JEE Main Exam 2015
(Paper & Solution)

Code – A

Date : 04-04-2015

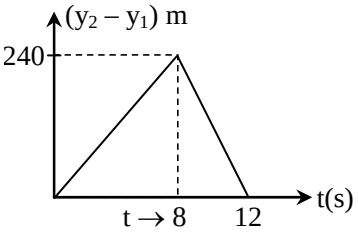
Part A – PHYSICS

Q.1 Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ m/s^2$)

(*The figures are schematic and not drawn to scale*)

(1)



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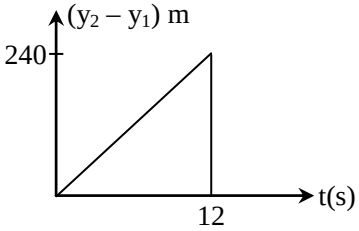

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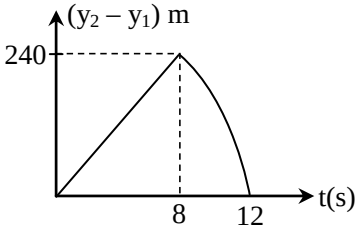

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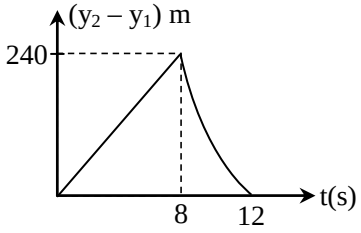


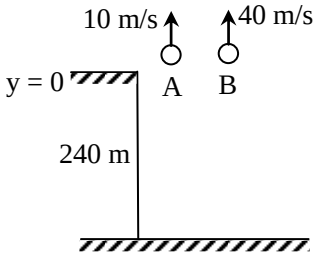
Ans. **[3]**

Sol.

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Motion in one Dimension, Level # 4A, Page No. 75, Q.12]

[JEE Advance, Chapter : Motion in one Dimension, Ex. # 5, Page No. 90, Q. 48]



(i) Time taken by A to reach the ground

$$-240 = 10t - 5t^2$$

or $5t^2 - 10t - 240 = 0$

$\therefore \quad t = 8\ s$

and time taken by B to reach the ground

$$-240 = 40t - 5t^2$$

$\therefore \quad t = 12\ s$

(ii) For $0 \le t \le 8$ sec

$y_1 = 10t - 5t^2$

$y_2 = 40t - 5t^2$

$y_2 - y_1 = 40t - 10t$

or $y_2 - y_1 = 30t$

For $0 \le t \le 8$ graph will be straight line

(iii) at $t = 8$ sec,

$y_1 = -240$ m

and $y_2 = 40(8) - 5(8)^2 = 0$

(iv) For $8 \le t \le 12$

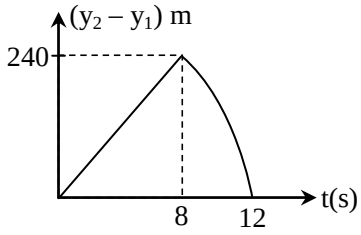
$y_1 = -240$ and

$y_2 = 40t - 5t^2$

$y_2 - y_1 = 40t - 5t^2 + 240$

or $y_2 - y_1 = -5t^2 + 40t + 240$

So graph will be parabolic with –ve slope



Q.2 The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is -

(1) 2 % (2) 3 %

(3) 1 % (4) 5 %

Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Advance, Chapter : Practical Physic, Ex. # 4, Page No. 110, Q. 7]

Measured length $L = 20.0$ cm

error in length $\Delta L = 1$ mm $= 0.1$ cm

For 100 oscillation time measured $t = 90$ s

error in this measured time $\Delta t = 1$ s

$t = nT$ ….(i)

(n = no. of oscillation, T = time period)

$\Delta t = n\Delta T$ ….(ii)

Equation (ii)/(i)

$$\frac{\Delta t}{t} = \frac{\Delta T}{T} \Rightarrow \frac{\Delta T}{T} = \frac{\Delta t}{t}$$

$$\Rightarrow \left(\frac{\Delta T}{T}\right) = \frac{1}{90}$$

Now $T = 2\pi\sqrt{\frac{L}{g}}$

$$g = \frac{4\pi^2 L}{T^2}$$

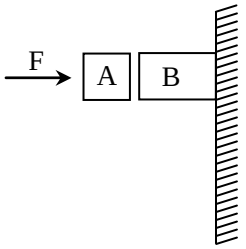
$$\frac{\Delta g}{g} = \left[\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right]$$

$$\frac{\Delta g}{g} = \left[\frac{0.1}{20.0} + 2\left(\frac{1}{90}\right)\right]$$

$$\% \text{ error} = \frac{\Delta g}{g} \times 100$$

$$\Rightarrow \left[\frac{1}{200} + \frac{1}{45}\right] \times 100 = 2.72\ \% \approx 3\%$$

Q.3 Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is -

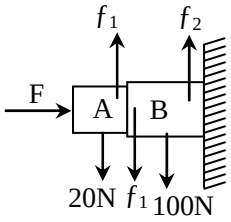


(1) 100 N (2) 80 N

(3) 120 N (4) 150 N

Ans. **[3]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Newton's Law of Motion & Friction, Level # 4B, Page No. 136, Q.1]
[JEE Advance, Chapter : Friction, Ex. # 2, Page No. 179, Q.12]



For equlibrium of A

$$f_1 = 20 \text{ N}$$

For equlibrium of B

$$f_1 + 100 = f_2$$

$$f_2 = 120 \text{ N}$$

Q.4 A particle of mass m moving in the x direction with speed 2v is hit by another particle of mass 2m moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to -

(1) 44 % (2) 50 %

(3) 56 % (4) 64 %

Ans. **[3]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Work, Power & Energy, Page No. 157, Solved Ex. 12]

$\because$ Loss of energy

$$\Delta E = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}[\vec{v}_2 - \vec{v}_1]^2$$

$$= \frac{1}{2}\frac{(m)(2m)}{m+2m}[2v\hat{i} - v\hat{j}]^2$$

$$= \frac{5}{3}mv^2$$

$$\% \text{ Loss} = \frac{\Delta E}{\frac{1}{2}m(2v)^2 + \frac{1}{2}2mv^2} \times 100$$

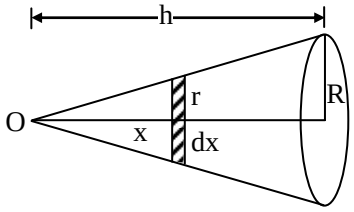
$$= \frac{\frac{5}{3}mv^2}{3mv^2} \times 100 \approx 56\%$$

Q.5 Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to -

(1) $\frac{h^2}{4R}$ (2) $\frac{3h}{4}$ (3) $\frac{5h}{8}$ (4) $\frac{3h^2}{8R}$

Ans. **[2]**

Sol.



Consider an element disc of radius r and thickness dx.

$\because$ $\frac{r}{x} = \frac{R}{h}$

$$r = \frac{R}{h}x$$

z_0 = distance of CM from O

$$= x_C = \frac{\int dm\, x}{M} = \frac{\int \rho(\pi r^2 dx).x}{M}$$

$$x_C = \frac{\pi\rho\int\left(\frac{R}{h}x\right)^2 dx.x}{M}$$

$$x_C = \pi.\frac{M}{\frac{1}{3}\pi R^2 h}.\frac{R^2}{h^2}\int_0^h x^3 dx$$

$$z_0 = x_C = \frac{3}{h^3}.\frac{h^4}{4} = \frac{3h}{4}$$

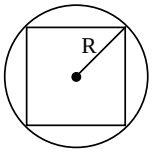
Q.6 From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its face is -

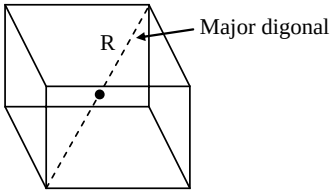
(1) $\frac{MR^2}{32\sqrt{2}\pi}$ (2) $\frac{MR^2}{16\sqrt{2}\pi}$

(3) $\frac{4MR^2}{9\sqrt{3}\pi}$ (4) $\frac{4MR^2}{3\sqrt{3}\pi}$

Ans. **[3]**

Sol.





$\because$ For cube of maximum volume distance from center of sphere to the corner of cube is equal to R.

By figure

$$\sqrt{3}\,a = 2R$$

$$a = \frac{2R}{\sqrt{3}} \text{ (side of cube)}$$

mass of cube

$$M' = \frac{M}{\frac{4}{3}\pi R^3}\left[\frac{8R^3}{3\sqrt{3}}\right] \quad (\because M' = \rho a^3)$$

$$M' = \frac{8M}{4\sqrt{3}\pi}$$

$\because$ MI of cube

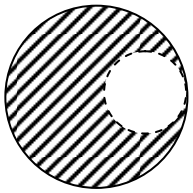
$$I = \frac{1}{6} M' a^2$$

$$= \frac{1}{6}\,\frac{8M}{4\sqrt{3}\pi}\left(\frac{2R}{\sqrt{3}}\right)^2$$

$$I = \frac{4MR^2}{9\sqrt{3}\pi}$$

Q.7 From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is -

(G = gravitational constant)



(1) $\frac{-GM}{2R}$ (2) $\frac{-GM}{R}$

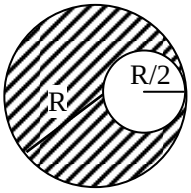
(3) $\frac{-2GM}{3R}$ (4) $\frac{-2GM}{R}$

Ans. **[2]**

Sol.

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : Gravitation, Ex. # 6, Page No. 204, Q. 9]



$\because$ Potential inside solid sphere is given by

$$V = -\frac{GM}{2R^3}[3R^2 - r^2]$$

potential at centre = potential due to large sphere – potential due to small sphere.

$$V = -\frac{GM}{2R^3}\left[3R^2 - \frac{R^2}{4}\right] + \frac{GM'}{2\left(\frac{R}{2}\right)^3}\left[3\left(\frac{R}{2}\right)^2 - 0\right]$$

$\because$ M′ = mass of small sphere

$$= \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

$$\therefore \quad V = -\frac{GM}{2R^3}\left(\frac{11R^2}{4}\right) + \frac{G(M/8)}{2R^3/8}\left[\frac{3}{4}R^2\right]$$

$$V = \frac{GM}{R}\left[-\frac{11}{8}+\frac{3}{8}\right]$$

$$V = -\frac{GM}{R}$$

Q.8 A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to -

(g = gravitational acceleration)

(1) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (2) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

(3) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (4) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

Ans. **[1]**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$ & $T_m = 2\pi\sqrt{\frac{\ell+\Delta\ell}{g}}$

$$Y = \frac{\left(\frac{F}{A}\right)}{\frac{\Delta\ell}{\ell}} \quad \Rightarrow \quad \Delta\ell = \frac{F\ell}{AY}$$

$$T_m = 2\pi\sqrt{\frac{\ell+\frac{F\ell}{AY}}{g}}$$

$$T_m = 2\pi\sqrt{\frac{\ell}{g}}\sqrt{\left(1+\frac{F}{AY}\right)}$$

$$\frac{T_m}{T} = \sqrt{1+\frac{F}{AY}} \Rightarrow \left(\frac{T_m}{T}\right)^2 = 1+\frac{F}{AY}$$

$$\left(\frac{T_m}{T}\right)^2 - 1 = \frac{F}{AY}$$

$$\frac{1}{Y} = \left[\left(\frac{T_m}{T}\right)^2 - 1\right]\frac{A}{F}$$

$$\frac{1}{Y} = \left[\left(\frac{T_m}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

Q.9 Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is -

(1) $T \propto e^{-R}$ (2) $T \propto e^{-3R}$

(3) $T \propto \frac{1}{R}$ (4) $T \propto \frac{1}{R^3}$

Ans. **[3]**

Sol. Energy per unit volume $\frac{U}{V} \propto T^4$

The process is adiabatic so dQ = 0

dQ = du + dw (from Ist law of thermodynamic)

dQ = 0

du + dw = 0

dw = – du

PdV = – du

given $P = \frac{1}{3}\frac{U}{V}$

$$\frac{1}{3}\frac{U}{V}\,dV = -\,du$$

$$\Rightarrow \quad \frac{du}{U} + \frac{1}{3}\frac{dv}{V} = 0$$

on integrating

$$\ell nU + \frac{1}{3}\,\ell n\,V = \text{const.}$$

$$\ell n\,(UV^{1/3}) = \text{const}$$

$$\Rightarrow \quad UV^{1/3} = \text{const.} \qquad ...(i)$$

given $\frac{U}{V} \propto T^4$

So $U \propto VT^4$

Substitute in equation (1)

$VT^4 V^{1/3} = \text{const}$

$T^4 V^{4/3} = \text{const.}$

$TV^{1/3} = \text{const}$

$V = \frac{4}{3}\pi R^3$

So $T\left(\frac{4}{3}\pi R^3\right)^{1/3} = \text{const}$

$TR = \text{const.}$

$\Rightarrow \boxed{T \propto \frac{1}{R}}$

Q.10 A solid body of constant heat capacity 1 J/ºC is being heated by keeping it in contact with reservoirs in two ways.

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat

In both the cases body is brought from initial temperature 100ºC to final temperature 200ºC. Entropy change of the body in the two cases respectively is -

(1) ln2, 4ln2 (2) ln2, ln2

(3) ln2, 2ln2 (4) 2ln2, 8ln2

Ans. **[2]**

Sol. $dQ = mCdT$ ….(i)

mC = heat capacity of body = 1 J/ºC

$dS = \frac{dQ}{T}$ ….(ii)

$dS \Rightarrow$ change in entropy

From (i) & (ii)

$dS = \frac{mCdT}{T}$

$dS = \frac{1 \times dT}{T}$

Integrating both side

$\int_{S_i}^{S_f} dS = \int_{T_i}^{T_f} \frac{dT}{T}$

$S_f - S_i = \Delta S = l\text{n}\frac{T_f}{T_i}$

in both case $T_f = 200$ K,

$T_i = 100$ K

$\therefore \Delta S$ in both case $= l\text{n}\frac{200}{100}$

$= l\text{n}2$

Note : In question temperature is given in °C but it should be in Kelvin (K) instead of °C

Q.11 Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is - $\left(\gamma = \frac{C_p}{C_v}\right)$

(1) $\frac{3\gamma+5}{6}$ (2) $\frac{3\gamma-5}{6}$

(3) $\frac{\gamma+1}{2}$ (4) $\frac{\gamma-1}{2}$

Ans. **[3]**

Sol. Average distance between two molecule is given by

$$\ell = \lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

$$\therefore \quad \ell = \frac{1 \qquad V}{\sqrt{2}\pi d^2 (\text{No. of molecules})}$$

or $\boxed{\ell \propto V}$ ....(1)

Average time for collision is

$$t = \frac{2\ell}{V_x} = \frac{2\sqrt{3}\ell}{V_{rms}}$$

$$t \propto \frac{2\sqrt{3}V}{\sqrt{\frac{3RT}{M}}} \propto \frac{V}{\sqrt{T}} \quad ....(2)$$

For adiabatic process

$$TV^{\gamma-1} = \text{const.}$$

$$\boxed{T \propto \frac{1}{V^{\gamma-1}}} \quad ....(3)$$

From (2) and (3)

$$t \propto \frac{V}{\left(\frac{1}{V^{\gamma-1}}\right)^{1/2}}$$

$$t \propto V \times V^{\frac{\gamma-1}{2}}$$

$$t \propto V^{\frac{\gamma+1}{2}}$$

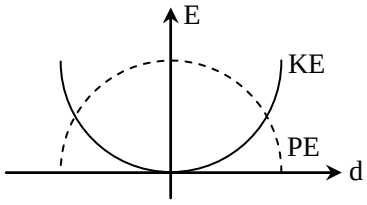
$\therefore$ On comparing with $t \propto V^q$

$\therefore$ $q = \frac{\gamma+1}{2}$

Q.12 For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ?

(*graphs are schematic and not drawn to scale*)

(1)



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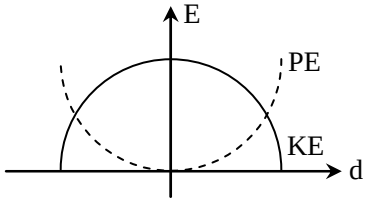

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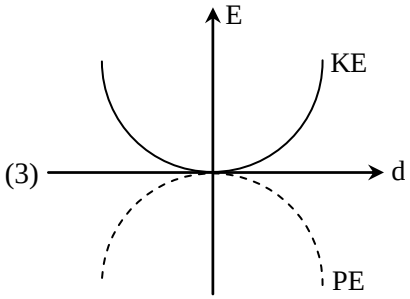

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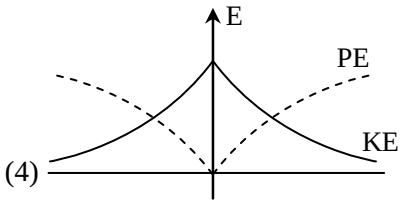


Ans. **[2]**

Sol.

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : S.H.M., Level # 1, Page No. 59, Q.31]

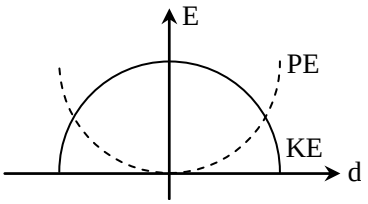
[JEE Advance, Chapter : S.H.M., Ex. # 4, Page No. 230, Q.4]

Potential energy = $\frac{1}{2}$ ky^2

where y is displacement

kinetic energy = $\frac{1}{2}k(A^2 - y^2)$

PE is minimum at $y = 0$



Q.13 A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as train passes him is (speed of sound = 320 ms^{-1}) close to -

(1) 6 % (2) 12 %

(3) 18 % (4) 24 %

Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Advance, Chapter : Doppler Effect, Page No. 256, Solved Ex. 1]

When train approaches the person apparent freq. $f_1 = \left(\frac{v}{v - v_s}\right) f$

Here $v \rightarrow$ speed of sound

$v_s \rightarrow$ Speed of train

$f \rightarrow$ original freq. of whistle

when train departs the person apparent freq

$$f_2 = \left(\frac{v}{v + v_s}\right) f$$

percentage change in freq.

$$\frac{f_1 - f_2}{f} \times 100\ \% = \left[\left(\frac{v}{v - v_s}\right) - \left(\frac{v}{v + v_s}\right)\right] \times 100\ \%$$

$$= \frac{2v\, v_s}{(v^2 - v_s^2)} \times 100\ \%$$

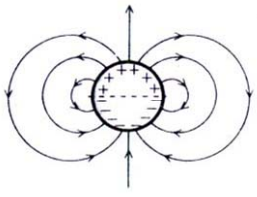
$(\because v^2 - v_s^2 \simeq v^2)$

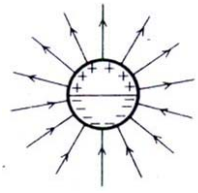
$$\simeq \frac{2v_s}{v} \times 100\ \% = \frac{2 \times 20}{320} \times 100\ \%$$

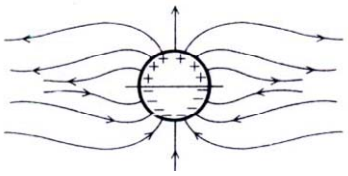
$$= 12.5\ \% \simeq 12\ \%$$

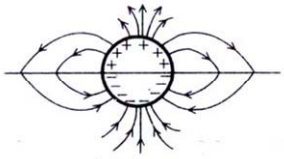
Q.14 A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in -

(*Figure are schematic and not drawn to scale*)

(1) 

(2) 

(3) 

(4) 

Ans. **[1]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Electrostatics, Level # 1, Page No. 32, Q.94]

Given charge system forming electric field dipoles. Electric field lines starts from positive charge & terminate at negative charge.

Q.15 A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then -

(1) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

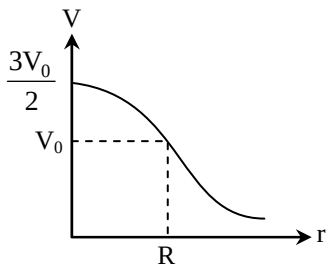
(2) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

(3) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(4) $2R < R_4$

Ans. **[3,4]**

Sol.



Given $V_0 = \frac{kq}{R}$

$\Rightarrow$ $kq = V_0R$

$r < R$, $V = \frac{kq}{2R^3}[3R^2 - r^2]$

$V = \frac{3}{2}\frac{kq}{R} - \frac{kqr^2}{2R^3}$

At $r = R_1$, $V = \frac{3V_0}{2}$

$\therefore$ $\frac{3V_0}{2} = \frac{3V_0}{2} - \frac{V_0R_1^2}{2R^2}$

$\Rightarrow$ $R_1 = 0$

At $r = R_2$, $V = \frac{5V_0}{4}$

$\frac{5V_0}{4} = \frac{3V_0}{2} - \frac{V_0R_2^2}{2R^2}$

$\Rightarrow$ $R_2 = \frac{R}{\sqrt{2}}$

For $r > R$, $V < V_0$

and given as $V = \frac{kq}{r} = \frac{V_0R}{r}$

At $r = R_3$, $V = \frac{3V_0}{4}$

$\frac{3V_0}{4} = \frac{V_0R}{R_3}$

$R_3 = \frac{4}{3}R$

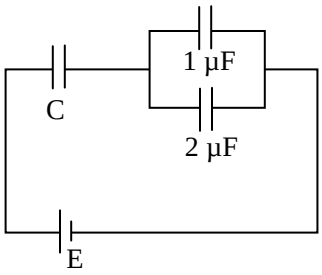
At $r = R_4$, $V = \frac{V_0}{4}$

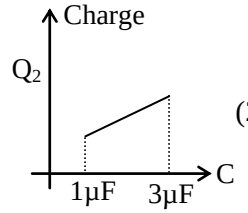
$\frac{V_0}{4} = \frac{V_0R}{R_4}$

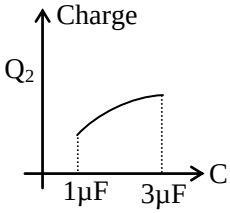
$R_4 = 4R$

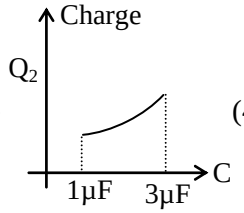
$\therefore$ $2R < R_4$

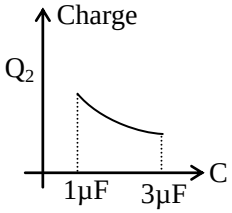
Q.16 In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by :(figures are drawn schematically and are not to scale)



(1)


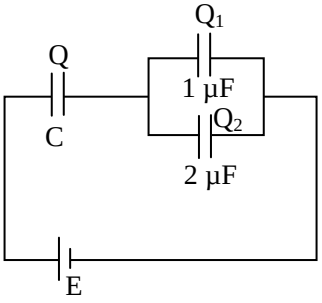
(2)


(3)


(4)


Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Capacitance, Level # 2, Q.21]



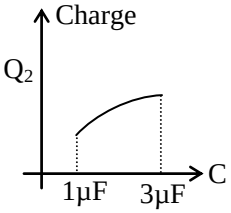
$Q = C_{eq}V = \left(\frac{C \times 3}{C+3}\right)E$

$= \frac{3CE}{C+3}$

$$Q_2 = \frac{2}{2+1}Q = \frac{2}{3}Q = \frac{2}{3} \times \frac{3CE}{C+3}$$

$$Q = \frac{2CE}{C+3} = \frac{2E}{1+\frac{3}{C}}$$

When C increase from the expression Q also increases slope of Q versus C is



$$\frac{dQ}{dC} = \frac{6E}{[C+3]^{+2}}$$

as C increases $\frac{dQ}{dC}$ decreases

Q.17 When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to :

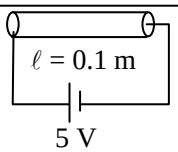
(1) 1.6×10^{-8} Ωm (2) 1.6×10^{-7} Ωm

(3) 1.6×10^{-6} Ωm (4) 1.6×10^{-5} Ωm

Ans. **[4]**

Sol. **Students may find similar question in CP exercise sheet :**

[JEE Main, Chapter : Current electricity, Level # 3, Q.18]



$E = \rho J$

$$\frac{V}{\ell} = \rho n e v_d$$

$$\rho = \frac{V}{\ell n e v_d}$$

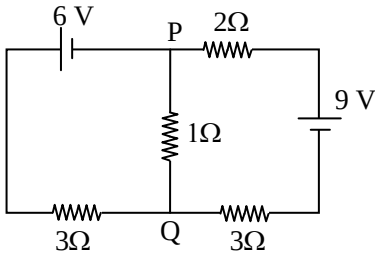
$$= \frac{5}{0.1 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}}$$

$= 0.156 \times 10^{-4}$

$= 1.56 \times 10^{-5}$

$= 1.6 \times 10^{-5}$ Ωm

Q.18



In the circuit shown, the current in the 1 Ω resistor is :

(1) 1.3 A, from P to Q

(2) 0 A

(3) 0.13 A, from Q to P

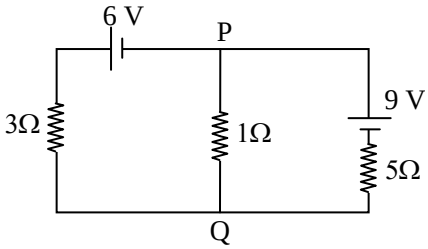
(4) 0.13 A, from P to Q

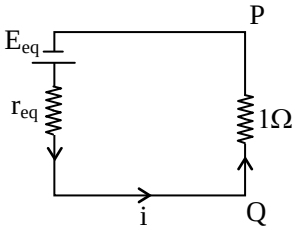
Ans. **[3]**

Sol. **Students may find similar question in CP exercise sheet :**

[JEE Main, Chapter : Current Electricity, Level # 4A, Q.19]

[JEE Advance, Chapter : Current Electricity, Ex. # 5, Q.45]





$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

$$= \frac{3\times 5}{3+5} = \frac{15}{8}$$

$$E_{eq} = \frac{\frac{E_1}{r_1} - \frac{E_2}{r_2}}{\frac{1}{r_1} - \frac{1}{r_2}}$$

$$= \frac{\frac{6}{3} - \frac{9}{5}}{\frac{1}{3} + \frac{1}{5}}$$

$$= \frac{2 - \frac{9}{5}}{\frac{5+3}{15}}$$

$$= \frac{\frac{1}{5}}{\frac{8}{15}} = \frac{1}{5}\times\frac{15}{8} = \frac{3}{8}$$

$$i = \frac{E_{eq}}{r_{eq} + 1}$$

$$= \frac{\frac{3}{8}}{\frac{15}{8}+1} = \frac{3}{23}$$

= 0.13 A from Q to P

Q.19 Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then :

(1) $\vec{F}_1 = \vec{F}_2 = 0$

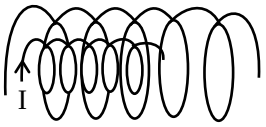
(2) $\vec{F}_1$ is radially inwards and $\vec{F}_2$ is radially outwards

(3) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$

(4) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$

Ans. **[1]**

Sol.



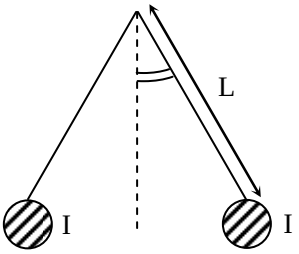
Inner coil is kept in uniform magnetic field produced by outer coil and we know current carrying coil in uniform magnetic experience no force.

$$\therefore \quad \vec{F}_1 = 0$$

and magnetic field outside the inner coil is approximate zero (assuming long solenoid)

$$\therefore \quad \vec{F}_2 = 0$$

Q.20



Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is :

(g = gravitational acceleration)

(1) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (2) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

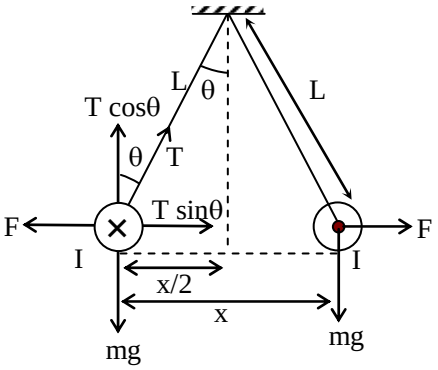
(3) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$ (4) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

Ans. **[2]**

Sol.

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : Magnetic Effect of Current, Solved Example 10, Page No. 172]



Consider equilibrium of length 'ℓ' of each wire

$T\cos\theta = mg$ ... (i)

$T\sin\theta = F$ ... (ii)

Divide eq (ii) with (i)

$$\tan\theta = \frac{F}{mg}$$

$$\tan\theta = \frac{\mu_0 I^2}{2\pi x \lambda g} \qquad \begin{cases} \sin\theta = \dfrac{x}{2L} \\ x = 2L\sin\theta \end{cases}$$

$$\tan\theta = \frac{\mu_0 I^2 \ell}{2\pi x\, mg}$$

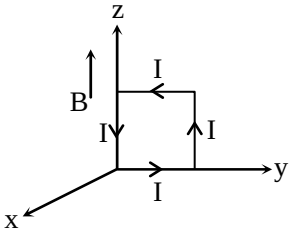
$$I^2 = \frac{\mu_0 I^2 \ell}{2\pi(2L\sin\theta)mg}$$

$$I^2 = \frac{4\pi L\lambda g\sin^2\theta}{\mu_0\cos\theta}$$

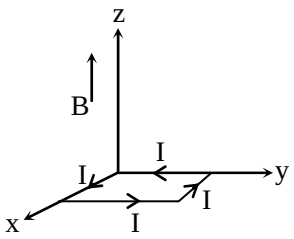
$$I = 2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$$

Q.21 A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientation as shown in the figures below :

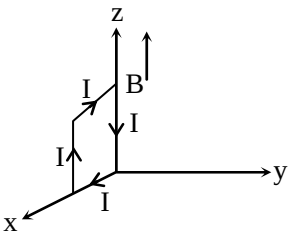
(a)



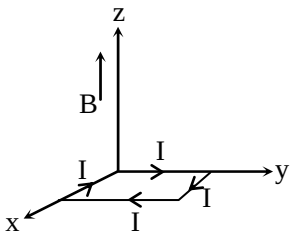
(b)



(c)



(d)



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium ?

(1) (a) and (b), respectively

(2) (a) and (c), respectively

(3) (b) and (d), respectively

(4) (b) and (c), respectively

Ans. **[3]**

Sol. (a) $\vec{M} = M\hat{i}$, $\vec{B} = B\hat{k}$

$\therefore$ $\vec{M} \perp \vec{B}$

$\therefore$ Torque $\neq 0$

$\therefore$ It is not in equilibrium position

(b) $\vec{M} = M\hat{k}$,

$\vec{B} = B\hat{k}$

$\therefore$ $\vec{M} \parallel \vec{B}$

$\therefore$ $U = -MB\cos0°$

$= -MB$ (min. potential energy)

$\therefore$ loop will be in stable equilibrium

(c) $\vec{M} = M(-\hat{j})$, $\vec{B} = B\hat{k}$

$\therefore \quad \vec{M} \perp \vec{B}$

$\therefore$ Torque $\neq 0$

$\therefore$ It is not in equilibrium position

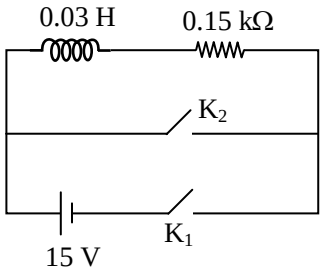
(d) $\vec{M} = M(-\hat{k})$, $\vec{B} = B\hat{k}$

$\therefore \quad U = -MB\cos 180°$

$= +MB$ (max. potential energy)

$\therefore$ It is in unstable equilibrium

Q.22 An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be : ($e^5 \cong 150$)



(1) 100 mA (2) 67 mA

(3) 6.7 mA (4) 0.67 mA

Ans. **[4]**

Sol.

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Electro magnetic Induction, Level # 4A, Q.14]

[JEE Advance, Chapter :Electro magnetic Induction, Ex. # 5, Q. 41]

When $k_1 \longrightarrow$ closed

steady state current $i_0 = \dfrac{V}{R}$

$= \dfrac{15}{0.15\times 1000}$

$= 0.1$ A

Now $k_1 \longrightarrow$ open, $k_2 \longrightarrow$ closed

$i = i_0 e^{-t/\tau}$,

$= 0.1\ e^{-\frac{1\text{ms}}{0.2\text{ms}}}$

$= 0.1\ e^{-5}$

$= \dfrac{0.1}{e^5}$

$= \dfrac{0.1}{150}$

$= 0.67$ mA

$\tau = \dfrac{L}{R} = \dfrac{0.03}{0.15\times 1000}$

$= 0.0002$ s

$= 0.2$ ms

Q.23 A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is :

(1) 1.73 V/m (2) 2.45 V/m

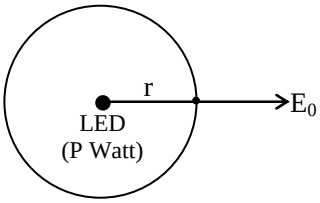
(3) 5.48 V/m (4) 7.75 V/m

Ans. **[2]**

Sol.

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Electro magnetic Wave, Level # 1, Q.20]



$I = \dfrac{P}{A} = \dfrac{1}{2}\varepsilon_0 E_0^2 c$

$= \dfrac{P}{4\pi r^2} = \dfrac{1}{2}\varepsilon_0 E_0^2 c$

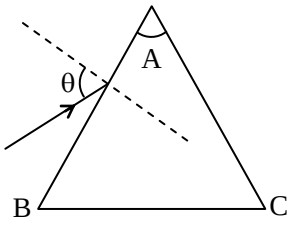
$E_0 = \sqrt{\dfrac{2P}{4\pi\varepsilon_0 r^2 c}}$

$= \sqrt{\dfrac{2\times 0.1\times 9\times 10^9}{1^2\times 3\times 10^8}}$

$= \sqrt{6}$

$= 2.45$ V/m

Q.24 Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided :



(1) $\theta > \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

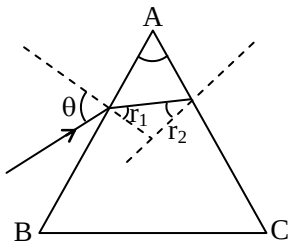
(2) $\theta < \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(3) $\theta > \cos^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(4) $\theta < \cos^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Ans. **[1]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Advance, Chapter : Prism, Deviation, Dispersion, Ex. # 2, Q. 3]



$\Rightarrow$ When we decrease θ, r_2 increases.

$\Rightarrow$ for transmission of light through AC

$r_2 < \theta_c$

$A - r_1 < \theta_c$

$A - \theta_c < r_1$ … (i)

$\sin(A-\theta_c) < \sin r_1$

snell law at AB

$\sin\theta = \mu\sin r_1$ … (ii)

from eq.(i) & (ii)

$$\sin\left(A-\sin^{-1}\frac{1}{\mu}\right) < \frac{\sin\theta}{\mu}$$

$$\theta > \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\frac{1}{\mu}\right)\right]$$

Q.25 On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam :

(1) becomes narrower

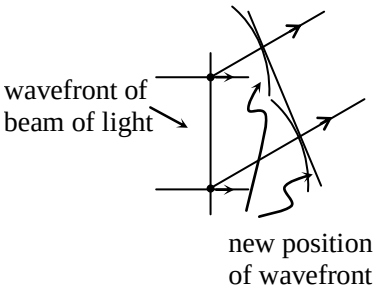
(2) goes horizontally without any deflection

(3) bends downwards

(4) bends upwards

Ans. **[4]**

Sol. When we go away from ground, refractive index of A get increase. Wavefront portion at higher attitude get retard whereas lower portion of wavefront proceed in medium at greater speed due to which wave get bend in upward direction



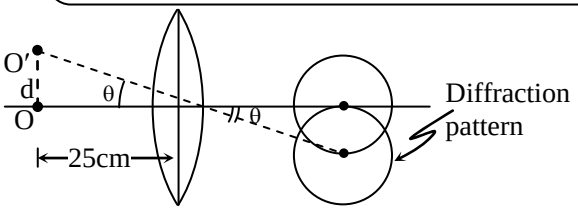
Q.26 Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is :

(1) 1 μm (2) 30 μm

(3) 100 μm (4) 300 μm

Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Refraction at curved Surface & Optical Instrument, Level # 4A, Q.5]



$\theta = \frac{1.22\lambda}{a}$

where a is diameter of human pupil.

Minimum distance between two object that can be resolved at $\lambda = 500$ nm

$d = 25 \times \theta$

$= \frac{25 \times 1.22\lambda}{a}$

put $\lambda = 500 \times 10^{-9}$,

$a = 0.5 \times 10^{-2}$ m

$d = 30\ \mu m$

Q.27 As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion :

(1) its kinetic energy increases but potential energy and total energy decrease

(2) kinetic energy, potential energy and total energy decrease

(3) kinetic energy decreases, potential energy increases but total energy remains same

(4) kinetic energy and total energy decrease but potential energy increases

Ans. **[1]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Atomic structure, Level # 3, Q.13]
[JEE Advance, Chapter : Atomic structure & Matter wave, Ex. # 6, Q. 8]

$T.E = -13.6 \frac{z^2}{n^2}$ eV as n↓ T.E.↓

$K.E = 13.6 \frac{z^2}{n^2}$ as n↓ K.E.↑

$P.E = -27.2 \frac{z^2}{n^2}$ as n↓ P.E.↓

Q.28 Match **List – I** (Fundamental Experiment) with **List – II** (its conclusion) and select the correct option from the choices given below the list :

	List – I		**List – II**
(A)	Franck-Hertz Experiment.	(i)	Particle nature of light
(B)	Photo-electric experiment	(ii)	Discrete energy levels of atom
(C)	Davison–Germer Experiment	(iii)	Wave nature of electron
		(iv)	Structure of atom

(1) (A)-(i) (B)-(iv) (C)-(iii)

(2) (A)-(ii) (B)-(iv) (C)-(iii)

(3) (A)-(ii) (B)-(i) (C)-(iii)

(4) (A)-(iv) (B)-(iii) (C)-(ii)

Ans. **[3]**

Sol. (A)-(ii) (B)-(i) (C)-(iii)

Q.29 A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are :

(1) 2 MHz only

(2) 2005 kHz, and 1995 kHz

(3) 2005 kHz, 2000 kHz and 1995 kHz

(4) 2000 kHz, and 1995 kHz

Ans. **[3]**

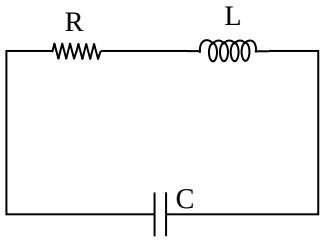
Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Communication system, Level # 1, Q.11]

In A.M. modulation the frequency of resultant signals are

ω_c, $\omega_c - \omega_m$ and $\omega_c + \omega_m$

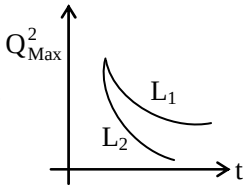
$\therefore$ 2000 KHz, 1995 kHz and 2005 kHz

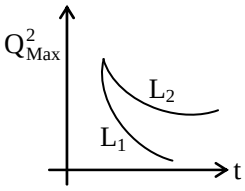
Q.30 A LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below :

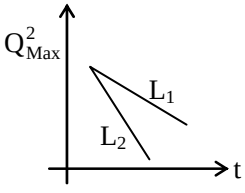


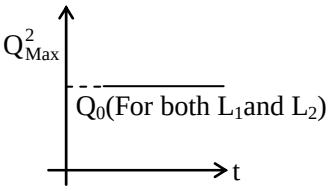
If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2($L_1 > L_2$) of L then which of the following represents this graph correctly ?

(plots are schematic and not drawn to scale)

(1)


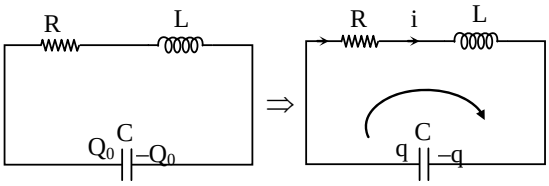
(2)


(3)


(4)


Ans. **[1]**

Sol.



At t = 0 At time t

When C is connected to L & R it start discharging. Let at time t charge on C is q and current flowing in circuit is i

using kirchoff law

$$iR + L\frac{di}{dt} - \frac{q}{c} = 0$$

$$i = -\frac{dq}{dt}$$

$$-\frac{dq}{dt}R - L\frac{d^2q}{dt^2} - \frac{q}{c} = 0$$

$$\frac{d^2q}{dt^2} + \frac{dq}{dt}\frac{L}{R} + \frac{q}{Rc} = 0$$

q is varying as it is similar to damped oscillation

$\therefore$ Amplitude of charge is Q_{max} vary as the following equation $Q_{max} = Q_0 e^{-\frac{Rt}{2L}}$

$$Q^2_{max} = Q_0^2 e^{-\frac{Rt}{L}}$$

time constant (τ) for decreasing of Q^2_{max} is $\frac{L}{R}$, higher the L, higher will be τ and thus slower the process of decreasing of Q^2_{max}

Part B – CHEMISTRY

Q.31 The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. Wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin ?

(1) $\frac{1}{103}$ (2) $\frac{1}{206}$

(3) $\frac{2}{309}$ (4) $\frac{1}{412}$

Ans. **[4]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Advance, Chapter : Redox & volumetric analysis, Ex. # 1, Q. 45]

$2C_8H_7SO_3Na + Ca^{2+} \rightarrow (C_8H_7SO_3)_2Ca + 2Na^+$

$\therefore$ from stoichiometry of the reaction

2 mole i.e. (206 × 2) g of $C_8H_7SO_3Na$ (resin) changes 1 mole of Ca^{2+}

i.e. 412 g of resin exchanges 1 mole of Ca^{2+}

$\therefore$ 1 g of resin exchanges $\frac{1}{412}$ mole of Ca^{2+}

So, maximum uptake of Ca^{2+} ions by the resin is $\frac{1}{412}$ mole per gram resin.

Q.32 Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29 Å. The radius of sodium atom is approximately :

(1) 1.86 Å (2) 3.22 Å

(3) 5.72 Å (4) 0.93 Å

Ans. **[1]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Solid state, Level # 4, Q. 8]
[JEE Advance, Chapter : Solid state, Ex. # 5, Q. 20]

For BCC crystal structure

$\sqrt{3}a = 4r$ and

$a = 4.29$ Å

$\therefore \quad r = \frac{\sqrt{3}}{4}a$

$= \frac{\sqrt{3}}{4} \times 4.29$ Å

$= 1.86$ Å

So, radius of sodium atom is 1.86 Å approx.

Q.33 Which of the following is the energy of a possible excited state of hydrogen ?

(1) + 13.6 eV (2) – 6.8 eV

(3) – 3.4 eV (4) + 6.8 eV

Ans. **[3]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Atomic structure, Level # 4, Q. 2]
[JEE Advance, Chapter : Atomic structure, Ex. # 5, Q. 29]

For hydrogen atom,

Energy of n^{th} level $= -\frac{13.6}{n^2}$ eV

For excited state, n = 2, 3, 4, ……

$\therefore$ for n = 2,

Energy $= -\frac{13.6}{2^2}$ eV = –3.4 eV.

Q.34 The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is :

(1) Ion – ion interaction

(2) Ion – dipole interaction

(3) London force

(4) Hydrogen bond

Ans. **[2]**

Sol. Ion-Ion interaction $\propto \frac{1}{r^2}$

Ion-Dipole interaction $\propto \frac{1}{r^3}$

London force $\propto \frac{1}{r^7}$

Hydrogen bond $\propto \frac{1}{r^4}$

Q.35 The following reaction is performed at 298 K.

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $NO_2(g)$ at 298 K?

($K_p = 1.6 \times 10^{12}$)

(1) $R(298)\, ln(1.6 \times 10^{12}) - 86600$

(2) $86600 + R(298)\, ln(1.6 \times 10^{12})$

(3) $86600 - \frac{ln(1.6\times10^{12})}{R(298)}$

(4) $0.5[2 \times 86{,}600 - R(298)\, ln(1.6 \times 10^{12})]$

Ans. **[4]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Advance, Chapter : Chemical equilibrium, Ex. # 4, Q. 6]

$$\Delta G^\circ_{rxn} = -RT\, \ell n\, K_p$$

and $\Delta G^\circ_{rxn} = 2\Delta G^\circ_{NO_2} - 2\Delta G^\circ_{NO} - \Delta G^\circ_{O_2}$

$\therefore\ 2\{\Delta G^\circ_{NO_2} - \Delta G^\circ_{NO}\} = -RT\, \ell n\, K_p \{\Delta G^\circ_{O_2} = 0\}$

or $2\{\Delta G^\circ_{NO_2} - 86600\} = -R(298)\, \ell n\, (1.6 \times 10^{12})$

$$\Delta G^\circ_{NO_2} = 0.5[2 \times 86600 - R(298)\, \ell n\, (1.6 \times 10^{12})]$$

Q.36 The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is :

(1) 32 (2) 64 (3) 128 (4) 488

Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Solution & Colligative properties, Level # 4, Q. 16]
[JEE Advance, Chapter : Solution, Ex. # 1, Q. 21]

$P^\circ = 185$ torr,

$P_S = 183$ torr

$W_{solute} = 1.2$ g, $W_{solvent} = 100$ g

$MM_{solute} = ?$ $MM_{solvent} = 58$ g/mol

$$\therefore \frac{P^o - P_S}{P_S} = \frac{W_{solute} \times MM_{solvent}}{W_{solvent} \times MM_{solute}}$$

$$\frac{185-183}{183} = \frac{1.2\times58}{100\times MM_{solute}}$$

$$MM_{solute} = \frac{1.2\times58\times183}{2\times100} \approx 64 \text{ g/mol.}$$

Q.37 The standard Gibbs energy change at 300 K for the reaction $2A \rightleftharpoons B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is [A] = $\frac{1}{2}$, [B] = 2 and [C] = $\frac{1}{2}$. The reaction proceeds in the :

[R = 8.314 J/K/mol, e = 2.718]

(1) Forward direction because $Q > K_C$

(2) Reverse direction because $Q > K_C$

(3) Forward direction because $Q < K_C$

(4) Reverse direction because $Q < K_C$

Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Advance, Chapter : Chemical equilibrium, Ex. # 2, Q. 20]

$$\Delta G^\circ_{rxn} = RT\, \ell n\, K_{eqm.}$$

$$2494.2 = -8.314 \times 300 \times \ell n(K_{eqm.})$$

$$\ell n(K_{eqm.}) = -1$$

or $K_{eqm.} = 2.718^{-1} = \frac{1}{2.718} \cong 0.37$

and, $Q = \frac{[B][C]}{[A]^2} = \frac{(2)\times\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2} = 4$

$\therefore$ $Q > K_{eqm.}$

i.e. reaction proceed in reverse direction

Q.38 Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is :

(at. Mass of Cu = 63.5 amu)

(1) 0 g (2) 63.5 (3) 2 g (4) 127 g

Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**

[JEE Main, Chapter : Electrochemistry, Level # 1, Q. 25]

[JEE Advance, Chapter : Electrochemistry, Ex. # 1, Q. 30]

$Cu^{2+} + 2e^- \rightarrow Cu$

So, $\frac{n_{e^-}}{2} = \frac{n_{Cu}}{1}$

and n_{e^-} = charge in Faraday = 2

$\therefore$ $\frac{2}{2} = \frac{W_{Cu}}{63.5}$

W_{Cu} = 63.5 g.

Q.39 Higher order (>3) reactions are rare due to :

(1) Low probability of simultaneous collision of all the reacting species

(2) Increase in entropy and activation energy as more molecules are involved

(3) Shifting or equilibrium towards reactants due to elastic collisions

(4) Loss of active species on collision

Ans. **[1]**

Sol. **Students may find same question in CP exercise sheet :**

[JEE Advance, Chapter : Chemical kinetics, Ex. # 1, Q. 32]

Higher order (>3) reactions are rare due to low probability of simultaneous collision of all the reacting species.

Q.40 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is :

(1) 18 mg (2) 36 mg

(3) 42 mg (4) 54 mg

Ans. **[1]**

Sol. **Students may find similar question in CP exercise sheet :**

[JEE Advance, Chapter : Basic concept of chemistry, Ex. # 4, Q. 7]

Valency factor of acetic acid is 1.

$\therefore$ Normality = Molarity

Now,

Initial mole of acetic acid = $(0.06 \times 50) \times 10^{-3}$

Final mole of acetic acid = $(0.042 \times 50) \times 10^{-3}$

$\therefore$ mole of acetic acid adsorbed

$= (0.06 - 0.042) \times 50 \times 10^{-3}$

$= 0.9 \times 10^{-3}$

$\therefore$ mass of acetic acid adsorbed

$= (0.9 \times 10^{-3} \times 60)$ g

= 0.054 g or 54 mg

$\therefore$ mass of acetic acid adsorbed per gram

$= \frac{54}{3}$ mg

= 18 mg

Q.41 The ionic radii (in Å) of N^{3-}, O^{2-} and F^{-} are respectively :

(1) 1.36, 1.40 and 1.71

(2) 1.36, 1.71 and 1.40

(3) 1.71, 1.40 and 1.36

(4) 1.71, 1.36 and 1.40

Ans. **[3]**

Sol. **Students may find similar question in CP exercise sheet :**

[JEE Main, Chapter : Periodic Table, Level # 1, Page No. 39, Q. 22]

[JEE Advance, Chapter : Periodic Table, Ex. # 3, Page No. 138, Q. 41]

Ion	N^{-3}	O^{-2}	F^{-}
Atomic no.	7	8	9
No. of electrons	10	10	10

Hence, these are isoelectronic.

In isoelectronic series as atomic number increases, radius decreases.

Order of radius is,

$N^{-3} > O^{-2} > F^{-}$

1.71 Å 1.40 Å 1.36 Å

Q.42 In the context of the Hall – Heroult process for the extraction of A*l*, which of the following statement is **false** ?

(1) CO and CO_2 are produced in this process

(2) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity

(3) Al^{3+} is reduced at the cathode to form Al

(4) Na_3AlF_6 serves as the electrolyte

Ans. **[4]**

Sol. **Students may find similar question in CP exercise sheet :**

[JEE Main, Chapter : Metallurgy, Level # 4, Page No. 136, Q. 1]

[JEE Advance, Chapter : Metallurgy, Example # 14, Page No. 118]

Hall – Heroult process,

Electrolyte : Fused Al_2O_3 mixed with Na_3AlF_6 and CaF_2

Cathode : $Al^{+3} + 3e^{-} \rightarrow Al$

Anode : $O^{-2} + C \rightarrow CO\uparrow + 2e^{-}$

or, $2O^{-2} + C \rightarrow CO_2\uparrow + 4e^{-}$

Incorrect statement is,

Na_3AlF_6 serves as the electrolyte.

Q.43 From the following statements regarding H_2O_2, choose the incorrect **statement** :

(1) It can act only as an oxidizing agent

(2) It decomposes on exposure to light

(3) It has to be stored in plastic or was lined glass bottles in dark

(4) It has to be kept away from dust

Ans. **[1]**

Sol. **Students may find similar question in CP exercise sheet :**

[JEE Main, Chapter : Hydrogen & Its compound, Level # 1, Page No. 143, Q. 23]

[JEE Advance, Chapter : Hydrogen & Its compound, Ex. # 2, Page No. 93, Q. 12]

H_2O_2 acts as both oxidising and reducing agent. In H_2O_2, 'O' is present in its intermediate oxidation state.

Q.44 Which one of the following alkaline earth metal sulphates has its hydration enthalpy grater than its lattice enthalpy ?

(1) $CaSO_4$

(2) $BeSO_4$

(3) $BaSO_4$

(4) $SrSO_4$

Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : s-block elements, Level # 3, Page No. 169, Q. 8]
[JEE Advance, Chapter : s-block elements, Ex. # 3, Page No. 211, Q. 1(C)]

Order of solubility in water,

$BeSO_4 > CaSO_4 > SrSO_4 > BaSO_4$

Hydration energy is inversely proportional to ionic size.

Be^{+2} smaller in size hence, $BeSO_4$ has greater hydration enthalpy than its lattice enthalpy.

Q.45 Which among the following is the most reactive?

(1) Cl_2 (2) Br_2 (3) I_2 (4) ICl

Ans. **[4]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Advance, Chapter : p-block elements, Ex. # 3, Page No. 55, Q. 38]

ICl is an interhalogen compound. Interhalogen compounds are more reactive than constituent halogens because bond present in interhalogen is polar while bond present in pure halogen is non-polar.

Q.46 Match the catalysts to the correct processes :

Catalyst	Process
(A) $TiCl_3$	(i) Wacker process
(B) $PdCl_2$	(ii) Ziegler – Natta Polymerization
(C) $CuCl_2$	(iii) Contact process
(D) V_2O_5	(iv) Deacon's process

(1) (A) – (iii), (B) – (ii), (C) – (iv), (D) – (i)

(2) (A) – (ii), (B) – (i), (C) – (iv), (D) – (iii)

(3) (A) – (ii), (B) – (iii), (C) – (iv), (D) – (i)

(4) (A) – (iii), (B) – (i), (C) – (ii), (D) – (iv)

Ans. **[2]**

Sol. $TiCl_3 \longrightarrow$ Ziegler – Natta polymerization

$PdCl_2 \longrightarrow$ Wacker process

$CuCl_2 \longrightarrow$ Deacon's process

$V_2O_5 \longrightarrow$ Contact process

(A) – (ii), (B) – (i), (C) – (iv), (D) – (iii)

Q.47 Which one has the highest boiling point?

(1) He (2) Ne (3) Kr (4) Xe

Ans. **[4]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : p-block elements, Level # 1, Page No. 33, Q. 84]
[JEE Advance, Chapter : p-block elements, Example # 2, Page No. 68, Q. 30]

Order of boiling point is -

He < Ne < Kr < Xe

Here, boiling point depends on strength of intermolecular bond. As the size increases London attraction force increases

Boiling point $\propto$ molecular weight $\propto$ Vander Waal's force of Attraction

Q.48 The number of geometric isomers that can exist for square planar [Pt (Cl) (py) (NH_3) $(NH_2OH)]^+$ is - (py = pyridine) :

(1) 2 (2) 3

(3) 4 (4) 6

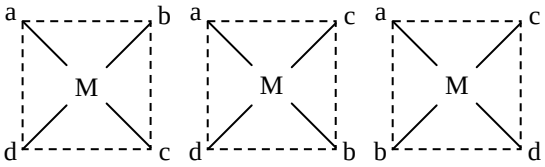
Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Co-ordination compounds, Level # 3, Page No. 109, Q. 17]
[JEE Advance, Chapter : Co-ordination compounds, Example # 6, Page No. 166, Q. 19]

[Pt (Cl) (py) (NH_3) $(NH_2OH)]^+$

General form is Mabcd.

Geometric isomers are -



Total geometric isomers = 3

Q.49 The color of $KMnO_4$ is due to :

(1) M → L Charge transfer transition

(2) d – d transition

(3) L → M charge transfer transition

(4) σ – σ* transition

Ans. **[3]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : d-block elements, Level # 3, Page No. 73, Q. 8]

$\underset{\text{Colourless}}{K^+}$ $\underset{\text{Coloured}}{MnO_4^-}$

Cause of colour in MnO_4^- is charge transfer from 'O' to 'Mn'.

$\underset{(L)}{\text{Ligand}} \longrightarrow \underset{(M)}{\text{Metal}}$ charge transfer transition

Q.50 **Assertion :** Nirtogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.

Reason : The reaction between nitrogen and oxygen requires high temperature.

(1) Both assertion and reason are correct, and the reason is the correct explanation for the assertion

(2) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion

(3) The assertion is incorrect, but the reason is correct

(4) Both the assertion and reason are incorrect

Ans. **[1]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : p-block elements, Level # 1, Page No. 28, Q. 4]
[JEE Advance, Chapter : p-block elements, Example # 2, Page No. 9, Q. 1]

Both N_2 and O_2 are less reactive at ordinary conditions. The reaction between nitrogen and oxygen requires high temperature.

Q.51 In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is :

(at. mass Ag = 108; Br = 80)

(1) 24 (2) 36 (3) 48 (4) 60

Ans. **[1]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Purification and Characterisation of Organic Compound, Level # 1, Q. 15]
[JEE Advance, Chapter : Practical Organic Chemistry, Example # 16]

$$\% \text{ of Br} = \frac{80}{188} \times \frac{\text{Weight of AgBr}}{\text{Weight of O.S.}} \times 100$$

$$= \frac{80}{188} \times \frac{141}{250} \times 100$$

$$= 24\%$$

Q.52 Which of the following compounds will exhibit geometrical isomerism?

(1) 1 – Phenyl – 2 - butene

(2) 3 – Phenyl – 1 - butene

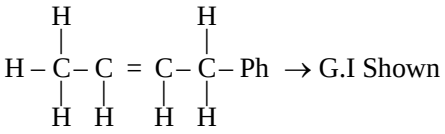
(3) 2 – Phenyl – 1 - butene

(4) 1, 1 – Diphenyl – 1 - propane

Ans. **[1]**

Sol. **Students may find same question in CP exercise sheet :**
[JEE Main, Chapter : Isomerism, Level # 2, Page No. 0, Q. 5]
[JEE Advance, Chapter : Isomerism, Example # 2, Page No. 0, Q. 51]

(1) 1 – Phenyl – 2- butene →



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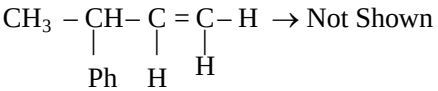
(2) 3 – Phenyl – 1- butene →



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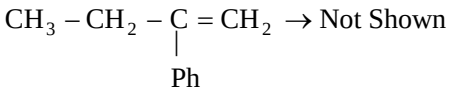
(3) 2 – Phenyl – 1- butene →



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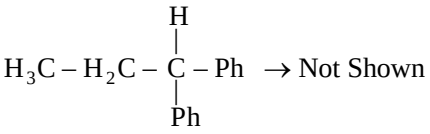
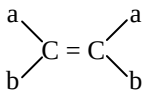
(4) 1, 1 – Diphenyl – 1- propane →



→ Not Shown

Conditions :

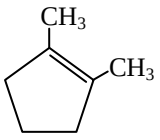
Two different groups or atoms attached with double bonded carbon atom.

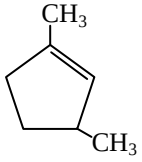


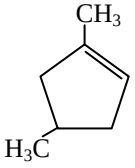
$a \neq b$

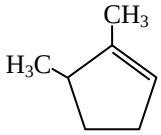
Q.53 Which compound would give

5–Keto–2–methyl hexanal upon ozonolysis ?

(1) 

(2) 

(3) 

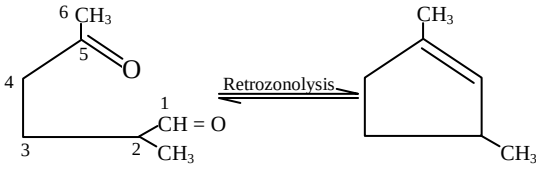
(4) 

Ans. **[2]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : Hydrocarbon, Level # 3, Q. 25]
[JEE Advance, Chapter : Carbonyl Comp., Example # 1, Q. 14]

5 – Keto – 2- methyl Hexanal

In ozonolysis alkene undergo oxidation to form carbonyl compounds



Q.54 The synthesis of alkyl fluorides is best accomplished by :

(1) Free radical fluorination

(2) Sandmeyer's reaction

(3) Finkelstein reaction

(4) Swarts reaction

Ans. **[4]**

Sol. **Students may find similar question in CP exercise sheet :**
[JEE Main, Chapter : HaloAlkane, Level # 1, Q. 8]
[JEE Advance, Chapter : Halogen Derrivative, Example # 1, Q. 10]

The synthesis of alkyl fluoride is best accomplished by Swarts Reaction :

$R - X \xrightarrow{AgF, HgF_2} R - F$

(X = Cl, Br, I)

When alkyl halide react with heavy metal fluoride then alkyl fluoride is prepared. Direct fluorination is not possible.

Q.55 In the following sequence of reactions :

Toluene $\xrightarrow{KMnO_4}$ A $\xrightarrow{SOCl_2}$ B $\xrightarrow[BaSO_4]{H_2/Pd}$ C,

The product C is :

(1) C_6H_5COOH (2) $C_6H_5CH_3$

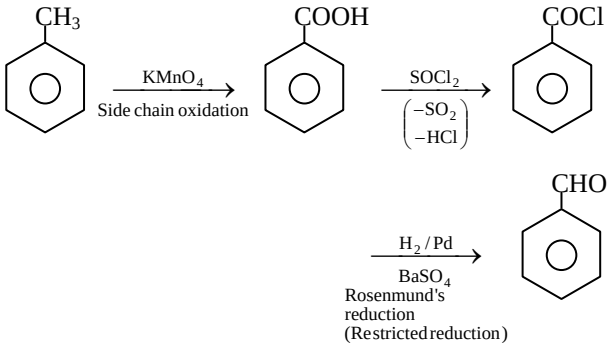
(3) $C_6H_5CH_2OH$ (4) C_6H_5CHO

Ans. **[4]**

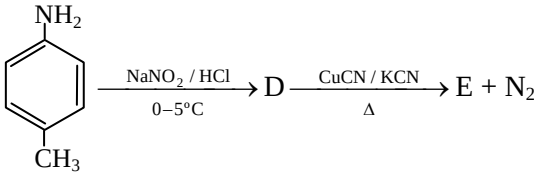
Sol. **Students may find similar question in CP exercise sheet :**

[JEE Main, Chapter : Oxygen Containing Substance-2, Level # 1, Q. 32]

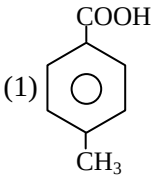
[JEE Advance, Chapter : Carboxylic acid, Example # 1, Q. 3]

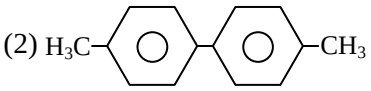


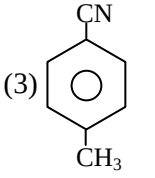
Q.56 In the reaction

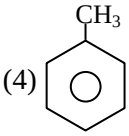


the product E is :

(1) 

(2) 

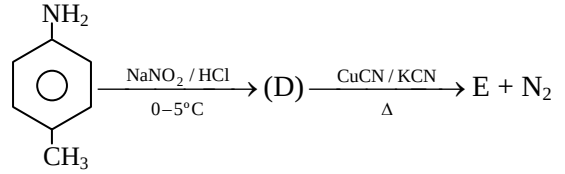
(3) 

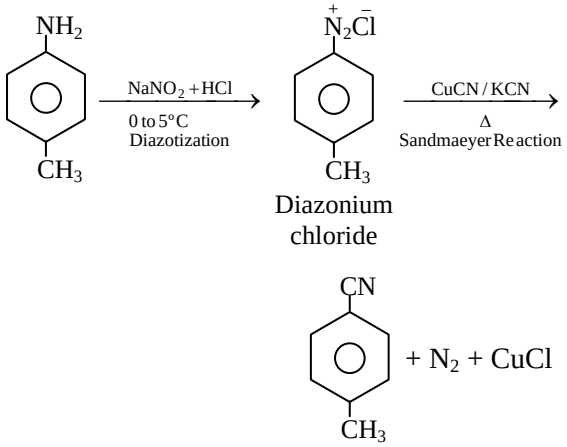
(4) 

Ans. **[3]**

Sol. **Students may find similar question in CP exercise sheet :**

[JEE Advance, Chapter : Carboxylic Acid, Example # 1, Q. 23]





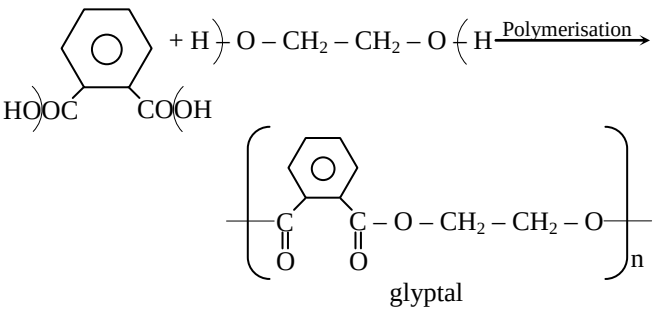
Q.57 Which polymer is used in the manufacture of paints and lacquers?

(1) Bakelite (2) Glyptal

(3) Polypropene (4) Poly vinyl chloride

Ans. **[2]**

Sol. Glyptal is used in manufacturing of paints and Lacquers



glyptal

Q.58 Which of the vitamins given below is water soluble?

(1) Vitamin C (2) Vitamin D

(3) Vitamin E (4) Vitamin K

Ans. **[1]**

Sol. Vitamin-C is water – soluble & it is continiously secrete through urine and that is why continious supply of vitamin C is necessary

Q.59 Which of the following compounds is not an antacid?

(1) Aluminium hydroxide

(2) Cimetidine

(3) Phenelzine

(4) Ranitidine

Ans. **[3]**

Sol. (1) Aluminium Hydroxide Ant acid

(2) Cimetidine Ant acid

(3) Phenalzine Sedative

(4) Ranitidine Ant acid

∴ Ans. is (3) Phenalzine

Q.60 Which of the following compounds is not colored yellow ?

(1) $Zn_2[Fe(CN)_6]$

(2) $K_3[Co(NO_2)_6]$

(3) $(NH_4)_3$ [As $(Mo_3 O_{10})_4$]

(4) $BaCrO_4$

Ans. **[1]**

Sol. Zn_2 [Fe $(CN)_6$] ⟶ White

K_3 [Co $(NO_2)_6$] ⟶ Yellow

$(NH_4)_3$ [As $(Mo_3O_{10})_4$] ⟶ Yellow

$BaCrO_4$ ⟶ Yellow

Part C – MATHEMATICS

Q.61 Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is :

(1) 219 (2) 256

(3) 275 (4) 510

Ans. **[1]**

Sol. **Students may find same question in CP Exercise Sheet :**
[JEE Main, Chapter : P&C, Level # 4(A), Page No. 18, Q. 121]

If n(A) = 4 elements

and n(B) = 2 elements

then n(A × B) = 4 × 2 = 8 elements

So, No. of subsets of A × B having 3 or more than 3 elements

$= {}^8C_3 + {}^8C_4 + ... + {}^8C_8$

$= 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$

$= 256 - 1 - 8 - 28$

$= 219$

Q.62 A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a :

(1) straight line parallel to x-axis

(2) straight line parallel to y-axis

(3) circle of radius 2

(4) circle of radius $\sqrt{2}$

Ans. **[3]**

Sol. **Students may find similar question in CP Exercise Sheet :**
[JEE Advance, Chapter : Complex Number, Exercise # 2, Page No. 26, Q. 5]

$|z_2| \neq 1$ (given)

$\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$ (given)

$\Rightarrow |z_1 - 2z_2| = |2 - z_1\bar{z}_2|$

$\Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$

(squaring both the sides)

$(z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 . z_2)$

$\Rightarrow z_1\bar{z}_1 + 4z_2\bar{z}_2 - 2z_2\bar{z}_1 - 2z_1\bar{z}_2$

$= 4 + z_1\bar{z}_1 z_2\bar{z}_2 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2$

$|z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2 |z_2|^2$

$\Rightarrow |z_1|^2 (|z_2|^2 - 1) = 4(|z_2|^2 - 1)$

$\Rightarrow |z_1|^2 = 4$ ($\because |z_2| \neq 1$)

$\Rightarrow |z_1| = 2$

$\Rightarrow z_1$ lies on circle centred at origin and radius = 2

Q.63 Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to :

(1) 6 (2) – 6

(3) 3 (4) – 3

Ans. **[3]**

Sol. **Students may find exactly same question in CP Exercise Sheet :**
[JEE Main, Chapter : Quadratic Equation, Level # 4(B), Page No. 33, Q. 21]
[JEE Advance, Chapter : Quadratic Equation, Exercise # 4, Page No. 52, Q. 16]

Let α and β be the roots of given equation

$x^2 - 6x - 2 = 0$

$\alpha + \beta = 6$ …(1)

$\alpha\beta = -2$ …(2)

$a_n = \alpha^n - \beta^n$ (given)

$\frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) + \alpha\beta(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$

$$= \frac{\alpha(\alpha^9 - \beta^9) + \beta(\alpha^9 - \beta^9)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha + \beta}{2}$$

$$= \frac{6}{2}$$

$$= 3$$

Q.64 If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to :

(1) (2, –1) (2) (–2, 1)

(3) (2, 1) (4) (–2, –1)

Ans. **[4]**

Sol. $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ (given)

$AA^T = 9I$ (given)

$$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} . \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\Rightarrow$ $a + 2b = -4$ …(1)

and $2a - 2b = -2$ …(2)

$+$

$3a = -6$

$a = -2$

$b = -1$

Q.65 The set of all values of λ for which the system of linear equations :

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution,

(1) is an empty set

(2) is a singleton

(3) contains two elements

(4) contains more than two elements

Ans. **[3]**

Sol. **Students may find similar question in CP Exercise Sheet :**
[JEE Main, Chapter : Determinant,
Level # 4(B), Page No. 150, Q. 9]

$$\left.\begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2 \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned}\right\} \text{(given system of equation)}$$

On simplification

$\Rightarrow$ $x_1(2 - \lambda) - 2x_2 + x_3 = 0$ …(1)

$2x_1 - (3 + \lambda)x_2 + 2x_3 = 0$ …(2)

$x_1 - 2x_2 + \lambda x_3 = 0$ …(3)

Homogenous system of equation

have nontrivial solution

iff. $\Delta = 0$

$$\Delta = \begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ 1 & -2 & \lambda \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} 1-\lambda & -2 & 1 \\ 1-\lambda & -(3+\lambda) & 2 \\ -(1-\lambda) & -2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda).\begin{vmatrix} 1 & -2 & 1 \\ 1 & -(3+\lambda) & 2 \\ -1 & -2 & \lambda \end{vmatrix} = 0$$

$(1 - \lambda)[\{1(-3\lambda - \lambda^2) + 4\} + 2\{(\lambda + 2)\}$

$+ 1\{-2 - (3 + \lambda)\}] = 0$

$(1 - \lambda)[-3\lambda - \lambda^2 + 4 + 2\lambda + 4 - 2 - 3 - \lambda] = 0$

$(1 - \lambda)(-\lambda^2 - 2\lambda + 3) = 0$

$(\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0$

$(\lambda - 1)(\lambda - 1)(\lambda + 3) = 0$

$\lambda = 1, 1, -3$

$\lambda = \{1, -3\}$

So, 2 values of λ

Q.66 The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is :

(1) 216 (2) 192

(3) 120 (4) 72

Ans. **[2]**

Sol. **Students may find similar question in CP Exercise Sheet :**
[JEE Main, Chapter : P&C, Level # 1, Page No. 113, Q. 112]
[JEE Advance, Chapter : P&C, Exercise # 1, Page No. 125, Q. 7]

Using digits $\rightarrow$ 3, 5, 6, 7, 8

Case-I

four digit numbers greater than 6000

$$\frac{3}{TH} \times \frac{4}{H} \times \frac{3}{T} \times \frac{2}{U} = 72$$

Case-II

five digit numbers

$$\frac{5}{TTH} \times \frac{4}{TH} \times \frac{3}{H} \times \frac{2}{T} \times \frac{1}{U} = 120$$

So, total numbers = Case-I + Case-II

= 72 + 120

= 192

Q.67 The sum of coefficients of integral powers of x in the binomial expansion of $(1-2\sqrt{x})^{50}$ is :

(1) $\frac{1}{2}(3^{50}+1)$ (2) $\frac{1}{2}(3^{50})$

(3) $\frac{1}{2}(3^{50}-1)$ (4) $\frac{1}{2}(2^{50}+1)$

Ans. **[1]**

Sol. **Students may find similar question in CP Exercise Sheet :**
[JEE Advance, Chapter : Binomial theorem, Level # 2, Page No. 101, Q. 9]

$$[1-2\sqrt{x}\,]^{50} = {}^{50}C_0 - {}^{50}C_1(2\sqrt{x}) + {}^{50}C_2(2\sqrt{x})^2 - \ldots + {}^{50}C_{50}(2\sqrt{x})^{50} \quad ...(1)$$

$$[1+2\sqrt{x}\,]^{50} = {}^{50}C_0 + {}^{50}C_1(2\sqrt{x}) + {}^{50}C_2(2\sqrt{x})^2 + \ldots + {}^{50}C_{50}(2\sqrt{x})^{50} \quad ...(2)$$

Both adding (1) and (2)

$$(1-2\sqrt{x})^{50} + (1+2\sqrt{x})^{50} = 2[{}^{50}C_0 + {}^{50}C_2(2\sqrt{x})^2 + {}^{50}C_4(2\sqrt{x})^4 + \ldots + {}^{50}C_{50}(2\sqrt{x})^{50}]$$

Put x = 1

$(1-2)^{50} + (1+2)^{50}$

= 2[sum of coeff. of integral powers of x]

$$\frac{1+3^{50}}{2} = S$$

Q.68 If m is the A.M. of two distinct real numbers ℓ and n(ℓ, n > 1) and G_1, G_2 and G_3 are three geometric means between ℓ and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals.

(1) $4\ell^2mn$ (2) $4\ell m^2n$

(3) $4\ell mn^2$ (4) $4\ell^2m^2n^2$

Ans. **[2]**

Sol. **Students may find similar question in CP Exercise Sheet :**
[JEE Advance, Chapter : Progression, Exercise # 1, Page No. 72, Q. 24]

$$\text{A.M.} = m = \frac{\ell + n}{2} \quad \ldots(1) \quad (\ell, n > 1)$$

ℓ, G_1, G_2, G_3, n are in G.P. (By definition)

$G_1 = \ell r$
$G_2 = \ell r^2$
$G_3 = \ell r^3$
$n = \ell r^4$

where

$$(c.r.) = r = \left(\frac{n}{\ell}\right)^{\frac{1}{4}}$$

$$\Rightarrow r^4 = \frac{n}{\ell}$$

$\Rightarrow \quad G_1^4 + 2G_2^4 + G_3^4$

$= (\ell r)^4 + 2(\ell r^2)^4 + (\ell r^3)^4$

$= \ell^4 r^4(1 + 2r^4 + r^8)$

$= \ell^4 . \left(\frac{n}{\ell}\right)\left(1 + \frac{2n}{\ell} + \frac{n^2}{\ell^2}\right)$

$= \ell^4 . \left(\frac{n}{\ell}\right)\left(1 + \frac{n}{\ell}\right)^2$

$= \ell.n.(\ell + n)^2$

$= \ell n.(2m)^2$

$= 4\ell nm^2$

Alter : check for 1, 2, 4, 8, 16

Q.69 The sum of first 9 terms of the series

$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots\ldots$ is :

(1) 71 (2) 96

(3) 142 (4) 192

Ans. **[2]**

Sol. **Students may find similar question in CP Exercise Sheet :**
[JEE Main, Chapter : Progression, Level # 3, Page No. 61, Q. 19]
[JEE Advance, Chapter : Progression, Exercise # 3, Page No. 77, Q. 1]

General Term . $(T_r) = \frac{1^3 + 2^3 + \ldots\ldots + r^3}{1 + 3 + 5 + \ldots\ldots + (2r-1)}$

$T_r = \frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2} = \frac{(r+1)^2}{4}$

$S_n = \sum_{r=1}^{n} T_r$

$S_9 = \frac{1}{4}\sum_{r=1}^{9}(r+1)^2$

$= \frac{1}{4}[2^2 + 3^2 + \ldots\ldots + 10^2]$

$= \frac{1}{4}[1^2 + 2^2 + \ldots\ldots + 10^2 - 1^2]$

$= \frac{1}{4}\left[\frac{10 \times 11 \times 21}{6} - 1\right]$

$= \frac{1}{4}[385 - 1] = \frac{1}{4} \times 384$

$= 96$

Q.70 $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to :

(1) 4 (2) 3

(3) 2 (4) $\frac{1}{2}$

Ans. **[3]**

Sol. **Students may find similar question in CP Exercise Sheet :**
[JEE Main, Chapter : Limit, Level # 4(A), Page No. 70, Q. 11]
[JEE Advance, Chapter : Limit, Exercise # 1, Page No. 50, Q. 25]

$\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$

$= \lim_{x \to 0} \frac{2\sin^2 x}{x^2} \times \frac{4x}{\tan 4x} \times \frac{1}{4} \times (3 + \cos x)$

$= 2 \times 1 \times \frac{1}{4} \times 4 = 2$

Q.71 If the function $g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx + 2, & 3 < x \le 5 \end{cases}$

is differentiable, then the value of k + m is :

(1) 2 (2) $\frac{16}{5}$ (3) $\frac{10}{3}$ (4) 4

Ans. **[1]**

Sol. Since function is differentiable at x = 3

it must be continuous at x = 3, also

f(3) = f(3 + h)

= f(3 – h)

$k\sqrt{3+1} = \lim_{h\to 0} m(3+h) + 2$

$= \lim_{h\to 0} k\sqrt{3-h+1}$

$2k = 3m + 2 = 2k$ …(1)

Now differentiable at x = 3

$f'(3+h) = m$

$f'(3-h) = \frac{d}{dx}(k\sqrt{x+1})$

$= k\,\frac{1}{2\sqrt{x+1}}$

at x = 3, $\frac{k}{4}$

hence $m = \frac{k}{4}$

$k = 4m$ …(2)

Solving (1) and (2) we get

$m = \frac{2}{5}, k = \frac{8}{5}$

$k + m = \frac{2}{5} + \frac{8}{5} = 2$

Q.72 The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) :

(1) does not meet the curve again

(2) meets the curve again in the second quadrant

(3) meets the curve again in the third quadrant

(4) meets the curve again in the fourth quadrant

Ans. **[4]**

Sol. Given curve $x^2 + 2xy - 3y^2 = 0$ is a pair of straight lines.

Lines are x – y = 0 & x + 3y = 0, normal at P (1, 1) will be perpendicular to line x – y = 0

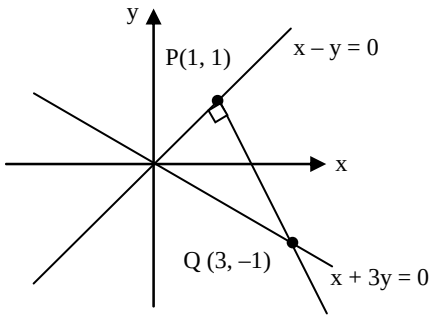
∴ N : x + y = 2

It will intersect the curve again at x + 3y = 0

∴ intersection point is obtained by solving

x + y = 2 & x + 3y = 0

i.e., Q (3, –1) in fourth quadrant.



Q.73 Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2.

If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then f(2) is equal to :

(1) – 8 (2) – 4 (3) 0 (4) 4

Ans. **[3]**

Sol. f(x) is poly. of degree 4 so

let $f(x) = a + bx + cx^2 + dx^3 + ex^4$

so $\lim_{x\to 0}\frac{a + bx + cx^2 + dx^3 + ex^4}{x^2} = 2$

a = 0, b = 0, c = 2

so $f(x) = 2x^2 + dx^3 + ex^4$ …(1)

As f(x) is extremum at x = 1, 2

so, $f'(x) = 4x + 3dx^2 + 4ex^3$

$f'(1) = 4 + 3d + 4e = 0$ …(2)

$f'(2) = 8 + 12d + 32e = 0$

$\Rightarrow$ $2 + 3d + 8e = 0$ …(3)

$2 - 4e = 0 \Rightarrow e = \frac{1}{2}$

$2 + 3d + 4 = 0$

$\Rightarrow d = -2$

From (1)

$f(x) = 2x^2 - 2x^3 + \frac{1}{2}x^4$

$f(2) = 8 - 16 + 8 = 0$

Q.74 The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals :

(1) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (2) $(x^4+1)^{\frac{1}{4}} + c$

(3) $-(x^4+1)^{\frac{1}{4}} + c$ (4) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

Ans. **[4]**

Sol. **Students may find same question in :**
[JEE Mains, Chapter : Integration, DPPS # 1, Page No. 3, Q. 32]

$$\int \frac{dx}{x^2(x^4+1)^{3/4}}$$

$$= \int \frac{dx}{x^2.x^3(1+x^{-4})^{3/4}} \quad \text{Put } 1 + x^{-4} = t^4$$

$$\int \frac{dx}{x^5(t^4)^{3/4}}\left(\frac{-t^3dt}{x^{-5}}\right) \quad -4x^{-5}dx = 4t^3dt$$

$$-\int 1.dt \quad dx = -\frac{t^3dt}{x^{-5}}$$

$$= -t + C$$

$$= -(1+x^{-4})^{1/4} + C$$

$$= -\frac{(x^4+1)^{1/4}}{x} + C$$

Q.75 The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)}dx$

is equal to :

(1) 2 (2) 4

(3) 1 (4) 6

Ans. **[3]**

Sol. **Students may find similar question in CP Exercise Sheet :**
[JEE Main, Chapter : Definite Integration, Level # 3, Page No. 72, Q. 35]

$$I = \int_2^4 \frac{\log x^2.dx}{\log x^2 + \log(6-x)^2}$$

Now by using the property

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$I = \int_2^4 \frac{\log(2+4-x)^2 dx}{\log(2+4-x)^2 + \log(6-(2+4-x))^2}$$

$$2I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(6-x)^2} + \frac{\log(6-x)^2}{\log(6-x)^2 + \log x^2}dx$$

$$= \int_2^4 1.dx = (x)_2^4$$

$$I = \frac{4-2}{2} = 1$$

Q.76 The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is :

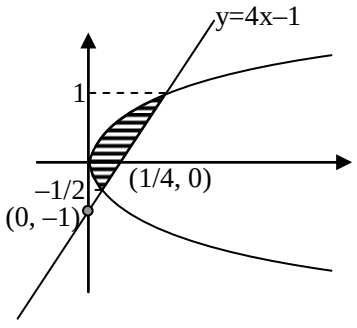
(1) $\frac{7}{32}$ (2) $\frac{5}{64}$ (3) $\frac{15}{64}$ (4) $\frac{9}{32}$

Ans. **[4]**

Sol. **Students may find similar question in CP Sheet :**
[JEE Advance, Chapter : Area under the curve, Ex. # 1, Page No. 80, Q.1]

We have

$y^2 \le 2x$, $y \ge 4x - 1$



by solving

$y^2 = 2x$,

$y = 4x - 1$

$(4x-1)^2 = 2x$

$16x^2 - 8x + 1 = 2x$

$16x^2 - 10x + 1 = 0$

$16x^2 - 8x - 2x + 1 = 0$

$8x(2x-1) - 1(2x-1) = 0$

$(2x-1)(8x-1) = 0$

$x = \frac{1}{2}, \frac{1}{8}$

at $x = \frac{1}{2}$, $y = 1$

at $x = \frac{1}{8}$, $y = -\frac{1}{2}$

Required area = area of line with y axis – area of parabola with y axis

$$= \int_{-1/2}^{1}\left(\left(\frac{y+1}{4}\right)-\frac{y^2}{2}\right)dy$$

$$= \frac{1}{4}\left(\frac{y^2}{2}+y\right)_{-1/2}^{1} - \frac{1}{2\times 3}\cdot(y^3)_{-1/2}^{1}$$

$$= \frac{1}{4}\left[\left(\frac{1}{2}+1\right)-\left(\frac{1}{8}-\frac{1}{2}\right)\right] - \frac{1}{6}\left[\left(1+\frac{1}{8}\right)\right]$$

$$= \frac{1}{4}\left(\frac{15}{8}\right)-\frac{1}{6}\times\frac{9}{8}$$

$$= \frac{15}{32}-\frac{3}{16} = \frac{9}{32}$$

Q.77 Let y(x) be the solution of the differential equation :

$(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \geq 1)$.

Then y(e) is equal to :

(1) e (2) 0

(3) 2 (4) 2e

Ans. **[3*]**

Sol. **Students may find similar question in CP Sheet :**

[JEE Main, Chapter : Differential Equation, Level # 1, Page No. 122, Q. 75]

In this question the condition $x \geq 1$ is wrong because when we divide by x log x then x can not be equal to 1, so the question is logically wrong.

$x \log x \frac{dy}{dx} + y = 2x \log x$ ….(1)

divide by x log x

$\frac{dy}{dx} + \frac{y}{x \log x} = 2$

above equation is linear differential equation

Now I.F. $= e^{\int \frac{dx}{x \log x}}$

$= e^{\log(\log x)} = \log x$

$y.\log x = \int 2.\log x.dx$

$y \log x = 2(x \log x - x) + C$ ….(2)

From given equation (1), $x \log x \frac{dy}{dx} + y = 2 \log x$

at x = 1, 0 + y = 0

y = 0

from equation (2) at x = 1, y = 0,

we get c = 2

again at x = e, y log e = 2 (e log e – e) + 2

y = 2

Q.78 The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0,41) and (41, 0) is :

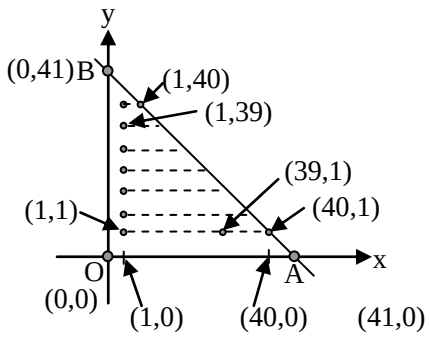
(1) 901 (2) 861 (3) 820 (4) 780

Ans. **[4]**

Sol. **Students may find similar question in CP Sheet :**

[JEE Main, Chapter : P & C, Level # 4B, Page No. 121, Q. 5]

[JEE Advance, Chapter : Straight line, Ex. # 4, Page No. 55, Q.8]



Total number of integral points lie interior of the triangle are

$\Rightarrow$ 39 + 38 + 37 + ……. + 3 + 2 + 1

$\Rightarrow$ 1 + 2 + 3 + ….. + 39

(it's a A.P.)

$\therefore$ Sum of n terms in A.P.

$\Rightarrow S_n = \frac{n}{2}(a + \ell) = \frac{39}{2}(1 + 39)$

$= 39 \times 20 = 780$

Alter : positive integral solutions of $x + y < 41$

$= {}^{40}C_2$

$= \frac{40 \times 39}{2} = 780$

Q.79 Locus of the image of the point (2, 3) in the line $(2x – 3y + 4) + k(x – 2y + 3) = 0$, $k \in R$, is a :

(1) Straight line parallel to x-axis

(2) Straight line parallel to y-axis

(3) Circle of radius $\sqrt{2}$

(4) Circle of radius $\sqrt{3}$

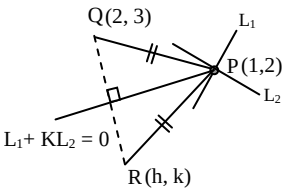
Ans. **[3]**

Sol. Line $\begin{cases} (2x-3y+4)+k(x-2y+3)=0 \\ L_1+KL_2=0 \end{cases}$

$\therefore$ $L_1 : 2x – 3y + 4 = 0$ ….(1)

$L_2 : x – 2y + 3 = 0$ ….(2)

Solve (1) & (2) $\Rightarrow$ P (1,2)



$\therefore$ Locus of point R is

PQ = PR

$\Rightarrow \sqrt{(2-1)^2+(3-2)^2} = \sqrt{(h-1)^2+(k-2)^2}$

$\Rightarrow 1 + 1 = h^2 + 1 – 2h + k^2 + 4 – 4k$

$\Rightarrow x^2 + y^2 – 2x – 4y + 3 = 0$

$\therefore$ radius of the circle is $\sqrt{1+4-3} = \sqrt{2}$

Q.80 The number of common tangents to the circles $x^2 + y^2 – 4x – 6y – 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **[3]**

Sol.

Students may find similar question in CP Sheet :

[JEE Main, Chapter : Circle, Level # 1, Page No.71, Q. 65, 69]

[JEE Advance, Chapter : Circle, Exercise # 2, Page No. 98, Q.40]

$x^2 + y^2 – 4x – 6y – 12 = 0$ …(1)

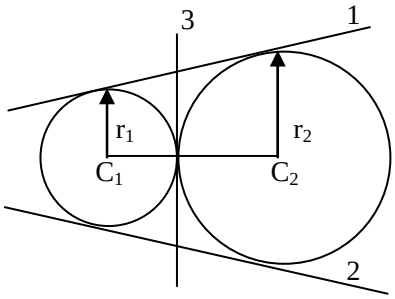
Centre C_1 (2, 3)

& radius $(r_1) = \sqrt{4+9+12} = 5$

& $x^2 + y^2 + 6x + 18y + 26 = 0$ …(2)

centre C_2 (–3, –9)

& radius $(r_2) = \sqrt{9+81-26} = 8$



$\therefore \quad C_1C_2 = \sqrt{(2+3)^2+(3+9)^2}$

$= \sqrt{25+144} = 13$

& $r_1 + r_2 = 5 + 8 = 13$

i.e. $C_1C_2 = (r_1 + r_2)$

So, Both circle touches each other externally

$\therefore$ Total number of common tangents are equal to 3

Q.81 The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is :

(1) $\frac{27}{4}$ (2) 18

(3) $\frac{27}{2}$ (4) 27

Ans. **[4]**

Sol. **Students may find similar question in CP Sheet :**

[JEE Main, Chapter : Ellipse, Level # 2, Page No. 123, Q. 1]

[JEE Advance, Chapter : Ellipse, Ex. # 2, Page No. 166, Q.25(D)]

$\frac{x^2}{9}+\frac{y^2}{5}=1$

$\because \quad a^2 = 9 \; ; \; b^2 = 5$

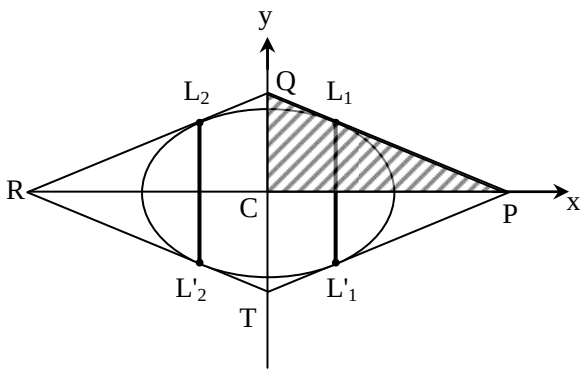
$\therefore$ eccentricity

$e = \sqrt{1-\frac{b^2}{a^2}}$

$= \sqrt{1-\frac{5}{9}}$

$= \frac{2}{3}$

$\therefore \quad L_1\left(ae, \frac{b^2}{a}\right) = \left(2, \frac{5}{3}\right)$



Equation of tangent at point L_1 is

$\Rightarrow \quad \frac{x(2)}{9}+\frac{y(5/3)}{5}=1$

$\Rightarrow \quad \frac{2x}{9}+\frac{y}{3}=1$

$\Rightarrow \quad 2x + 3y = 9$

$\therefore$ coordinates of the points P & Q are

$\Rightarrow \quad \left(\frac{9}{2},0\right)$ & (0, 3) respectively

$\therefore$ (Area of quadrilateral PQRT)

= 4 (Area of ΔPCQ)

$= 4\left\{\frac{1}{2}(CP)(CQ)\right\}$

$= 2\left(\frac{9}{2}\right)(3) = 27$

Q.82 Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is :

(1) $x^2 = y$ (2) $y^2 = x$

(3) $y^2 = 2x$ (4) $x^2 = 2y$

Ans. **[4]**

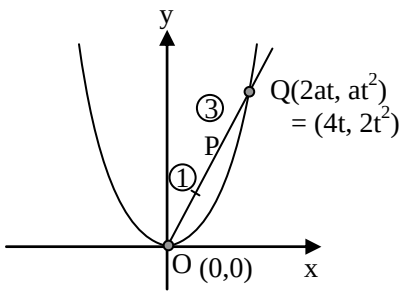
Sol. **Students may find similar question in CP Sheet :**

[JEE Main, Chapter : Parabola , Level # 4(B), Page No. 108, Q. 13]

$x^2 = 8y$ ....(1)

given $\frac{OP}{PQ}=\frac{1}{3}$

Let P(h, k) $\Rightarrow$ locus ?



By (1) 4a = 8 $\Rightarrow$ a = 2

$\therefore$ By section formula

$$p\begin{cases} h=\frac{1(4t)+3(0)}{1+3}=t & .....(2) \\ k=\frac{1(2t^2)+3(0)}{1+3}=\frac{t^2}{2} & .....(3) \end{cases}$$

From (2) & (3)

$K=\frac{h^2}{2}$

$\therefore$ Required locus is $x^2 = 2y$

Q.83 The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane x – y + z = 16, is :

(1) $2\sqrt{14}$ (2) 8

(3) $3\sqrt{21}$ (4) 13

Ans. **[4]**

Sol. **Students may find similar question in CP Sheet :**

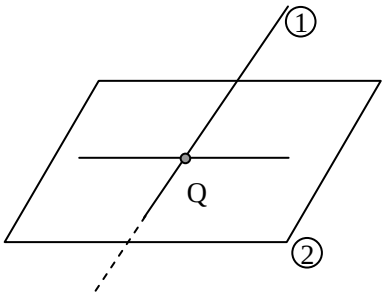
[JEE Main, Chapter : 3D, Level # 2, Page No. 81, Q. 8]

[JEE Advance, Chapter : 3D, Ex. # 5, Page No. 216, Q.8]

P(1, 0, 2)

$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=\lambda$ (let) ….(1)

x – y + z = 16 …(2)



let, any point lies on the line is

Q(3λ + 2, 4λ –1, 12λ +2)

$\because$ Point Q lies on the plane, then by(2)

$\Rightarrow$ (3λ + 2) – (4λ –1) + (12λ + 2) = 16

$\Rightarrow$ 11λ + 5 =16

$\Rightarrow$ λ = 1

$\therefore$ Q(5, 3, 14)

$\therefore$ Distance between the points P & Q is

$PQ=\sqrt{(5-1)^2+(3-0)^2+(14-2)^2}$

$=\sqrt{16+9+144}=13$

Q.84 The equation of the plane containing the line 2x – 5y + z = 3 ; x + y + 4z = 5 and parallel to the plane, x + 3y + 6z = 1, is :

(1) 2x + 6y + 12z = 13

(2) x + 3y + 6z = –7

(3) x + 3y + 6z = 7

(4) 2x + 6y + 12z = – 13

Ans. **[3]**

Sol. Equation of the plane which containing the line 2x –5y +z = 3 ; x + y + 4z = 5 is

$\Rightarrow$ (2x – 5y + z – 3) + λ (x + y + 4z – 5) = 0 …. (1)

$\because$ Plane (1) is parallel to the plane

x + 3y + 6z = 1 ….. (2)

$\therefore \frac{(2+\lambda)}{1}=\frac{(\lambda-5)}{3}=\frac{(4\lambda+1)}{6}$

by (1) & (2)

$\Rightarrow 6+3\lambda=\lambda-5$

$\Rightarrow 2\lambda = -11$

$\Rightarrow \lambda = -\frac{11}{2}$

Put in (1)

$\Rightarrow (2x - 5y + z - 3) - \frac{11}{2}(x + y + 4z - 5) = 0$

$\Rightarrow 4x - 10y + 2z - 6 - 11x - 11y - 44z + 55 = 0$

$\Rightarrow -7x - 21y - 42z + 49 = 0$

$\Rightarrow x + 3y + 6z = 7$

Q.85 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is :

(1) $\frac{2\sqrt{2}}{3}$ (2) $\frac{-\sqrt{2}}{3}$

(3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{3}}{3}$

Ans. **[1]**

Sol. **Students may find similar question in CP Sheet :**
[JEE Main, Chapter : Vector , Level # 4(A), Page No. : 45, Q. : 14]
[JEE Advance, Chapter : Vector, Ex. # 2, Page No. 210, Q.20]

$\because (\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow -\vec{c}\times(\vec{a}\times\vec{b}) = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

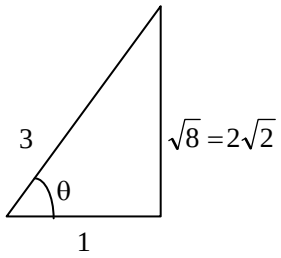
$\Rightarrow -\{\vec{a}(\vec{c}.\vec{b}) - \vec{b}(\vec{c}.\vec{a})\} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \vec{b}(\vec{c}.\vec{a}) - \vec{a}(\vec{c}.\vec{b}) = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \vec{b}(\vec{c}.\vec{a}) - \vec{a}\{\vec{c}.\vec{b} + \frac{1}{3}|\vec{b}||\vec{c}|\} = \vec{0}$

$\because \vec{a}$ & $\vec{b}$ are two non – zero non collinear vectors, then –

$$\Rightarrow \begin{cases} \vec{c}.\vec{a} = 0 \\ \vec{c} \perp \vec{a} \end{cases} \text{ and } \begin{cases} \vec{c}.\vec{b} + \frac{1}{3}|\vec{b}||\vec{c}| = 0 \\ |\vec{c}||\vec{b}|\cos\theta = -\frac{1}{3}|\vec{b}||\vec{c}| \\ \cos\theta = \frac{-1}{3} \end{cases}$$



$\therefore \sin\theta = \frac{2\sqrt{2}}{3}$

Q.86 If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is :

(1) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (2) $55\left(\frac{2}{3}\right)^{10}$

(3) $220\left(\frac{1}{3}\right)^{12}$ (4) $22\left(\frac{1}{3}\right)^{11}$

Ans. **[1*]**

Sol. Conceptual mistake in question due to two reasons

1. Examiner has assumed that one particular bag will have exactly 3 balls.
2. Answer will come if bags and balls are considered as different.

Total ways = $(3)^{12}$

Fav. ways = ${}^{12}C_3 \times (2)^9$

$\therefore$ Prob. $= \frac{{}^{12}C_3 \times (2)^9}{(3)^{12}}$

$= \frac{\frac{12\times11\times10}{6}\times(2)^9}{(3)^{12}}$

$= \frac{55\times(2)^{11}}{(3)^{12}}$

$= \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

Q.87 The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is

(1) 16.8 (2) 16.0

(3) 15.8 (4) 14.0

Ans. **[4]**

Sol. **Students may find similar question in CP Sheet :**
[JEE Main, Chapter : Statistics , Level # 2, Page No. 151, Q. 8]

given that $\bar{x} = 16$ and n = 16

$\because \bar{x} = \dfrac{\sum_{n=1}^{16} x}{n}$

$\Rightarrow \sum_{n=1}^{n=16} x = 256$

One observation 16 is deleted and three new observations 3, 4, 5 are added

$\therefore$ Sum of new observations

$\sum X = 256 - 16 + (3 + 4 + 5) = 252$

New mean $\bar{X} = \dfrac{\sum X}{n}$

$\bar{X} = \dfrac{252}{18} = 14$

Q.88 If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, AB : BC, is :

(1) $\sqrt{3}:1$

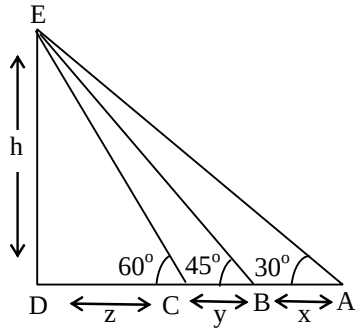
(2) $\sqrt{3}:\sqrt{2}$

(3) $1:\sqrt{3}$

(4) 2 : 3

Ans. **[1]**

Sol.



Let height of the tower is h

In ΔEDC,

$\tan 60^\circ = \dfrac{h}{z}$

$\Rightarrow z = \dfrac{h}{\sqrt{3}}$

and in ΔEDB,

$\tan 45^\circ = \dfrac{h}{y+z}$

$\Rightarrow h = y + z$

$\Rightarrow y = h - \dfrac{h}{\sqrt{3}} = \left(\dfrac{\sqrt{3}-1}{\sqrt{3}}\right)h$

In ΔEDA

$\tan 30^\circ = \dfrac{h}{x+y+z} \Rightarrow x + y + z = \sqrt{3}h$

$\Rightarrow x = \sqrt{3}h - \dfrac{h}{\sqrt{3}} - h + \dfrac{h}{\sqrt{3}}$

$x = (\sqrt{3}-1)h$

$\therefore \dfrac{x}{y} = \dfrac{\sqrt{3}}{1}$

Q.89 Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$,

where $|x| < \dfrac{1}{\sqrt{3}}$, then a value of y is :

(1) $\dfrac{3x - x^3}{1-3x^2}$ (2) $\dfrac{3x + x^3}{1-3x^2}$

(3) $\dfrac{3x - x^3}{1+3x^2}$ (4) $\dfrac{3x + x^3}{1+3x^2}$

Ans. **[1]**

Sol. **Students may find similar question in CP Sheet :**
[JEE Advance, Chapter : 3D, Ex. # 2,
Page No. 210, Q.20]

$\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\Rightarrow \tan^{-1}y = \tan^{-1}x + 2\tan^{-1}x \left(\because |x| < \frac{1}{\sqrt{3}}\right)$

$\Rightarrow \tan^{-1}y = 3\tan^{-1}x$

$\Rightarrow \tan^{-1}y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$y = \frac{3x-x^3}{1-3x^2}$

Q.90 The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) $s \wedge \sim r$ (2) $s \wedge (r \wedge \sim s)$

(3) $s \vee (r \vee \sim s)$ (4) $s \wedge r$

Ans. **[4]**

Sol. $\Rightarrow \sim(\sim s \vee (\sim r \wedge s))$

$= s \wedge \sim(\sim r \wedge s)$

$= s \wedge (r \vee \sim s)$

$= (s \wedge r) \vee (s \wedge \sim s)$

$= (s \wedge r) \vee c$

$= s \wedge r$