
FORMULA DERIVATION - POLYGENIC MODEL

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0.1 ALGORITHM

Model:

$$y = X\beta + \epsilon$$

Transform:

$$\hat{\beta} = \frac{X^T X \beta}{n} + \frac{X^T \epsilon}{n}$$

$$E(\hat{\beta}) = \frac{X^T X}{n} \beta = \Omega \beta$$

$$\text{Var}(\hat{\beta}) = \frac{X^T}{n} \text{Var}(y) \left(\frac{X^T}{n}\right)^T = \frac{X^T X}{n^2} = \frac{\Omega}{n}$$

where

$$\hat{\beta} \sim \text{MVN}(\Omega \beta, \Omega/n)$$

$$\beta_{k_j} \sim N(0, \sigma_k^2)$$

(+ prior distribution for σ_k^2)

Joint distribution is

$$\prod_{k=1}^K \prod_{j=1}^{n_k} p(y|\beta_{k_j}, \sigma_k^2) \times \prod_{k=1}^K \prod_{j=1}^{n_k} p(\beta_{k_j}|\sigma_k^2) \times \prod_{k=1}^K p(\sigma_k^2|\alpha_k, \tau^{-1})$$