# Excercise 4
# Implementing a centralized agent

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## 1 Solution Representation

### 1.1 Variables

Instead of representing a vehicle's journey as a sequence of tasks, we chose to represent it as a sequence of *pickup* and *delivery* actions. Each task $t \in \mathcal{T}$ has one action of both types (1). This accounts for the fact that a vehicle can carry multiple tasks at a time if there are two pickups in a row. The variables (2) define the first pickup of each vehicle $v \in \mathcal{V}$. If the variable is `null` this means the vehicle does not accomplish any actions.

$$\mathcal{P} = \{pickup(t) : t \in \mathcal{T}\},\ \mathcal{D} = \{delivery(t) : t \in \mathcal{T}\},\ \mathcal{A} = \mathcal{P} \cup \mathcal{D} \tag{1}$$

$$\forall v \in \mathcal{V} :\ firstPickup(v) \in \mathcal{P} \cup \{\texttt{null}\} \tag{2}$$

$$\forall a \in \mathcal{A} :\ nextAction(a) \in \mathcal{A} \cup \{\texttt{null}\} \tag{3}$$

$$\forall a \in \mathcal{A} : vehicle(a) \in \mathcal{V};\ \ \forall a \in \mathcal{A} : time(a) \in \mathbb{N} \tag{4}$$

A vehicles journey is completely defined by its $firstPickup$ and the variables (3) where again the `null` signifies that a vehicle has no further actions to perform. We will call the sequence of actions of a vehicle its action chain. All travels are made on the shortest possible path.

The variables (4) help us state the constraints. The *vehicle* variables define which vehicle carries out a certain action. This can be derived from $firstPickup$ at the start of the action chain defined by (3). The second variable can also be derived from the action chains. It simply gives the rank of each action in the chain. Both derivations are more formally stated in the next paragraph.

### 1.2 Constraints

As explained before, the action chain of a vehicle defines the *time* and the *vehicle* for each action:

$$firstPickup(v) = a \Rightarrow vehicle(a) = v;\quad nextAction(b) = c \Rightarrow vehicle(c) = vehicle(b) \tag{5}$$

$$firstPickup(v) = a \Rightarrow time(a) = 1;\quad nextAction(b) = c \Rightarrow time(c) = time(b) + 1 \tag{6}$$

Additionally, the same vehicle must *pickup* and *deliver* a task (9). It has to pickup the task before it delivers it (10) and **each task must be picked up and delivered**.

$$\forall a \in \mathcal{A} : nextAction(a) \neq a \tag{7}$$

$$nextAction(a) = \texttt{null} \Rightarrow a \in \mathcal{D} \text{ and } nextAction(\texttt{null}) = \texttt{null} \tag{8}$$

$$\forall t \in \mathcal{T} : vehicle(pickup(t)) = vehicle(delivery(t)) \tag{9}$$

$$\forall t \in \mathcal{T} : time(pickup(t)) < time(delivery(t)) \tag{10}$$

$$\forall t \in \mathcal{T}\ \exists \{pickup(t), delivery(t)\} \subset \mathcal{A} \tag{11}$$

$$\forall a \in \mathcal{A}\ \exists v \in \mathcal{V} : vehicle(a) = v \tag{12}$$

Last but not least, at all times $\tau$ a vehicle $v$ can never carry more weight than its capacity.

$$carriedTasks(\tau, v) = \{t \in \mathcal{T} : vehicle(pickup(t)) = v \wedge time(pickup(t)) < \tau \wedge time(delivery(t)) > \tau\}$$

$$\forall \tau \in \mathbb{N}, \forall v \in \mathcal{V} : \sum_{t \,\in\, carriedTasks(\tau,v)} weight(t) \leq capacity(v)$$

### 1.3 Objective function

The goal of the company is to maximise the reward. Because all tasks have to be delivered, all rewards will be earned and the overall reward is constant. Thus the objective function we want to minimise is the cost of the overall assignment $\mathcal{S}$. We define $dist(a, b)$ to be the shortest distance between the associated cities of actions $a, b \in \mathcal{A}$, $dist(a, \texttt{null}) = 0$, $start(v)$ is the initial postion and $cost(v)$ the cost per kilometre of vehicle $v$.

$$cost(\mathcal{S}) = \sum_{v \in \mathcal{V}} dist(start(v), firstPickup(v)) \cdot cost(v) + \sum_{a \in \mathcal{A}} dist(a, nextAction(a)) \cdot cost(vehicle(a))$$

## 2 Stochastic optimization

### 2.1 Initial solution

Our initial solution is already a valid, greedy solution. Therefore, our program returns a correct assignment at all times, even if there is no computation time. The initial solution is computed by appending consecutively the *pickup* and *delivery* pair of each task to the action chain of the vehicle that is the closest (and has enough capacity). This distance is calculated between the last position in the action chain of each vehicle and the *pickup* location. Therefore in the initial solution the vehicles don't carry multiple tasks at once.

### 2.2 Generating neighbours

We generate neighbors of a current assignment $\mathcal{S}$ by applying two stochastic operators. For both of them, we randomly choose a vehicle $v$. Then, for the first set of neighbors, we remove both actions $p, d$ for a random task $t$ from the action chain of $v$. The neighbors result from inserting $p, d$ into the action chains of all other neighbors. This yields a lot of possibilities because the $p, d$ can be inserted in many ways into the new action chain as long as the capacity and the $p$ before $d$ constraint is respected. The second set of neighbors is obtained from reordering the actions in $v$'s chain without removing or inserting. Still the reordering has to respect the capacity and the $p$ before $d$ constraint.

### 2.3 Stochastic optimization algorithm

As proposed in the paper[1] we use stochastic local search to find a better solution than the initial one. Thereby we generate neighbors at each iteration and with probability $p$, we take the least costly neighbor as a new solution.

We made a critical addition to the algorithm that avoids getting stuck in local minima. In fact, if there are two pickups at the same location, this yields two solutions depending on the order of the pickups. However, the solutions are completely equivalent in terms of cost. Therfore, each time a new solution is chosen, we add this solution to a set called `formerSolutions`. In addition, the algorithm is not allowed to subsequently choose a solution with the **same cost** as one that is already in the set. In that manner it has to keep exploring new solutions with new and possibly lower cost. At the end the overall minimum that was ever visited is returned.

[1] Radu Jurca, Nguyen Quang Huy and Michael Schumacher *Finding the Optimal Delivery Plan: Model as a Constraint Satisfaction Problem* 2006-2007: Intelligent Agents course

# 3 Results

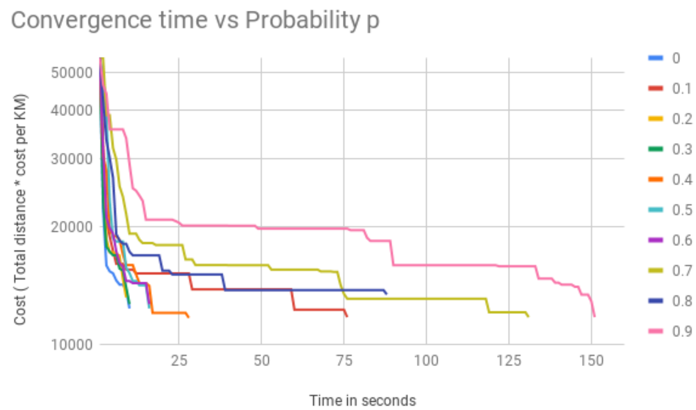
## 3.1 Experiment 1: Model parameters

### 3.1.1 Setting

Default configuration given in handout (30 tasks, 4 cars). Probability $p$ between 0 and 0.9

### 3.1.2 Observations

As we can see in the graph on the right, a higher probability $p$ value takes more time to converge towards an optimal solution than lower $p$ values. This makes sense because with a high probability $p$ it is likely that the algorithm choose a new solution more often. Thus more solutions are generated.




Other than the time to converge to a good solution, the cost of the final solution itself (given enough time) does not seem to depend on the probability $p$. Indeed, after 3 minutes, all plans cost between 10,000 and 13,000 which is a relatively small range.

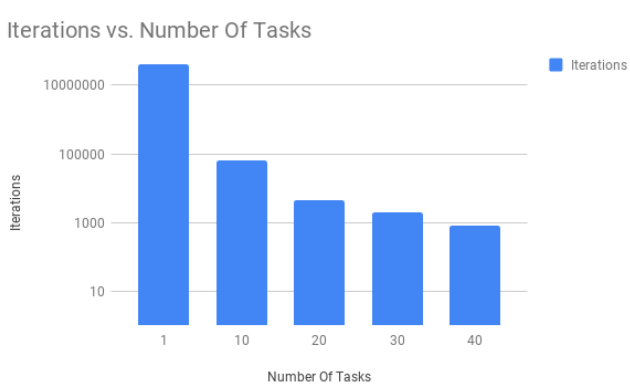
## 3.2 Experiment 2: Different configurations

### 3.2.1 Setting

Default configuration with tasks from 1 to 40 and number of cars from 1 to 4.

### 3.2.2 Observations

Changing the number of cars has very little influence on the cost of a plan. Indeed, with one car (and 30 tasks) we were able to find a plan with a cost of 12'451 and with four cars we found a plan with a cost of 11'703. The reason for this small difference in cost is that in the better solution with four cars only one car is actually delivering tasks. This is because there is no notion of time efficiency to deliver all tasks, only the distance travelled is relevant. It is therefore normal that you can obtain similar results using only one car and that plans with multiple cars are unfair (some drivers will not deliver any tasks).




In the graph on the left we see that the complexity of the algorithm increases with the number of tasks. This is to be expected as in every iteration of the algorithm we calculate the possible neighbors as explained in part 2.2. Finding all possible combinations of pickups and deliveries of a plan takes time $\mathcal{O}(n^3)$, where $n$ is the number of tasks in the plan.

The complexity of our algorithm is however not dependent on the number of vehicles because tasks are not evenly distributed between cars (most cars take constant time $\mathcal{O}(n^3) = \mathcal{O}(1)$ per iteration as no tasks are given to them $n = 0$).