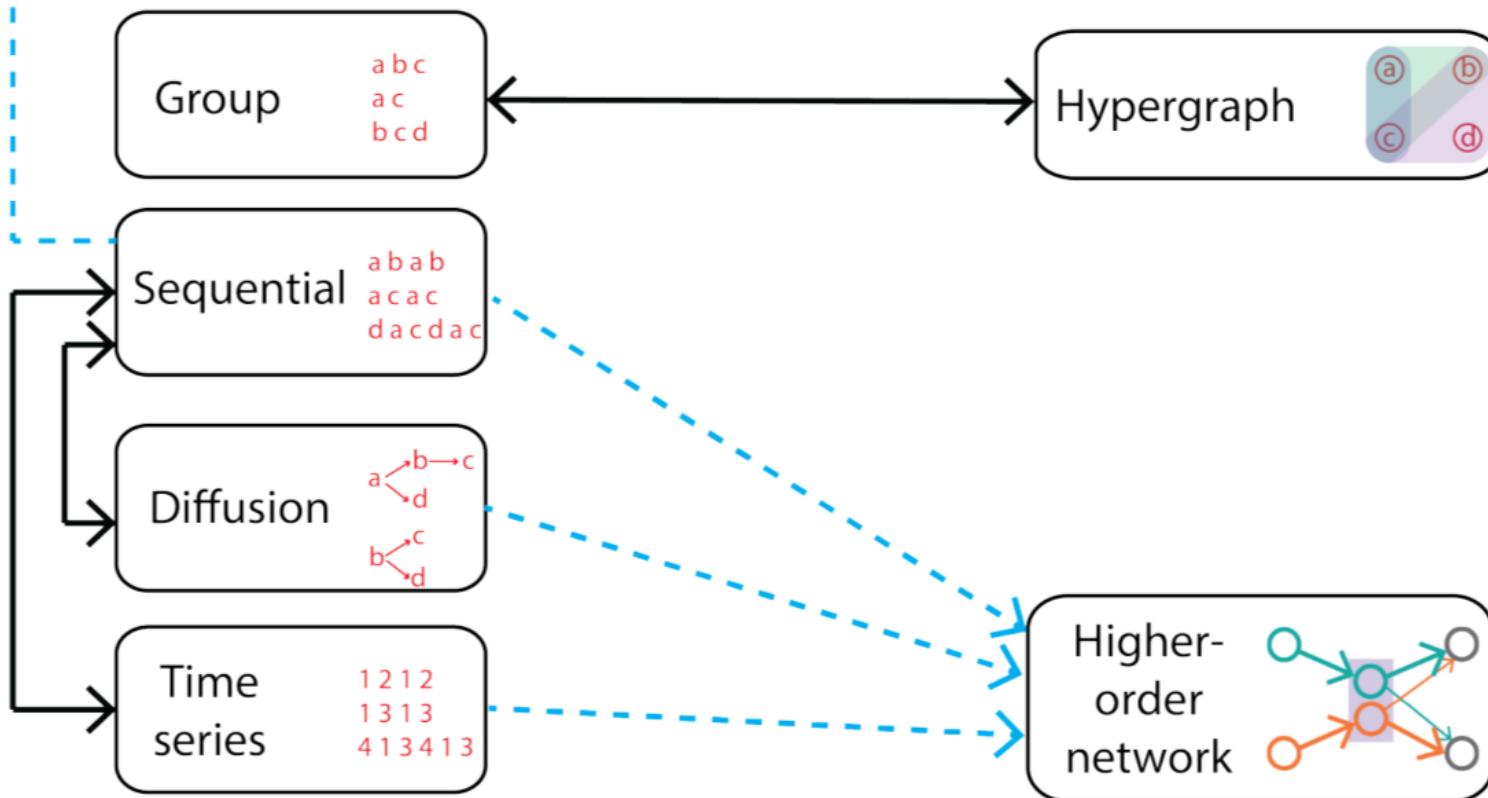
Transaction 3	 
Transaction 4	 
Transaction 5	   



Higher-order relationships



Higher-order relationships



Higher-order network

Representing higher-order dependencies in networks

Higher-order network

Fixed-order network

First-order Markov



Second-order Markov



Variable orders in HON

Fixed-order

Variable-order

Assuming a fixed order beyond the second order becomes impractical because “*higher-order Markov models are more complex*” due to combinatorial explosion

--- Rosvall et al. (Nature Comm. 2014)

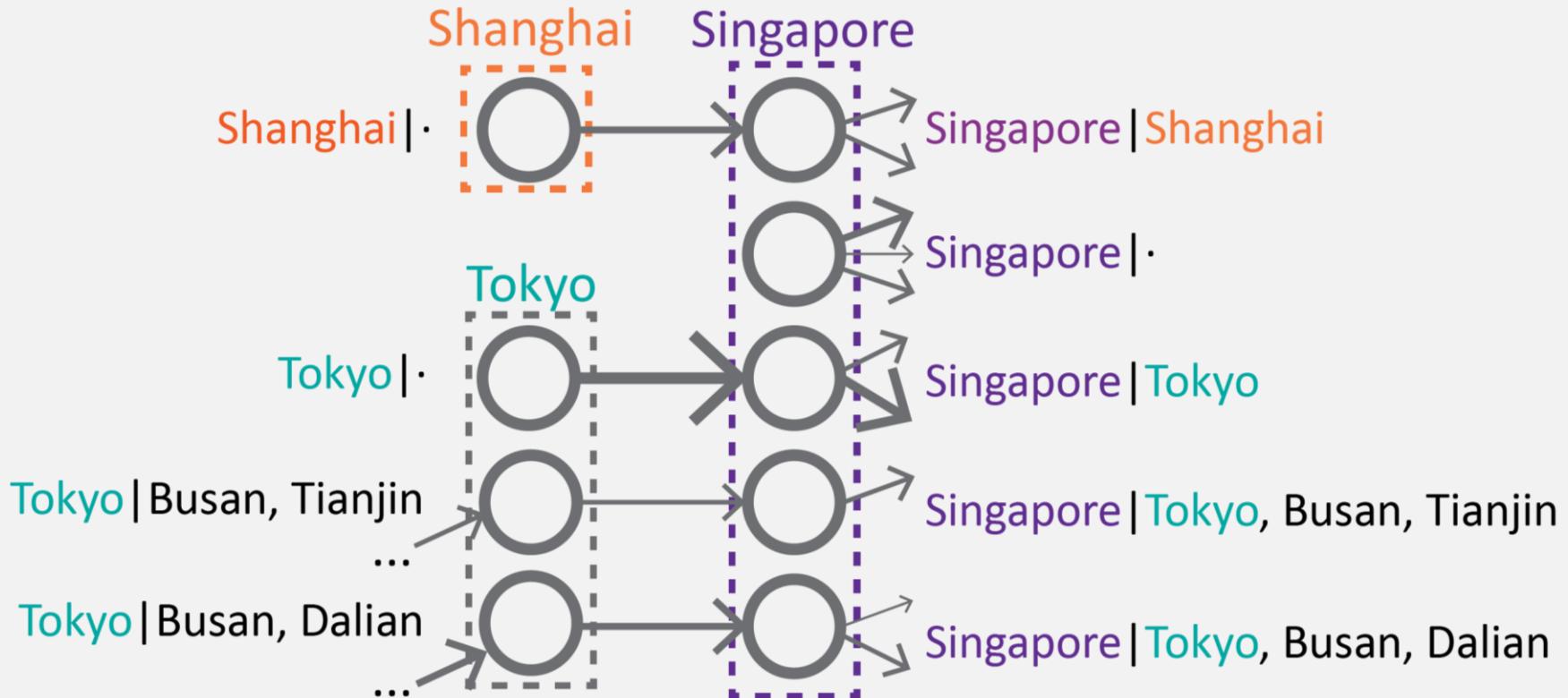
Relatively easier to build

Accurate: use higher-order when necessary

Scalable: use lower-order when sufficient

Variable orders in HON

Variable orders of dependencies in HON

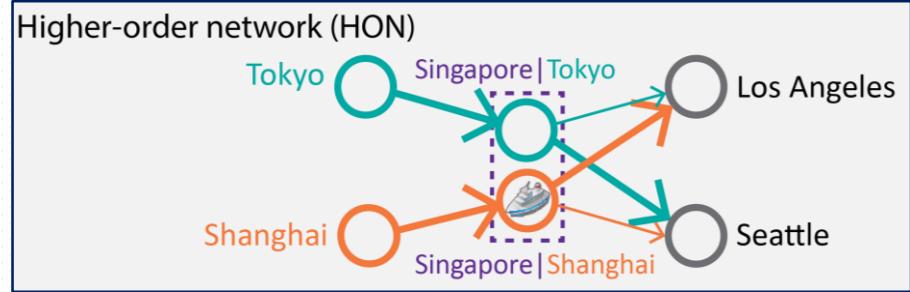
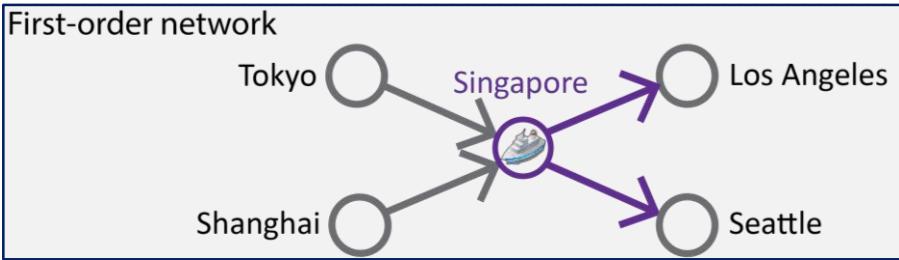


Scalable for big data

Compatible with existing tools

Conventionally: every node represents a single entity (location, state, etc.)

Now: break down nodes into higher-order nodes that carry different dependency relationships



$$P(X_{t+1} = i_{t+1} | X_t = i_t) = \frac{W(i_t \rightarrow i_{t+1})}{\sum_j W(i_t \rightarrow j)}$$

$$P(X_{t+1} = j | X_t = (i|h)) = \frac{W(i|h \rightarrow j)}{\sum_k W(i|h \rightarrow k)}$$

Only change the node labeling

Takeaways

Higher-order network is:

More **accurate** in capturing dynamics in raw data.

More **scalable** than fixed-order networks.

Compatible with existing network algorithms.

Limitations:

Multiple parameters: maximum order & minimum support.

Costly to build for very high orders.

Higher-order network optimized revision

Parameter-free, scalable for arbitrarily high order

HON construction workflow



- Sequential data
- Which nodes to split into higher-order nodes, and how high the orders are
- Connect nodes representing different orders
- Use HON like the conventional network for analyses

HON

Raw data

A B C A B C A C B

HON

Raw data

A B C A B C A C B



Build observation

Build all 1st-order

A -> B: 2
A -> C: 1
B -> C: 2
C -> A: 2
C -> B: 1

Build all 2nd-order

A|C -> B: 1
A|C -> C: 1
B|A -> C: 2
C|B -> A: 2
C|A -> B: 1

Build all 3rd-order

A|C.B -> B: 1
A|C.B -> C: 1
B|A.C -> C: 1
C|B.A -> A: 2
C|A.C -> B: 1

HON

Raw data

A B C A B C A C B

Build all 1st-order

A -> B: 2
A -> C: 1
B -> C: 2
C -> A: 2
C -> B: 1

Build all 2nd-order

A|C -> B: 1
A|C -> C: 1
B|A -> C: 2
C|B -> A: 2
C|A -> B: 1

Build all 3rd-order

A|C.B -> B: 1
A|C.B -> C: 1
B|A.C -> C: 1
C|B.A -> A: 2
C|A.C -> B: 1

Build distribution

Build all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33

Build all 2nd-order

A|C -> B: 0.5
A|C -> C: 0.5
B|A -> C: 1
C|B -> A: 0.67
C|A -> B: 0.33

Build all 3rd-order

A|C.B -> B: 0.5
A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33

HON

Raw data

A B C A B C A C B

Build observation

Build all 1st-order

A -> B: 2
A -> C: 1
B -> C: 2
C -> A: 2
C -> B: 1

Build all 2nd-order

A|C -> B: 1
A|C -> C: 1
B|A -> C: 2
C|B -> A: 2
C|A -> B: 1

Build all 3rd-order

A|C.B -> B: 1
A|C.B -> C: 1
B|A.C -> C: 1
C|B.A -> A: 2
C|A.C -> B: 1

Build distribution

Build all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33

Build all 2nd-order

A|C -> B: 0.5
A|C -> C: 0.5
B|A -> C: 1
C|B -> A: 0.67
C|A -> B: 0.33

Build all 3rd-order

A|C.B -> B: 0.5
A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33

Rule growing

Grow all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33

Grow all 2nd-order

A|C -> B: 0.5
A|C -> C: 0.5
B|A -> C: 1
C|B -> A: 0.67
C|A -> B: 0.33

Grow all 3rd-order

A|C.B -> B: 0.5
A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33

HON

Raw data

A B C A B C A C B



Build observation

Build all 1st-order

A -> B: 2
A -> C: 1
B -> C: 2
C -> A: 2

Build all 2nd-order

A|C -> B: 1
A|C -> C: 1
B|A -> C: 2
C|B -> A: 2

Build all 3rd-order

A|C.B -> B: 1
A|C.B -> C: 1
B|A.C -> C: 1
C|B.A -> A: 2

$$D_{KL}(ExtDistr || Distr) \leq \frac{NewOrder}{\log_2(1 + \sum C[ExtSource][*])}$$

A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33

A|C -> C: 0.5
B|A -> C: 1
C|B -> A: 0.67
C|A -> B: 0.33

A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33



Rule growing

Grow all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33



Grow all 2nd-order

A|C -> B: 0.5
A|C -> C: 0.5
B|A -> C: 1
C|B -> A: 0.67
C|A -> B: 0.33



Grow all 3rd-order

A|C.B -> B: 0.5
A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33

HON

Raw data

A B C A B C A C B

Build observation

Build all 1st-order

A -> B: 2
A -> C: 1
B -> C: 2
C -> A: 2
C -> B: 1

Build all 2nd-order

A|C -> B: 1
A|C -> C: 1
B|A -> C: 2
C|B -> A: 2
C|A -> B: 1

Build all 3rd-order

A|C.B -> B: 1
A|C.B -> C: 1
B|A.C -> C: 1
C|B.A -> A: 2
C|A.C -> B: 1

Build distribution

Build all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33

Build all 2nd-order

A|C -> B: 0.5
A|C -> C: 0.5
B|A -> C: 1
C|B -> A: 0.67
C|B -> B: 0.33

Build all 3rd-order

A|C.B -> B: 0.5
A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33

Rule growing

Grow all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33

Grow all 2nd-order

A|C -> B: 0.5
A|C -> C: 0.5
B|A -> C: 1
C|B -> A: 0.67
C|B -> B: 0.33

Grow all 3rd-order

A|C.B -> B: 0.5
A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33

Actual dependencies

C|B -> A: 0.67
C|A -> B: 0.33



HON

Raw data

A B C A B C A C B

$\Theta(Order^2 \times RawDataSize)$

Build observation

Build all 1st-order

A -> B: 2
A -> C: 1
B -> C: 2
C -> A: 2
C -> B: 1

Build all 2nd-order

A|C -> B: 1
A|C -> C: 1
B|A -> C: 2
C|B -> A: 2
C|A -> B: 1

Build all 3rd-order

A|C.B -> B: 1
A|C.B -> C: 1
B|A.C -> C: 1
C|B.A -> A: 2
C|A.C -> B: 1

Build distribution

Build all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33

Build all 2nd-order

A -> B: 0.5
A -> C: 0.5
B -> C: 1
B -> A: 0.67
B: 0.33

Build all 3rd-order

A|C.B -> B: 0.5
A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33

Storage cost

Rule growing

Grow all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33

Grow all 2nd-order

A -> B: 0.5
A -> C: 0.5
B -> C: 1
B -> A: 0.67
B: 0.33

Grow all 3rd-order

A|C.B -> B: 0.5
A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33

5th order

4th order

3rd order

2nd order

1st order

Actual dependencies

HON

Raw data

Build observation

Build distribution

Storage cost

Rule growing

A B C A B C A C B

$\Theta(Order \times RawDataSize)$

Build all 1st-order

A -> B: 2
A -> C: 1
B -> C: 2
C -> A: 2
C -> B: 1

Build all 2nd-order

A|C -> B: 1
A|C -> C: 1
B|A -> C: 2
C|B -> A: 2
C|A -> B: 1

Build all 3rd-order

A|C.B -> B: 1
A|C.B -> C: 1
B|A.C -> C: 1
C|B.A -> A: 2
C|A.C -> B: 1

Build all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33

Build all 2nd-order

-> B: 0.5
-> C: 0.5
> C: 1
> A: 0.67
B: 0.33

Build all 3rd-order

A|C.B -> B: 0.5
A|C.B -> C: 0.5
B|A.C -> C: 1
C|B.A -> A: 0.67
C|A.C -> B: 0.33

5th order

4th order

3rd order

2nd order

1st order

Actual dependencies

A B C A B C A C B

Build observation

A -> B: 2

A -> C: 1

B -> C: 2

C -> A: 2

Build all 1st-orderBuild all 2nd-orderBuild all 3rd-order

A|C -> B: 1

A|C -> C: 1

B|A -> C: 2

C|B -> A: 2

A|C.B -> B: 1

A|C.B -> C: 1

B|A.C -> C: 1

C|B.A -> A: 2

$$\begin{aligned}
 \max(D_{KL}(ExtDistr||Distr)) &= \max\left(\sum_{i \in Distr} P_{ExtDistr}(i) \times \log_2 \frac{P_{ExtDistr}(i)}{P_{Distr}(i)}\right) \\
 &= 1 \times \log_2 \frac{1}{\min(P_{Distr}(i))} + 0 + 0 + \dots \\
 &= -\log_2(\min(P_{Distr}(i)))
 \end{aligned}$$

A -> B: 0.0/

A -> C: 0.33

B -> C: 1

C -> A: 0.67

C -> B: 0.33

A|C -> B: 0.0/

A|C -> C: 0.5

B|A -> C: 1

C|B -> A: 0.67

C|A -> B: 0.33

A|C.B -> B: 0.0/

A|C.B -> C: 0.5

B|A.C -> C: 1

C|B.A -> A: 0.67

C|A.C -> B: 0.33

HON

Raw data

A B C A B C A C B

Build observation

A \rightarrow B: 2

A \rightarrow C: 1

B \rightarrow C: 2

C \rightarrow A: 2

Build all 1st-order

Build all 2nd-order

Build all 3rd-order

A|C \rightarrow B: 1

A|C \rightarrow C: 1

B|A \rightarrow C: 2

C|B \rightarrow A: 2

A|C.B \rightarrow B: 1

A|C.B \rightarrow C: 1

B|A.C \rightarrow C: 1

C|B.A \rightarrow A: 2

$$-\log_2(\min(P_{Distr}(i))) \leq \frac{NewOrder}{\log_2(1 + \sum C[ExtSource][*])}$$

A \rightarrow C: 0.33

B \rightarrow C: 1

C \rightarrow A: 0.67

C \rightarrow B: 0.33

A|C \rightarrow C: 0.5

B|A \rightarrow C: 1

C|B \rightarrow A: 0.67

C|A \rightarrow B: 0.33

A|C.B \rightarrow C: 0.5

B|A.C \rightarrow C: 1

C|B.A \rightarrow A: 0.67

C|A.C \rightarrow B: 0.33

Rule growing

Grow all 1st-order

A \rightarrow B: 0.67

A \rightarrow C: 0.33

B \rightarrow C: 1

C \rightarrow A: 0.67

C \rightarrow B: 0.33

Grow all 2nd-order

A|C \rightarrow B: 0.5

A|C \rightarrow C: 0.5

B|A \rightarrow C: 1

C|B \rightarrow A: 0.67

C|A \rightarrow B: 0.33

Grow all 3rd-order

A|C.B \rightarrow B: 0.5

A|C.B \rightarrow C: 0.5

B|A.C \rightarrow C: 1

C|B.A \rightarrow A: 0.67

C|A.C \rightarrow B: 0.33

HONOR

Raw *data*

A B C A B C A C B

HONOR

Raw data

A B C A B C A C B



Build observation

Build all 1st-order

A -> B: 2

A -> C: 1

B -> C: 2

C -> A: 2

C -> B: 1

HONOR

Raw data

A B C A B C A C B



Build observation

Build all 1st-order

A -> B: 2

A -> C: 1

B -> C: 2

C -> A: 2

C -> B: 1



Build distribution

Build all 1st-order

A -> B: 0.67

A -> C: 0.33

B -> C: 1

C -> A: 0.67

C -> B: 0.33

HONOR

Raw data

A B C A B C A C B



Build observation

Build all 1st-order

A -> B: 2
A -> C: 1
B -> C: 2
C -> A: 2
C -> B: 1



Build distribution

Build all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33



Rule growing

Grow all 1st-order

A -> B: 0.67
A -> C: 0.33
B -> C: 1
C -> A: 0.67
C -> B: 0.33



HONOR

Raw data

A B C A B C A C B

Build observation

Build all 1st-order

A -> B: 2

A -> C: 1

B -> C: 2

C -> A: 2

C -> B: 1

Build 2nd-order on demand

A|C -> B: 1

A|C -> C: 1

B|A -> C: 2

C|B -> A: 2

C|A -> B: 1

Build distribution

Build all 1st-order

A -> B: 0.67

A -> C: 0.33

B -> C: 1

C -> A: 0.67

C -> B: 0.33

Build 2nd-order on demand

A|C -> B: 0.5

A|C -> C: 0.5

B|A -> C: 1

C|B -> A: 0.67

C|A -> B: 0.33

Rule growing

Grow all 1st-order

A -> B: 0.67

A -> C: 0.33

B -> C: 1

C -> A: 0.67

C -> B: 0.33

Grow 2nd-order on demand

A|C -> B: 0.5

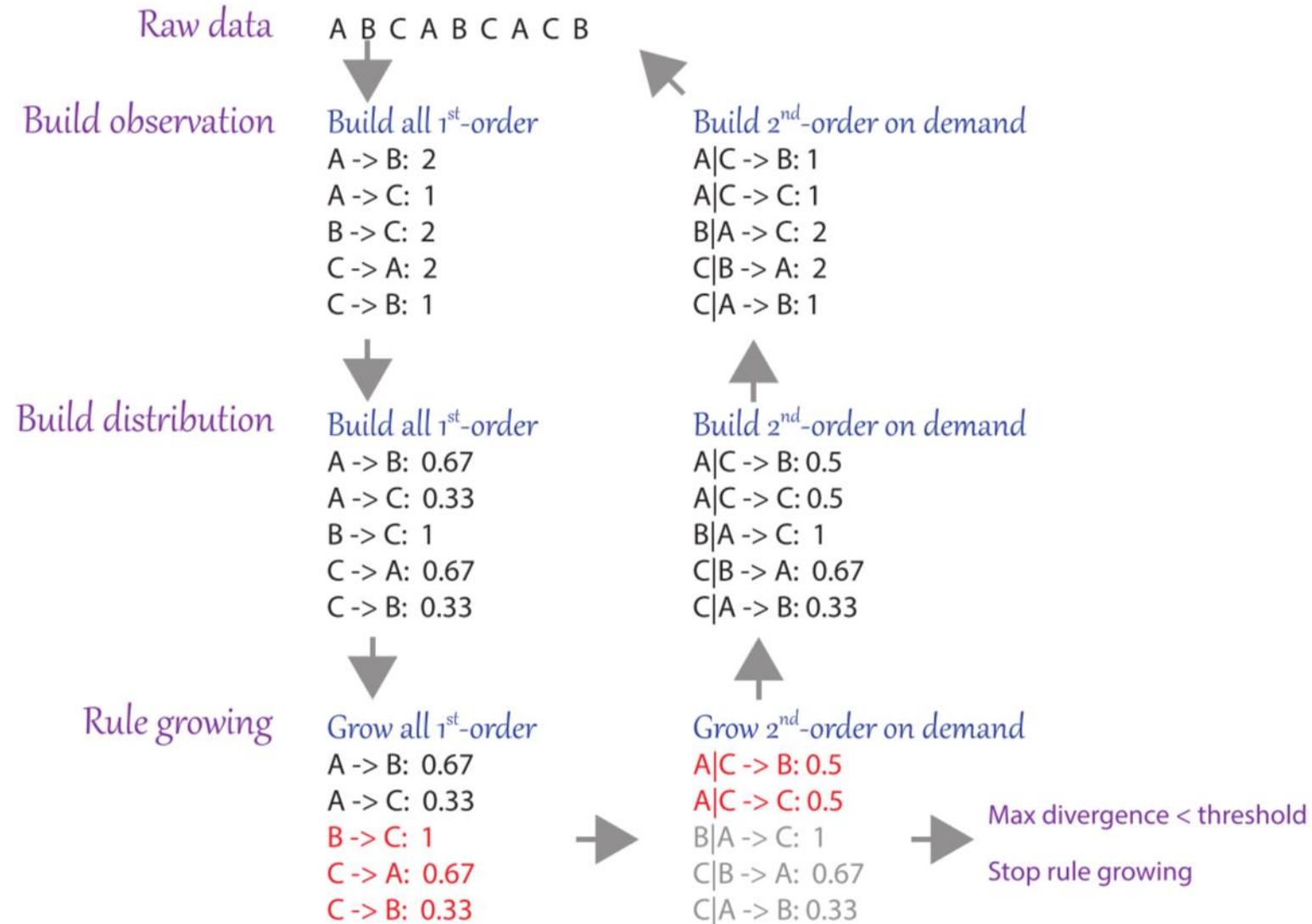
A|C -> C: 0.5

B|A -> C: 1

C|B -> A: 0.67

C|A -> B: 0.33

HONOR



Takeaways

HONOR is:

Parameter-free version of HON.

More scalable for big data

Supports arbitrarily high order.

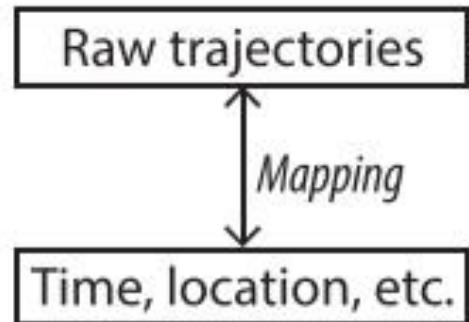
Lazy evaluation reduces actual search space.

HONVis

Visualization & interactive exploration software

HoNVis framework

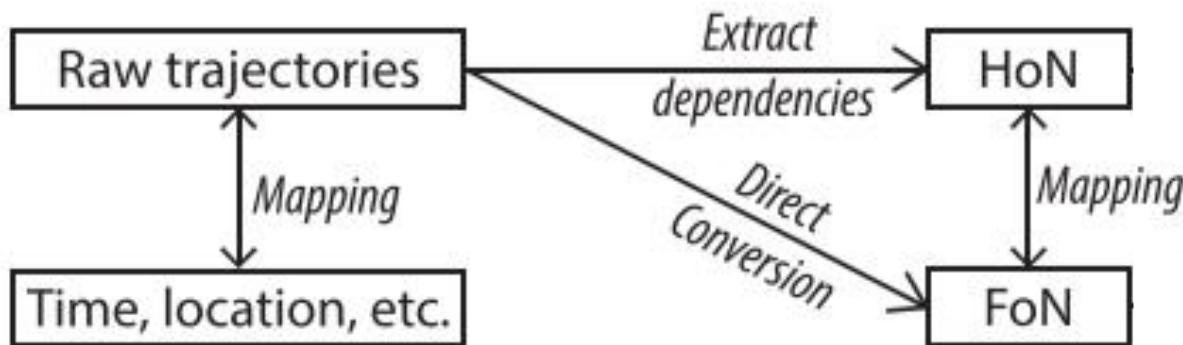
Input



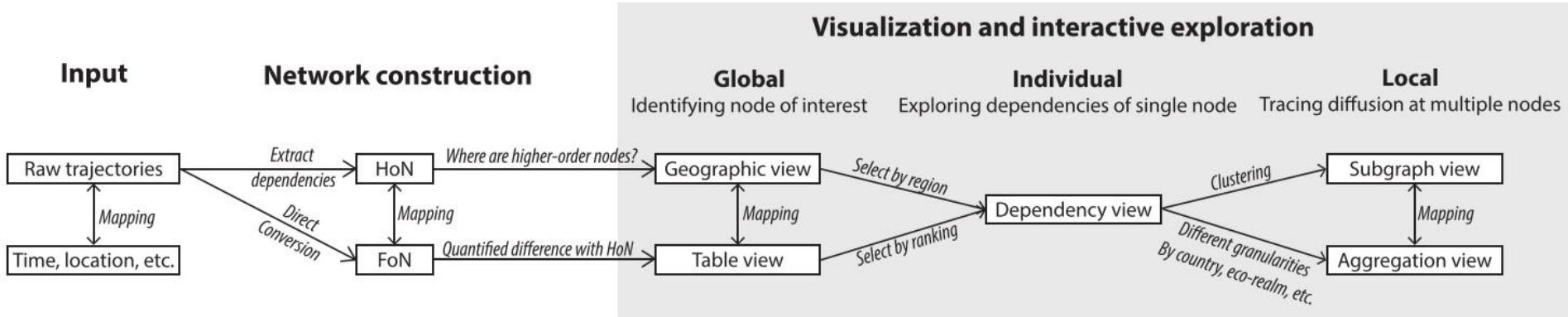
HoNVis framework

Input

Network construction



HoNVis framework



HoNVis interface

Geographic View

Node Weight

Prefer High Weight

Edge Straightening

Dependency View

Destination Order

Minimum Probability

Minimum Weight

Point Size

Label Margin

Subgraph View

Trace Forward

Parameter Control Panel

Point Size

Label Margin

Aggregation View

Hide View

Exact Group

Use Subgraph

Node Group

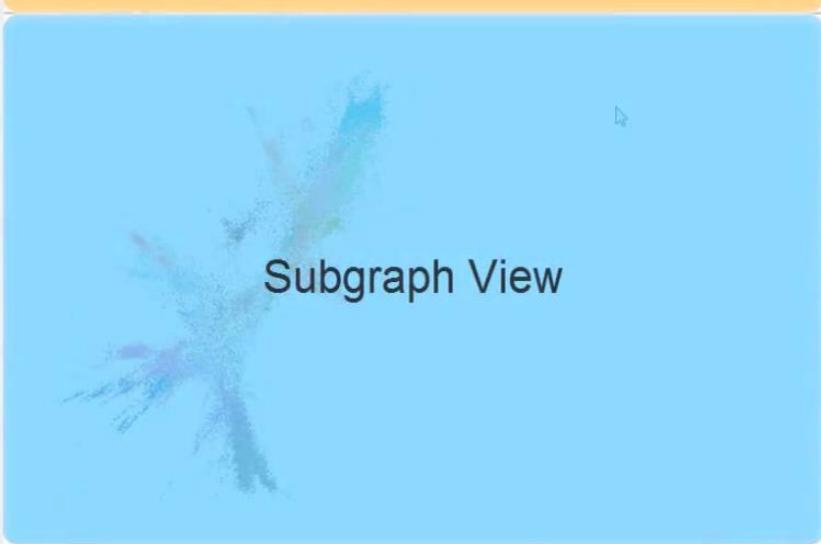
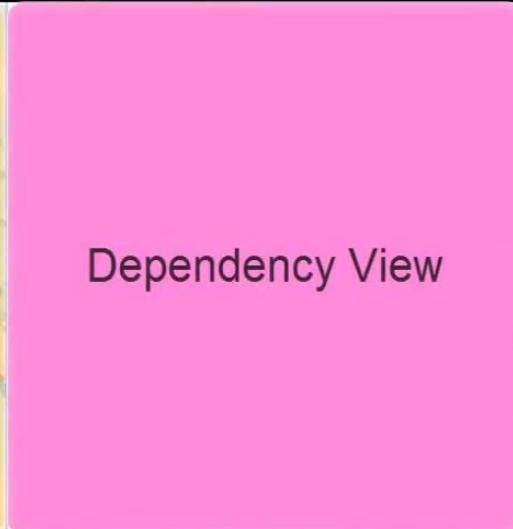
Node Weight

Display In-Group Edges

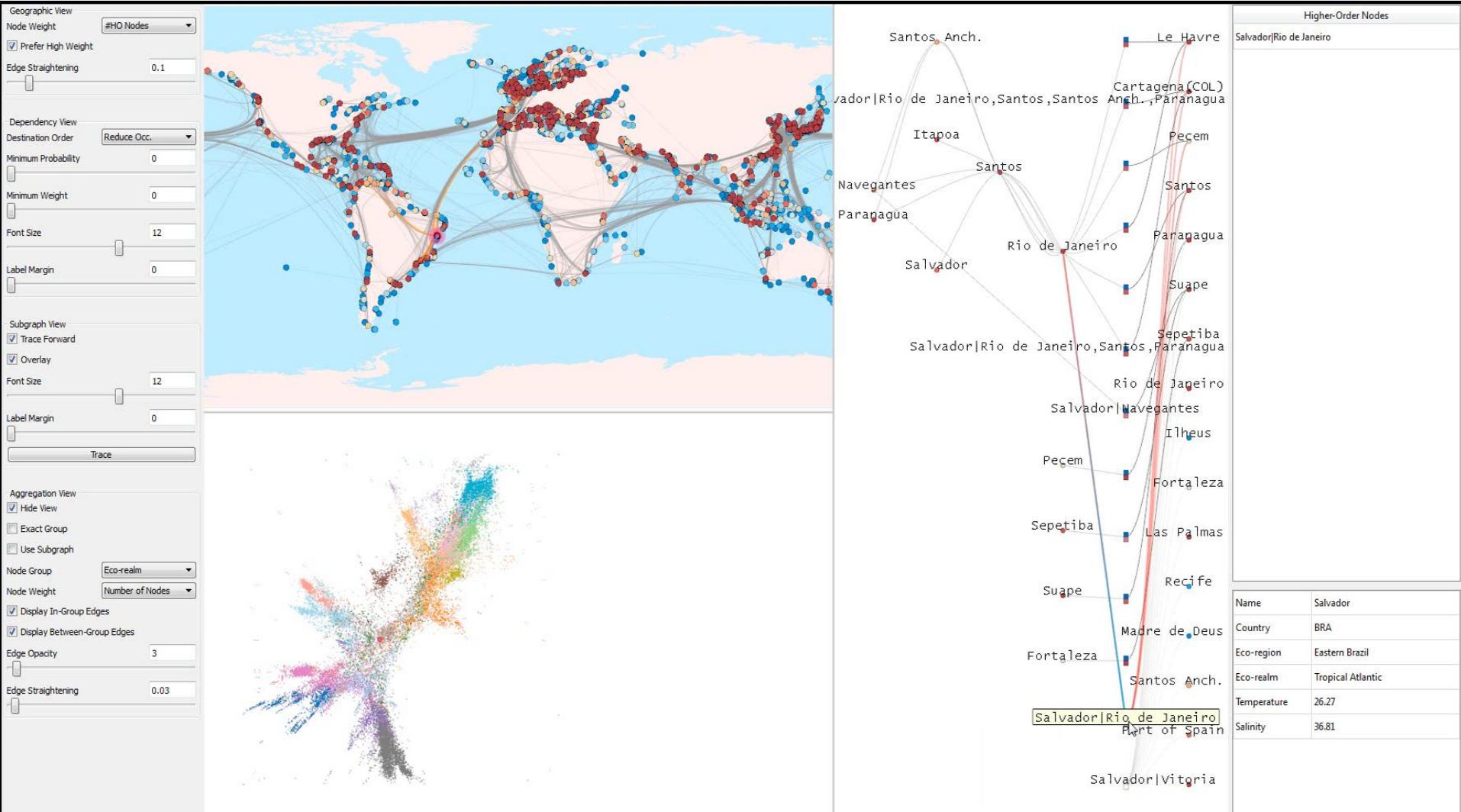
Display Between-Group Edges

Edge Opacity

Edge Straightening



Visualization & interactive exploration



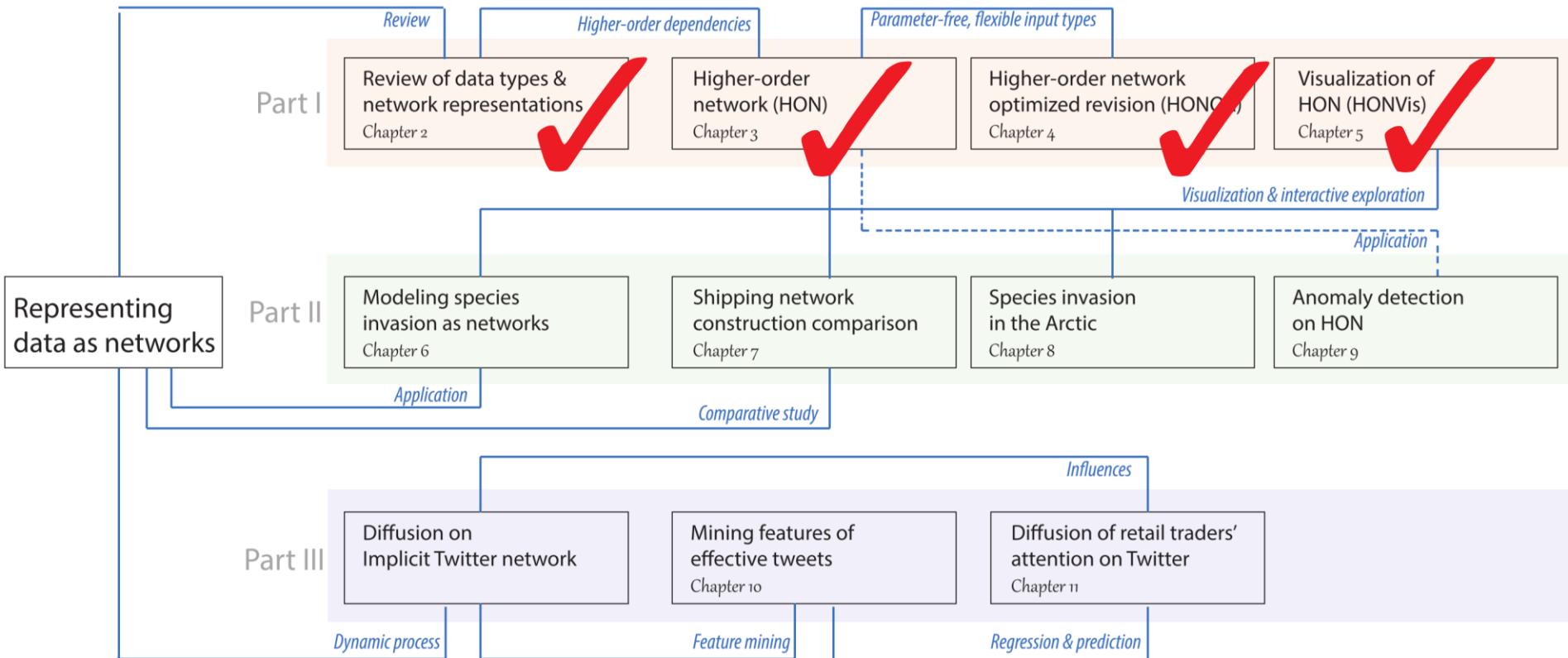
Takeaways

HONVis is:

The **first** visualization software for HON.

Facilitates interactive explorations.

Overview



Part II

Insights in real-world applications

Species invasion network

Non-indigenous species risk assessment &
prediction framework (NIS-RAPS)

Invasive species

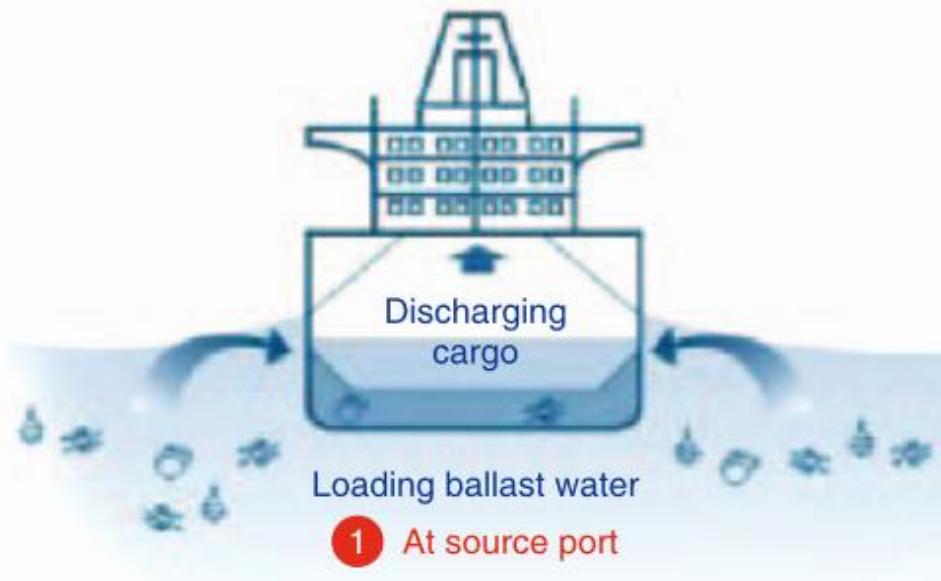


Zebra mussels @ Great Lakes
Clogging water pipes, attach to boats

**\$120 billion / year
damage & control costs**



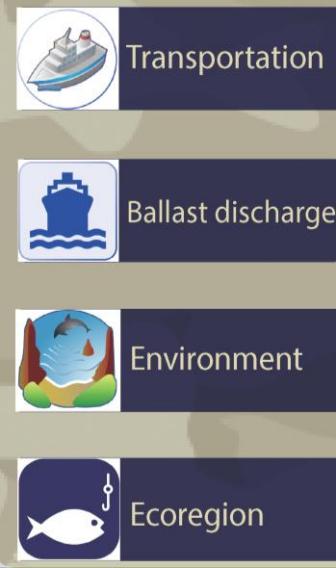
Ship-borne species invasion



Ship-borne species invasion

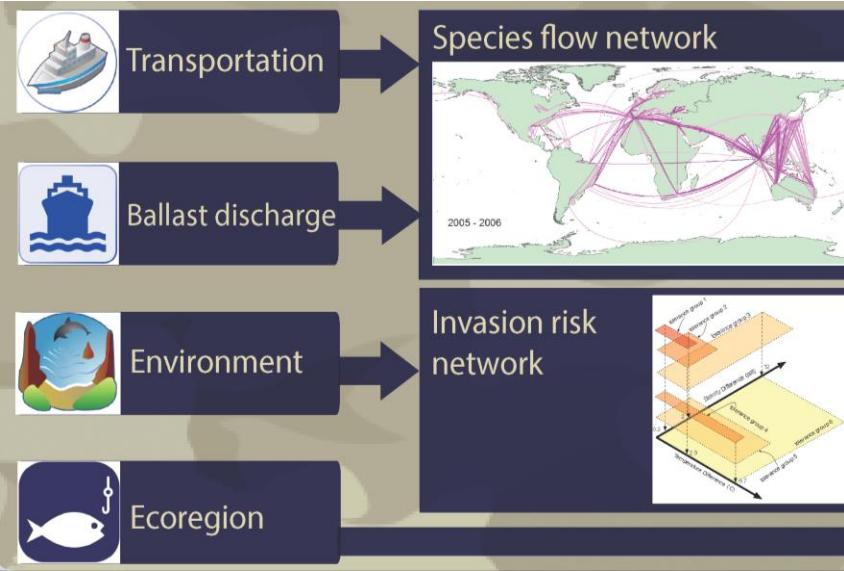


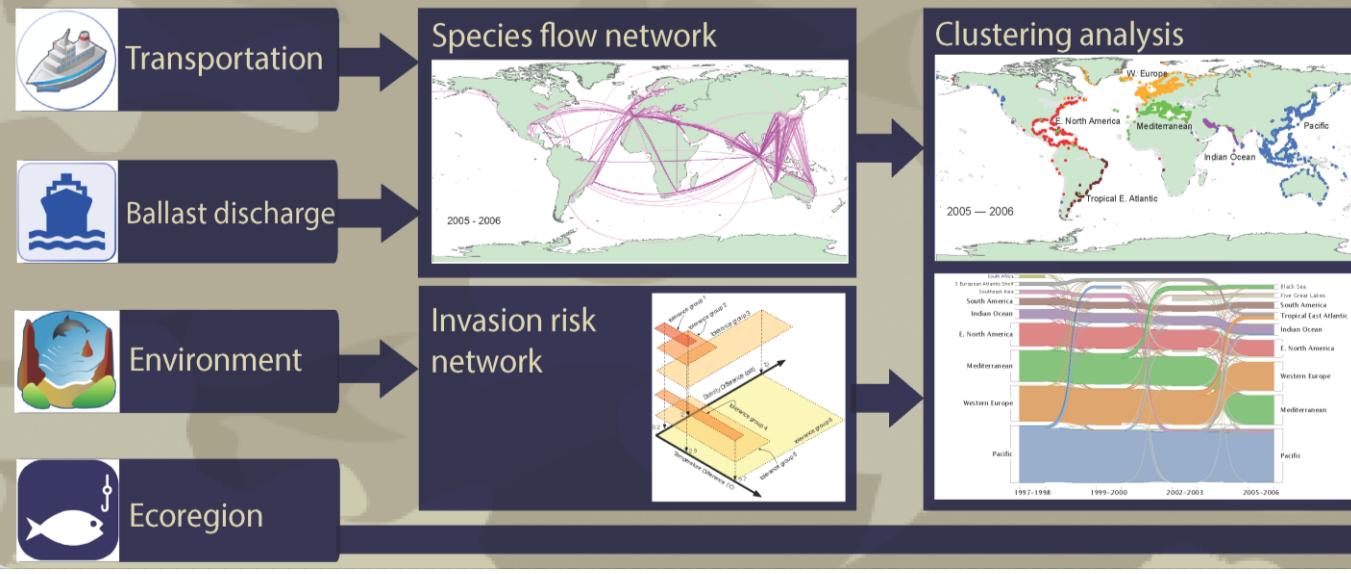
-  Transportation
-  Ballast discharge
-  Environment
-  Ecoregion

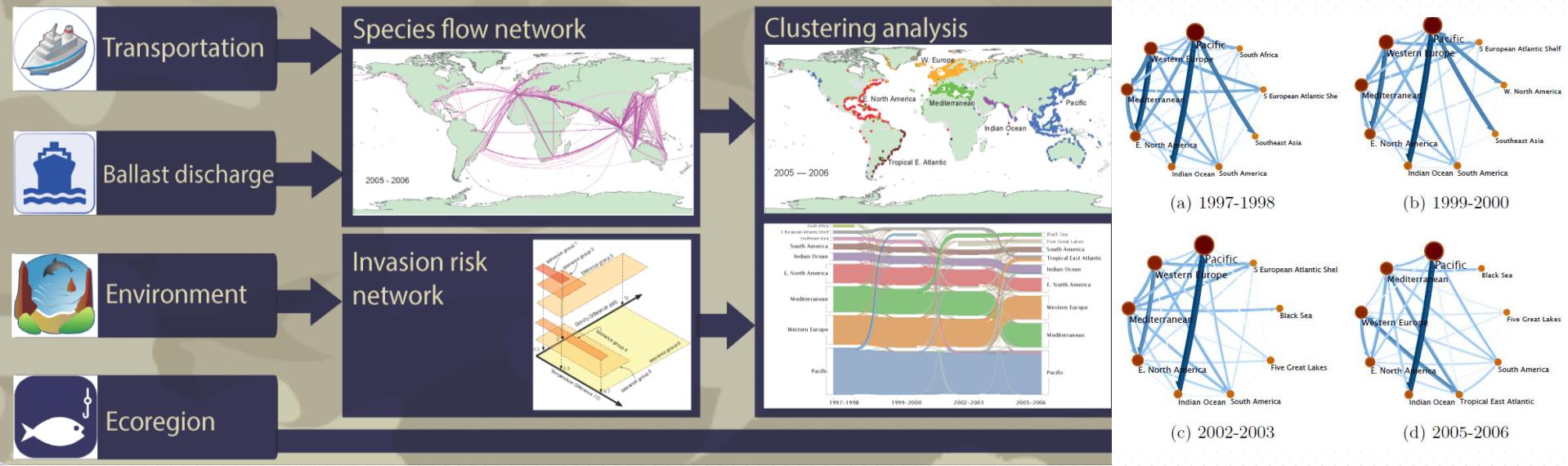


Probability of vessel v introducing species from port i to port j

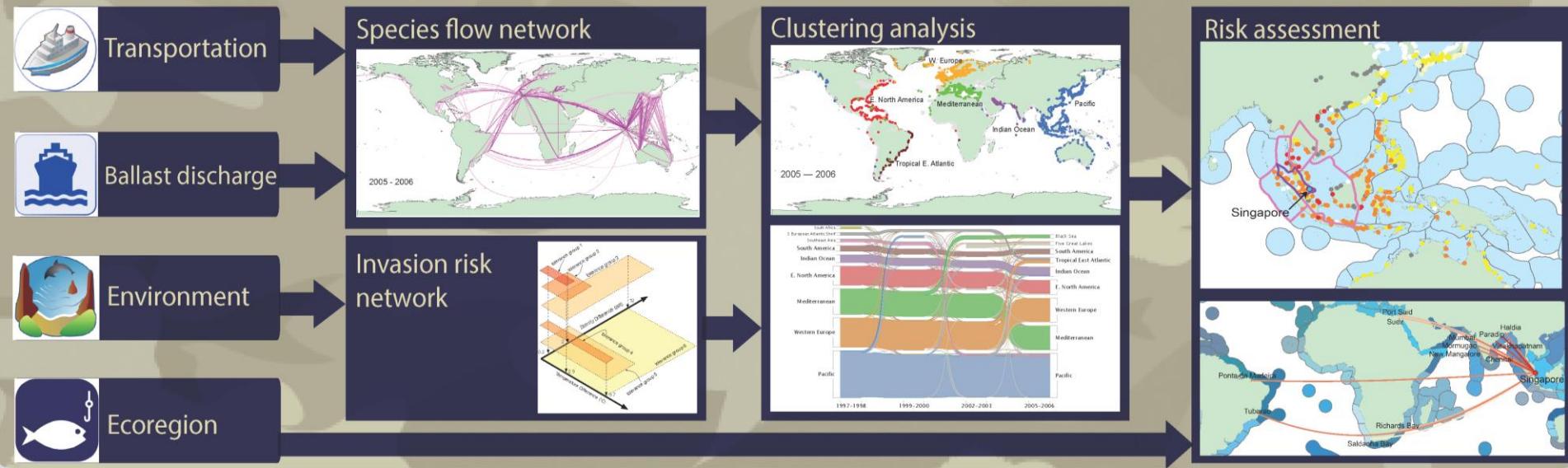
$$p_{ij}^{(v)} = \frac{\rho_{ij}^{(v)}}{\text{Mgmt efficacy}} \frac{(1 - e^{-\lambda D_{ij}^{(v)}})}{\text{Ballast discharge}} \frac{e^{-\mu \Delta t_{ij}^{(v)}}}{\text{Mortality}}$$







* Clustering uses MapEquation by Rosvall et al. (2008)



Takeaways

NIS-RAPS:

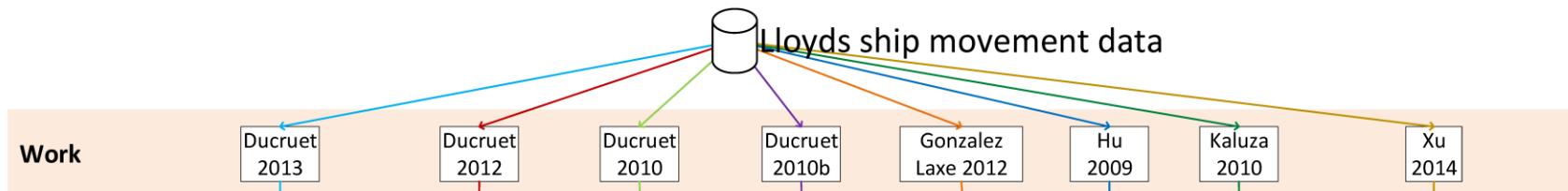
Integrates multiple sources of data.

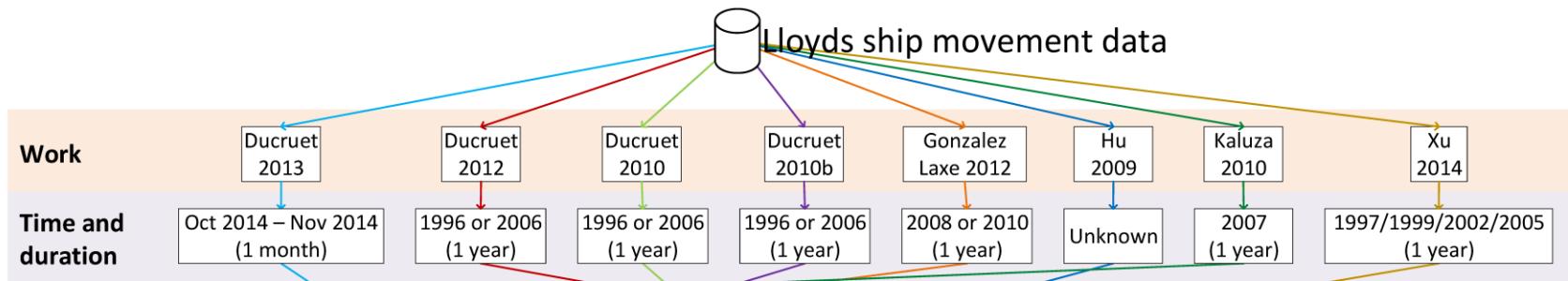
A network approach for invasive species modeling.

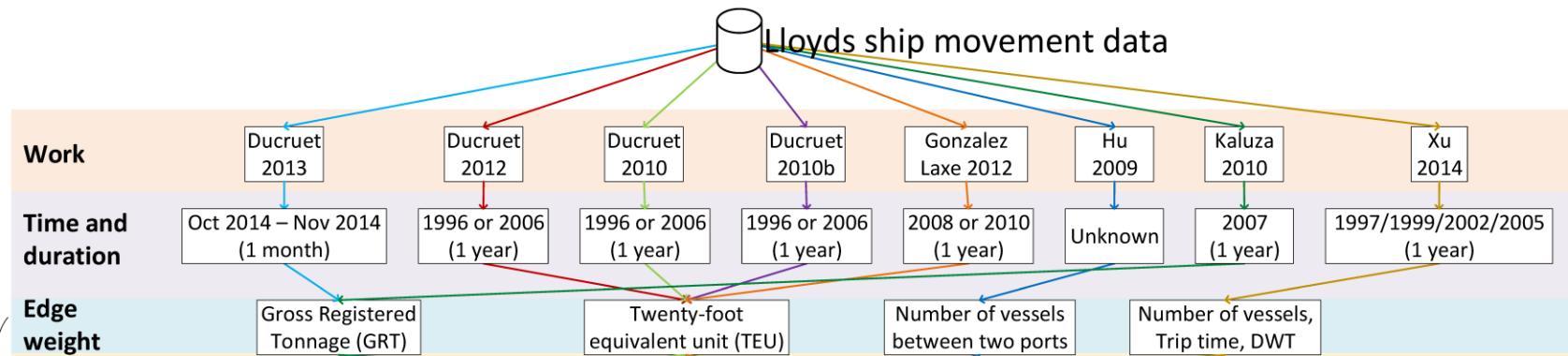
Provides insights to inform policy makers.

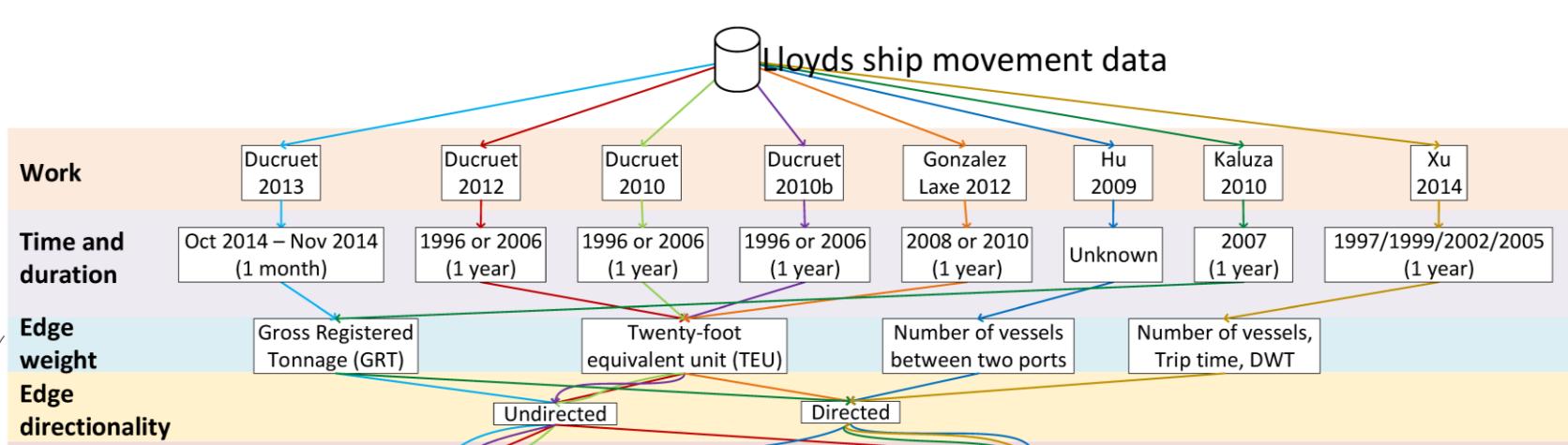
Shipping network construction

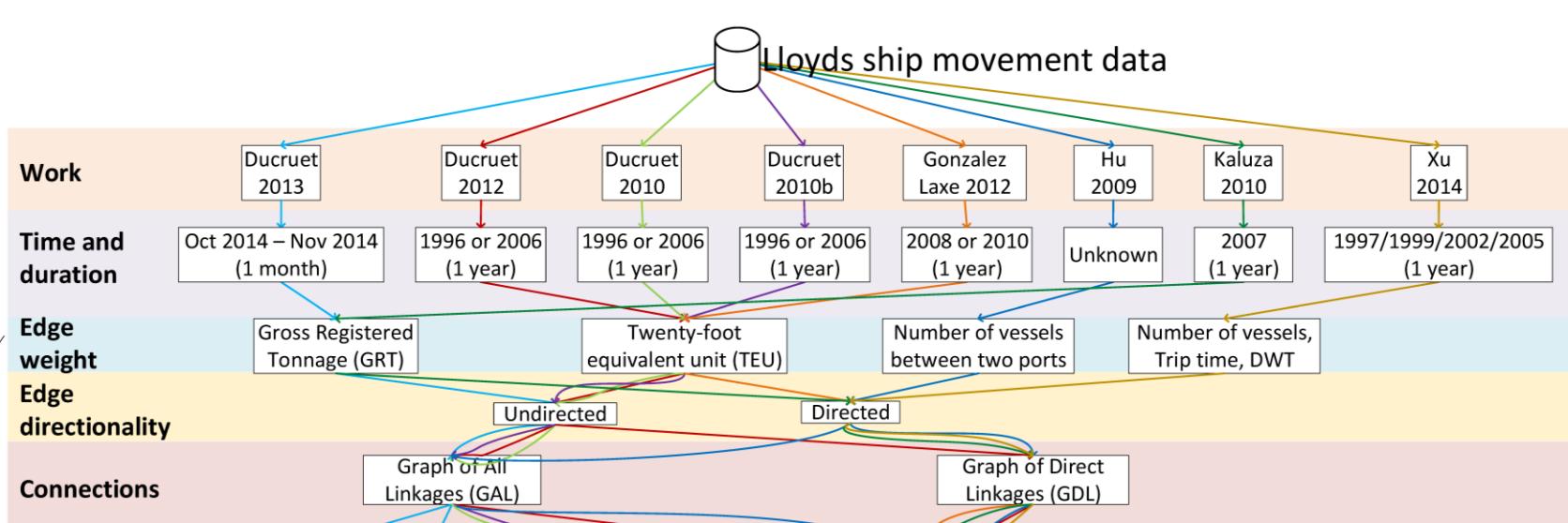
How does network construction choices
influence network properties and analysis results?

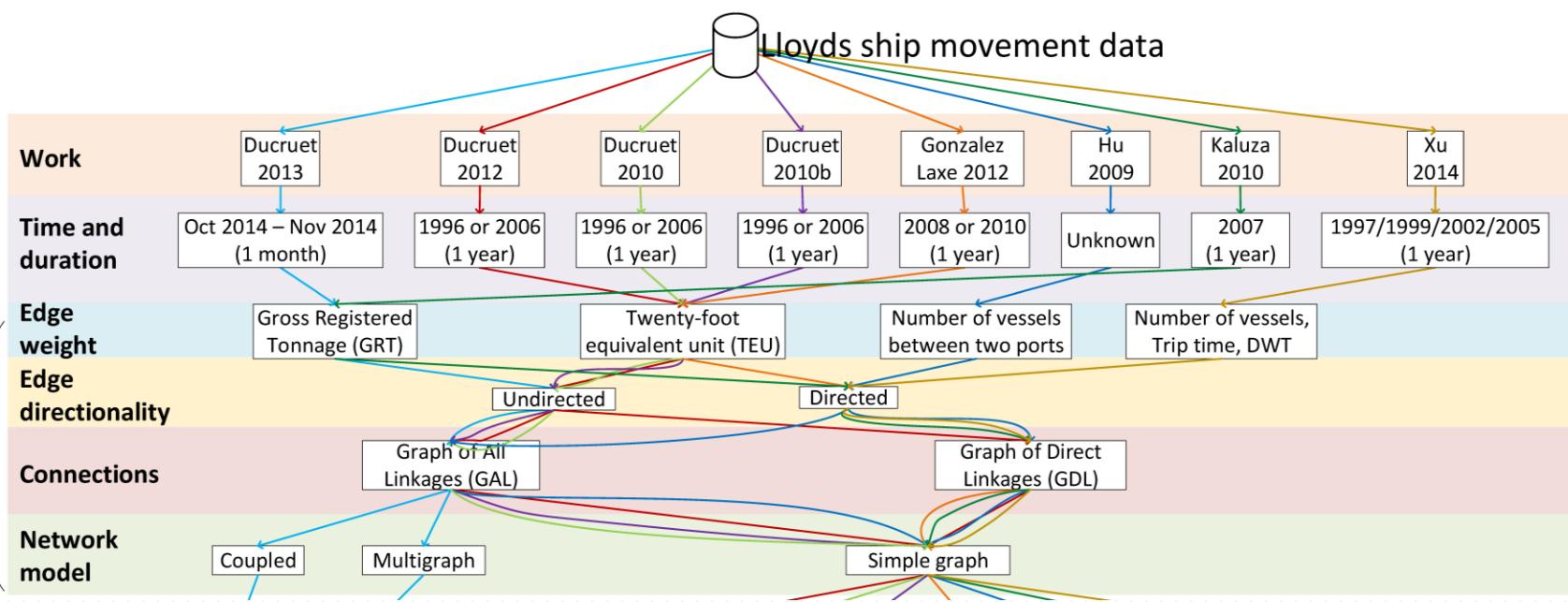


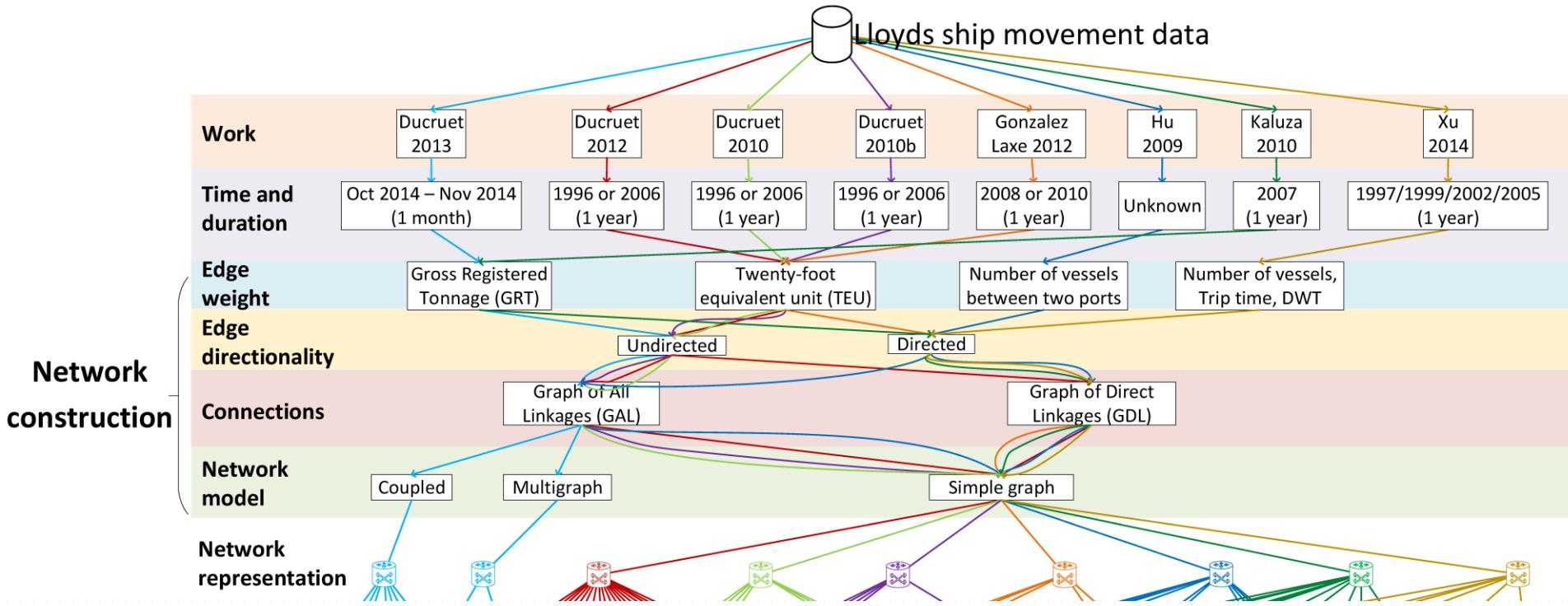


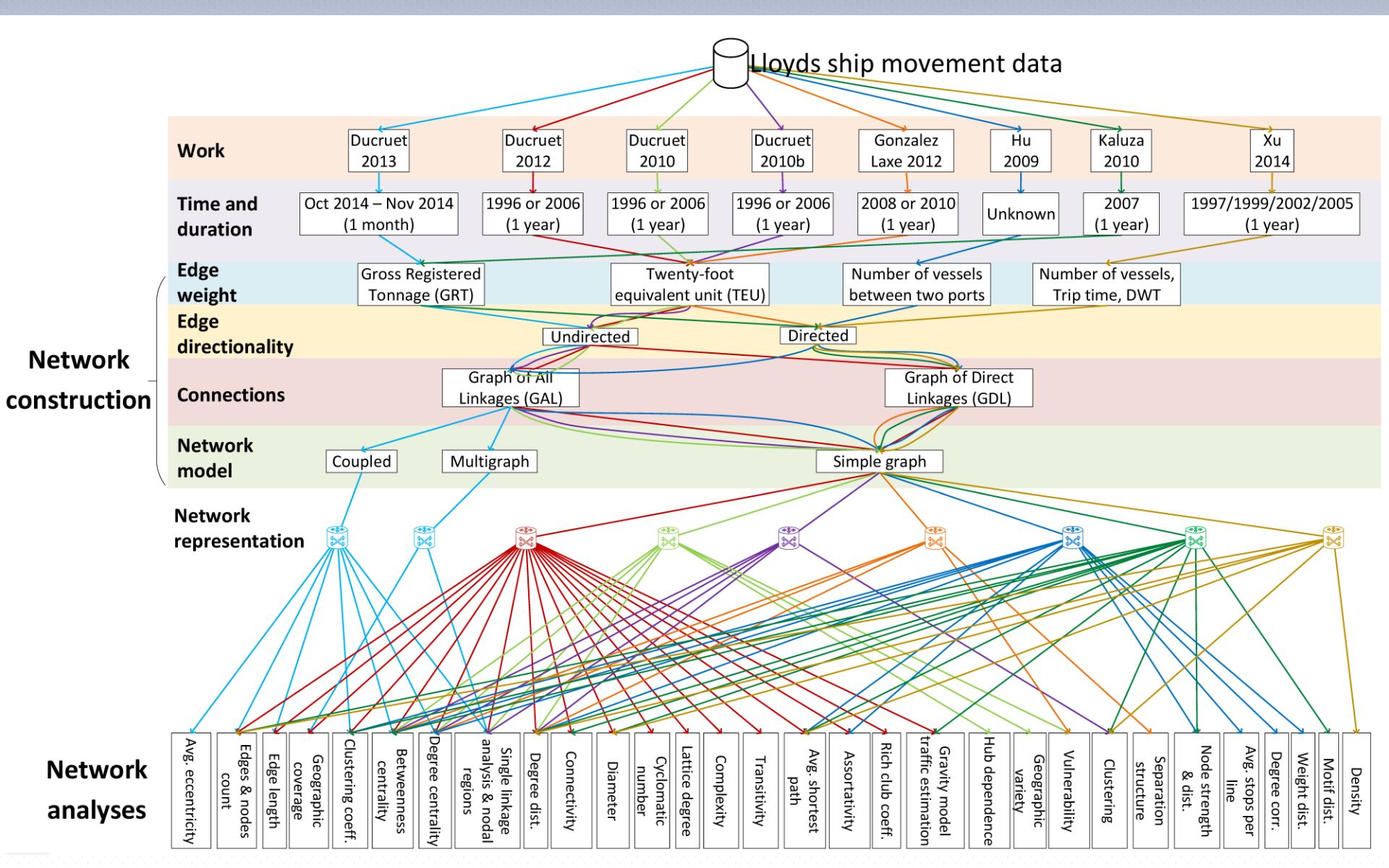










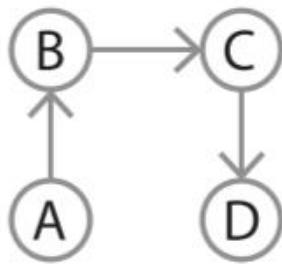


Network linkage mechanisms

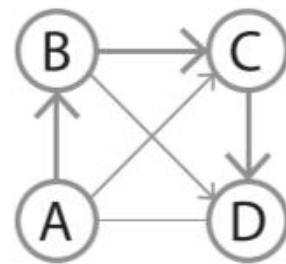
Raw trajectory

A B C D

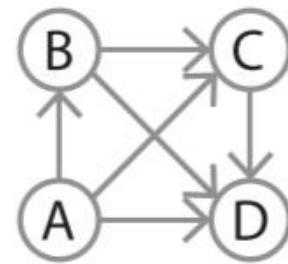
Direct linkage



Weighted indirect linkage



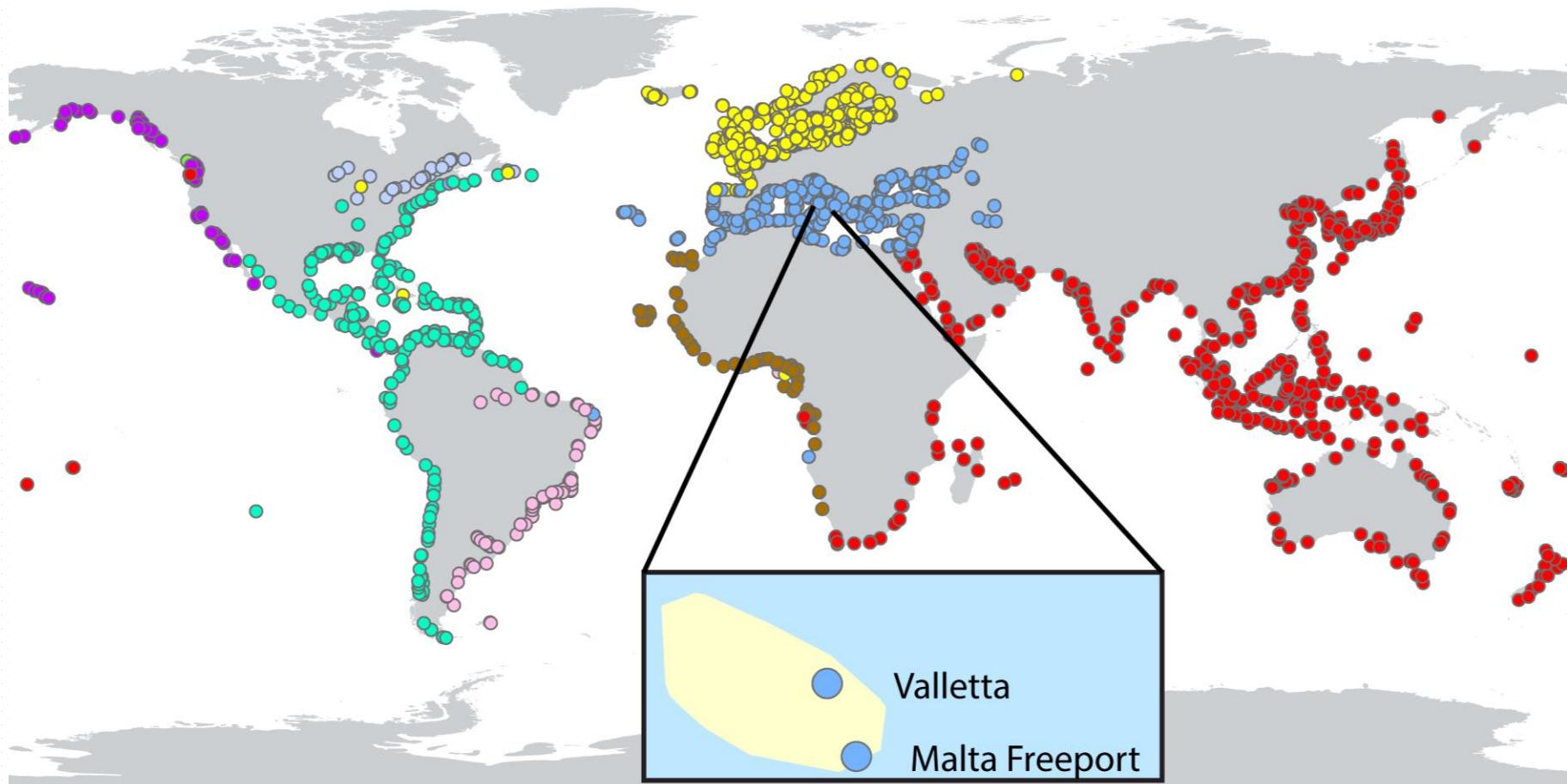
Indirect linkage



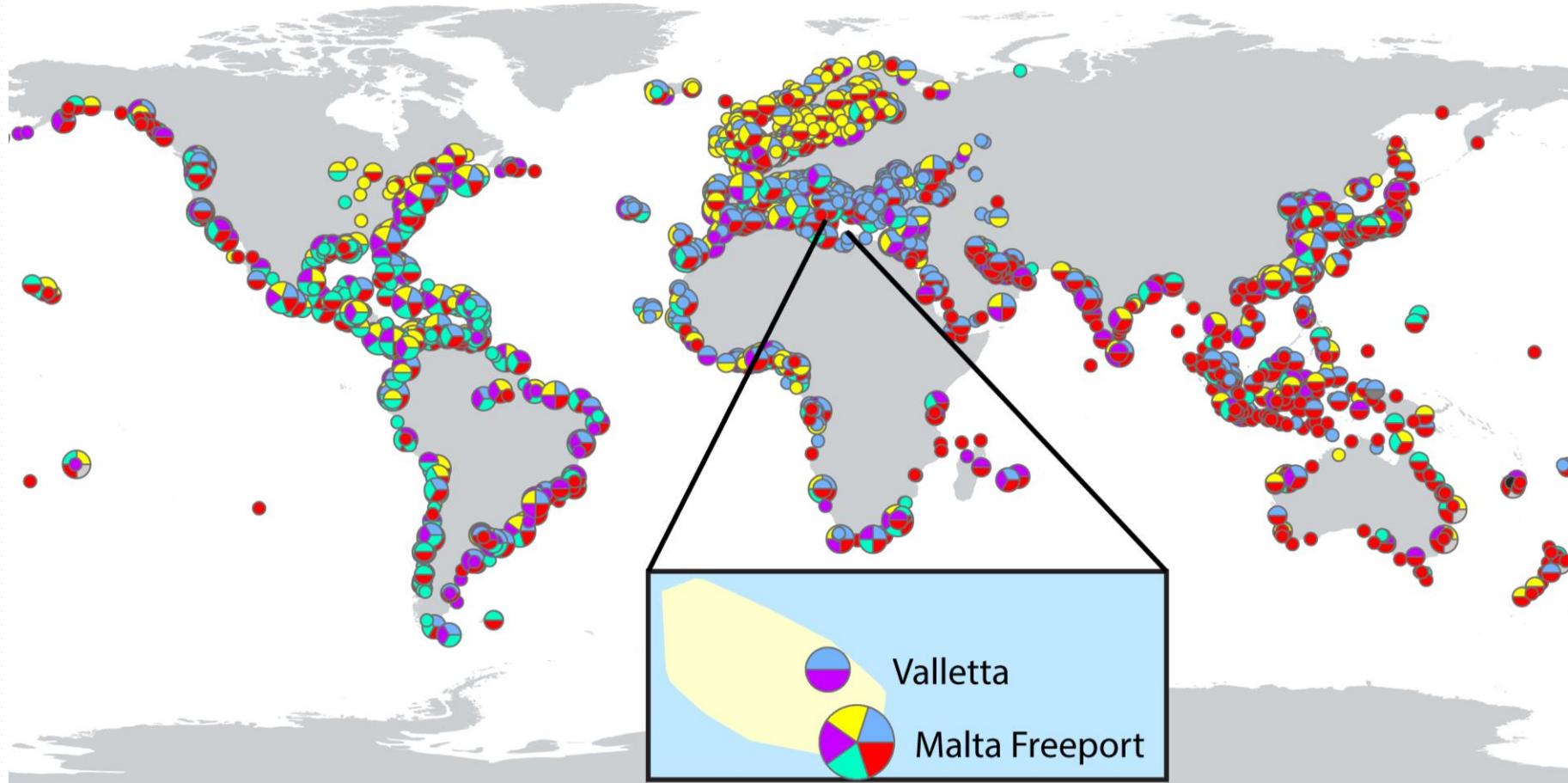
Network linkage mechanisms

	Direct Linkage	Indirect Linkage
num_of_nodes	3.60E+3	=
num_of_edges	1.32E+5	<
density	2.05E-2	<
average_degree	7.35E+1	<
highest_degree	1.28E+3	<
generalized_clustering_coefficient	5.48E-1	<
transitivity	2.96E-1	<
avg_shortest_path	2.65	>
diameter	8	>
radius	4	>

Clustering: first-order network



Clustering: higher-order network



Takeaways

Global shipping traffic is:

Imbalanced in directionality – directed network

Unevenly distributed shipping frequency & traffic – weighted network

Higher-order movement patterns – higher-order network

Other important factors include

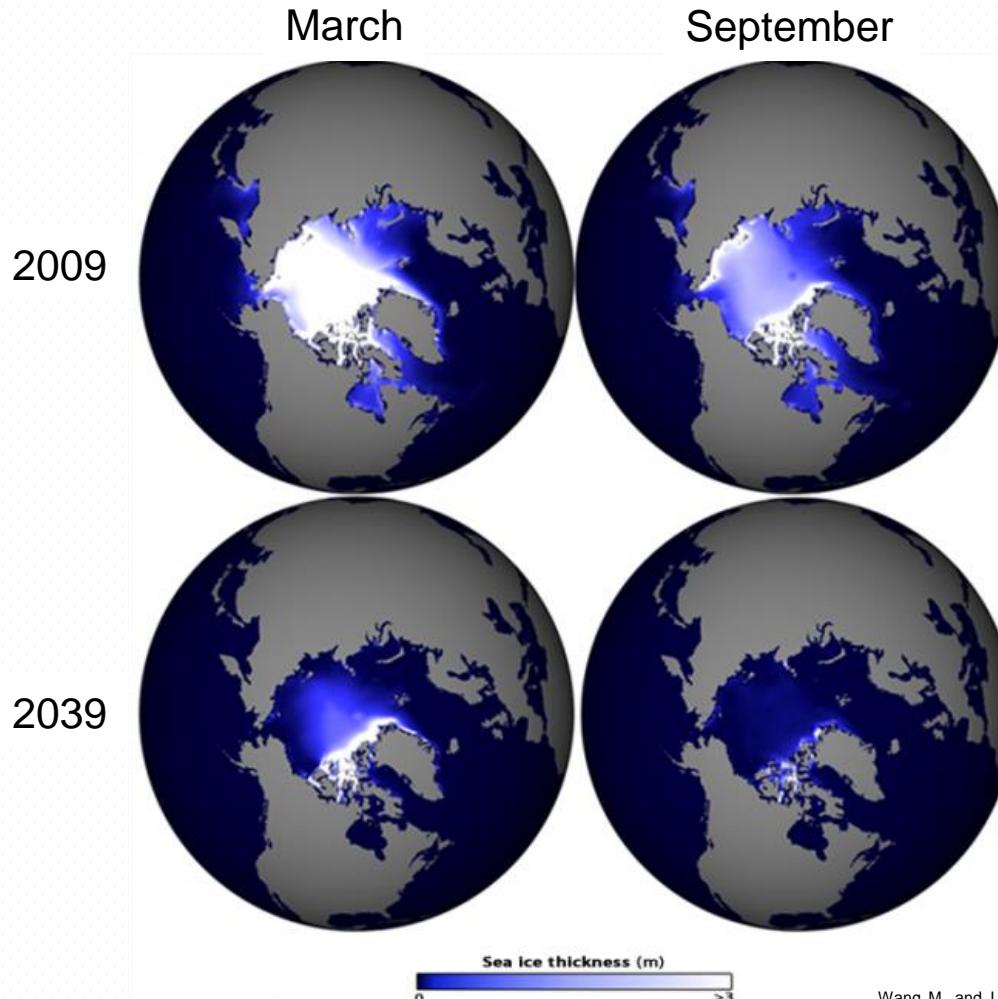
Linkage mechanisms, time window, seasonality, evolution

Considerations when representing shipping traffic
as network, or reporting analysis results

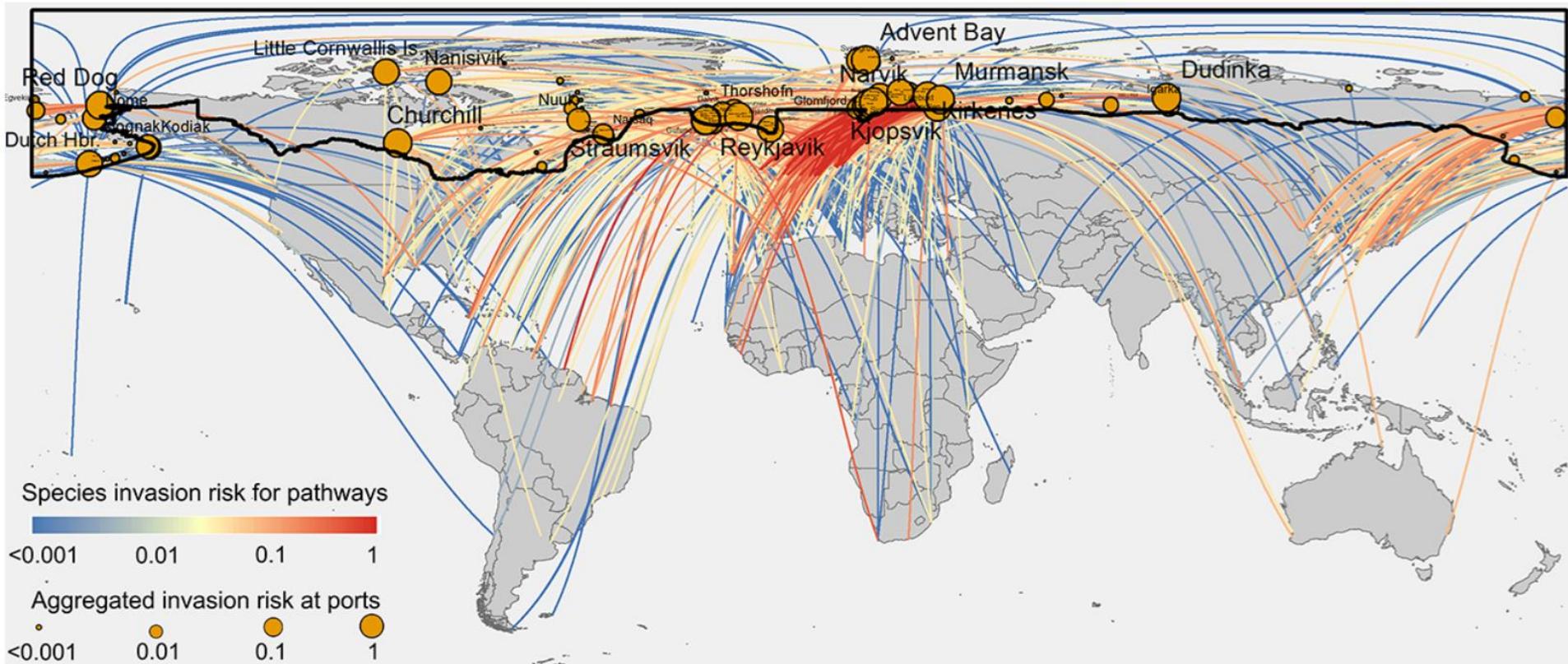
Species invasion in the Arctic

Introduction to Arctic ports & diffusion among Arctic ports

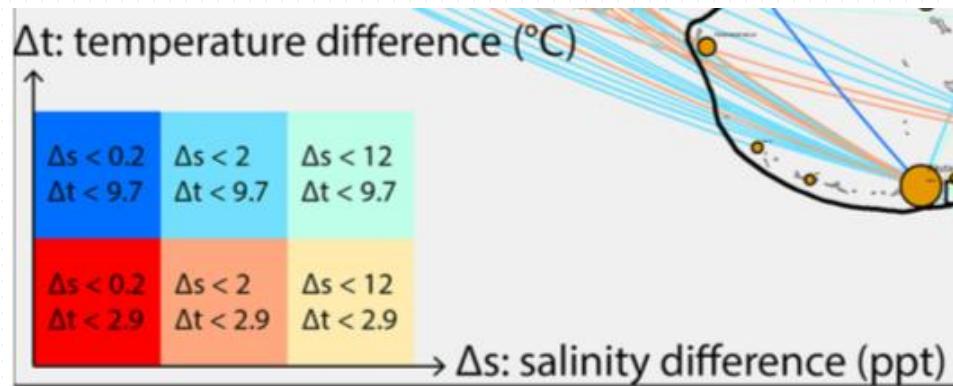
The melting Arctic sea ice



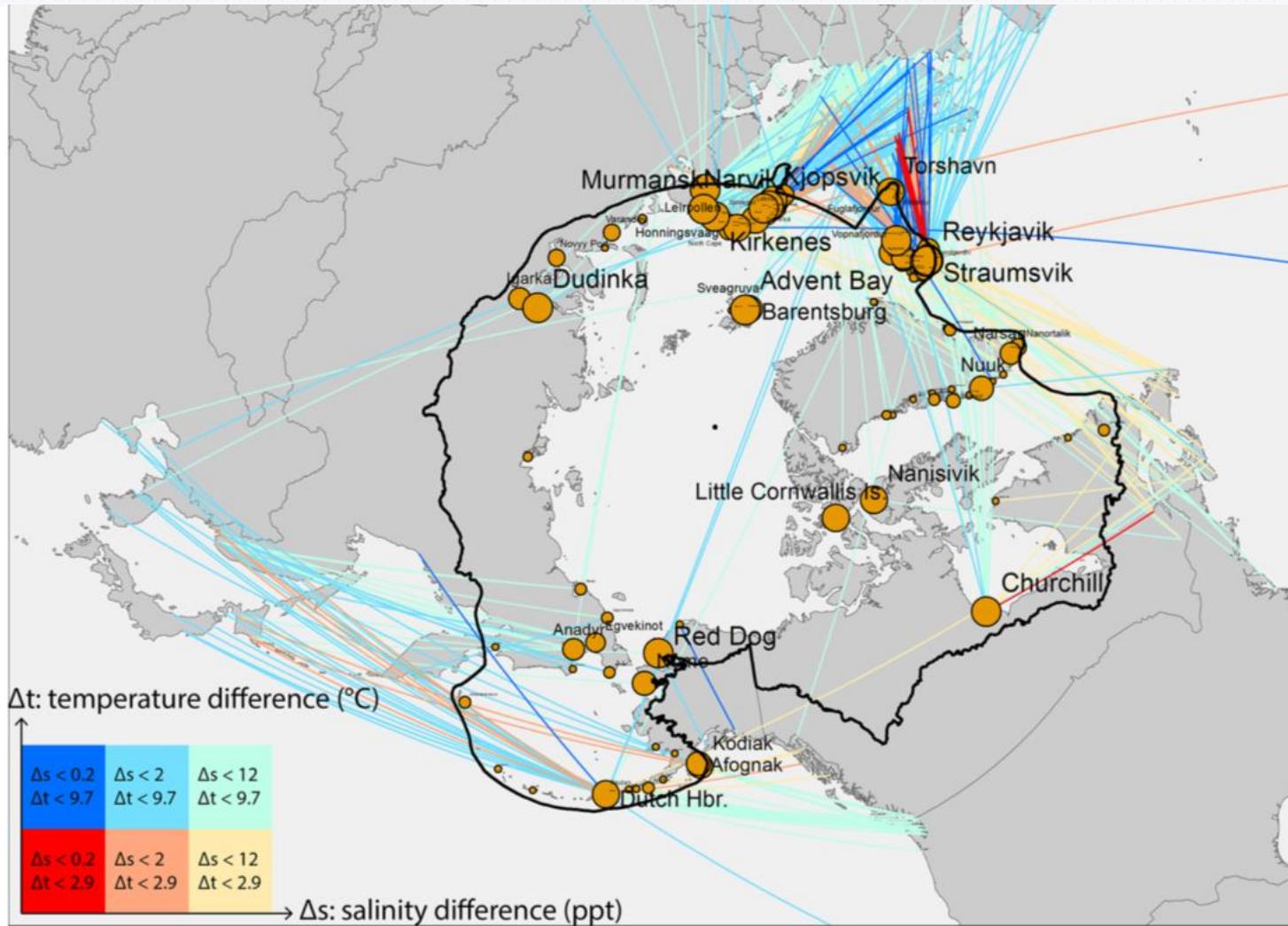
Species introduction pathways to the Arctic



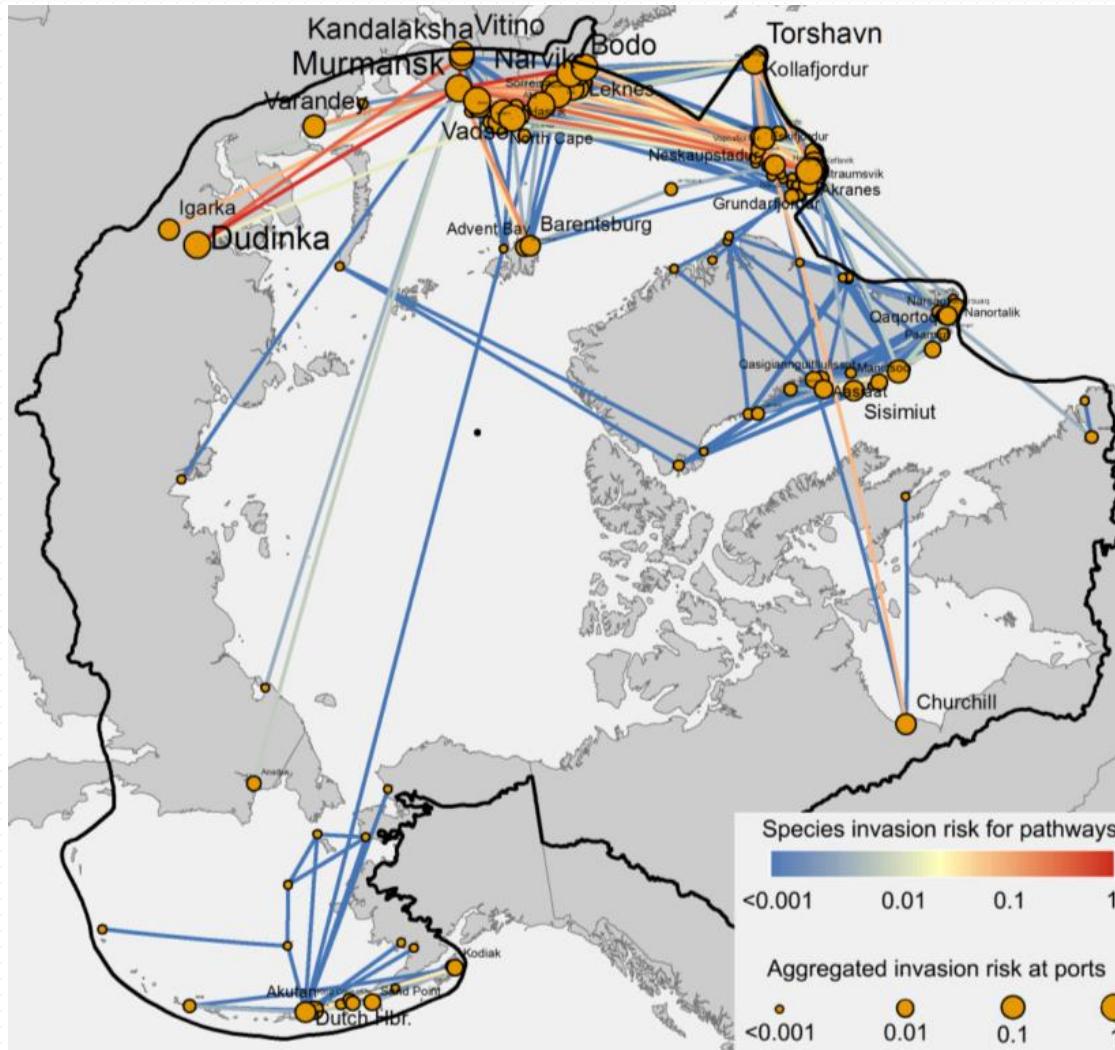
Environmental tolerance

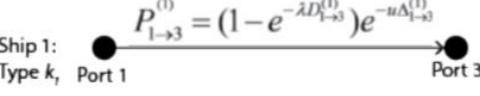
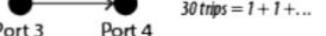
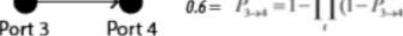
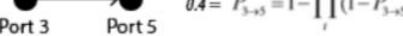
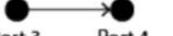
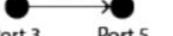


Environmental tolerance

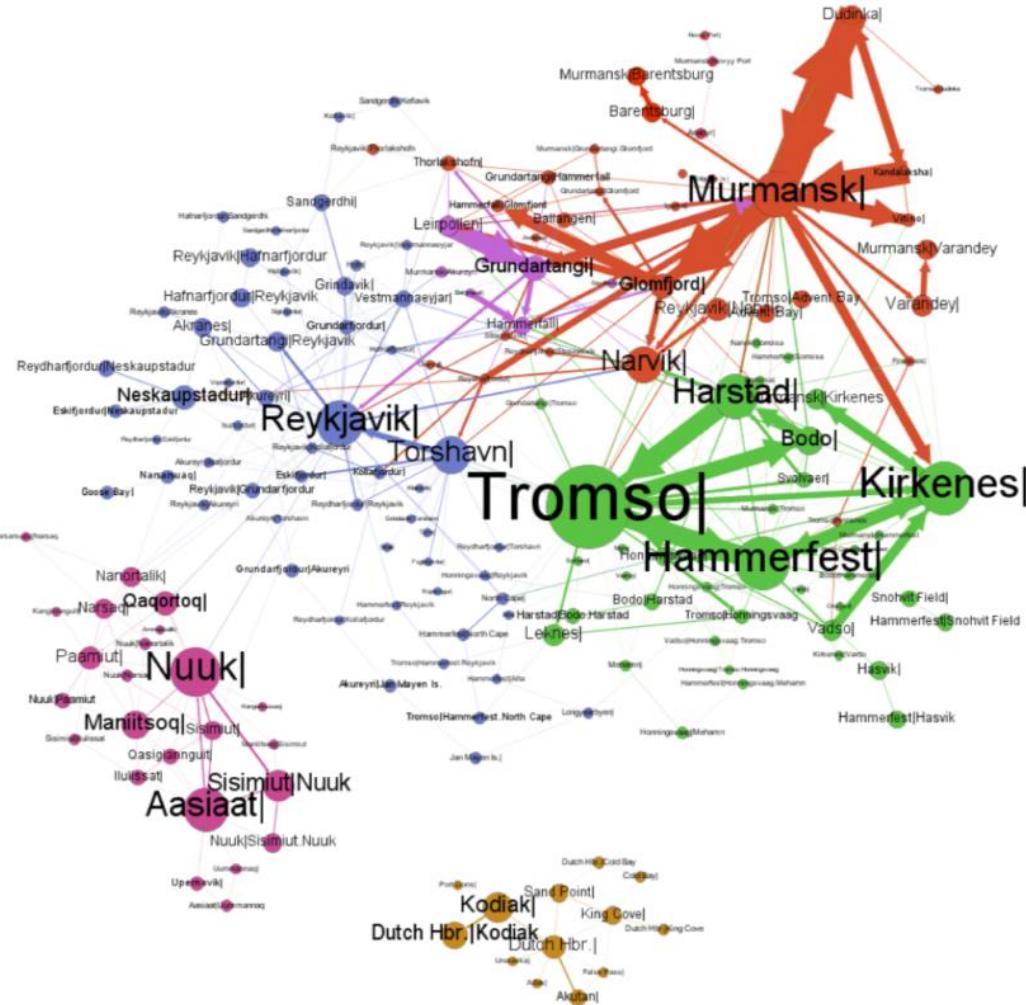


Species diffusion within the Arctic

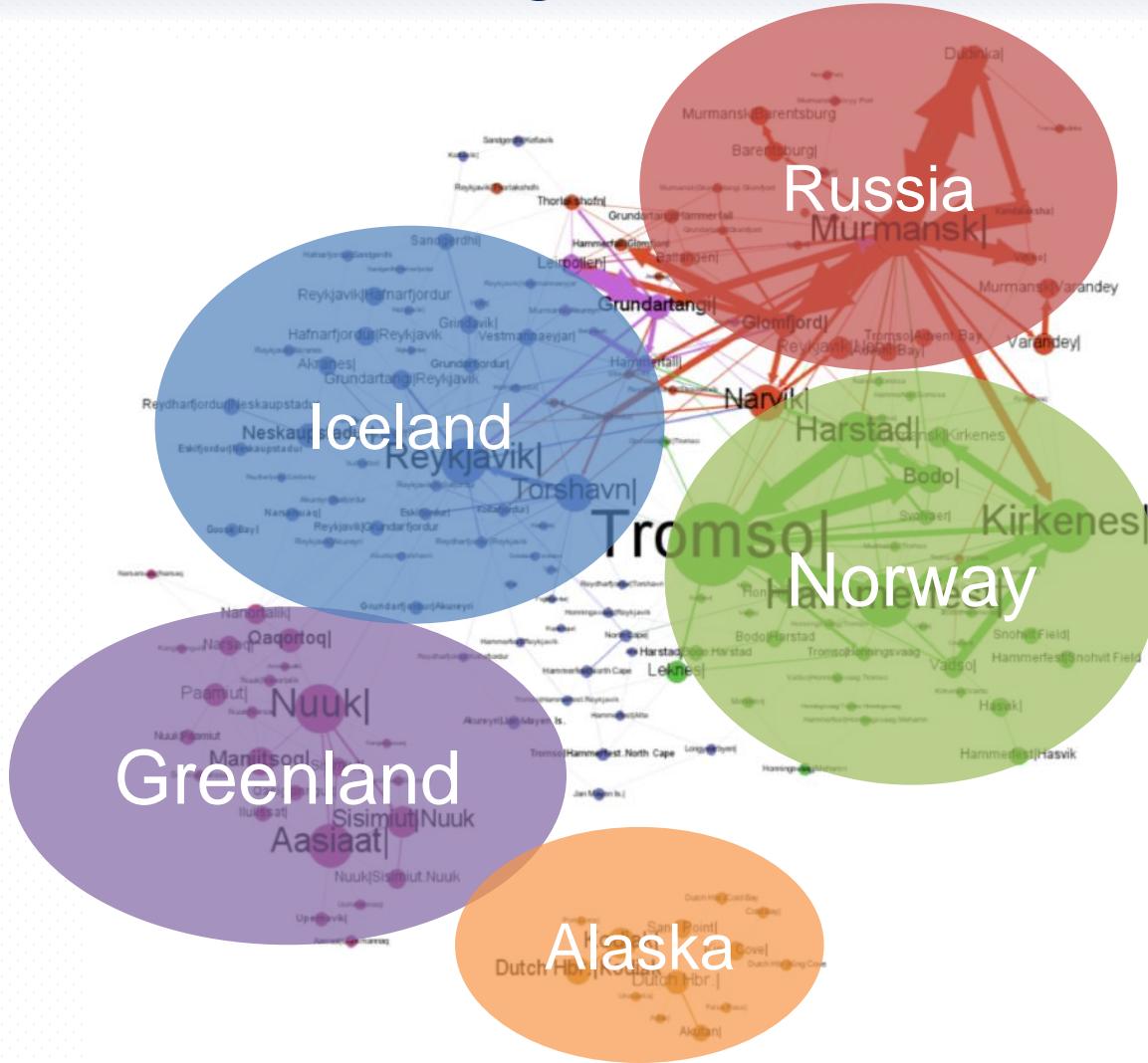


	HON for ship movements	HON for species flow (SF-HON)
Input	<p>Ship 1: </p> <p>Ship 2: </p>	<p>Ship 1: </p> <p>Type k_1: Port 1 $\xrightarrow{D_n \Delta t_n}$ Port 3 $\xrightarrow{D_n \Delta t_n}$ Port 4</p> <p>Ship 2: </p> <p>Type k_2: Port 2 $\xrightarrow{D_n \Delta t_n}$ Port 3 $\xrightarrow{D_n \Delta t_n}$ Port 5</p>
Influence per trip	<p>Ship 1: </p>	<p>Ship 1: </p> <p>Type k_1: Port 1 $\xrightarrow{P^{(1)}_{1→3} = (1 - e^{-λ D_{1→3}^{(1)}}) e^{-u Δ_{1→3}^{(1)}}}$ Port 3</p>
Counting subsequences	<p></p> <p></p> <p></p>	<p></p> <p></p> <p></p>
Normalization	<p></p> <p></p>	<p></p> <p></p>
Rule extraction terminating condition	<p>Minimum support = 10</p> <p></p> <p></p>	<p>Minimum support = 0.2</p> <p></p> <p></p>

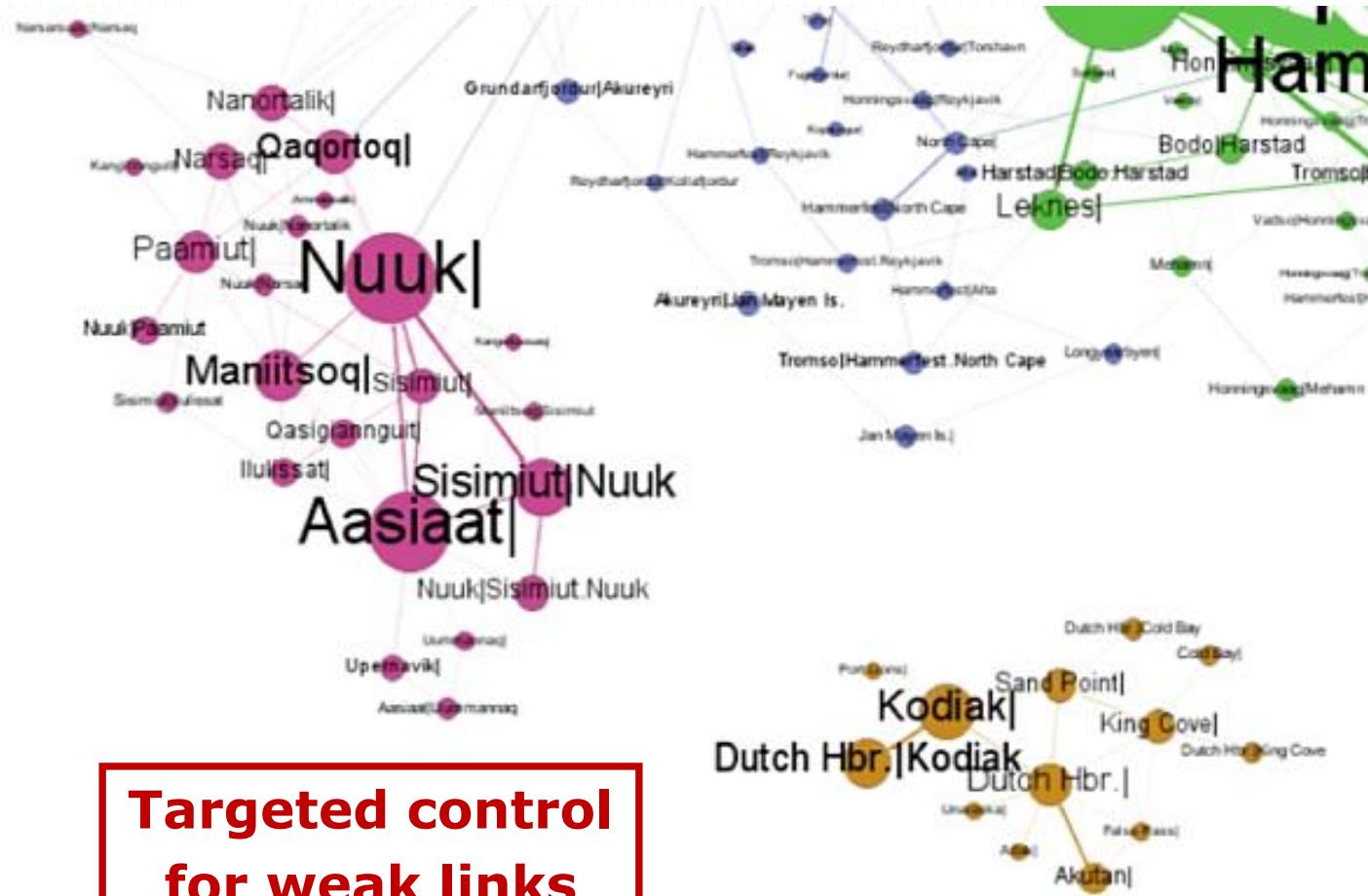
Species flow higher-order network



Species flow higher-order network



Species flow higher-order network



Targeted control for weak links

Case studies

Soft shell clam



Scientific name: *M. arenaria*

Temperature tolerance: -2 – 18 °C

Salinity tolerance: 28 – 35 PSU

Red king crab



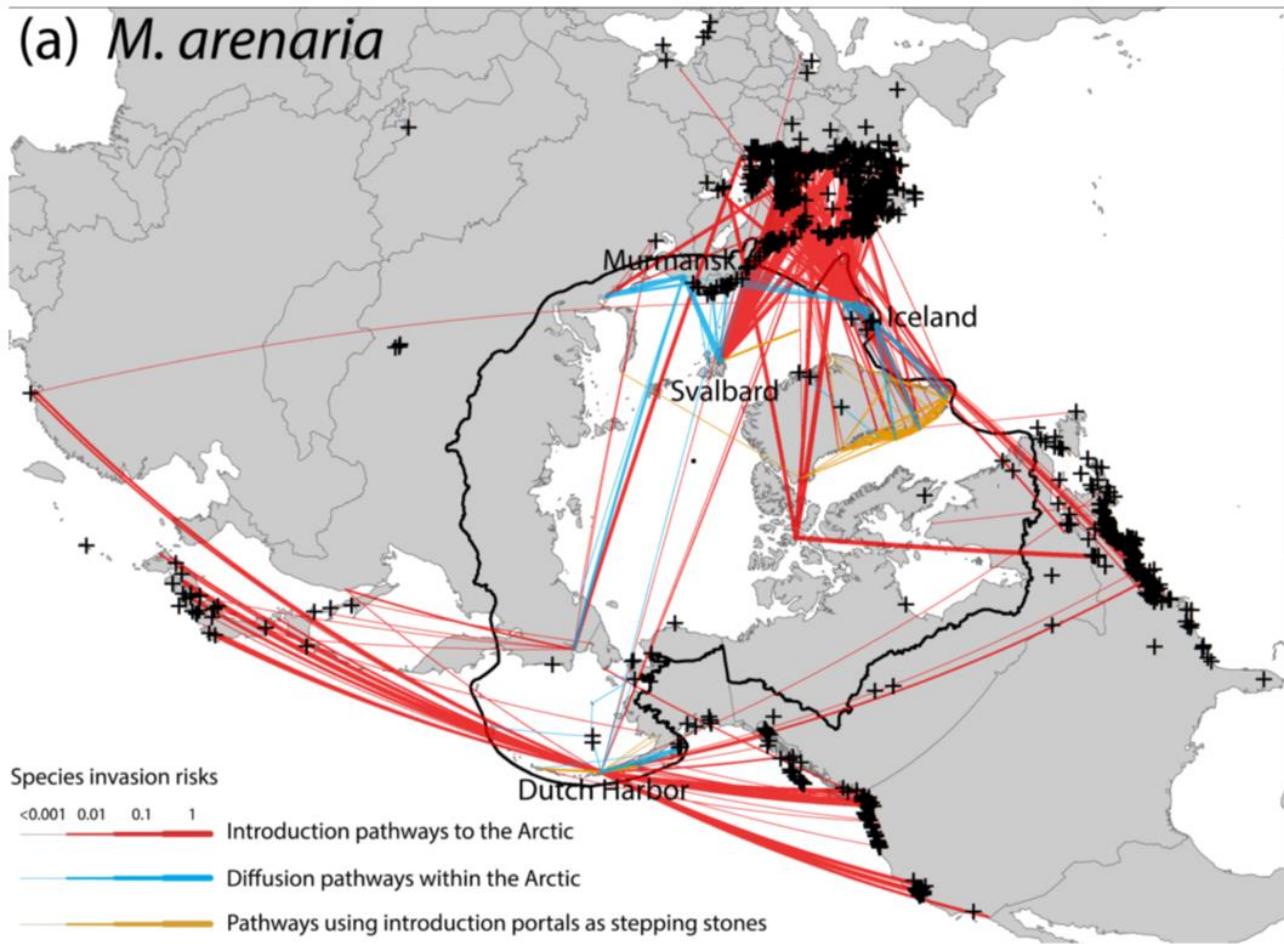
Scientific name: *P. camtschaticus*

Temperature tolerance: -2 – 18 °C

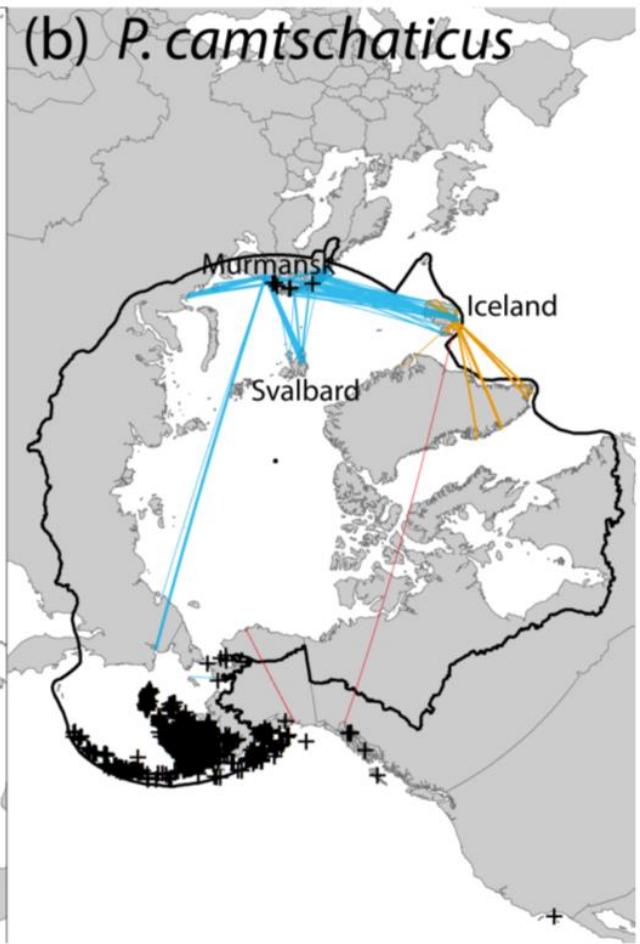
Salinity tolerance: 28 – 35 PSU

Case studies

(a) *M. arenaria*



(b) *P. camtschaticus*



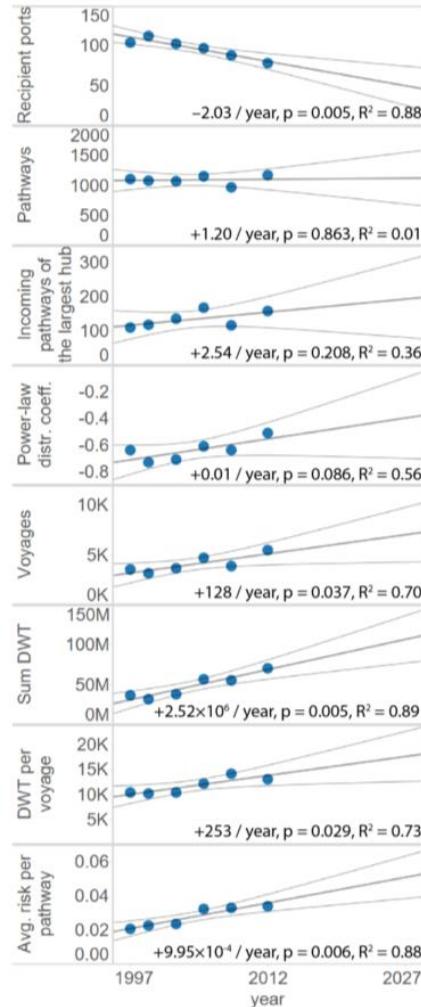
Risk projection

Decreasing recipient ports

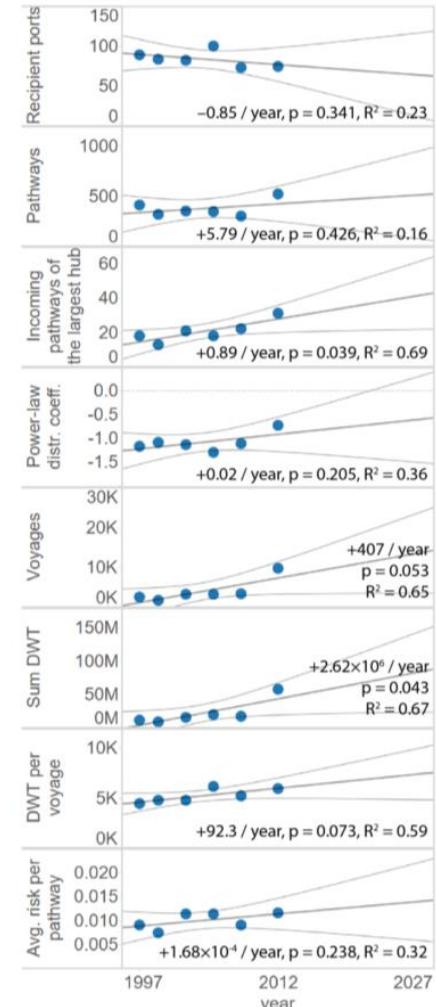
~~Same # of pathways~~
Emergence of hubs
~~Increased connections at hubs~~

“Rich gets richer”

Introduction to the Arctic

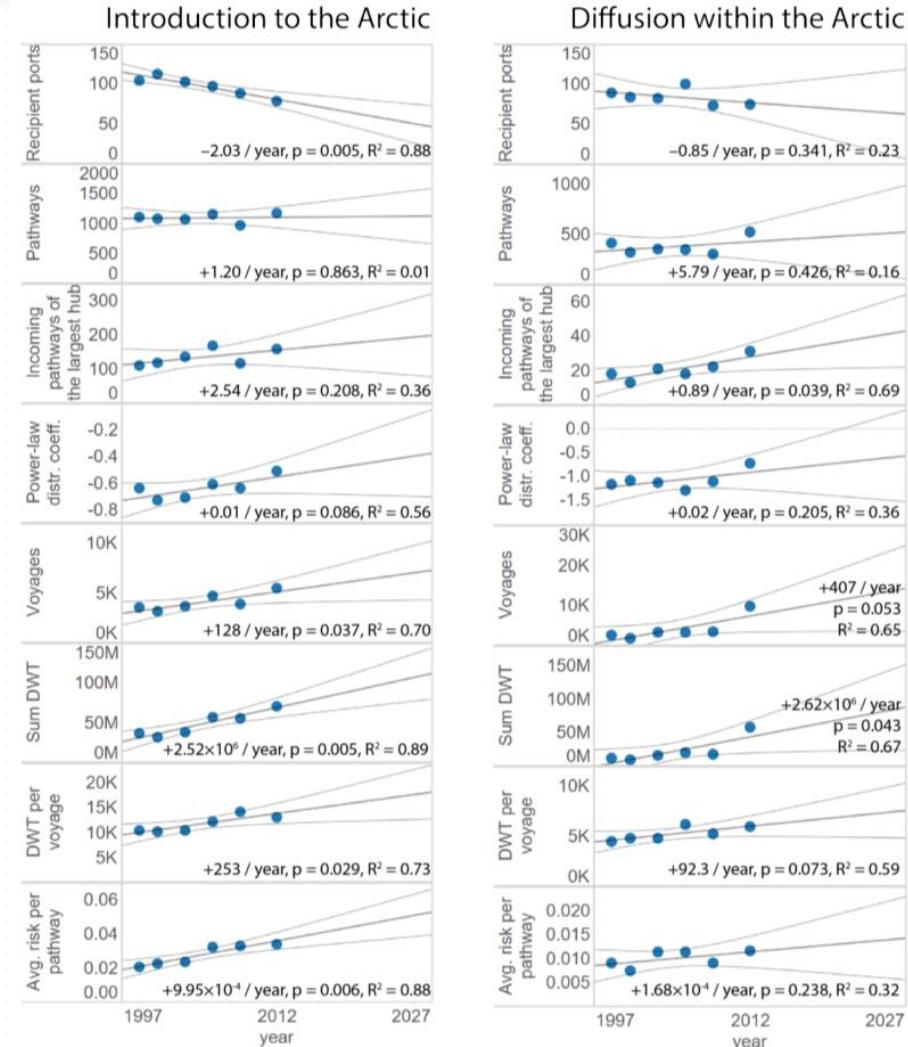


Diffusion within the Arctic

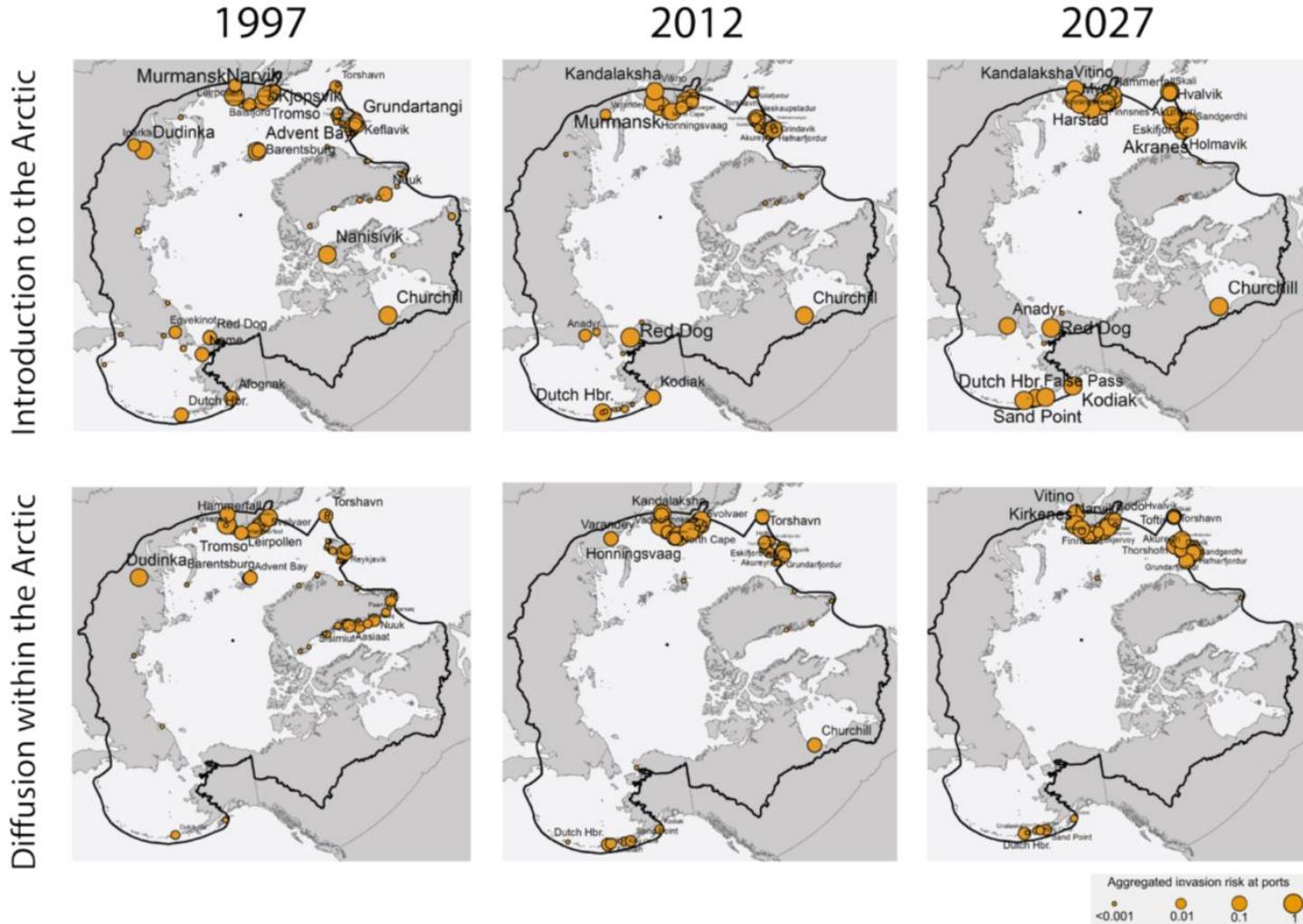


Risk projection

- Decreasing recipient ports
- Same # pathways
- Emergence of hubs**
- Increased connections at hubs
- “Rich gets richer”
- Increasing voyages
- Increasing sum of shipping
- Increasing risk**
- Increasing average ship size
- Increasing risk per pathway

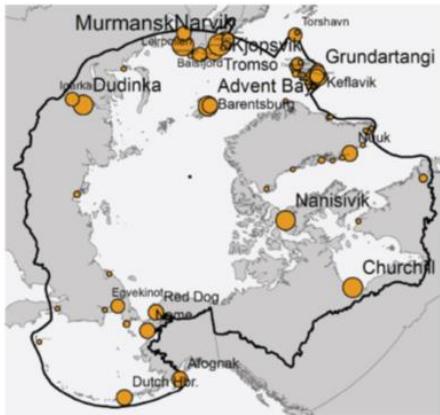


Risk projection

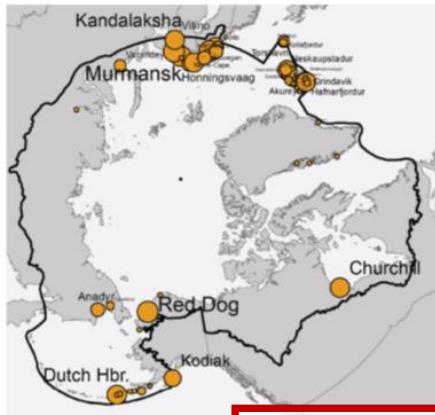


Risk projection

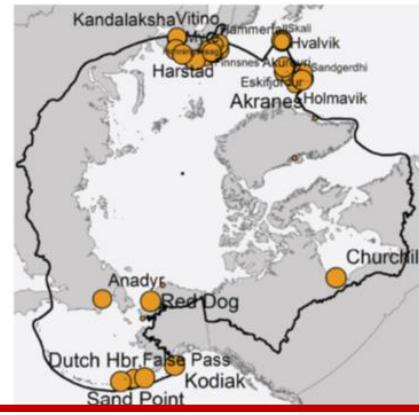
1997



2012

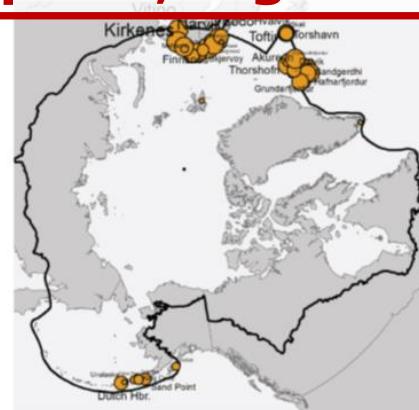
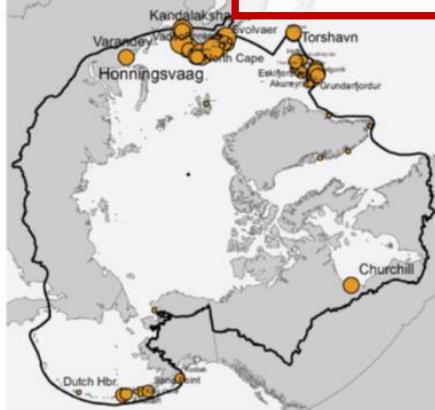
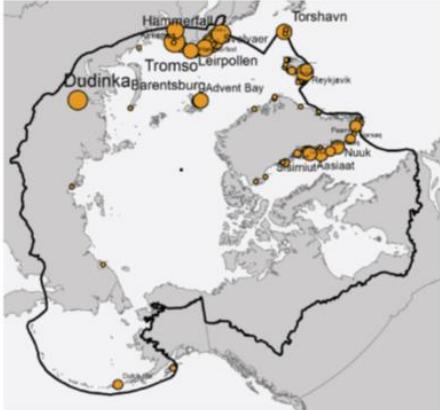


2027



Fewer ports, higher risks

Diffusion within the Arctic



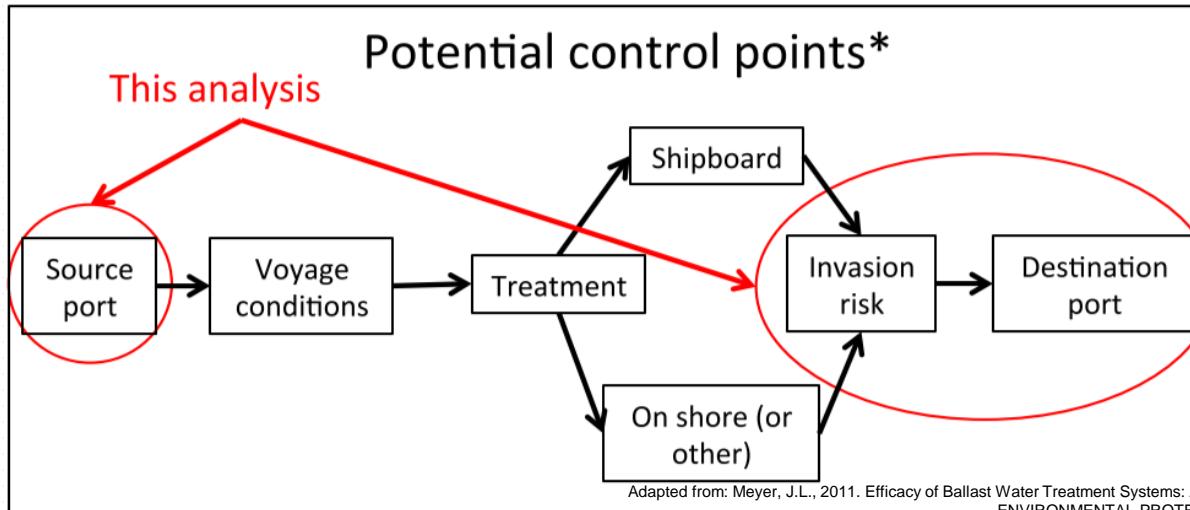
Aggregated invasion risk at ports

<0.001	0.01	0.1	1
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Takeaways

Arctic species invasion

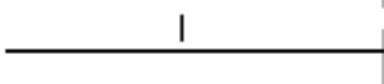
- Increasing risk
- Emergence of hubs
- Targeted controls



Anomaly detection

Unveiling higher-order anomalies with HON

Anomaly detection with dynamic network

Time frames: 

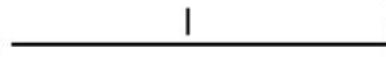
Trajectory 1: f a c d g f b c e g

Trajectory 2: f a c e g f b c d g

Trajectory 3: f b c d g f a c e g

Trajectory 4: f b c e g f a c d g

Anomaly detection with dynamic network

Time frames: 

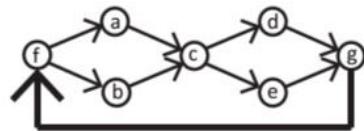
Trajectory 1: f a c d g f b c e g

Trajectory 2: f a c e g f b c d g

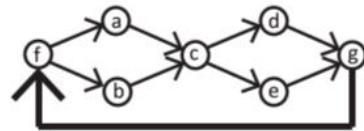
Trajectory 3: f b c d g f a c e g

Trajectory 4: f b c e g f a c d g

First-order network



Higher-order network



Anomaly detection with dynamic network

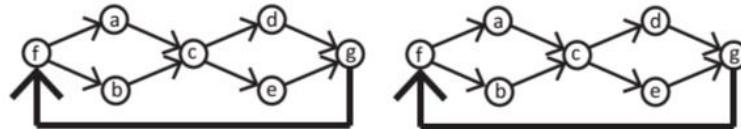
Time frames:

I

II

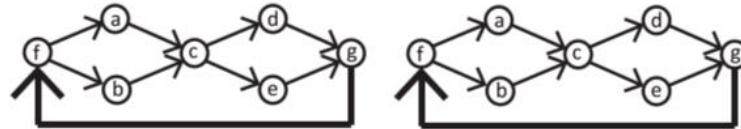
Trajectory 1: f a c d g f b c e g | f a c e g f b c d g
Trajectory 2: f a c e g f b c d g | f a c d g f b c e g
Trajectory 3: f b c d g f a c e g f b c e g | f a c d g
Trajectory 4: f b c e g f a c d g f b c d g f a c e g

First-order network



$$D(G_1, G_2) = 0$$

Higher-order network

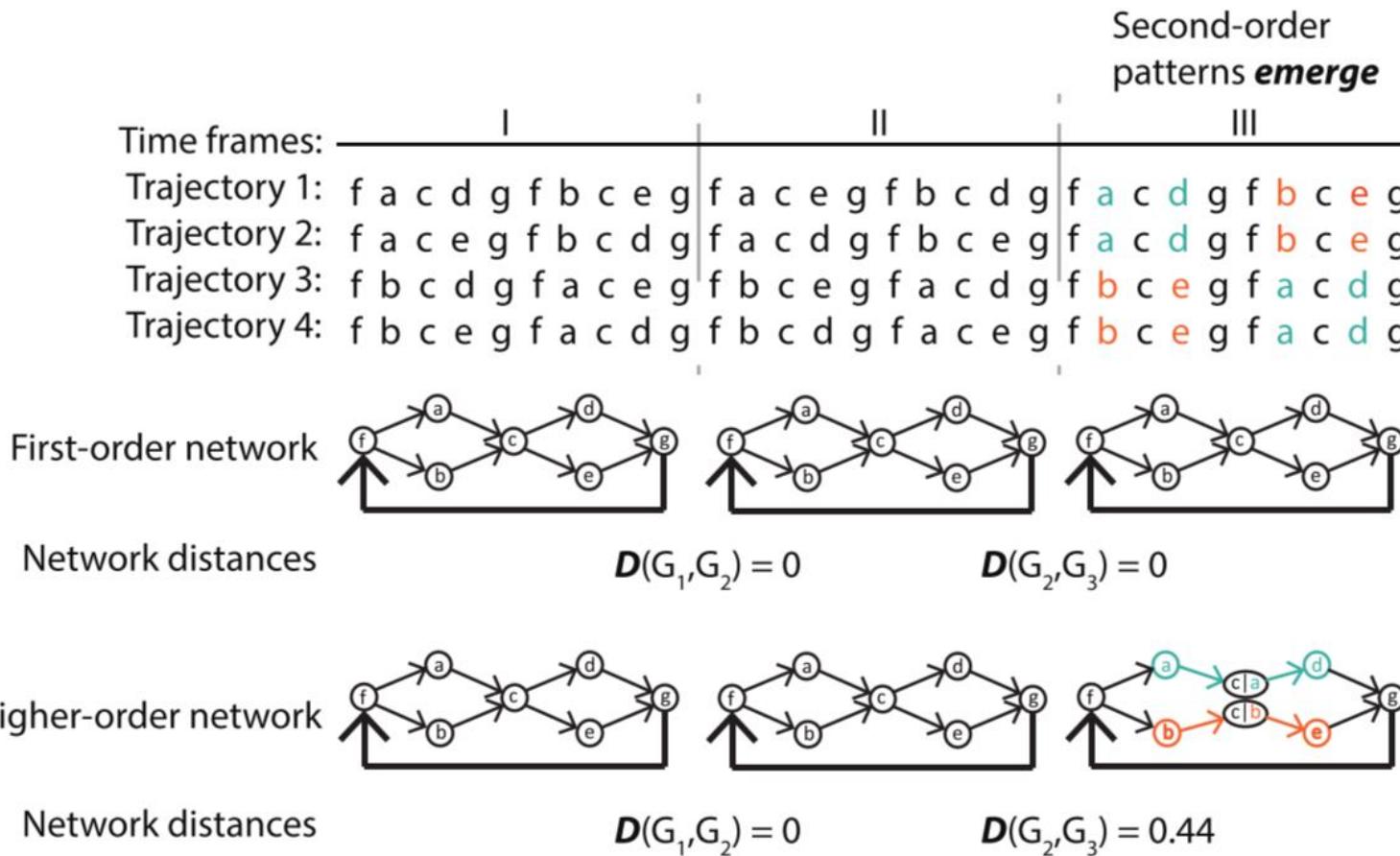


Network distances

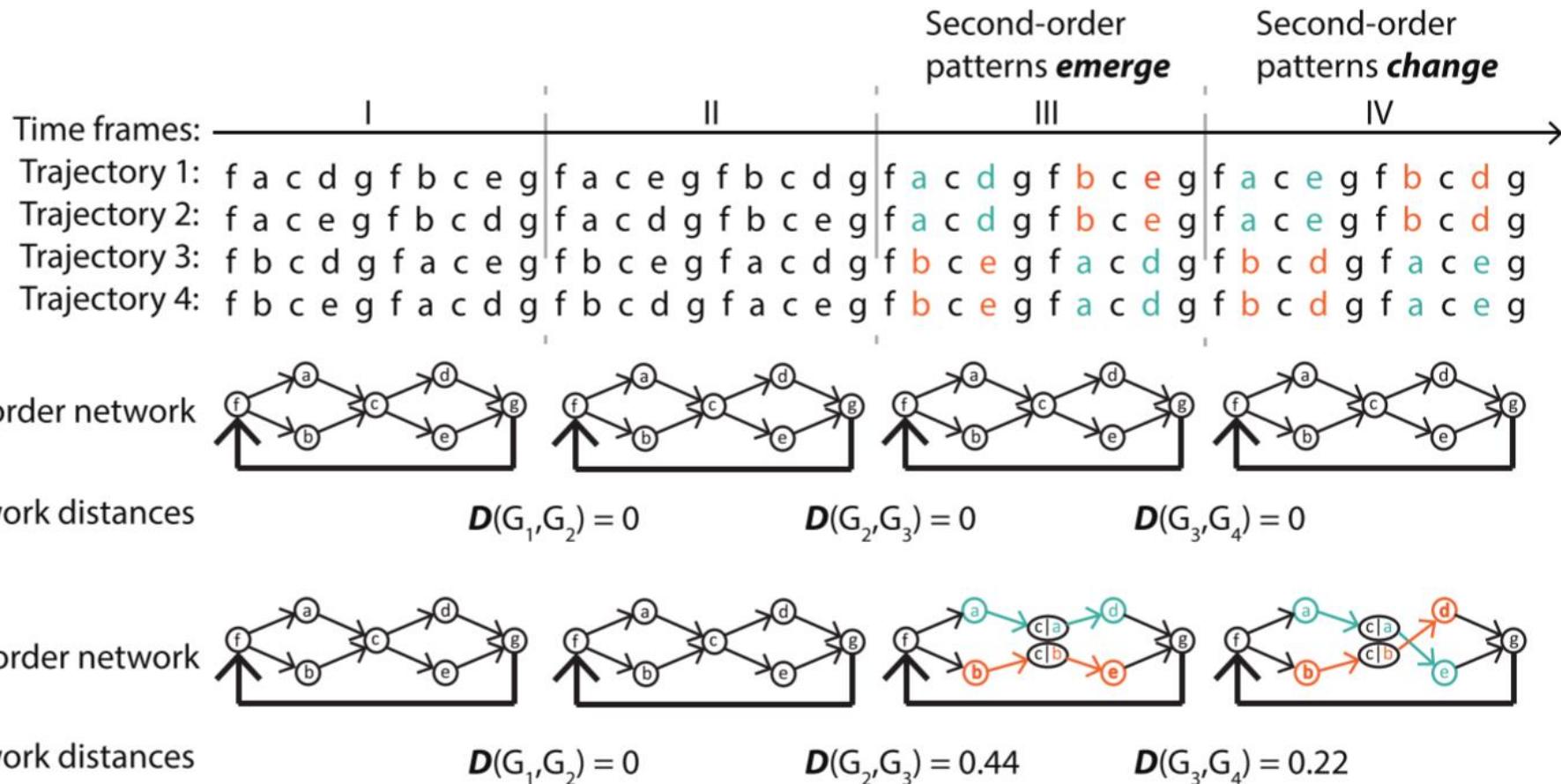
$$D(G_1, G_2) = 0$$

$$D(G, H) = \frac{\sum_{u,v \in V} \frac{|w_E^G(u,v) - w_E^H(u,v)|}{\max(w_E^G(u,v) - w_E^H(u,v))}}{|E_G \cup E_H|}$$

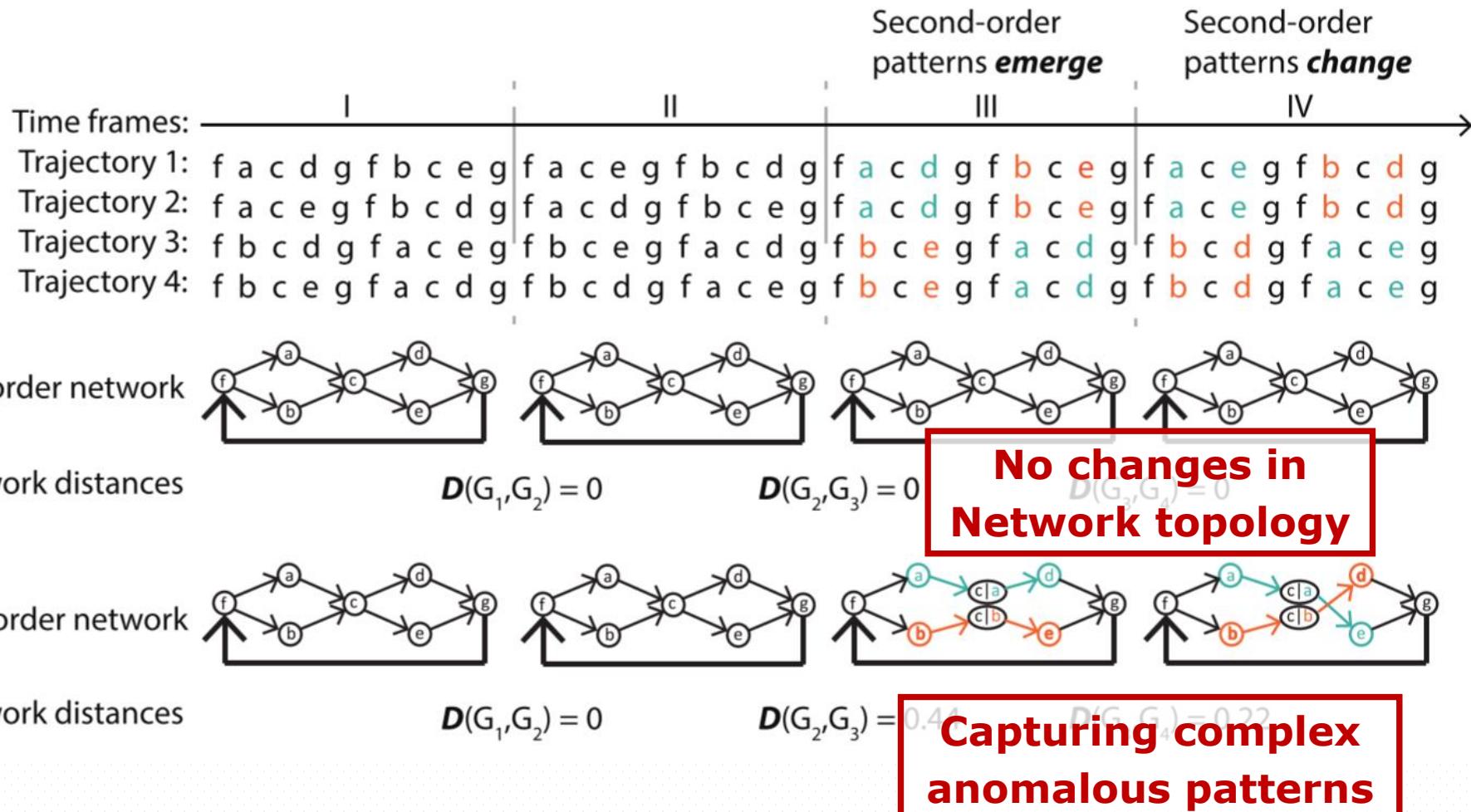
Anomaly detection with dynamic network



Anomaly detection with dynamic network



Anomaly detection with dynamic network

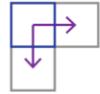


Synthetic data with 11 billion movements

100,000 ships, each moving 100 steps;
11 scenarios, each repeating 100 times;
Total: 11,000,000,000 movements

First order

$t = [1, 100]$
Random walking
right and down



$t = [101, 200]$
Add first order
@ cell 00, 03, 06

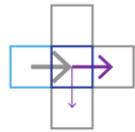


$t = [201, 300]$
Change first order
@ cell 00, 03, 06

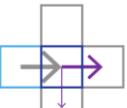


Second order

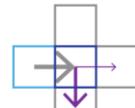
$t = [301, 400]$
Add second order
@ cell 28



$t = [401, 500]$
Add second orders
@ cell 31, 35



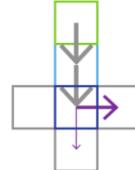
$t = [501, 600]$
Change second orders
@ cell 31, 35



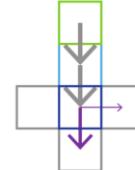
103

Third order

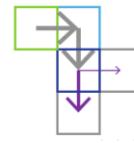
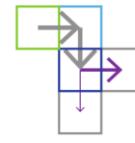
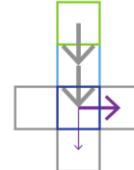
$t = [601, 700]$
Add third order
@ cell 81



$t = [701, 800]$
Add third orders
@ cell 84, 87

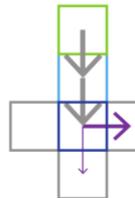


$t = [801, 900]$
Change third orders
@ cell 84, 87

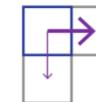
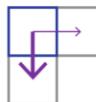
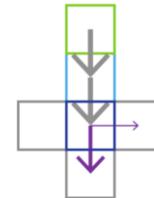


Mixed order

$t = [901, 1000]$
Add mixed orders
@ cell 59

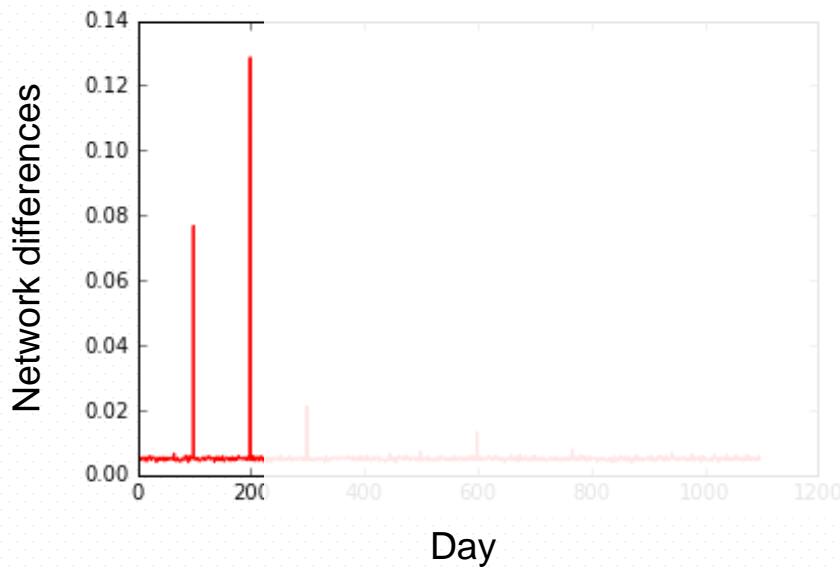


$t = [1001, 1100]$
Change mixed orders
@ cell 59

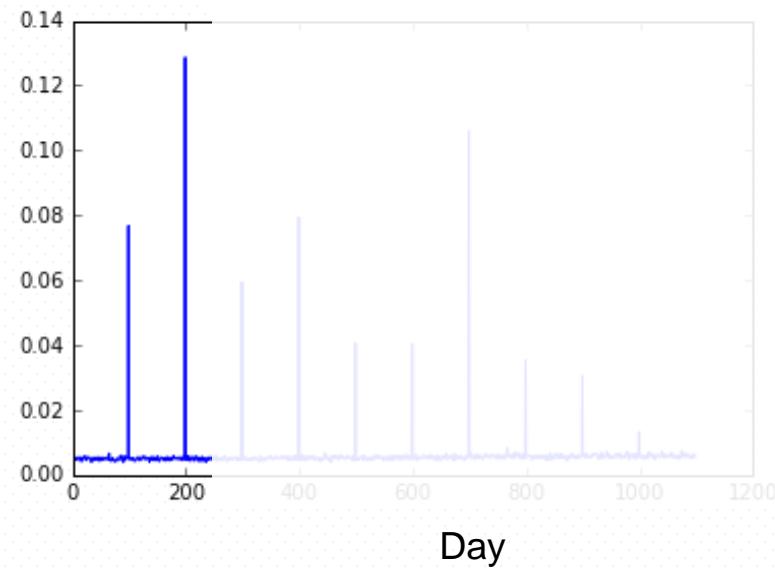


Higher-order anomalies captured by HON

First-order network



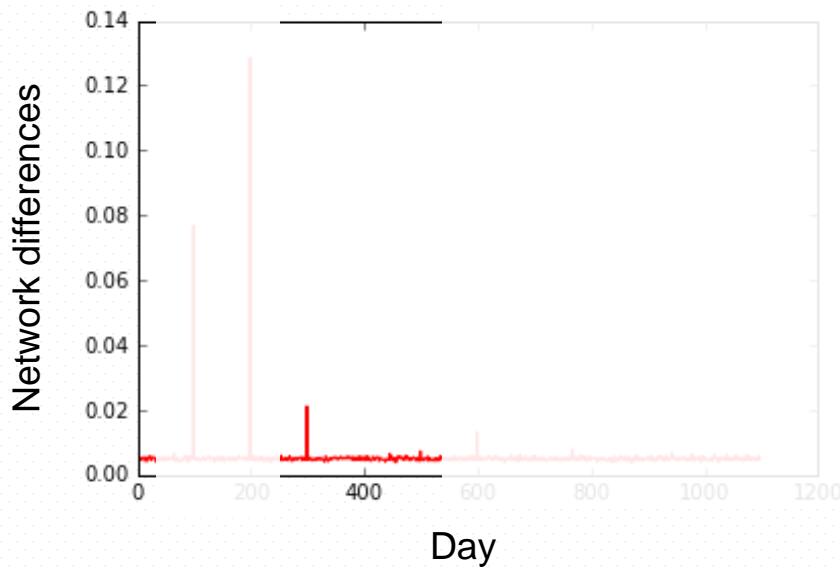
HON



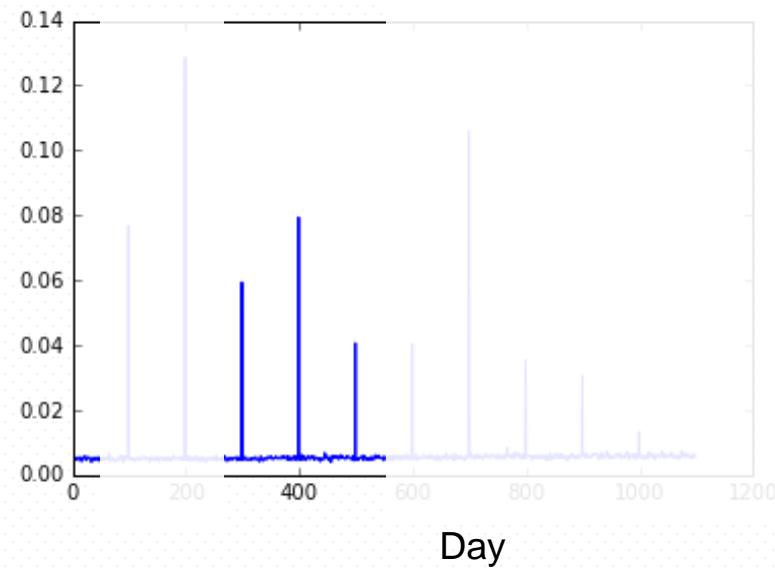
Injection and alternation of 1st order dependencies

Higher-order anomalies captured by HON

First-order network



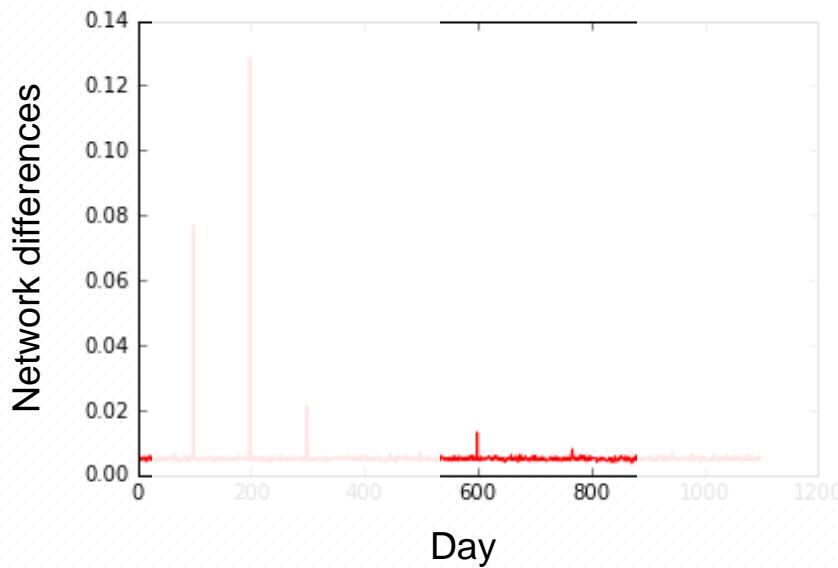
HON



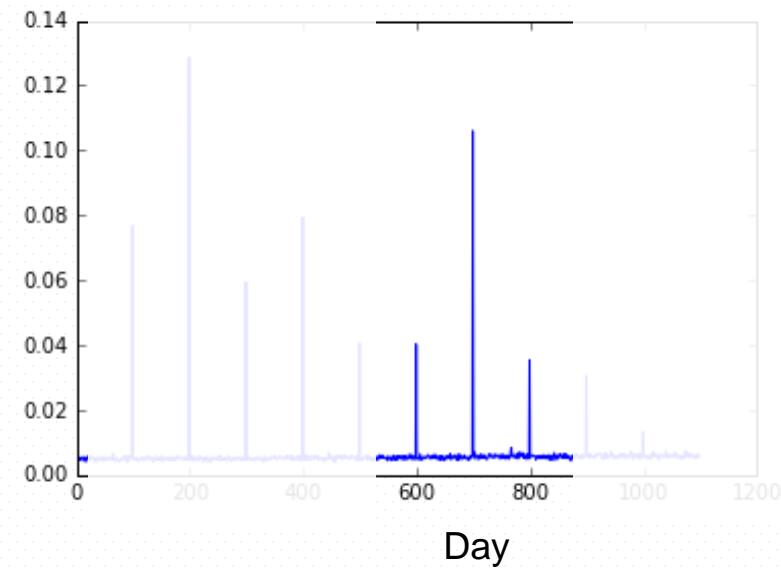
Injection and alternation of 2nd order dependencies

Higher-order anomalies captured by HON

First-order network



HON

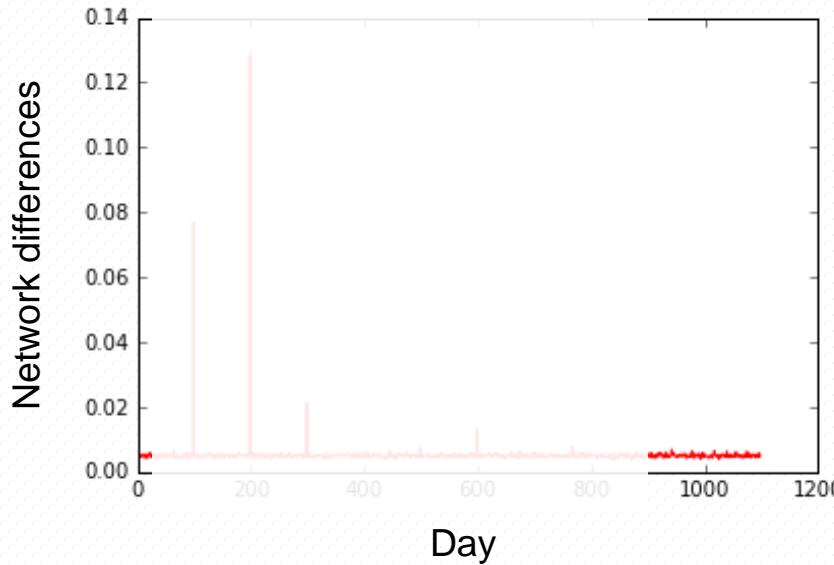


Injection and alternation of 3rd order dependencies

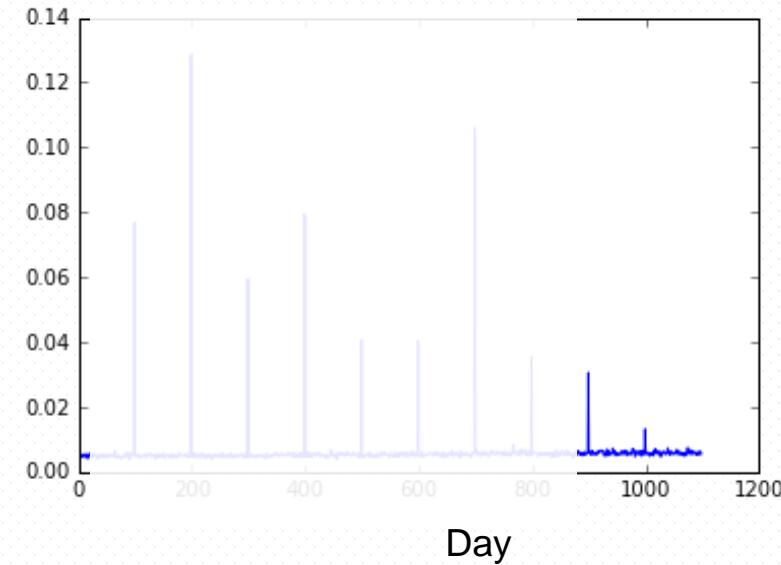
Higher-order anomalies captured by HON

Fails to capture certain anomalies

First-order network



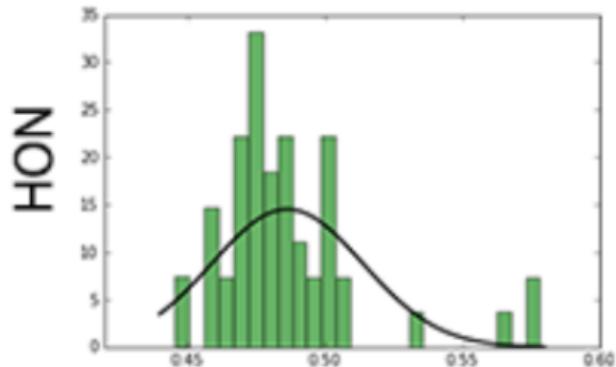
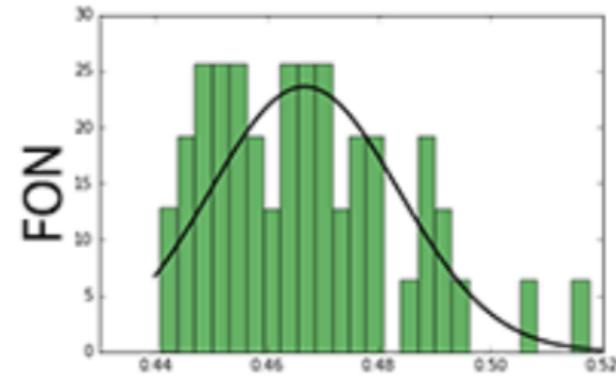
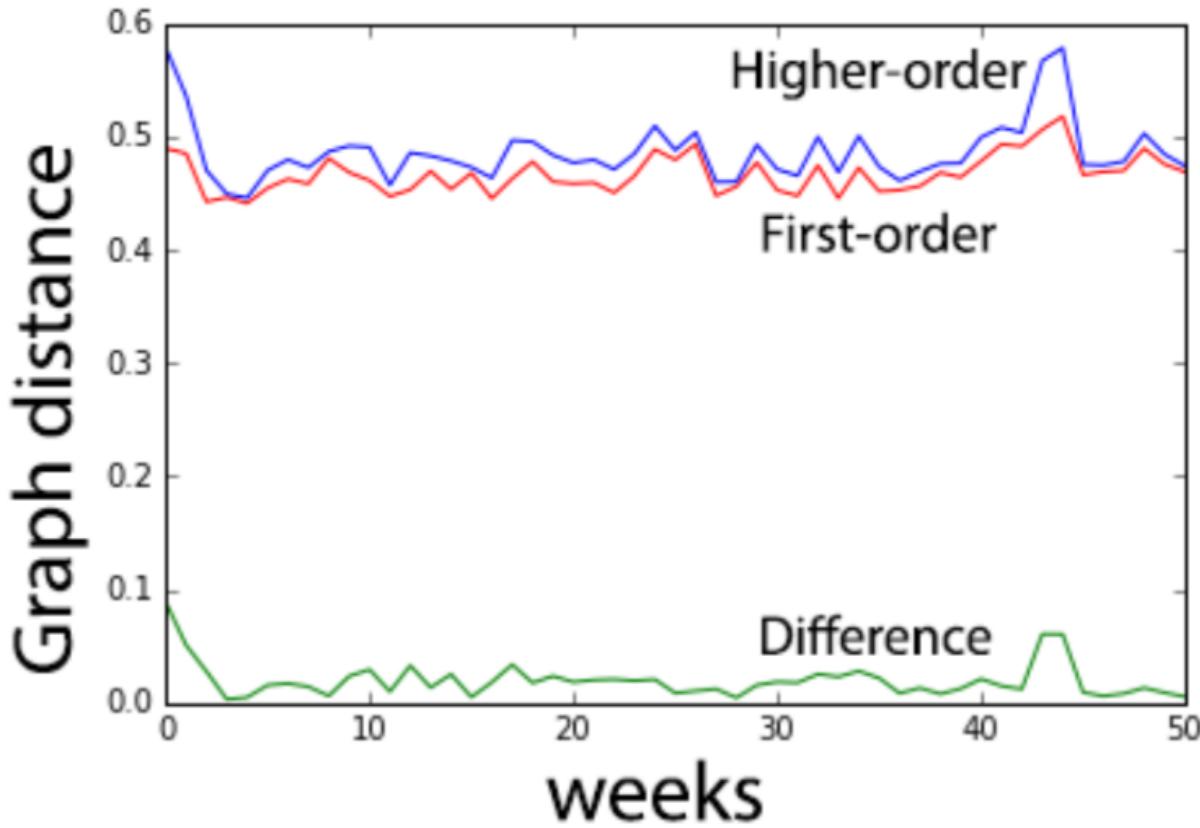
HON



Injection and alternation of higher-order dependencies

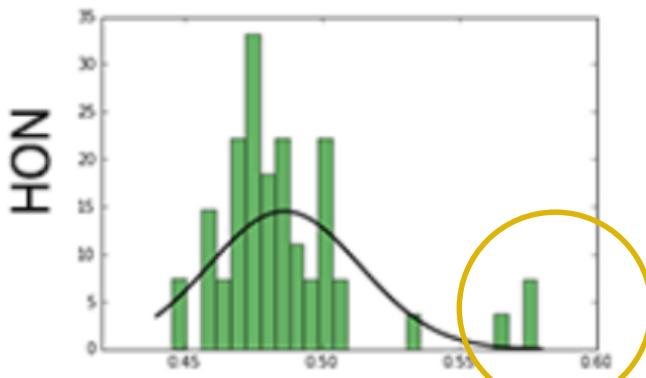
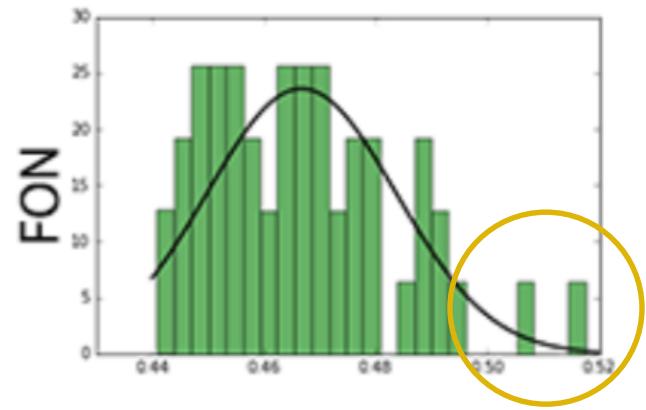
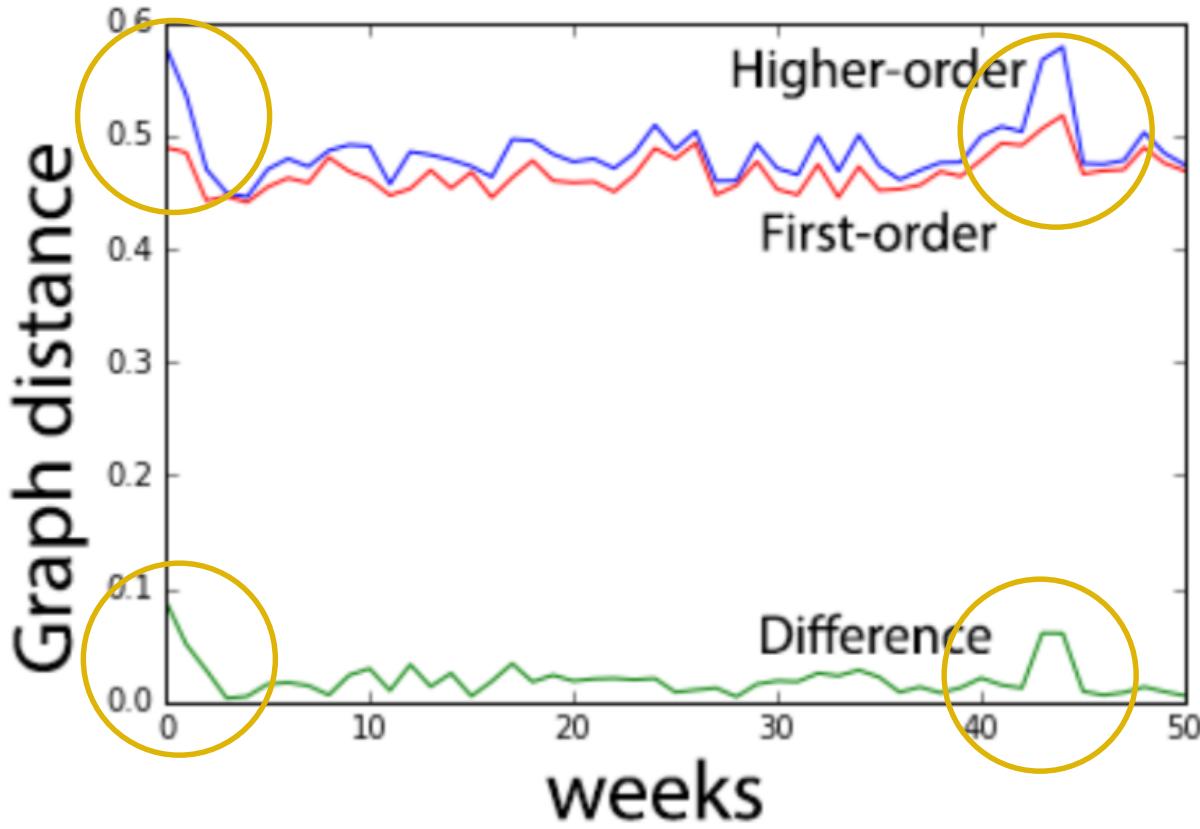
Higher-order anomalies captured by HON

Porto Taxi GPS trajectory data, 1 year



Higher-order anomalies captured by HON

Amplifying anomalous signals



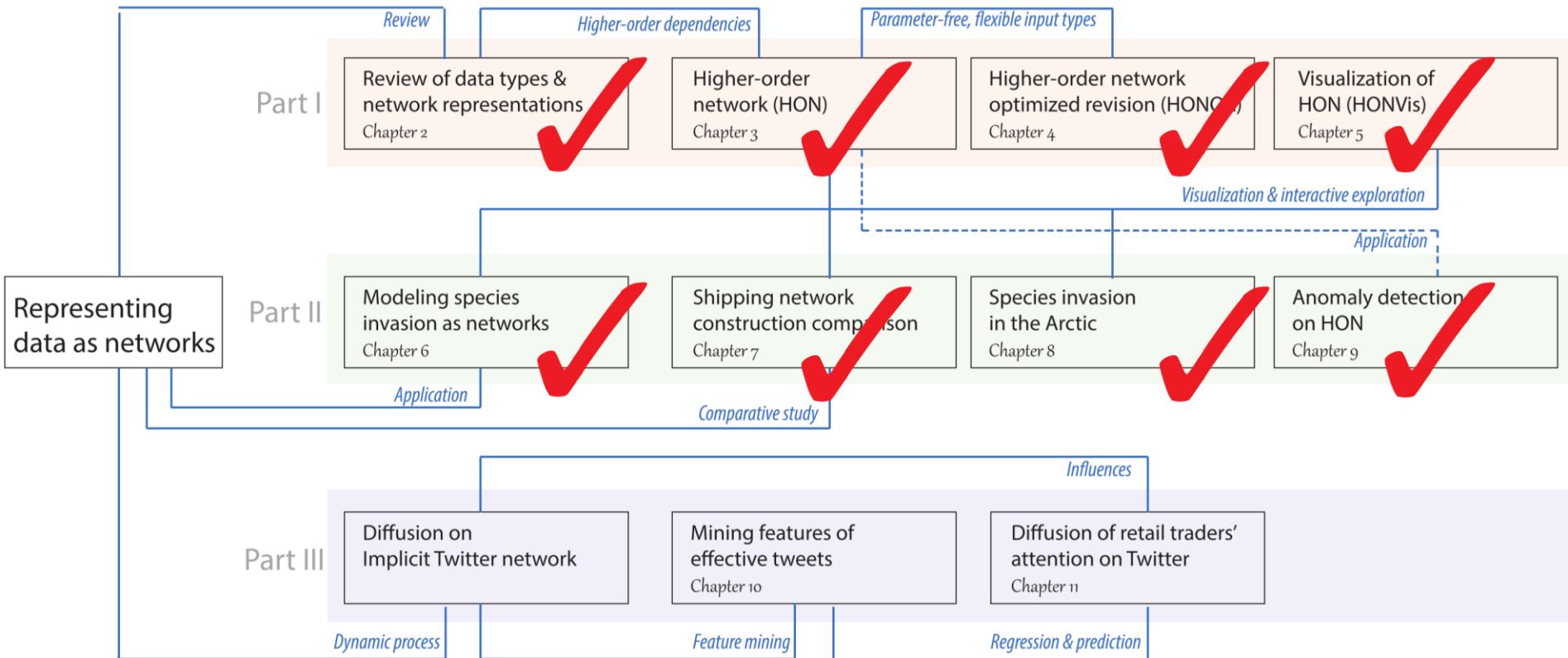
Takeaways

Anomaly detection on dynamic HON

Unveils higher-order anomalies that are otherwise ignored

Amplifies anomaly signals

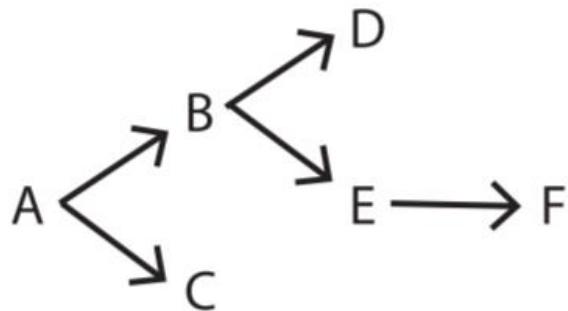
Overview



Discussions

Flexible inputs

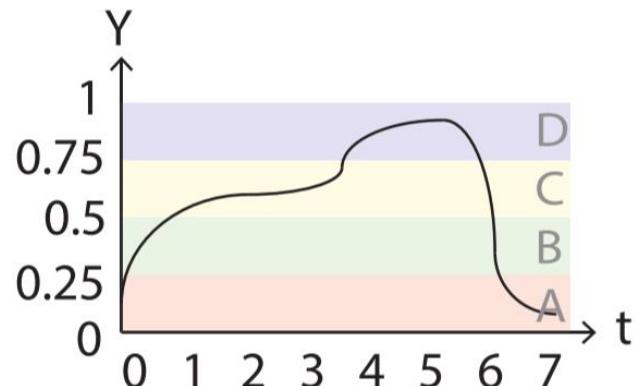
Raw diffusion data Observations



AB
AC
BD
BE
EF
ABD
ABE
BEF
ABEF

Flexible inputs

Raw time series data



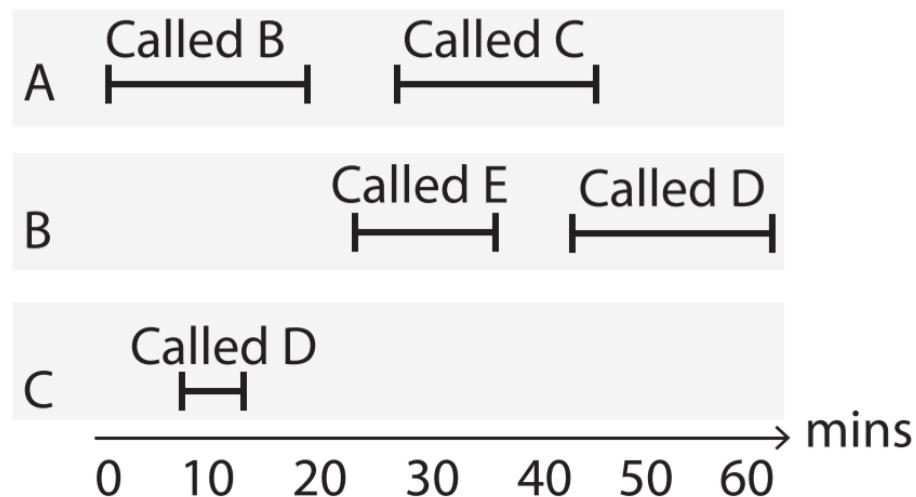
A C C C D D B A

Observations

A C
C D
D B
B A
A C D
C D B
D B A
A C D B
C D B A
A C D B A

Flexible inputs

Raw pairwise interaction
temporal data



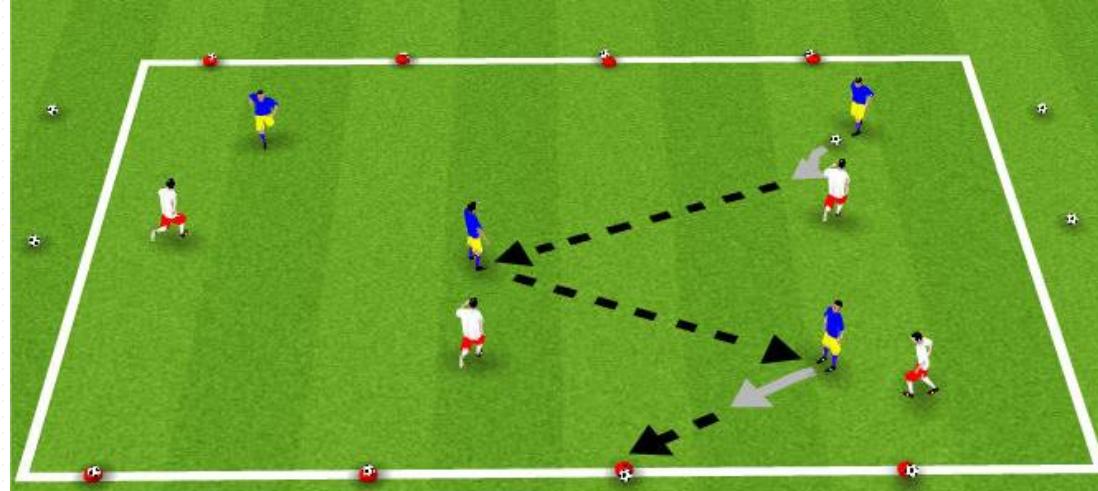
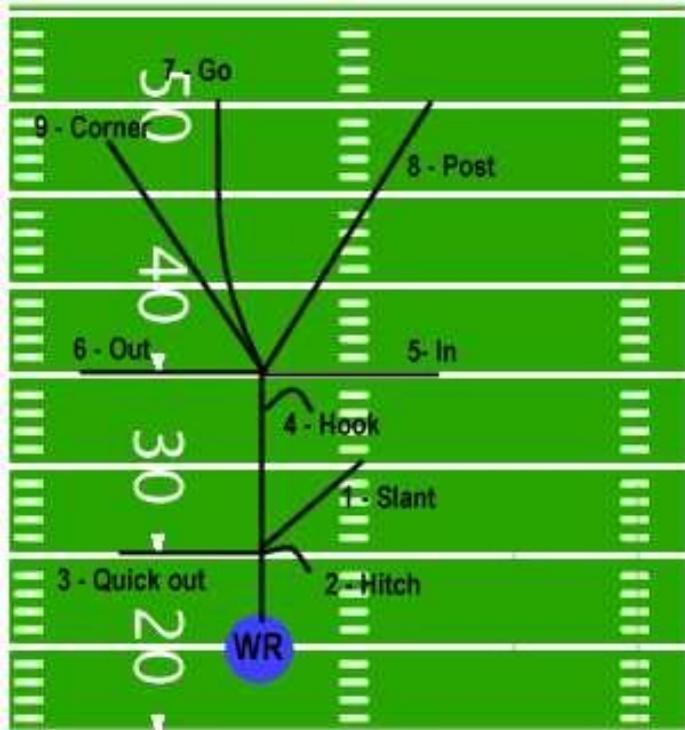
Observations

A B
A C
B E
B D
C D
C B
A B E

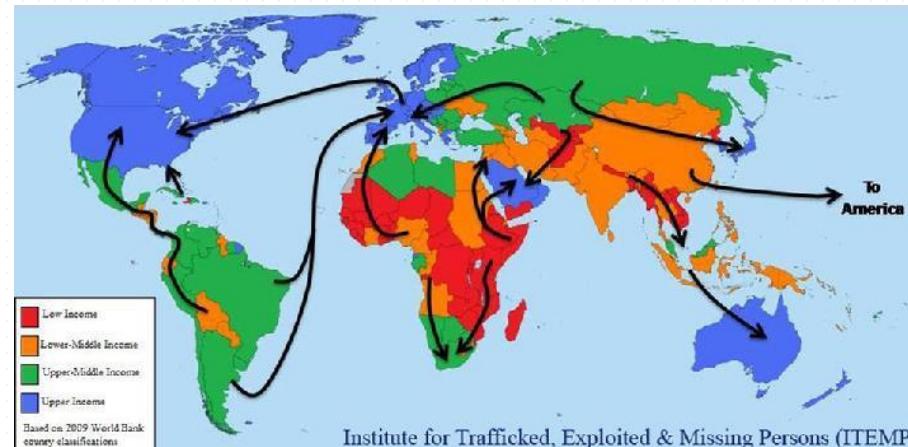
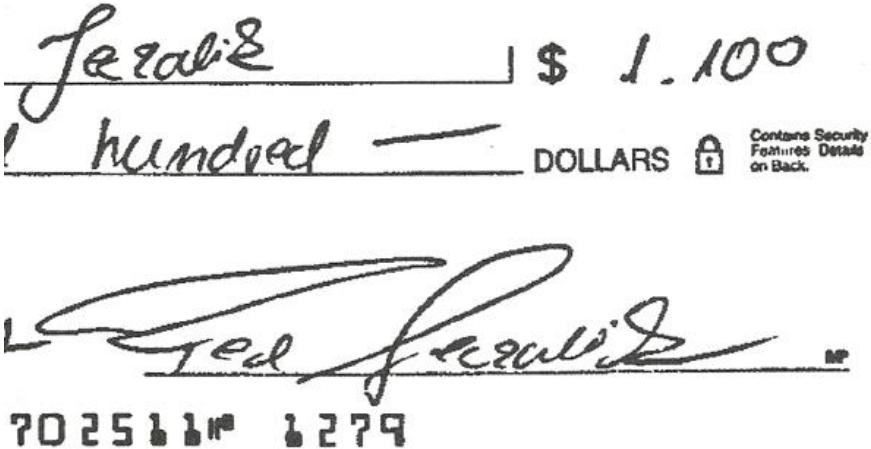
Varieties of data

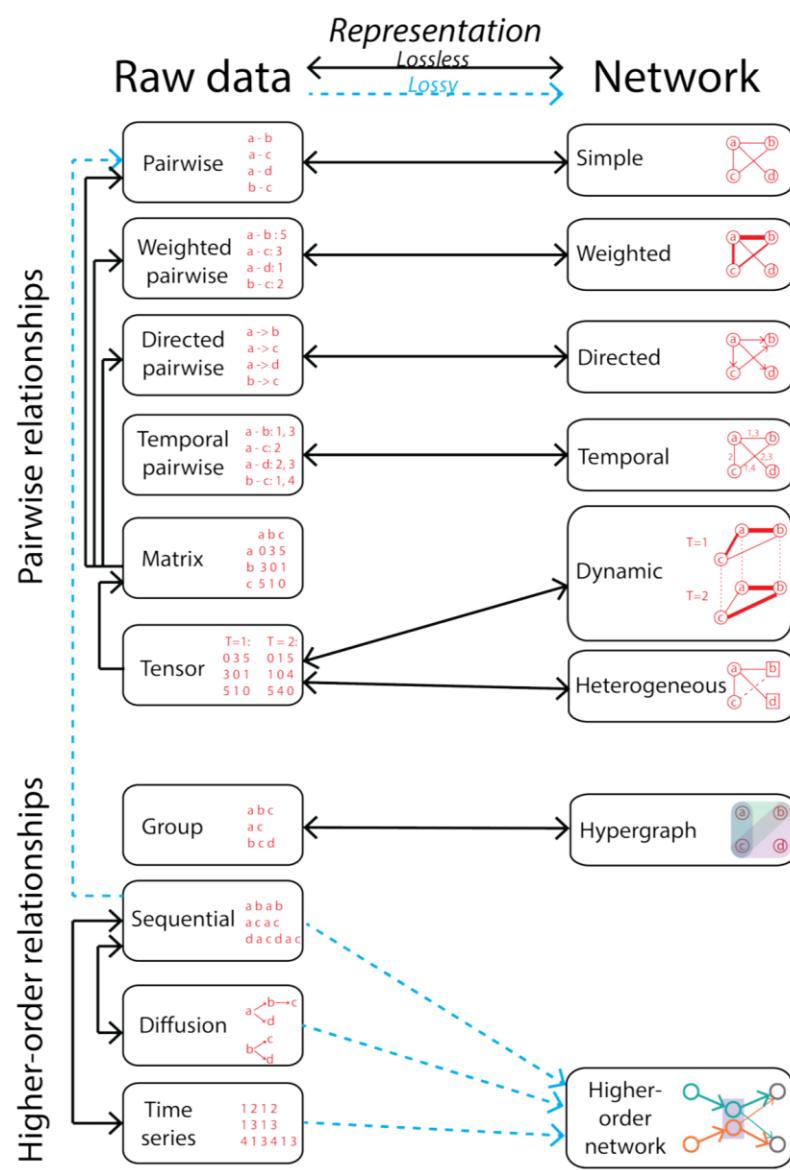
Application fields	Input Trajectories	Nodes	Edges
Transportation	Ship trajectories	Ports	Ship traffic
Computer network	Clickstreams	Web pages	Web traffic
Human interactions	Phone call or message cascades	People	Information flow
Human behavior	Human movements	POIs	Traffic
Healthcare	Patient records	Diseases	Disease evolutions
NLP	Sentences	Words	# word pairs

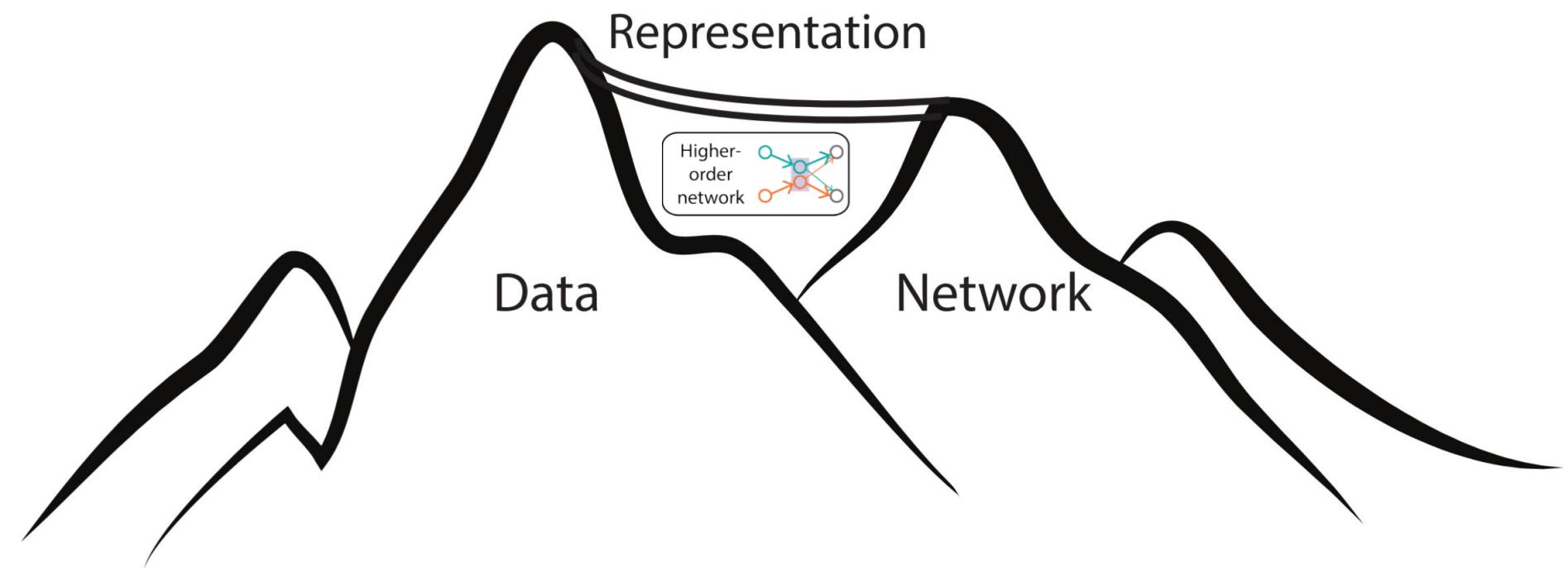
Other potential applications



Other potential applications







Research outputs

As leading student author:

- HON, published @ *Science advances*: **Jian Xu**, Thanuka L. Wickramarathne, and Nitesh V. Chawla. "Representing higher-order dependencies in networks." 2, no. 5 (2016): e1600028.
- HoNVis, published @ *IEEE PacificVis*: Jun Tao, **Jian Xu**, Chaoli Wang, and Nitesh V. Chawla. "HoNVis: Visualizing and Exploring Higher-Order Networks."
- HoNVis, demo published @ *IEEE IoT/I*: Jian Xu, Jun Tao, Nitesh V. Chawla and Chaoli Wang. "Visual Analytics of Higher-order Dependencies in Sensor Data"
- Species invasions, published @ *ACM SIGKDD*: **Jian Xu**, Thanuka L. Wickramarathne, Nitesh V. Chawla, Erin K. Grey, Karsten Steinhaeuser, Reuben P. Keller, John M. Drake, and David M. Lodge. "Improving management of aquatic invasions by integrating shipping network, ecological, and environmental data: data mining for social good."
- Retail diffusion: under review @ *Journal of Management Science*: Nitesh Chawla, Zhi Da, **Jian Xu**, and Mao Ye. *Catching fire: the diffusion of retail attention on twitter*.
- Effective tweeting: under review @ *ASONAM*: **Jian Xu**, Nitesh Chawla.
- Arctic species invasion: to submit to *Nature Communications*. Jian Xu, Salvatore Curasi, Erin Grey, Nitesh Chawla and David Lodge. "Species introduction and diffusion in the Arctic through global shipping: risk assessment and projection"
- Anomaly detection with HON: to submit to *ICDM*. Jian Xu, Nitesh Chawla.

Research outputs

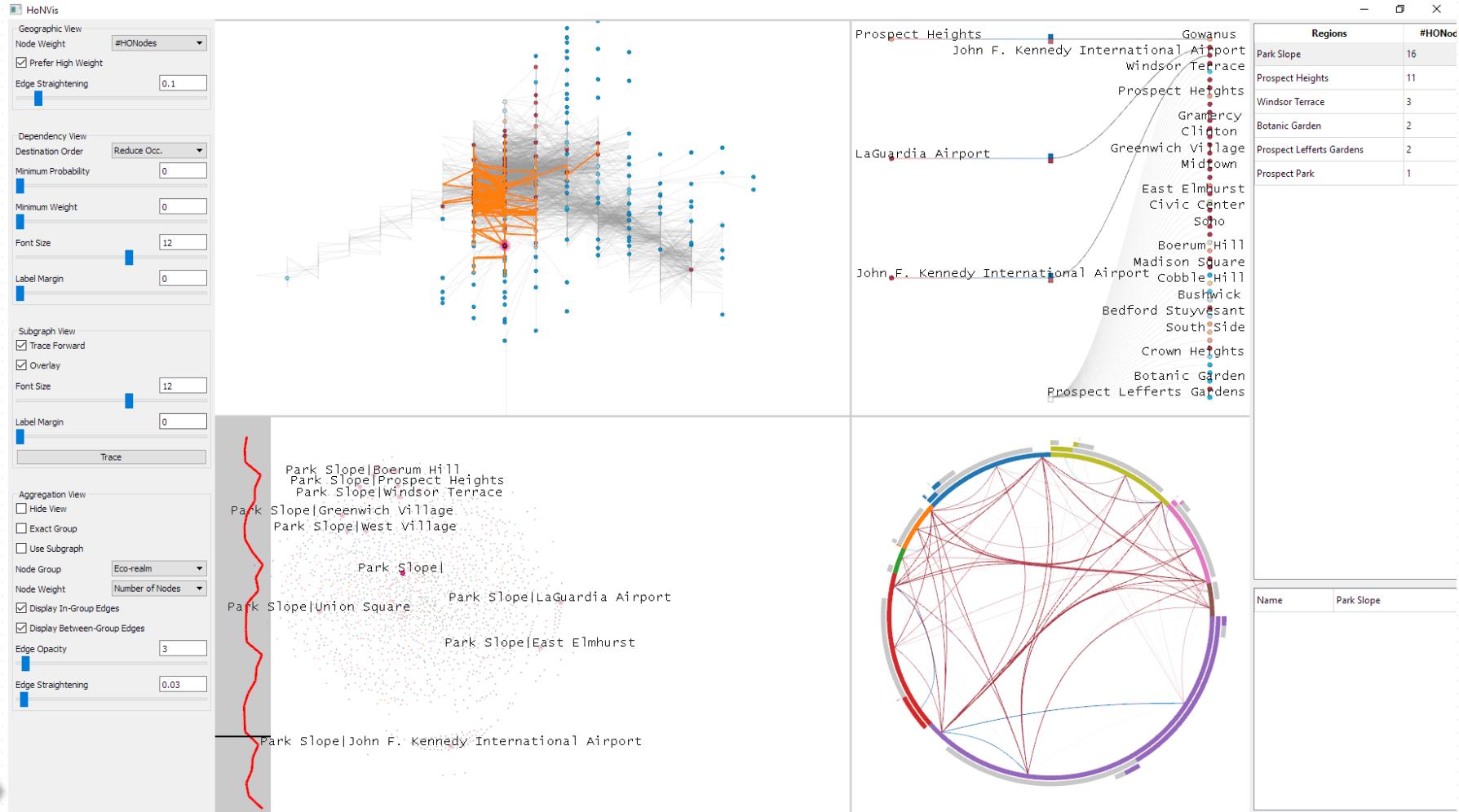
Other published collaborative work:

- Structural diversity, published @ *ACM SIGKDD*: Yuxiao Dong, Reid A. Johnson, **Jian Xu** and Nitesh V. Chawla. "Structural Diversity and Homophily: A Study Across More Than One Hundred Big Networks"
- Temporal motifs, published @ *IEEE Transaction on Systems, Man and Cybernetics*. Zhang, Yi-Qing, Xiang Li, **Jian Xu**, and Athanasios V. Vasilakos. "Human interactive patterns in temporal networks."

Other work in progress:

- HONVis extension: adding the time dimension, and the anomaly detection module.
- Comparative analysis of different network representations of global shipping.

HoNVis for dynamic HON & anomaly detection



For the community

The website for the 'Higher-Order Network' project features a header with navigation links: Overview, Algorithm, Applications, Code, Visualization, Paper, and Acknowledgement. Below the header is a 'Higher-Order Network' section with a diagram showing four nodes (B, M, Y, X) connected by directed edges. A large arrow points from this diagram to the text 'Count number of pairwise interactions as edge weights'. The main title 'Higher-order network' is displayed prominently, with the subtitle 'Capturing higher-order dependencies in big data' underneath. A large circular graphic on the left contains the number '85'. A red 'CLICK TO BEGIN' button is located in the center. Two callout boxes provide additional information: 'In the top 5% of all research outputs scored by Altmetric' and 'High Attention Score compared to outputs of the same age (97th percentile)'.

Higher-Order Network

Overview Algorithm Applications Code Visualization

Paper Acknowledgement

Higher-Order Network

B → M → Y

B → M → X

B → M → Y

B → M → Y

Count number of pairwise interactions as edge weights

Higher-order network

Capturing higher-order dependencies in big data

CLICK TO BEGIN

85

In the top 5% of all research outputs scored by Altmetric

High Attention Score compared to outputs of the same age (97th percentile)

A GitHub commit history for the user 'xyjprc' is shown. The commit message is 'committed on GitHub Update'. The commit history includes the following files: applications, cl-HON, data, figs, pyHON, and README.md.

xyjprc committed on GitHub Update

applications

cl-HON

data

figs

pyHON

README.md



Acknowledgements: committee

Prof. Nitesh Chawla, *chair*



Prof. David Lodge



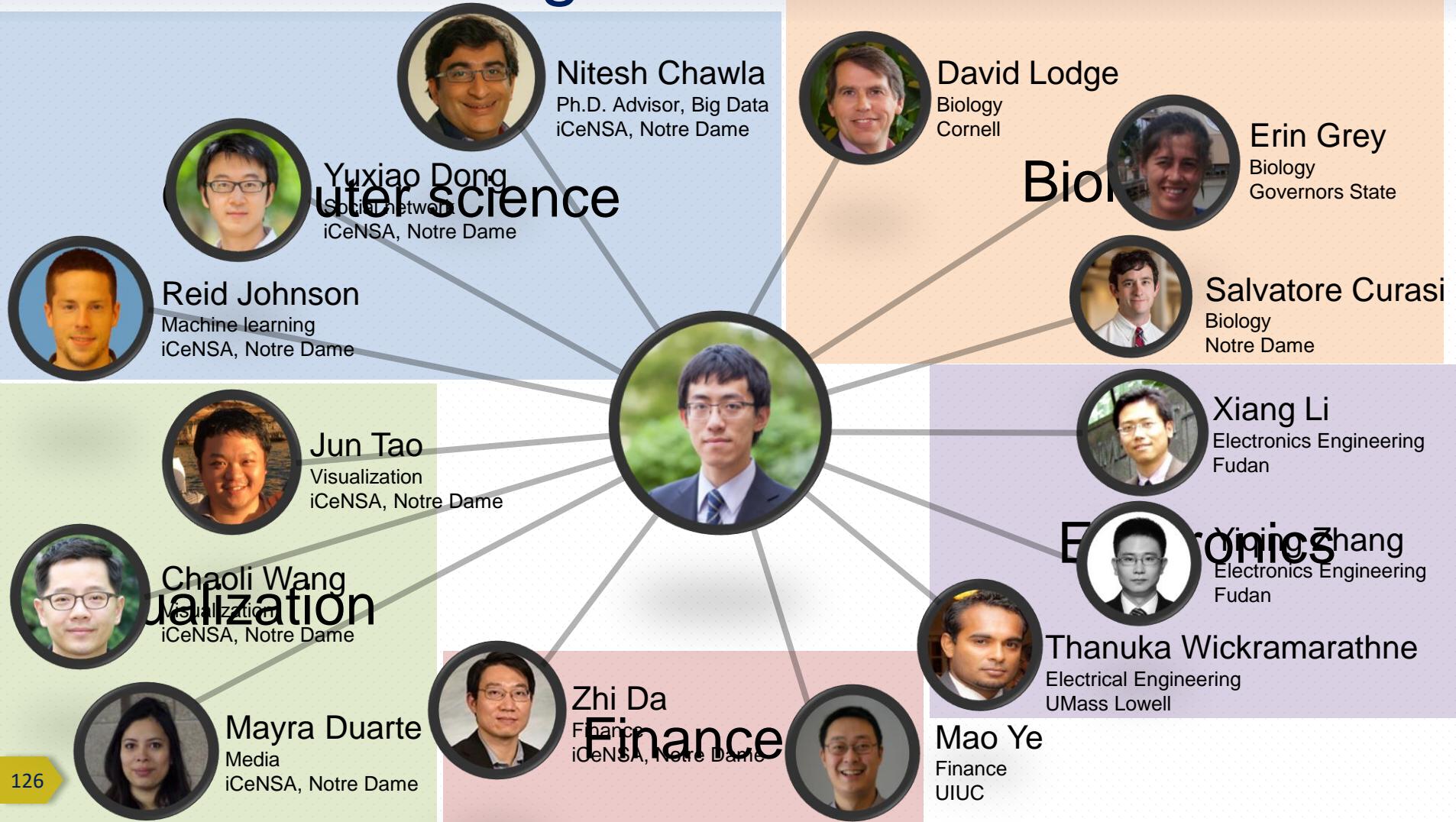
Prof. Tijana Milenkovic



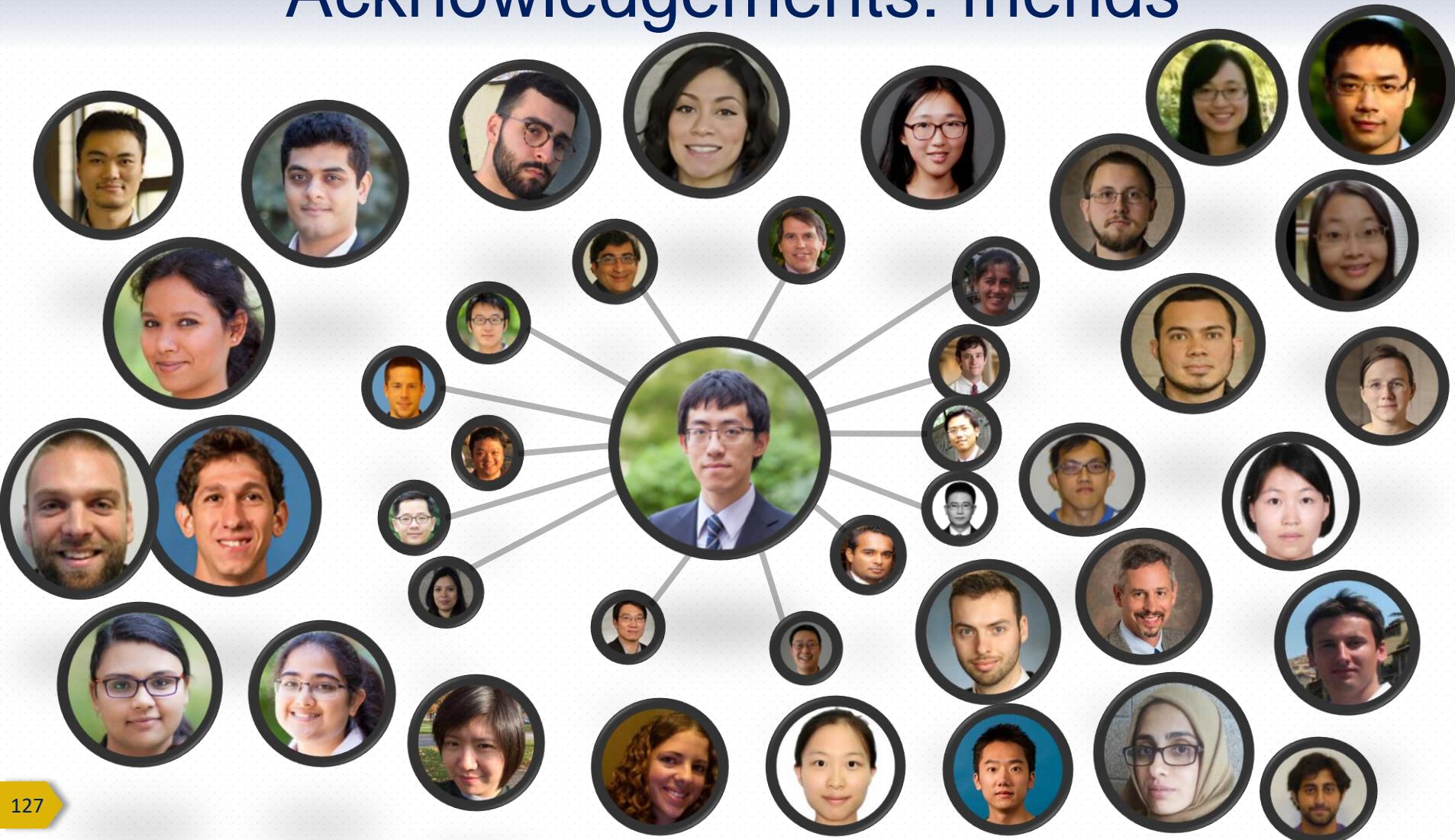
Prof. Zoltan Torontzkai



Acknowledgements: collaborators



Acknowledgements: friends



Acknowledgements: Funding



NETWORK
SCIENCE
CTA



Thank you!

Jian Xu

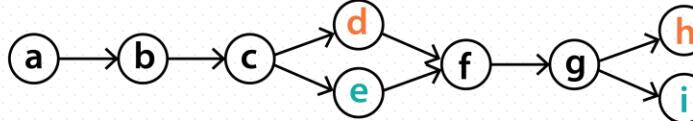
Appendix

References

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Comparison with VOM

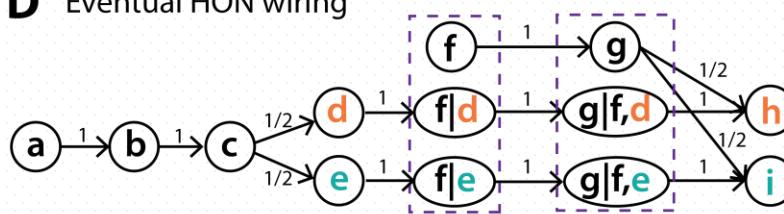
A True connections of ports



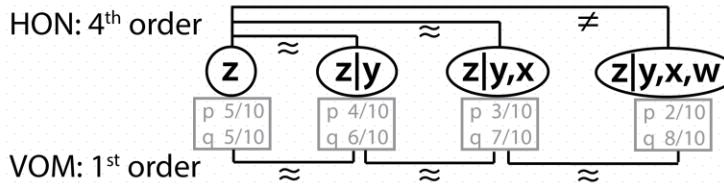
B Trajectories

Ship-1	a	b	c	d	f	g	h
Ship-2	b	c	d	f	g	h	
Ship-3	a	b	c	e	f	g	i
Ship-4	b	c	e	f	g	i	

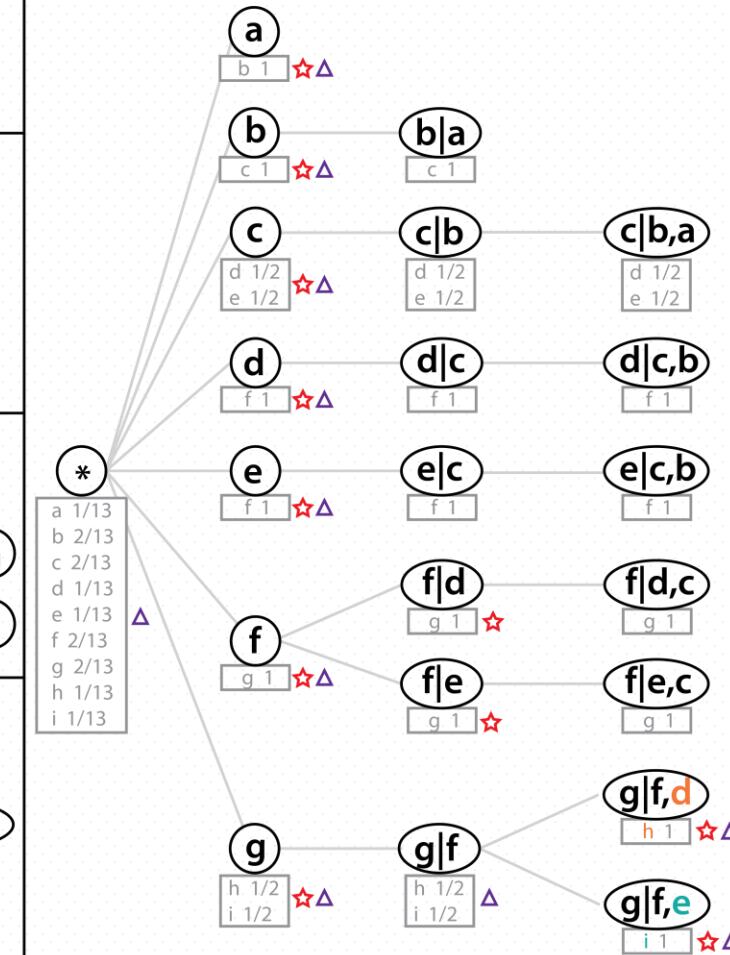
D Eventual HON wiring



E HON rule growing vs VOM pruning



C Rules extracted by HON and VOM



Higher-order network

Scalability

Scalability

Network representation (global shipping data)	Number of edges	Number of nodes	Network density	Clustering time (mins)	Ranking time (s)
Conventional first-order	31,028	2,675	4.3×10^{-3}	4	1.3
Fixed second-order	116,611	19,182	3.2×10^{-4}	73	7.7
HON, max order two	64,914	17,235	2.2×10^{-4}	45	4.8
HON, max order three	78,415	26,577	1.1×10^{-4}	63	6.2
HON, max order four	83,480	30,631	8.9×10^{-5}	67	7.0
HON, max order five	85,025	31,854	8.4×10^{-5}	68	7.6

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Scalability

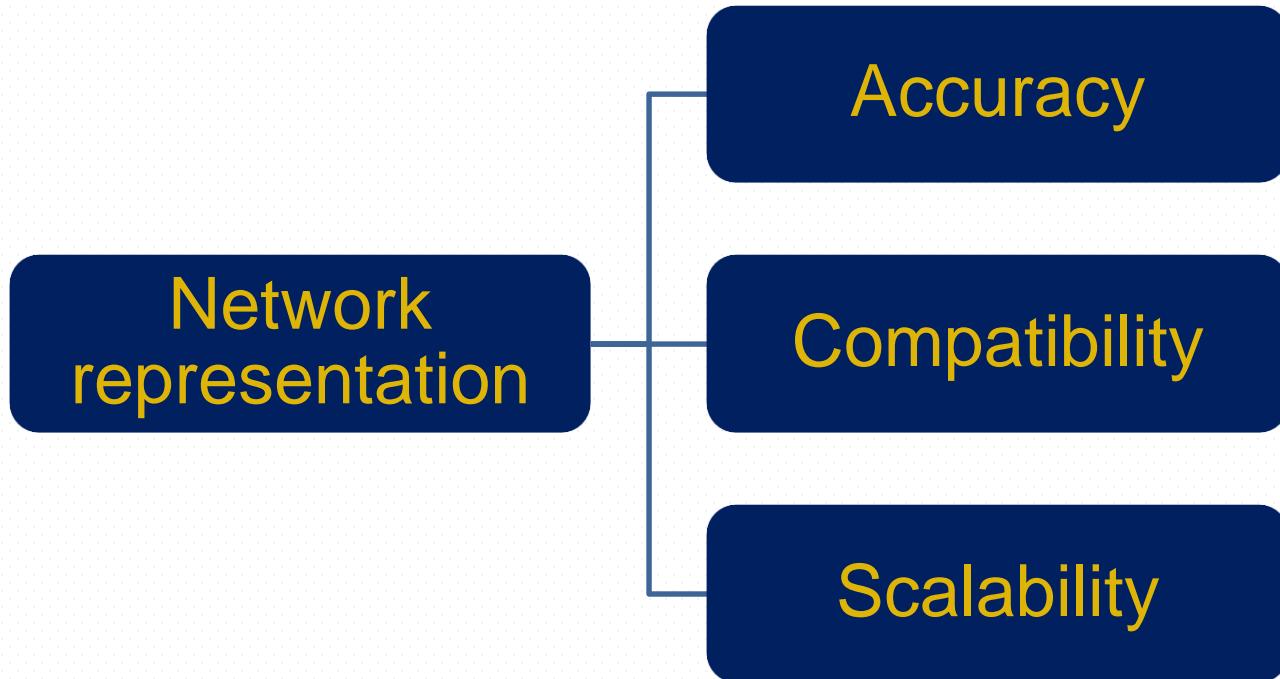
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* Using MapEquation with 1000 iterations

** Using PageRank

Goals

How shall we represent such big data derived from complex system as networks, and accurately capture higher-order dependencies?



Higher-order dependencies revealed by HON

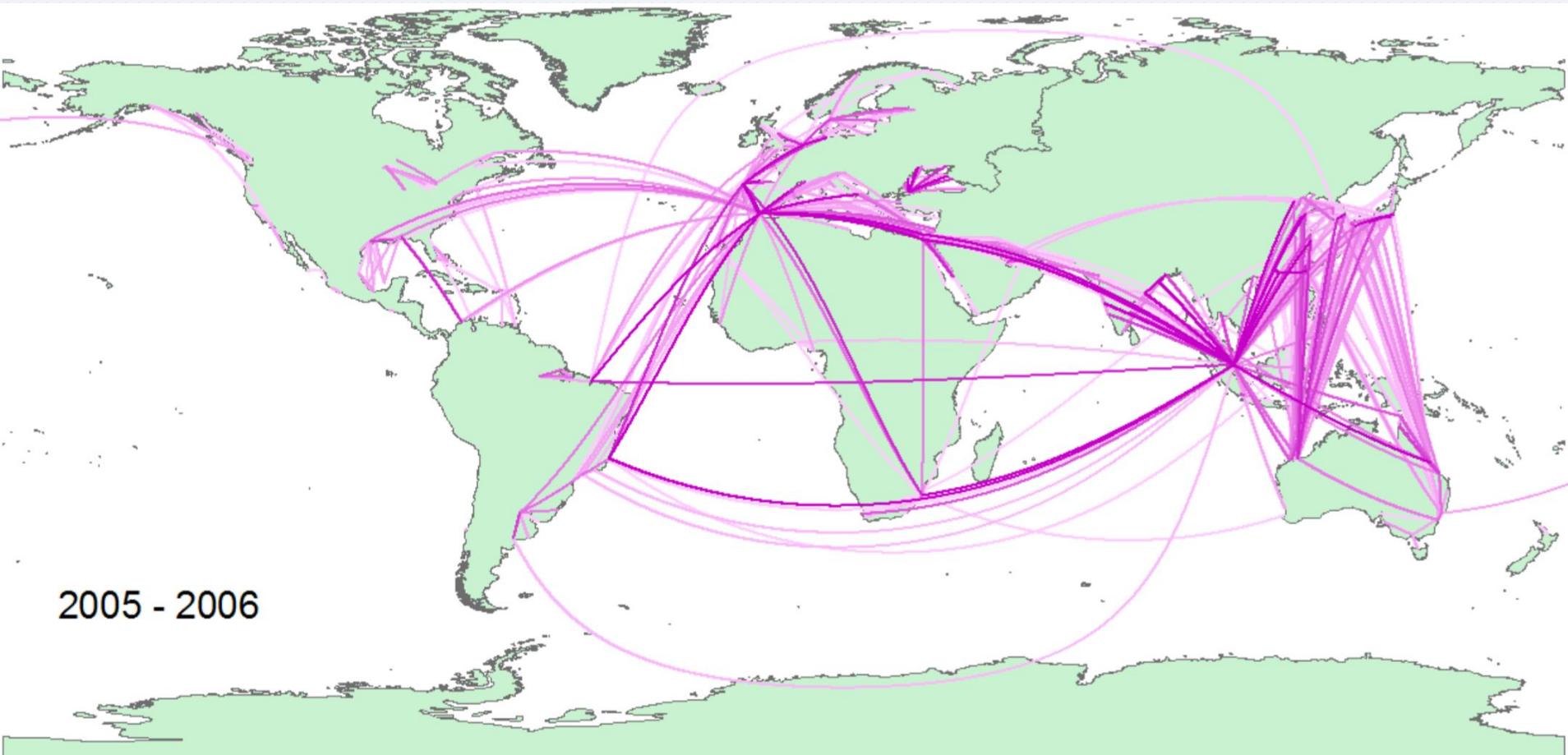
Data	# Records	Inject known variable-order dependencies
Synthetic	10,000,000	10 second-order 10 third-order 10 fourth-order

- Effectiveness: correctly captures all 30 of the higher-order dependencies
- Accuracy: does not extract false dependencies beyond the fourth order
- Compactness: determines that all other dependencies are first-order

Clustering: higher-order network

- 45% of ports belong to **more than one cluster**
- 44 ports (1.7% of all) belong to **five clusters**
 - New York, Shanghai, Hong Kong, Gibraltar, Hamburg, etc.
- Panama Canal belongs to **six clusters**
- Highlighting ports that may be invaded by species from multiple regions

Ship-borne species diffusion pathways



Ranking on clickstream network

Pages that gain PageRank scores	ΔPageRank	Pages that lose PageRank scores	ΔPageRank
South Bend Tribune - Home.	+0.0119	KTUU - Home.	-0.0057
Hagerstown News / obituaries - Front.	+0.0115	KWCH - Home.	-0.0031
South Bend Tribune - Obits - 3rd Party.	+0.0112	Imperial Valley Press - Home.	-0.0011
South Bend Tribune / sports / notredame - Front.	+0.0102	Hagerstown News / sports - Front.	-0.0005
Aberdeen News / news / obituaries - Front.	+0.0077	Imperial Valley Press / classifieds / topjobs - Front.	-0.0004
WDBJ7 - Home.	+0.0075	Gaylord - Home.	-0.0004
KY3 / weather - Front.	+0.0075	WDBJ7 / weather / web-cams - Front.	-0.0004
Hagerstown News - Home.	+0.0072	KTUU / about / meetnewsteam - Front.	-0.0003
Daily American / lifestyle / obituaries - Front.	+0.0054	Smithsburg man faces more charges following salvag	-0.0003
WDBJ7 / weather / closings - Front.	+0.0048	KWCH / about / station / newsteam - Front.	-0.0003
WSBT TV / weather - Front.	+0.0041	South Bend Tribune / sports / highschoolsports - Front.	-0.0003
Daily American - Home.	+0.0036	Hagerstown News / opinion - Front.	-0.0002
WDBJ7 / weather / radar - Front.	+0.0036	WDBJ7 / news / anchors-reporters - Front.	-0.0002
WDBJ7 / weather / 7-day-planner - Front.	+0.0031	Petoskey News / news / obituaries - Front.	-0.0002
WDBJ7 / weather - Front.	+0.0019	KWCH / news - Front.	-0.0002

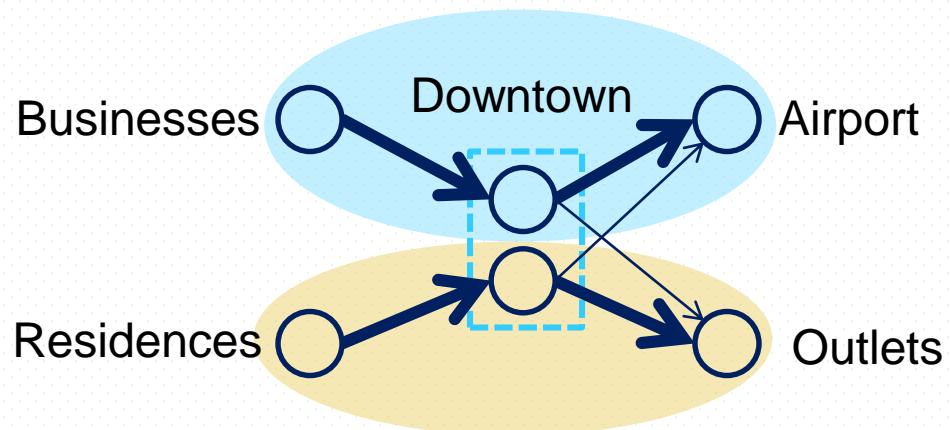
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South Bend Tribune - Obits - 3rd Party.	+0.0112	Imperial Valley Press - Home.	-0.0011
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Aberdeen News / news / obituaries - Front.	+0.0077	Imperial Valley Press / classifieds / topjobs - Front.	-0.0004
WDBJ7 - Home.	+0.0075	Gaylord - Home.	-0.0004
KY3 / weather - Front.	+0.0075	WDBJ7 / weather / web-cams - Front.	-0.0004
Hagerstown News - Home.	+0.0072	KTUU / about / meetnewsteam - Front.	-0.0003
Daily American / lifestyle / obituaries - Front.	+0.0054	Smithsburg man faces more charges following salvag	-0.0003
WDBJ7 / weather / closings - Front.	+0.0048	KWCH / about / station / newsteam - Front.	-0.0003
WSBT TV / weather - Front.	+0.0041	South Bend Tribune / sports / highschoolsports - Front.	-0.0003
Daily American - Home.	+0.0036	Hagerstown News / opinion - Front.	-0.0002
WDBJ7 / weather / radar - Front.	+0.0036	WDBJ7 / news / anchors-reporters - Front.	-0.0002
WDBJ7 / weather / 7-day-planner - Front.	+0.0031	Petoskey News / news / obituaries - Front.	-0.0002
WDBJ7 / weather - Front.	+0.0019	KWCH / news - Front.	-0.0002

No changes
to the ranking algorithm

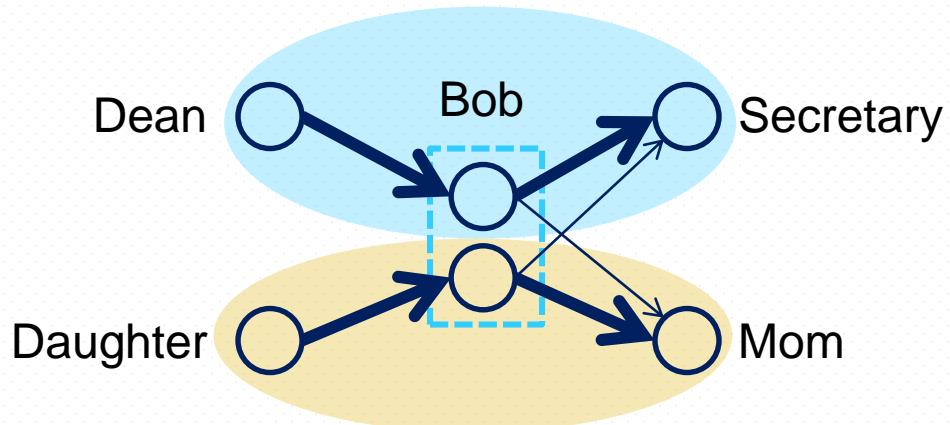
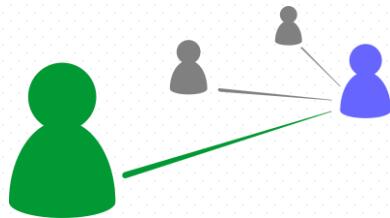
Interdisciplinary applications

Urban planning & Event detection



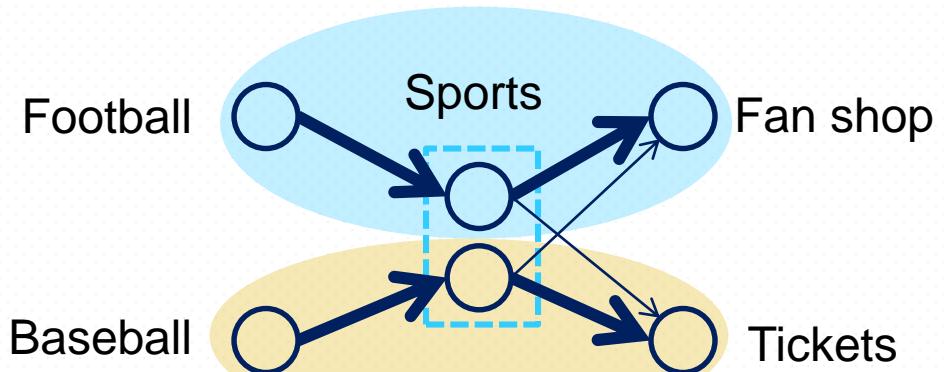
Interdisciplinary applications

Social network & Information diffusion



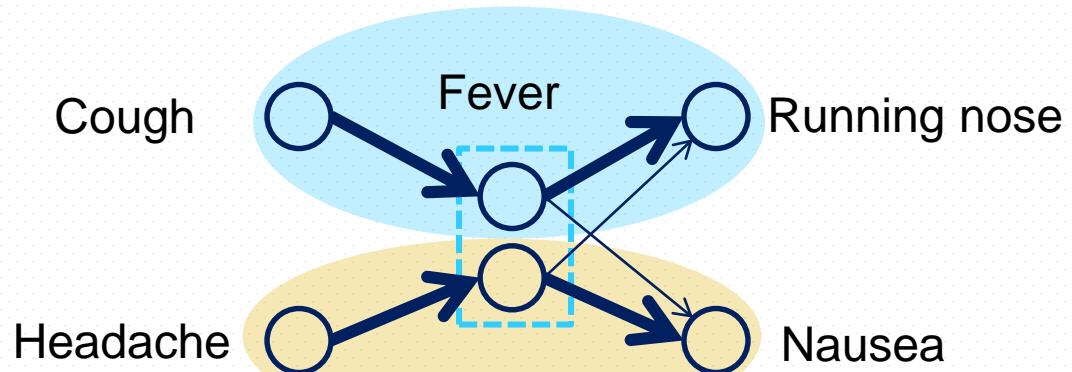
Interdisciplinary applications

Web optimization,
advertising,
network security

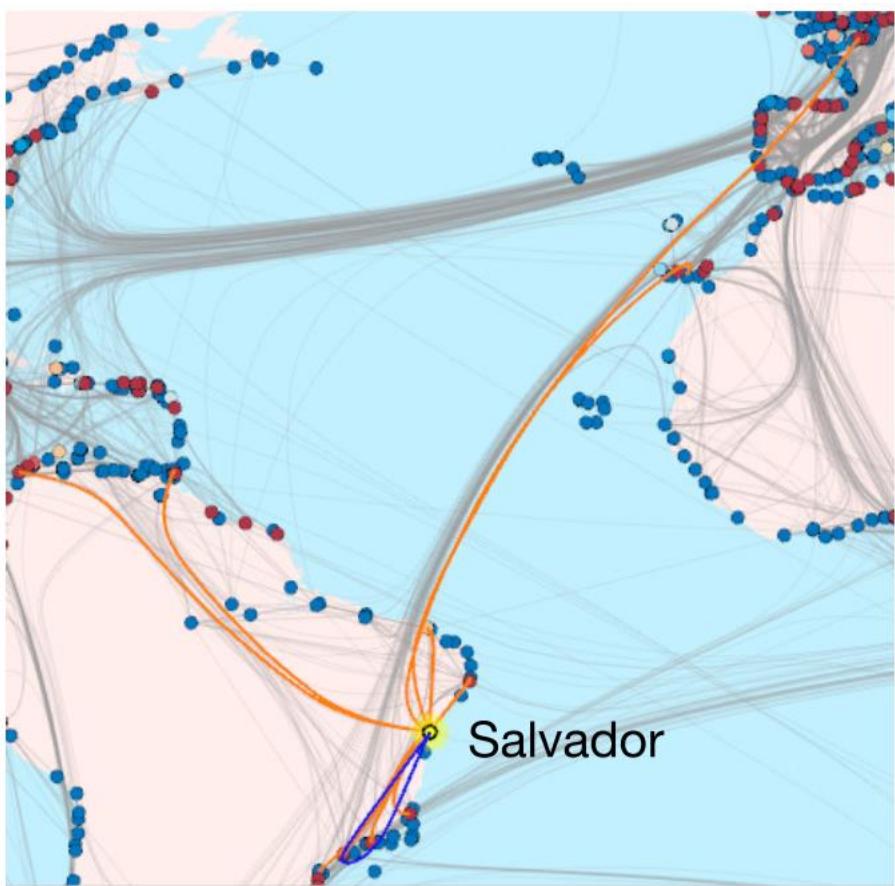


Interdisciplinary applications

Healthcare,
Epidemics monitoring,
Gene tech

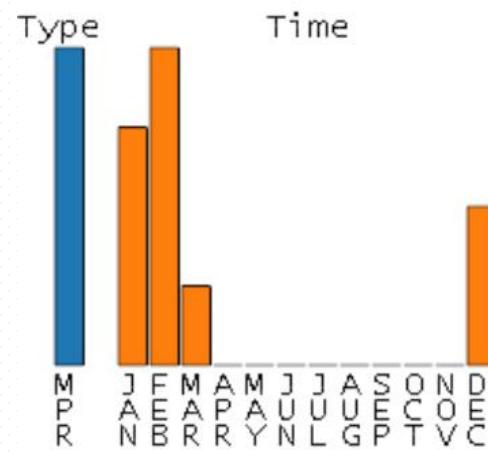


Explore geographically & rank by features

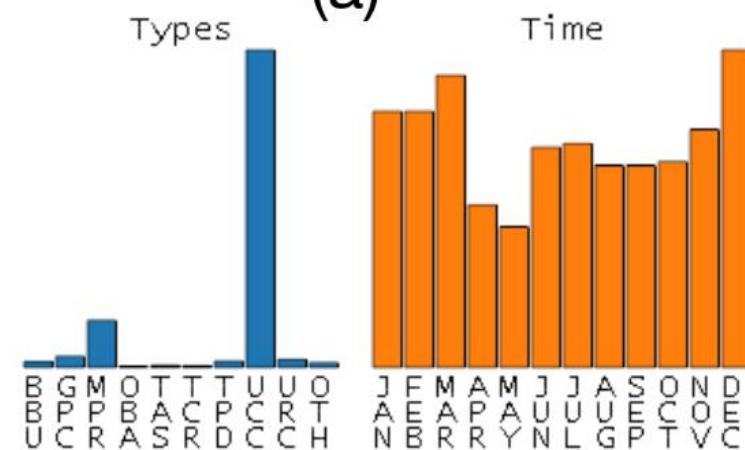


Ports	#HO Nodes
Suape	19
Vitoria	19
Salvador	13
Tubarao	6
Praia Mole	5
Portocel	2
Ponta do Ubu	2
Aratu	2
Recife	2
Madre de Deus	1
Cabedelo	1
Ilheus	1
Maceio	1
Jubarte Field	1

View dependencies & underlying metadata



(a)



Santos Anch.

Itapoia

Navegantes
Paranagua

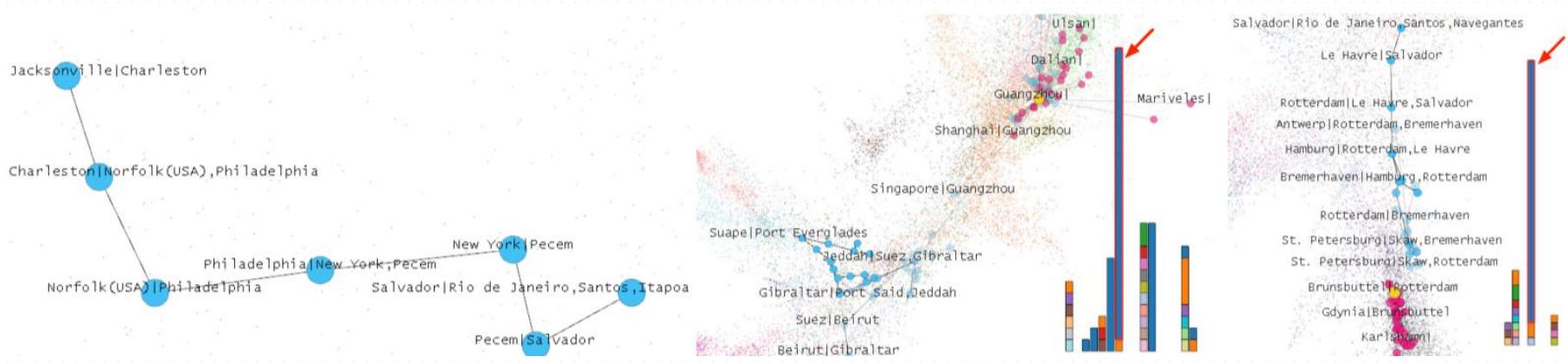
Salvador

Rio de Janeiro

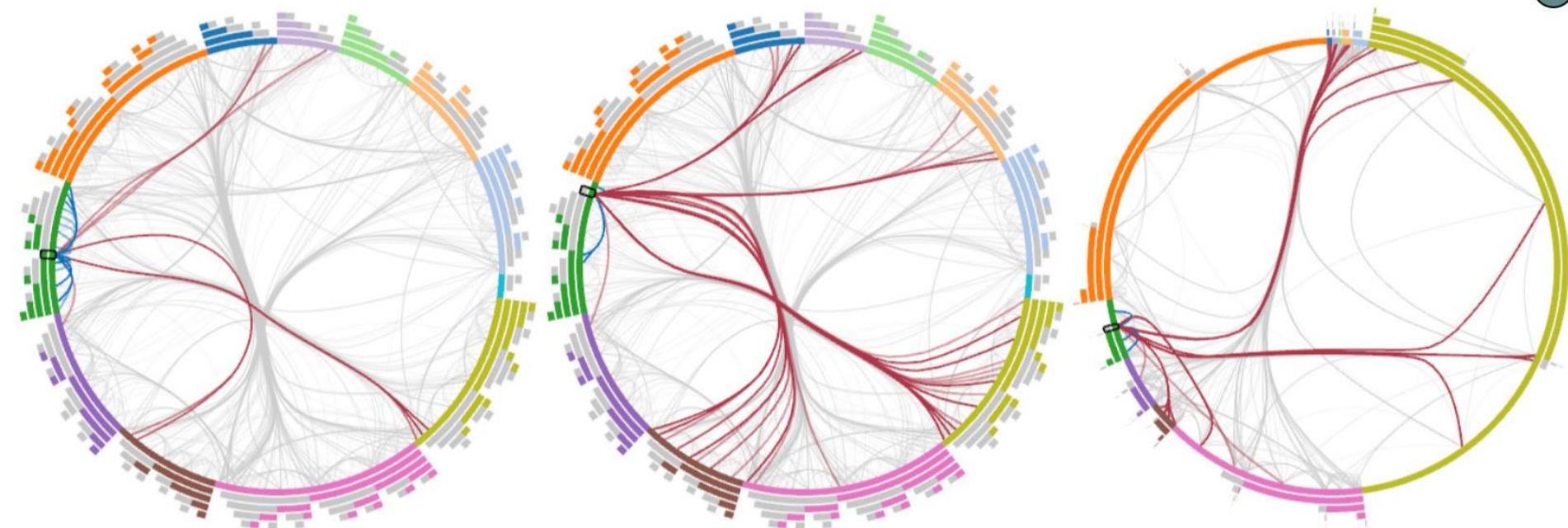
Santos



Track diffusion on the network



Aggregate at different granularities



Clustering

- *Walktrap*: “Random walks on a graph tend to get ‘trapped’ into densely connected parts corresponding to communities.” (Pons & Latapy 2006)

Ranking

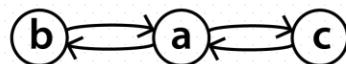
- *PageRank*: “The simplified version corresponds to the standing probability distribution of a random walk on the graph of the Web.” (Page et al. 1999)

Influence on dynamics

Ship-1	b	a	b	a	b	a	b	a	b
Ship-2		b	a	b	a	b	a	b	b
Ship-3	a	c	a	c	a	c	a	c	c

← Training * Testing →

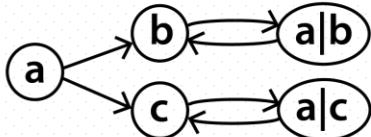
First-order network



b	a	b	✓
b	a	c	✗
c	a	b	✗

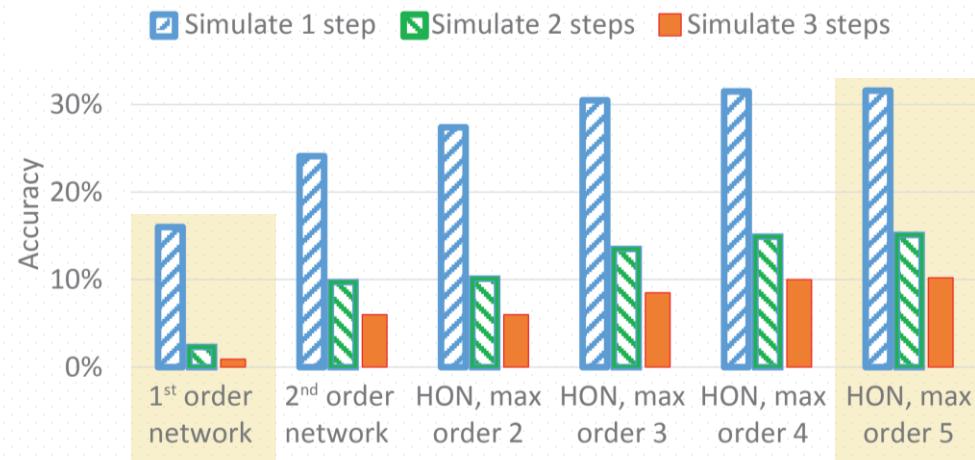
← Prediction → 1/3 correct

HON



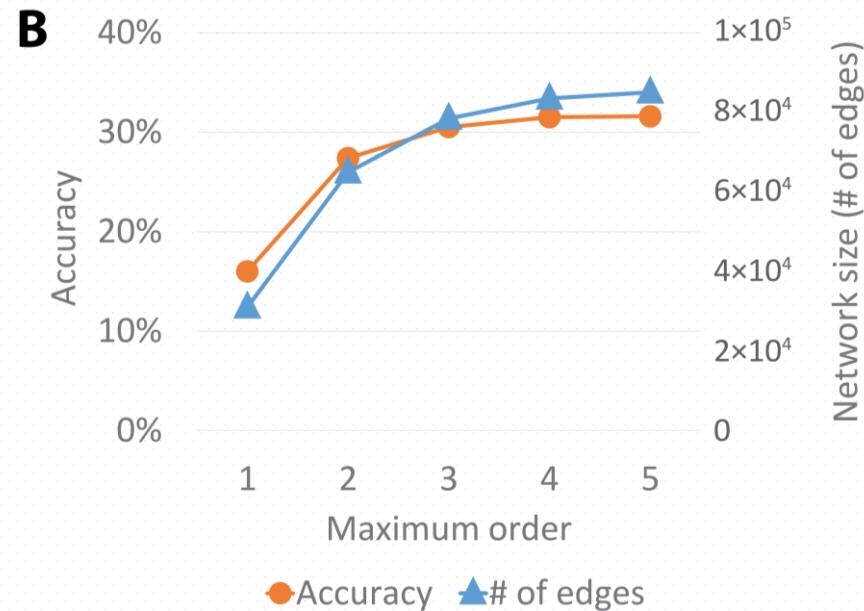
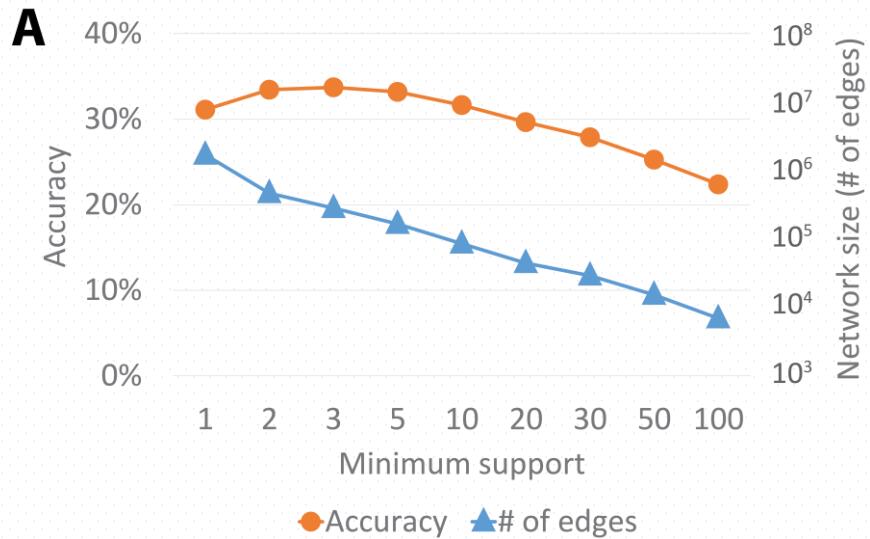
b	a	b	✓
b	a	b	✓
c	a	c	✓

← Prediction → 3/3 correct



Network representation	Number of edges	Number of nodes	Network density	Prob. of returning after two steps	Prob. of returning after three steps	Entropy rate (bits)	Clustering time (mins)	Ranking
Conventional first-order	31,028	2,675	4.3×10^{-3}	10.7%	1.5%	3.44	4	1.3
Fixed second-order	116,611	19,182	3.2×10^{-4}	42.8%	8.0%	1.45	73	7.7
HON, max order two	64,914	17,235	2.2×10^{-4}	41.7%	7.3%	1.46	45	4.8
HON, max order three	78,415	26,577	1.1×10^{-4}	45.9%	16.4%	0.90	63	6.2
HON, max order four	83,480	30,631	8.9×10^{-5}	48.9%	18.5%	0.68	67	7.0
HON, max order five	85,025	31,854	8.4×10^{-5}	49.3%	19.2%	0.63	68	7.6

Parameter sensitivity



Comparison with VOM

	HON	VOM	In HON but not in VOM	In VOM but not in HON
0th order	0	3,029	0	3029
1st order	31,028	31,028	0	0
2nd order	32,960	35,288	427	2,755
3rd order	15,642	21,536	550	6,444
4th order	4,632	8,973	302	4,643
5th order	763	2,084	23	1,344
Total	85,025	101,938	1,302	18,215

- **Global shipping data.** This data made available by Lloyd's Maritime Intelligence Unit (LMIU) contains ship movement information such as vessel_id, port_id, sail_date and arrival_date. Our experiments are based on a recent LMIU data set that spans one year from May 1st, 2012 to April 30th, 2013, totaling 3,415,577 individual voyages corresponding to 65,591 ships that move among 4,108 ports and regions globally. A minimum support of 10 is used to filter out noise in the data.
- **Clickstream data.** This data made available by a media company contains logs of users clicking through web pages that belong to 50 news web sites owned by the company. Fields of interest include user_ip, pagename and time. Our experiments are based on the clickstream records that span two months from December 4th, 2012 to February 3rd, 2013, totaling 3,047,697 page views made by 179,178 distinct IP addresses on 45,257 web pages. A minimum support of 5 is used to filter out noise in the data. Clickstreams that are likely to be created by crawlers (abnormally long clickstreams / clickstreams that frequently hit the error page) are omitted.
- **Retweet data.** This data (50) records retweet history on Weibo (a Chinese microblogging website), with information about who retweets whose messages at what time. The data was crawled in 2012 and there are 23,755,810 retweets recorded, involving 1,776,950 users.

Synthetic data. We created a trajectory data set (data and code are available at <https://github.com/xyjprc/hon>) with known higher-order dependencies to verify the effectiveness of the rule extraction algorithm. In the context of shipping, we connect 100 ports as a 10×10 grid, then generate trajectories of 100,000 ships moving among these ports. Each ship moves 100 steps, yielding 10,000,000 movements in total. Normally each ship has equal probabilities of going up/down/left/right on the grid in each step (with wrapping, e.g., going down at the bottom row will end up in the top row); we use additional higher-order rules to control the generation of ship movements. For example, a second-order rule can be defined as whenever a ship comes from Shanghai to Singapore, instead of randomly picking a neighboring port of Singapore for the next step, the ship has 70% chance of going to Los Angeles and 30% chance of going to Seattle. We predefine 10 second-order rules like this, and similarly 10 third-order rules, 10 fourth-order rules, and no other higher-order rules, so that movements that have variable orders of dependencies are generated. To test the rule extraction algorithm, we set the maximum order as five to see if the algorithm will incorrectly extract false rules beyond the fourth order which we did not define; we set minimum support as five for patterns to be considered as rules.

Higher-order network

Algorithm

How can we tell if this network representation
more accurately captures the pattern in raw data?

A

- Convert all first-order rules into edges

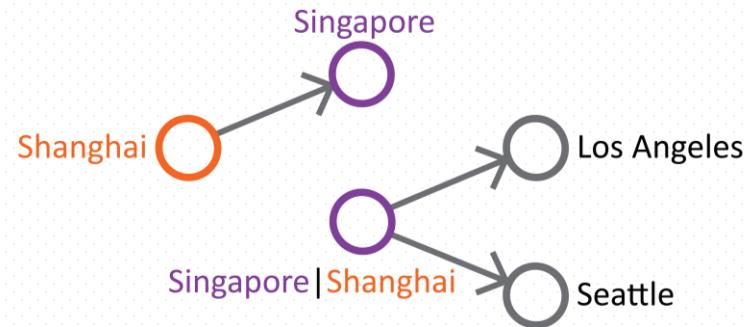


A

- Convert all first-order rules into edges

B

- Convert higher-order rules
- Add higher-order nodes when necessary



A

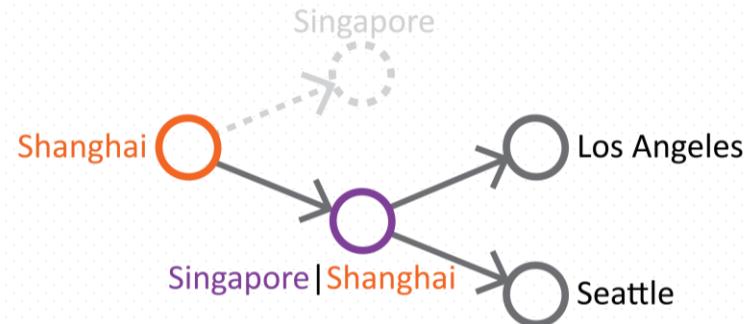
- Convert all first-order rules into edges

B

- Convert higher-order rules
- Add higher-order nodes when necessary

C

- Rewire edges
- The edge weights are preserved



A

- Convert all first-order rules into edges

B

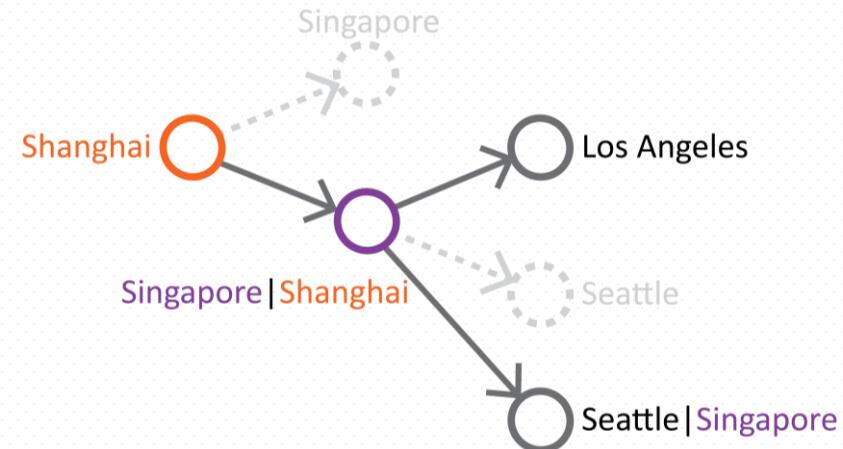
- Convert higher-order rules
- Add higher-order nodes when necessary

C

- Rewire edges
- The edge weights are preserved

D

- Rewire remaining edges



Higher-order network

Effectiveness

Higher-order dependencies revealed by HON

Data	# Records	Dependencies revealed	Similar observations
Ship movement	3,415,577	Up to 5 th order	N/A
Clickstream	3,047,697	Up to 3 rd order	<i>“... appear to saturate at $k = 3$ for Yahoo... browsing behavior across websites is definitely not Markovian but can be captured reasonably well by a not-too-high order Markov chain.”</i> --- Chierichetti et al. (2012)
Retweet	23,755,810	N/A	Assuming the second order has <i>“marginal consequences for disease spreading”</i> --- Rosvall et al. (2014)

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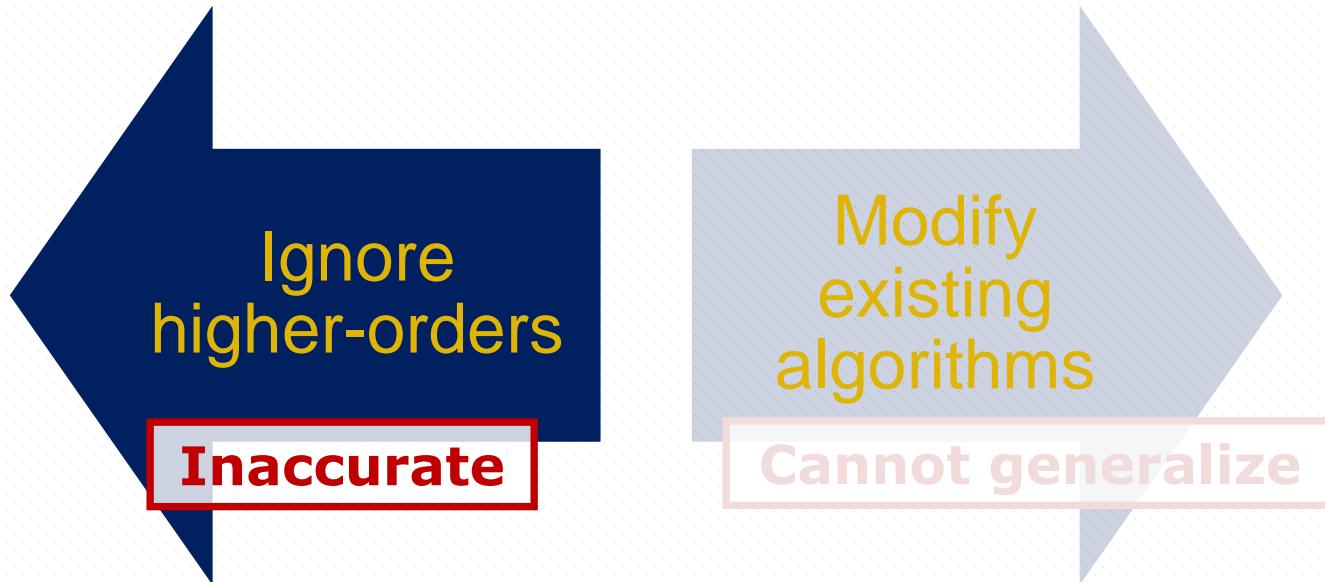
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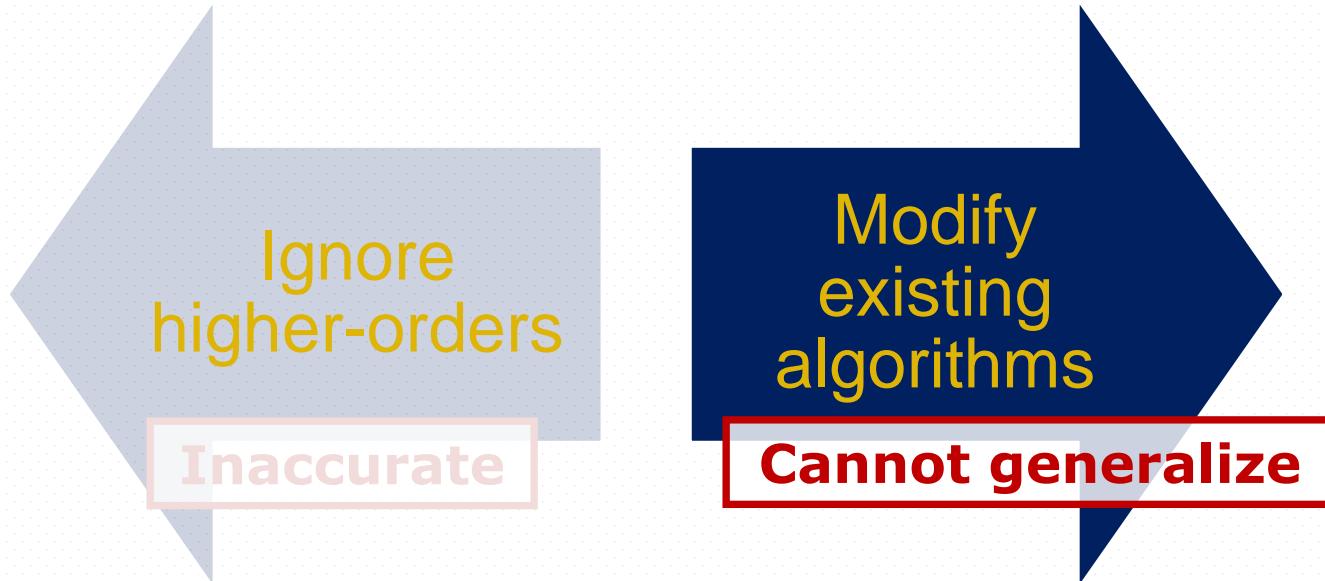
Higher-order dependencies revealed by HON

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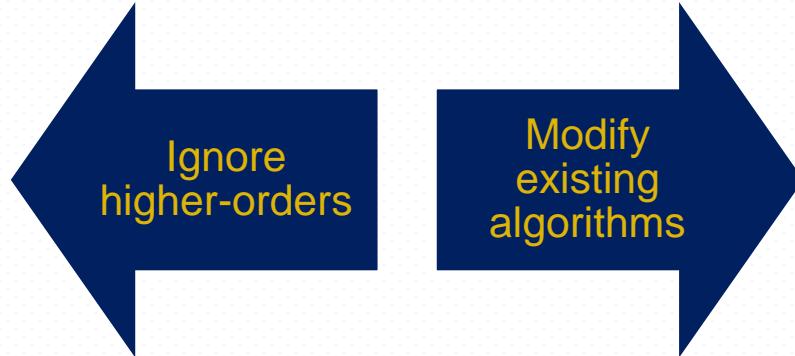
Existing approaches



Existing approaches



Existing approaches



Higher-order network

- ✓ Accurate representation
- ✓ Generalizes to existing algorithms

Influence on dynamics

Ship-1 b a b a b a

Ship-2 **b** **a** **b** **a**

Ship-3 a c a c a

← Training →

Influence on dynamics

Ship-1 b a b a b a

Ship-2 b a b a

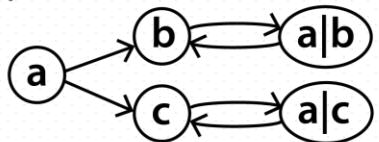
Ship-3 a c a c a

← Training →

First-order network



HON

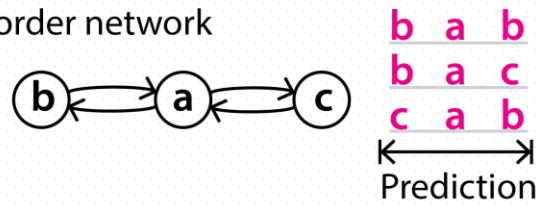


Influence on dynamics

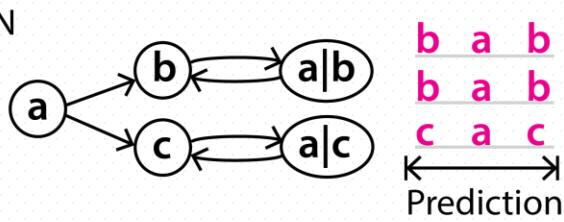
Ship-1	b	a	b	a	b	a	b	a	b
Ship-2		b	a	b	a	b	a	b	
Ship-3		a	c	a	c	a	c	a	c

← Training * Testing →

First-order network



HON



Influence on dynamics

Ship-1	b	a	b	a	b	a	b	a	b
Ship-2		b	a	b	a	b	a	b	
Ship-3		a	c	a	c	a	c	a	c

← Training * Testing →

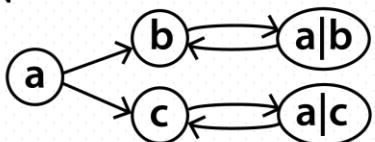
First-order network



b	a	b	✓
b	a	c	✗
c	a	b	✗

← Prediction → 1/3 correct

HON

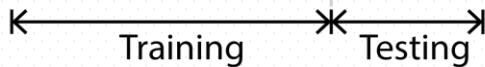


b	a	b	✓
b	a	b	✓
c	a	c	✓

← Prediction → 3/3 correct

Influence on dynamics

Ship-1	b	a	b	a	b	a	b	a	b
Ship-2		b	a	b	a	b	a	b	
Ship-3		a	c	a	c	a	c	a	c



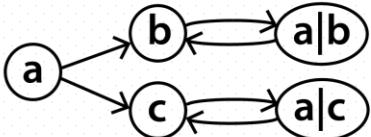
First-order network



b	a	b	✓
b	a	c	✗
c	a	b	✗

← Prediction → 1/3 correct

HON



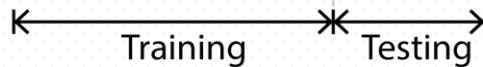
b	a	b	✓
b	a	b	✓
c	a	c	✓

← Prediction → 3/3 correct



Influence on dynamics

Ship-1	b	a	b	a	b	a	b	a	b
Ship-2		b	a	b	a	b	a	b	b
Ship-3		a	c	a	c	a	c	a	c



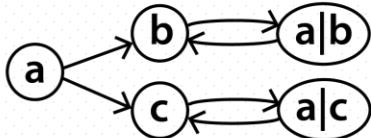
First-order network



b	a	b	✓
b	a	c	✗
c	a	b	✗

← Prediction → 1/3 correct

HON



b	a	b	✓
b	a	b	✓
c	a	c	✓

← Prediction → 3/3 correct



**Higher accuracy
in simulating real movements**

Higher-order network

Application: ranking

Web page access behaviors for server optimization and advertising

Ranking on clickstream network

User 1 WDBJ7 home → View photo → WDBJ7 home → ...

User 2 WDBJ7 home → View photo → Upload photo → ...

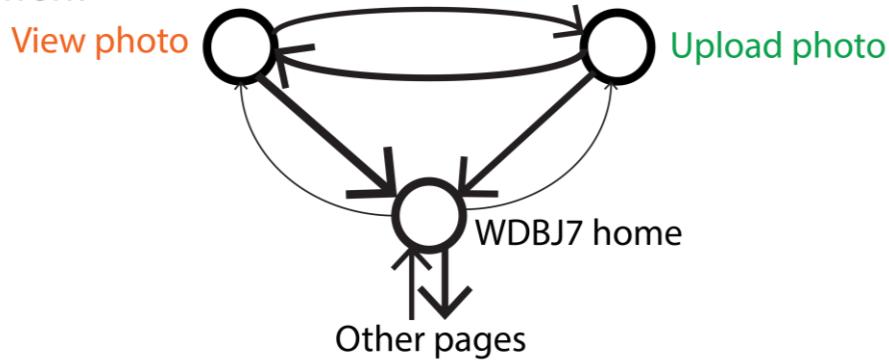
User 3 View photo → Upload photo → View photo → ...

User 4 WDBJ7 home → Upload photo → WDBJ7 home → ...

.....

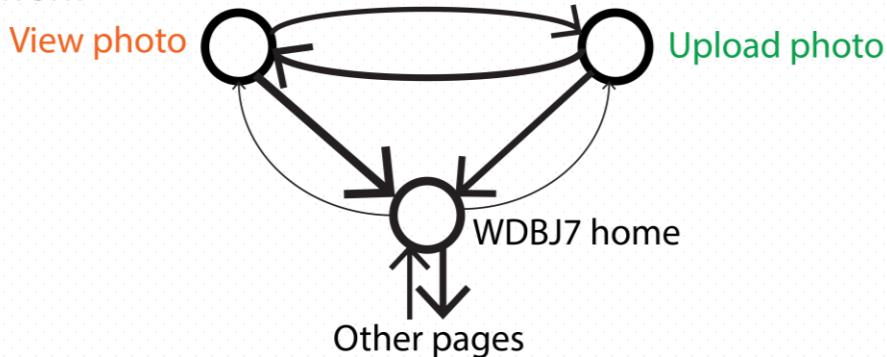
Ranking on clickstream network

First-order network



Ranking on clickstream network

First-order network



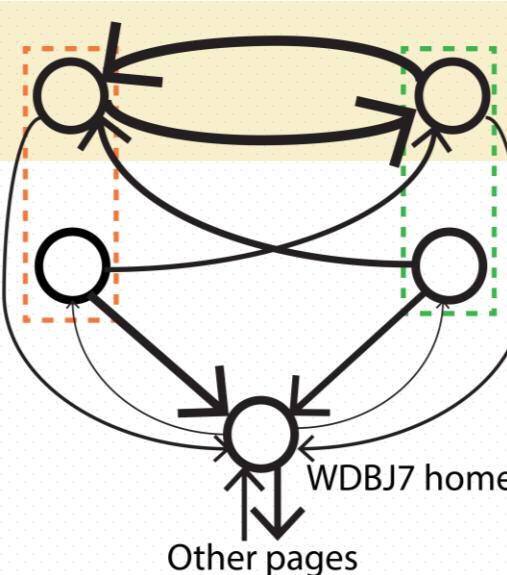
HON

View photo|Upload photo

Upload photo|View photo

View photo

Upload photo

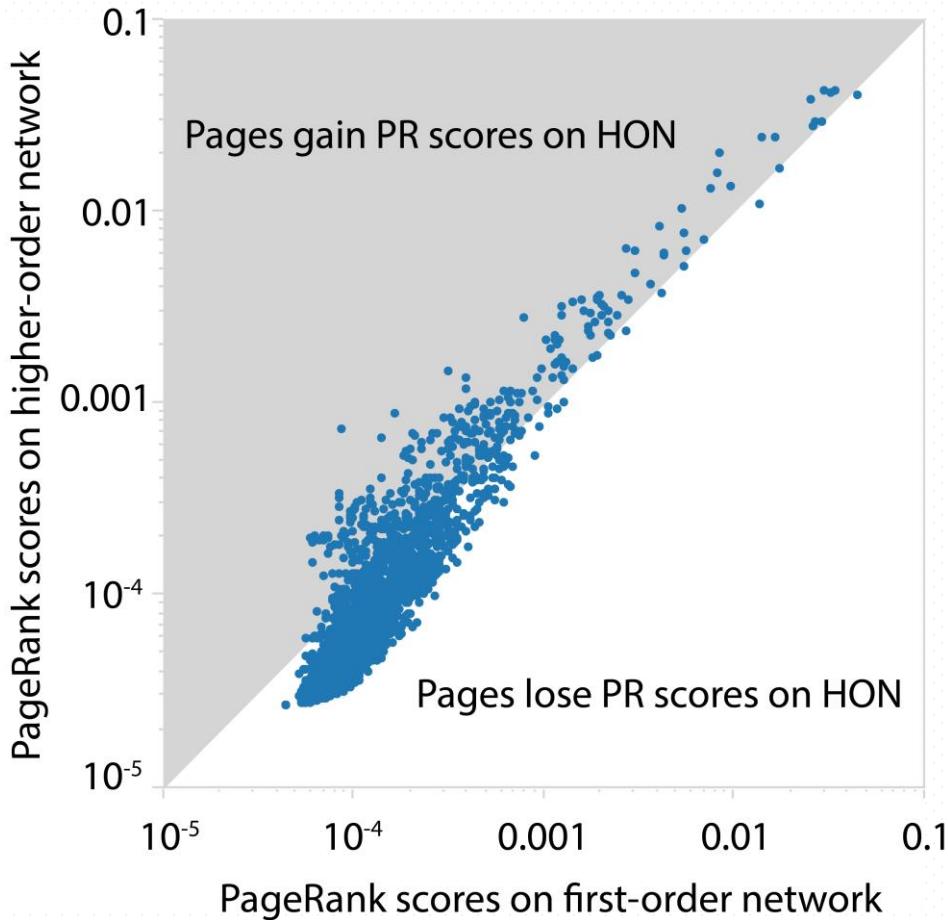


Ranking on clickstream network

Simulate 1 step Simulate 2 steps Simulate 3 steps



Ranking on clickstream network



- 26% pages show more than 10% changes in ranking
- More than 90% pages lose PageRank scores, while a few pages gain significant scores

**No changes
to the ranking algorithm**

Algorithm 3 HONOR rule extraction algorithm. Given the raw sequential data T , extracts arbitrarily high orders of dependencies, and output the dependency rules R . Optional parameters include $MaxOrder$, $MinSupport$, and $ThresholdMultiplier$

```

1: define global  $C$  as nested counter
2: define global  $D, R$  as nested dictionary
3: define global  $SourceToExtSource$ ,  $StartingPoints$  as dictionary
4:
5: function EXTRACTRULES( $T$ , [ $MaxOrder$ ,  $MinSupport$ ,  $ThresholdMultiplier = 1$ ])
6:   global  $MaxOrder$ ,  $MinSupport$ ,  $Aggresiveness$ 
7:   BUILDFIRSTORDEROBSERVATIONS( $T$ )
8:   BUILDFIRSTORDERDISTRIBUTIONS( $T$ )
9:   GENERATEALLRULES( $MaxOrder$ ,  $T$ )
10:
11:  function BUILDFIRSTORDEROBSERVATIONS( $T$ )
12:    for  $t$  in  $T$  do
13:      for ( $Source$ ,  $Target$ ) in  $t$  do
14:         $C[Source][Target] += 1$ 
15:         $IC.add(Source)$ 
16:
17:  function BUILDFIRSTORDERDISTRIBUTIONS( $T$ )
18:    for  $Source$  in  $C$  do
19:      for  $Target$  in  $C[Source]$  do
20:        if  $C[Source][Target] < MinSupport$  then
21:           $C[Source][Target] = 0$ 
22:        for  $Target$  in  $C[Source]$  do
23:          if  $C[Source][Target] > 0$ 
24:             $D[Source][Target] = C[Source][Target]/(\sum C[Source][*])$ 
25:
26:  function GENERATEALLRULES( $MaxOrder$ ,  $T$ )
27:    for  $Source$  in  $D$  do
28:      ADDTORULES( $Source$ )
29:      EXTENDRULE( $Source$ ,  $Source$ , 1,  $T$ )
30:
31:  function KLDTHRESHOLD( $NewOrder$ ,  $ExtSource$ )
32:    return  $ThresholdMultiplier \times NewOrder/\log_2(1 + \sum C[ExtSource][*])$ 

```

Algorithm 3 (continued)

```
33: function EXTENDRULE(Valid, Curr, order, T)
34:   if Order  $\leq$  MaxOrder then
35:     ADDTORULES(Source)
36:   else
37:     Distr = D[Valid]
38:     if  $-\log_2(\min(Distr[*].vals)) < \text{KLDTHRESHOLD}(order + 1)$ , Curr then
39:       ADDTORULES(Valid)
40:     else
41:       NewOrder = order + 1
42:       Extended = EXTENDSOURCE(Curr)
43:       if Extended =  $\emptyset$  then
44:         ADDTORULES(Valid)
45:       else
46:         for ExtSource in Extended do
47:           ExtDistr = D[ExtSource]
48:           divergence = KLD(ExtDistr, Distr)
49:           if divergence  $> \text{KLDTHRESHOLD}(NewOrder, ExtSource)$  then
50:             EXTENDRULE(ExtSource, ExtSource, NewOrder, T)
51:           else
52:             EXTENDRULE(Valid, ExtSource, NewOrder, T)
53:
54: function ADDTORULES(Source):
55:   for order in  $[1..len(Source) + 1]$  do
56:     s = Source[0 : order]
57:     if not s in D or len(D[s]) == 0 then
58:       EXTENDSOURCE(s[1:])
59:     for t in C[s] do
60:       if C[s][t]  $> 0$  then
61:         R[s][t] = C[s][t]
62:
63: function EXTENDSOURCE(Curr)
64:   if Curr in SourceToExtSource then
65:     return SourceToExtSource[Curr]
66:   else
67:     EXTENDOBSERVATION(Curr)
68:     if Curr in SourceToExtSource then
69:       return SourceToExtsource[Curr]
70:     else
71:       return  $\emptyset$ 
```

Algorithm 3 (continued)

```
72: function EXTENDOBSERVATION(Source)
73:   if length(Source) > 1 then
74:     if not Source[1 :] in ExtC or ExtC[Source] =  $\emptyset$  then
75:       EXTENDOBSERVATION(Source[1 :])
76:     order = length(Source)
77:     define ExtC as nested counter
78:     for Tindex, index in StartingPoints[Source] do
79:       if index - 1  $\leq$  0 and index + order < length(T[Tindex]) then
80:         ExtSource = T[Tindex][index - 1 : index + order]
81:         ExtC[ExtSource][Target] += 1
82:         StartingPoints[ExtSource].add((Tindex, index - 1))
83:       if ExtC =  $\emptyset$  then
84:         return
85:       for S in ExtC do
86:         for t in ExtC[s] do
87:           if ExtC[s][t] < MinSupport then
88:             ExtC[s][t] = 0
89:             C[s][t] += ExtC[s][t]
90:             CsSupport =  $\sum$  ExtC[s][*]
91:             for t in ExtC[s] do
92:               if ExtC[s][t] > 0 then
93:                 D[s][t] = ExtC[s][t]/CsSupport
94:                 SourceToExtSource[s[1 :]].add(s)
95:
96: function BUILDSOURCETOEXTSOURCE(order)
97:   for source in D do
98:     if len(source) = order then
99:       if len(source) > 1 then
100:         NewOrder = len(source)
101:         for starting in [1..len(source)] do
102:           curr = source[starting :]
103:           if not curr in SourceToExtSource then
104:             SourceToExtSource[curr] =  $\emptyset$ 
105:           if not NewOrder in SourceToExtSource[curr] then
106:             SourceToExtSource[curr][NewOrder] = {}
107:             SourceToExtSource[curr][NewOrder].add(source)
```

Rank	Risk of single-step direct invasion	Risk of multi-step indirect invasion
1	Murmansk, RUS	Tromso, NOR
2	Tromso, NOR	Reykjavik, ISL
3	Dudinka, RUS	Murmansk, RUS
4	Glomfjord, NOR	Hammerfest, NOR
5	Hammerfest, NOR	Nuuk, GRL
6	Kirkenes, NOR	Kirkenes, NOR
7	Grundartangi, ISL	Harstad, NOR
8	Harstad, NOR	Dutch Harbor, USA
9	Hammerfall, NOR	Grundartangi, ISL
10	Bodo, NOR	Aasiaat, GRL

The method also adapts to

Transportation



Flow of information



Evolution of diseases







Sequential data

①

Ship-001: ..., Tokyo, Singapore, Los Angeles, ...
Ship-002: ..., Shanghai, Singapore, Seattle, ...

⋮

Count subsequences of various orders

Singapore → Los Angeles: 60
Singapore → Seattle: 65
Shanghai → Singapore → Los Angeles: 30
Shanghai → Singapore → Seattle: 5

⋮

