0.1 Algorithm

Model:

$$y = X\beta + \epsilon$$

Transform:

$$\hat{\beta} = \frac{X^T X\beta}{n} + \frac{X^T \epsilon}{n}$$

$$E(\hat{\beta}) = \frac{X^T X}{n}\beta = \Omega\beta$$

$$Var(\hat{\beta}) = \frac{X^T}{n} Var(y)(\frac{X^T}{n})^T = \frac{X^T X}{n^2} = \frac{\Omega}{n}$$

where

$$\hat{\beta} \sim MVN(\Omega\beta, \Omega/n)$$

$$\beta_{k_j} \sim N(0, \sigma_k^2)$$

(+ prior distribution for σ_k^2)
Joint distribution is

$$\prod_{k=1}^{K}\prod_{j=1}^{n_k} p(y|\beta_{k_j}, \sigma_e^2) \times \prod_{k=1}^{K}\prod_{j=1}^{n_k} p(\beta_{k_j}|\sigma_k^2) \times \prod_{k=1}^{K} p(\sigma_k^2|\alpha_k, \tau^{-1})$$

Which could be written as

$$\underbrace{\prod_{k=1}^{K}\prod_{j=1}^{n_k} p(\hat{\beta}_{k_j}|\beta_{k_j}, \sigma_e^2)}_{Part1} \times \underbrace{\prod_{k=1}^{K}\prod_{j=1}^{n_k} p(\beta_{k_j}|\sigma_k^2)}_{Part2} \times \underbrace{\prod_{k=1}^{K} p(\sigma_k^2|\alpha_k, \tau^{-1})}_{Part3}$$

when y is normalized and $\sigma_e^2 = 1$

$$Part1 \propto exp[-\frac{1}{2}(\hat{\beta} - \Omega\beta)^T (\Omega/n)^{-1} (\hat{\beta} - \Omega\beta)]$$

$$Part1 \propto \prod_{k=1}^{K}\prod_{j=1}^{n_k} exp[-\frac{1}{2}(n\hat{\beta}_{k_j}\Omega^{-1}_{k_j,k_j} - 2n\hat{\beta}_{k_j}\beta_{k_j} + n\beta^2_{k_j}\Omega_{k_j,k_j} + 2n\beta_{k_j}(\Sigma\beta_{-k_j}\Omega_{k_j,-k_j}))]$$

$$Part2 \propto \prod_{k=1}^{K}\prod_{j=1}^{n_k} \sigma_k^{-1} exp[-\frac{1}{2}\beta^2_{k_j}\sigma_k^{-2}]$$

Part3:

The full conditional posterior distribution of β_{k_j} is

$$p(\beta_{k_j}) \propto exp[-\frac{1}{2}(\beta_{k_j}^2\sigma_k^{-2} + n\hat{\beta}_{k_j}^2\Omega_{k_j,k_j}^{-1} - 2n\hat{\beta}_{k_j}\beta_{k_j} + n\beta_{k_j}^2\Omega_{k_j,k_j} + 2n\beta_{k_j}(\Sigma\beta_{-k_j}\Omega_{k_j,-k_j}))]$$