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# **FORMULA DERIVATION - POLYGENIC MODEL**

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## 0.1 ALGORITHM

Model:

$$y = X\beta + \epsilon$$

Transform:

$$\hat{\beta} = \frac{X^T X \beta}{n} + \frac{X^T \epsilon}{n}$$

$$E(\hat{\beta}) = \frac{X^T X}{n} \beta = \Omega \beta$$

$$\text{Var}(\hat{\beta}) = \frac{X^T}{n} \text{Var}(y) \left(\frac{X^T}{n}\right)^T = \frac{X^T X}{n^2} = \frac{\Omega}{n}$$

where

$$\hat{\beta} \sim MVN(\Omega\beta, \Omega/n)$$

$$\beta_{k_j} \sim N(0, \sigma_k^2)$$