State Space LSTM for NLP

T. Kirscher, Y. Remmache, L. Morisset

ENSAE - IP Paris

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Introduction

- Significance of LSTMs in sequence modeling
  - LSTMs' ability to capture long-range dependencies
  - Applications in natural language processing (NLP)
- Interpretability advantages of state space models
- **Motivation**: Integrating interpretability of state space models with the power of LSTMs
- **Reference Paper**: [1] *State Space LSTM Models with Particle MCMC Inference*, Xun Zheng et al., CoRR, 2017

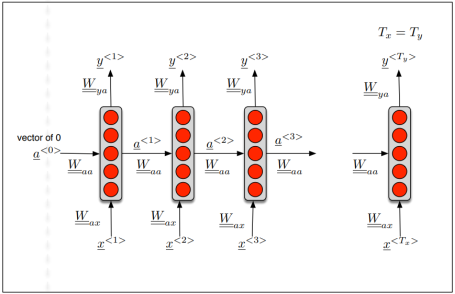
Overview

Table of content

Reccurent Neural Network: a type of neural network designed to process sequential data

Forward propagation (one hidden layer):

$$s_t = g_1(W_{ss}s_{t-1} + W_{sx}x_t + b_s)$$
$$\hat{y}_t = g_2(W_{ys}s_t + b_y)$$



Back Propagation Through Time (BPTT) and Vanishing Gradient

To train an RNN we need to back-propagate through layers and through time.

$$\frac{\partial \mathcal{L}}{\partial W_{ss}} = \sum_{t=1}^{t} \frac{\partial \mathcal{L}^{(t)}}{\partial W_{ss}}$$

$$\frac{\partial \mathcal{L}^{(t)}}{\partial W_{ss}} = \sum_{k=0}^{t} \left(\prod_{i=k+1}^{t} \frac{\partial s_i}{\partial s_{i-1}} \right) \frac{\partial s_k}{\partial W_{ss}}$$

Vanishing gradient appends when $|\frac{\partial s_i}{\partial s_{i-1}}| < 1$:

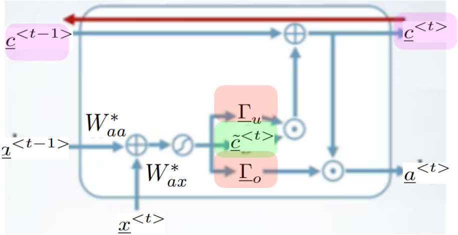
- Contributions from faraway steps vanish and don't affect the training
- Difficult to learn long-range dependencies

LSTM: a type of RNN introduced to learn long-term dependencies

Simplified LSTM:

We add a new path: the **memory cell** c_t

- The candidate cell value $\tilde{c}_t$ is added to the memory cell if the update gate Γ_u is open
- The hidden state is output if the output gate Γ_o is open



LSTM vs. SSM

Model	Pros	Cons
LSTM	Capture long-term dependencies	Not interpretable
SSM	Interpretable	Cannot capture long-term dependencies due to the Markovian nature of the latent variable process

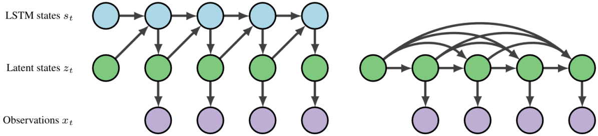
Table: Comparison of LSTM and SSM

SSL: a combination of SSM and LSTM models

Main idea: we place an LSTM on the latent space to simulate the states z_t. We then generate the observations x_t from a SSM.

Generative process:

1. $s_t = LSTM(s_{t-1}, z_{t-1})$ (LSTM part)
2. $z_t | z_{1:t-1} \sim p_\omega(z_t | z_{1:t-1}) =: p_\omega(z; g(s_t))$ (LSTM part)
3. $x_t | z_t \sim p_\phi(x; h(z_t))$ (SSM part)



(a) Full model with explicit LSTM states. (b) Model after collapsing LSTM states.

Figure 1: Generative process of SSL.

Background

Topical SSL:

- $x = (x_1, \ldots, x_t, \ldots, x_T)$ is a text sequence of length T
- $z = (z_1, \ldots, z_t, \ldots, z_T)$ is a sequence of unobserved topics characterizing the words in x

Example:

- $x =$ ("I", "was", "born", "in", "Palaiseau", "in", "1999")
- $z =$ ("Pronoun", "Verb", "Verb", "Preposition", "City", "Preposition", "Date")

Goal: create a generative model. Given a sequence $(x_1, \ldots, x_t)$, we want to predict $(x_{t+1}, \ldots, x_T)$.

Generative Model

x_t and z_t are discrete variables:

- $x_t \in \{1, \ldots, N\}$
- $z_t \in \{1, \ldots, K\}$

Where N is the size of the vocabulary and K is the number of topics. The SSL model can be written as:

1. $z_t | z_{1:t-1} \sim \text{Categorical}(\text{softmax}(Ws_t + b))$ $(LSTM)$
2. $x_t | z_t \sim \text{Categorical}(\phi_{z_t})$ (SSM)

Where:

- $\text{softmax}(Ws_t + b) = [p_\omega(z_t = 1 | z_{1:t-1}), \ldots, p_\omega(z_t = K | z_{1:t-1})]'$
- $\phi_{z_t} = [p_\phi(x_t = 1 | z_t), \ldots, p_\phi(x_t = N | z_t)]'$

Dataset used, pre processing and technical aspects

Dataset:

- IMBD dataset [2] : 25,000 movie reviews, keep $n = 200$ reviews
- Consider the 5000 "top words": $N = 5000$
- Truncation of the sequences: $T = 170$
  - $T_{\text{train}} = 100$
  - $T_{\text{test}} = 70$

LSTM:

- 64 hidden neurons
- Add a dropout layer with $p = 0.2$
- Optimizer: Adam
- z_t variables are one hot encoded in LSTM part

Particle Gibbs: a way to sample from the variational distribution in EM algorithm

Main idea:

We combine stochastic EM algorithm with backpropagation to train the SSL model. For each text and at each iteration of the EM algorithm, we sample $z^*_{1:T_{\text{train}}}$ from the optimal variational distribution in order to compute the MLE of the SSM and the LSTM. To sample $z^*_{1:T_{\text{train}}}$ we use an algorithm called **Particle Gibbs**.

Particle Gibbs Inference

Overview

- Sequential Monte Carlo (SMC) method
- Particle Gibbs inference for sampling from joint posterior
- No factorization assumptions

Particle Gibbs Inference

Algorithm

Algorithm Inference with Particle Gibbs

Require: P: number of particules, T: length of the sequence

Initialize $z_0^p = z_0$ and $\alpha_0^p = \frac{1}{P}$ for $p = 1, \dots, P$
for $t = 1, \dots, T$ **do**
    Fix reference path: set $a_{t-1}^1 = 1$ and $z_{1:t}^1 = z_{1:t}^*$ from the previous iteration
    for $p = 2, \dots, P$ **do**
        Sample ancestors $a_{t-1}^p \sim \alpha_{t-1}$
    end for
    for $p = 2, \dots, P$ **do**
        Sample particles $z_t^p \sim \gamma_t^p$ and set $z_{1:t}^p = (z_{1:t-1}^{a_{t-1}^p}, z_t^p)$
    end for
    for $p = 1, \dots, P$ **do**
        Compute normalized weights α_t^p
    end for
end for
Sample $r \sim \alpha_T$
return the particle path $z_{1:T}^{a_T^r}$

Particle Gibbs Inference

Recall:

- $\text{softmax}(Ws_t + b) = [p_\omega(z_t = 1|z_{1:t-1}), \ldots, p_\omega(z_t = K|z_{1:t-1})]'$
- $\phi_{z_t} = [p_\phi(x_t = 1|z_t), \ldots, p_\phi(x_t = N|z_t)]'$

Let $\phi = [\phi_1, \ldots, \phi_K] \in \mathbb{R}^{N \times K}$, such as $\phi[i,j] = p_\phi(x_t = i|z_t = j)$. Here:

- $\alpha_t = p(x_t|z_{1:t-1}) \propto \sum_{k=1}^{K} \text{softmax}(Ws_t + b)[k] \odot \phi_k$: unormalized distribution on $\{1, \ldots, N\}$
- $\gamma_t = p(z_t|z_{1:t-1}, x_t) \propto \text{softmax}(Ws_t + b) \odot \phi[x_t, 1:K]$: unormalized distribution on $\{1, \ldots, K\}$

Training Loop

Algorithm Training Loop for Topical SSL

Require: S_x: set of training sequences, P: number of particules, K: number of topics, N: size of the vocabulary, n: number of training samples, n_epochs: number of epochs

```
for i = 1, ..., n do
    z*_{1:T_train} ~ i.i.d U{1, ..., K}
end for
for epoch = 1, ..., n_epochs do
    S_{z*_{1:T_train}} = [ ]
    for i = 1, ..., n do
        Compute z*_{1:T_train} by Particle Gibbs using previous z*_{1:T_train}
        Add z*_{1:T_train} to S_{z*_{1:T_train}}
        Train LSTM with z*_{1:T_train} by back-propagation
        Compute the MLE of the SSM on S_x[1 : i] and S_{z*_{1:T_train}}[1 : i]
    end for
end for
```

- **for** $i = 1, \ldots, n$ **do**
  - $z^*_{1:T_{\text{train}}} \sim$ i.i.d $\mathcal{U}\{1, \ldots, K\}$
- **end for**
- **for** epoch $= 1, \ldots,$ n_epochs **do**
  - $S_{z^*_{1:T_{\text{train}}}} = [\,]$
  - **for** $i = 1, \ldots, n$ **do**
    - Compute $z^*_{1:T_{\text{train}}}$ by Particle Gibbs using previous $z^*_{1:T_{\text{train}}}$
    - Add $z^*_{1:T_{\text{train}}}$ to $S_{z^*_{1:T_{\text{train}}}}$
    - Train LSTM with $z^*_{1:T_{\text{train}}}$ by back-propagation
    - Compute the MLE of the SSM on $S_x[1:i]$ and $S_{z^*_{1:T_{\text{train}}}}[1:i]$
  - **end for**
- **end for**

Implementation Details

- All the algorithms coded in `pytorch`
    - GPU usage for LSTM
    - CPU usage for other parts
- n_epochs $= 5$
- $P = 10$
- $K \in \{10, 50, 100\}$
- Evaluation metric: perplexity

Perplexity curve for the LSTM model

Baseline

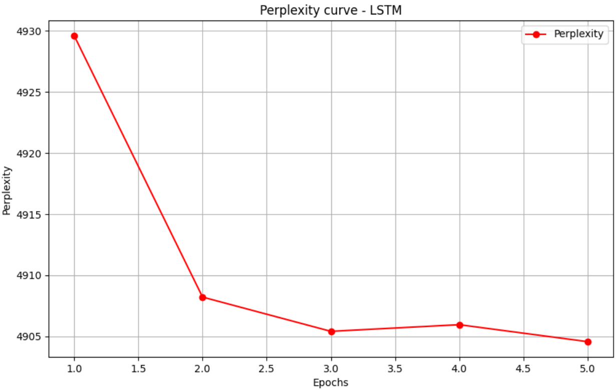
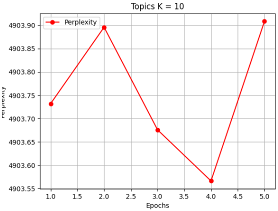
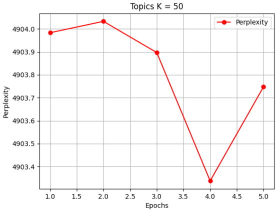


Figure: Perplexity through epochs for standard LSTM

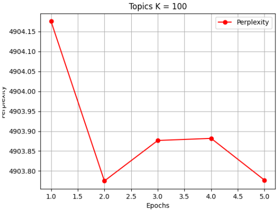
Perplexity curves for the SSL model



10 Topics



50 Topics



100 Topics

Topics Visualization (Interpretability)

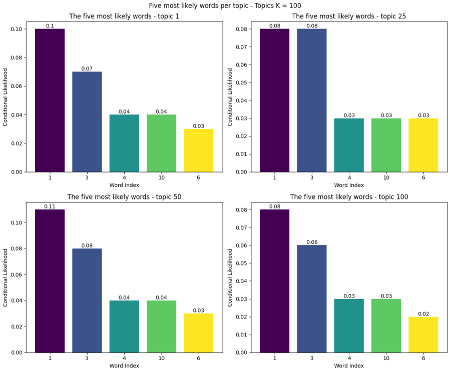


Figure: Latent Topics Visualization

Key insight: Conditionally on topic 50, the third word in the vocabulary is the second word most likely to appear, with a conditional probability of 0.08.

Conclusion: pros and cons of SSL

Pros:

- Similar performances to the LSTM model on our dataset. In the article, LSTM provides better results, but the amount of data is much greater
- Results are interpretable (main advantage)

Cons:

- Computational complexity: assuming that all internal operations can be performed in constant time (which is not the case), the algorithmic complexity of the Particle Gibbs algorithm is $\mathcal{O}(P + T(P-1)^2 P)$ for a single observation
- Concretely, with the same set-up: about 5 minutes to run one epoch with LSTM versus about 4 hours with SSL
- Choice of K: K too high can lead to overfitting whereas K too low can lead to underfitting

References

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Q & A

Thank you!
Questions?