SIMULTANEOUS GO VIA QUANTUM COLLAPSE

YASHA SAVELYEV

ABSTRACT. In this note we describe a simultaneous version of the classical game Go or S-Go for short. Which means that players move simultaneously without knowledge of the other player's move. In particular the classical Komi rule is removed, as well as the Ko rule. S-Go is a not quiet perfect information game, because the other player's move is not known on the given turn. To resolve the natural issues that can occur with simultaneity we use ideas inspired by quantum mechanics, but instead of a dice roll there is a certain deterministic "quantum state" reduction, in turn inspired by ideas of the physicist Roger Penrose. In particular state evolution is deterministic. A theoretical connection of S-Go with Go is also given via a theorem.

1. INTRODUCTION

The game Go is an ancient and highly interesting game of local to global geometric principles, with a very large diversity of possible game states. For the most part it also has transparent, simple and beautiful rules. However like all turn based games in existence it suffers from time asymmetry between "black" and "white" players. In Go this is somewhat addressed by a rather ad-hock rule called *komi*: whereby the white player which moves second is given a specific point advantage. But the trouble of time asymmetry goes deeper than one inelegant rule. It results in counter-intuitive local pathologies in game states, for example *Ko* situation, where after a capture of a stone the opposing player may immediately recapture, but is forbidden by an extra ad-hock rule, to avoid an infinite loop situation. While this sounds innocuous it may be rather counterintuitive and artificially shut down a player's global strategy. I won't go very deep into examining time asymmetry in Go, since the main purpose of this note is simply to introduce another interesting possibility.

We are going to introduce a time symmetric or simultaneous version of Go, or S-Go for short, which aside from fixing the time asymmetry greatly expands the allowed number of game states. The idea for the time symmetry that we introduce is inspired by quantum mechanics, augmented by the objective reduction ideas of Roger Penrose in [1], [2]. But the game is still very simple [1], transparent and deterministic, in the sense that the game state is completely determined by the action of the players. Moreover existing programs can be easily modified for S-Go, and it is even possible to play on a physical board, although as we shall see it may be cumbersome. This game S-Go is not a completely ad-hoc creation, it is in a sense part of a broader theoretic framework that we describe in the last section of the paper.

It should be noted that there are existing simultaneous versions of chess, however as far as I am aware, they all either use initiative which is either decided randomly at each moment or at the beginning of the match, which then changes each turn, and so are not truly time symmetric. Or instead of initiative introduce some random element. So this is

[1] At least it is simple if one leaves turn resolution to a computer, on which the game will be simulated.

qualitatively very different from what we do here. Chess does have a simultaneous variant in the sense of this paper, in fact we construct in the last section a simultaneazation functor *Sym*, from all games to simultaneous games. However the game $Sym(Chess)$ is unplayably complex, just from the point of view of representing all the information of game states. The remarkable phenomenon is that the game $Sym(Go)$ has a rather simple, and very playable refinement which is our S-Go, and existence of this refinement can be traced to local-global nature of the game Go.

Recently, some years after earlier versions of this note first appeared on arXiv, a somewhat similar idea was developed by André Ranchin [3], in a sense that it also uses ideas of quantum mechanics to enrich Go. Although Ranchin's version is not simultaneous, moreover the implementation seems rather different, there are most definitely some connections.

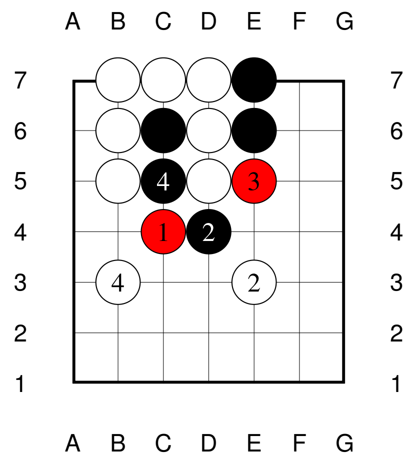
Some fundamental advances in machine learning recently lead to computer Go agents winning against stellar human opponents, particular alphaGo winning against Lee Sedol. This is remarkable, but at the same time computer agents still lag against human opponents in games with imperfect information, like Poker. It is interesting to speculate how alphaGo and its cousins would do against a human opponent in S-Go, which is both vastly more complicated and is an imperfect information game.

2. Game rules

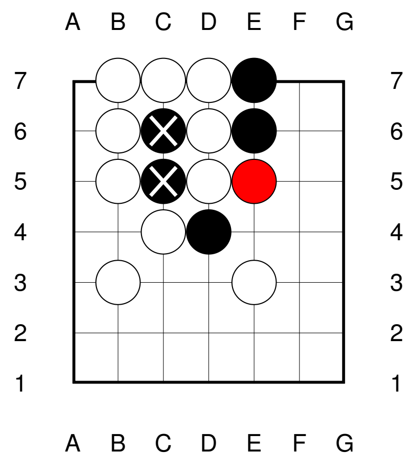
The game is played on a standard Go board, with two players that we call Black and White. As in Go both players move by placing a black or white stone on the board, but in S-Go they do this on the same turn without knowledge of the others move. This necessitates introduction of three new rules in S-Go, while removing the Komi rule, Ko rule and the no suicide rule.

First new rule is that if the players place stone in the same position a red stone is instead placed there. A red stone is in a sense simultaneously both white and black. By the first rule its state resolves to plain white or black the next turn, including the turn on which it is created, it is used to unambiguously eliminate liberties of a group of black respectively white stones, (i.e. used to capture the corresponding group). It resolves to black if used to capture a white group and to white if used to capture a black group. We explain what means unambiguously shortly. Our convention is that red stones instead of being captured resolve to white or black, if surrounded by black respectively white stones. This is to make the game behave more like classical Go.

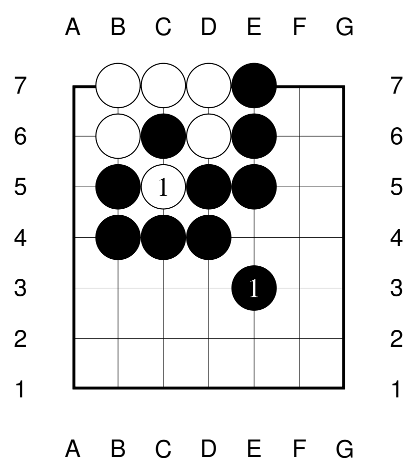
Black and White both move to C4 on turn 1. This game looks lost for Black. Here is a possible continuation which is not meant to be perfect play, we just want to show how the first rule is applied.



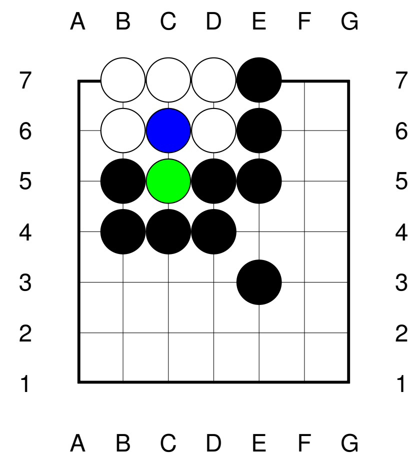
The resolved game state on turn 4. Note that C4 resolves to white. Black C5 is suicide, this is allowed.



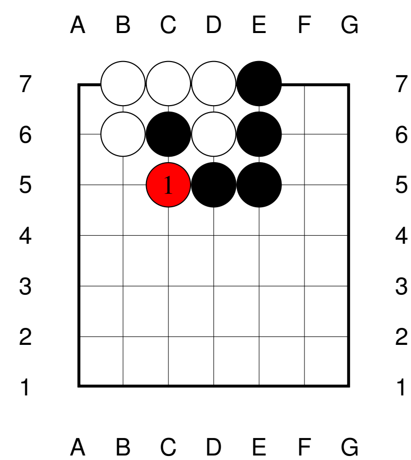
Suicidal captures. Here we give a first example of an entangled state. White to C5, is this a suicide or a capture? In classical Go it is a capture move, black C6 is removed and white C5 is alive. In S-Go a new ambiguous or entangled state is created, we need a new rule to tell us what to do. This will be rule 2 that will follow. In this case this rule says Black C6 is both dead and alive, and is replaced by a Blue stone to indicate its ambiguous state. White C5 is likewise dead and alive and is replaced by a green stone.



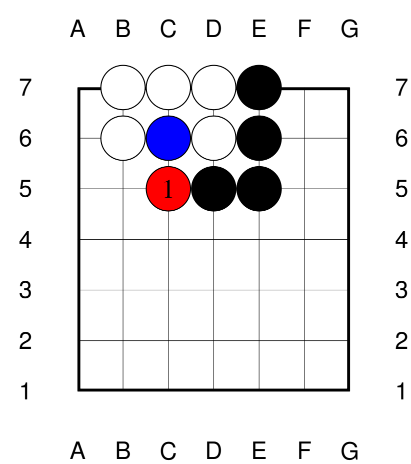
Entangled state.

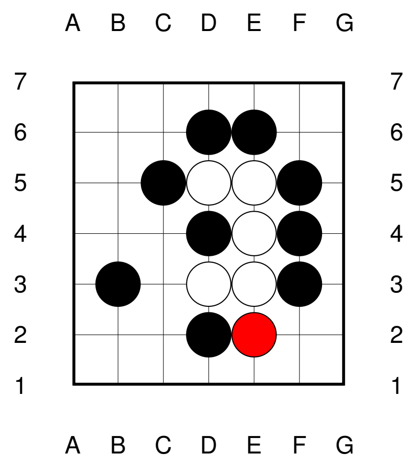


Ambiguous saves. Here is another basic example of an entangled state. Simultaneous move to C5. Is this a capture? Again in S-Go a new ambiguous or entangled state is created. Rule 2 again says that black C6 is both dead and alive, and is replaced by a blue stone. We say that C5 is an *ambiguous save by Black*, similarly for White.



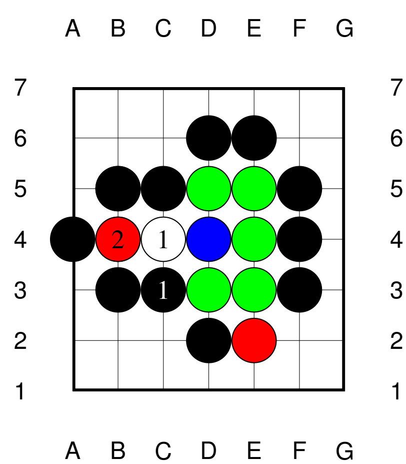
Entangled state.



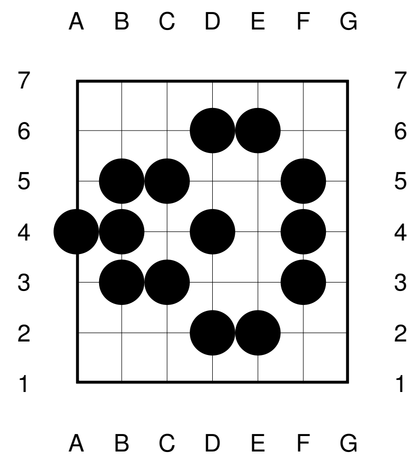


2.1. **Dual capture.** Here is another basic example of an entangled game state. In this position on the right White attempts capture at C4, while Black attempts capture at C3. As this is classically ambiguous the second rule of S-Go gets invoked. Under simultaneous capture situation of black and white groups neither group is captured when the two groups that would be captured reduce each others liberty. The groups that would be captured are left in place, they are mutually "entangled". To distinguish them from ordinary white respectively black groups we use colored green, respectively blue stones. We shall call them blue, respectively green stones, or entangled stones for both. Red, blue stones are jointly called *ambiguous black stones*, and red green stones are are jointly called *ambiguous white stones*, together they are called *ambiguous stones*. Such a game state will be called *entangled*.

Entangled state. The figure on the right is the resulting entangled state. The fate of these entangled groups is ultimately resolved by the same principle as in the first rule, and is the content of rule 3. First a definition, given a group G of stones, we get a graph with vertices stones, and edges between vertices whenever the corresponding stones are horizontally or vertically adjacent. Let us call a group G of stones a ***solid group*** if the corresponding graph is connected. Then the next time some stones in a solid group G of blue stones are used in capturing a group of (possibly ambiguous) white stones, (but not the stones in the entangled pair group of G), the stones in G are resolved to be black. Additionally if some red stones were used in this capture, they are also resolved to be black. The resolution of green stones is analogous. After some solid group of ambiguous stones is resolved on a turn, it may mean another capture must happen on the same turn, which can in turn resolve another group, and so on. We shall say how to reduce such states completely shortly. In the case of the position on the right, it completely resolves to a classical Go state, on turn 2.



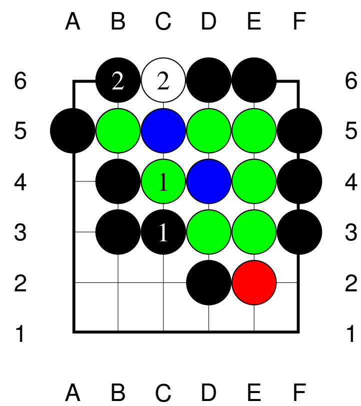
Reduced position on turn 2.



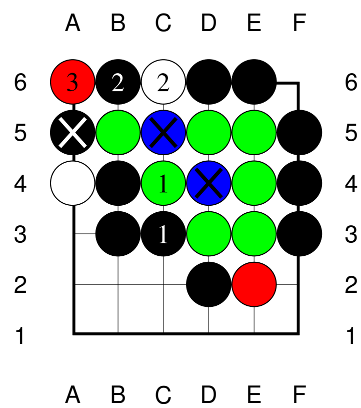
2.2. **Statement of the second rule.** The second rule of S-Go: *The entanglement rule*, gets invoked either in dual capture, ambiguous save or suicidal capture as illustrated above. For example in case of dual capture, from the point of view of Black second rule is invoked on a turn whenever a would be capture by Black (possibly using ambiguous red, blue stones) of a group G of stones, uses (for elimination of liberty) a group of stones, some of which would be captured on the same turn by White. Or if the capture move by Black of G is uses a red stone R s.t. R is also used by White in a capture move this same turn. Note that R may be created on this turn. The ambiguous capture moves (p,p') as above, meaning Black to p, White to p' (possibly same as p) will be called *entangling moves*. They create a pair of entangled groups of green-blue stones G,G', as well as possibly a red stone if $p = p'$. In other words the groups G,G' are the groups of stones that are ambiguously captured by (p,p'). We say in this case that G,G' are entangled by (p,p').

Naturally suicidal captures are special cases of dual captures. In the case of an ambiguous save, in the corresponding move (p,p'), $p = p'$, so that this creates a red stone R. If this is an ambiguous save by say White, then the entangled groups are G,R where G is the corresponding group of green stones, and R is red. We now describe some nested entangled states, and then state the third rule of S-Go.

2.3. **Nested entangled game states.** We may have a form of nested entanglement. Consider the game on the right. On turn 2 the position does not reduce to a classical state, instead we get a higher or nested entangled state with new ambiguous stones at C5, B5. The third rule will tell under what conditions such states can be reduced or partially reduced. We explain this via some examples first, and then give a more formal description.

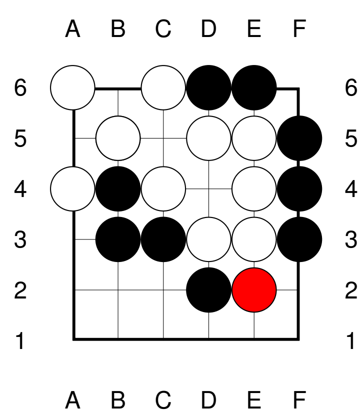


On the right, move 3 is a would be capture of Black A5, B6 using the ambiguous B5, the first stage of rule 3, analogously to rule 1 tells us that in this case B5 gets resolved to white, and move 3 is resolved to a capture move. To be consistent white C6 should likewise be resolved to a capture move, otherwise we get a state on the board which makes no sense. This is exactly what rule 3 will say. So C5 is captured. But then green group on the right was involved in a capture, so by stage one of rule 3 is resolved to white, (and hence unambiguously alive). And so again to be consistent, Black C3 is resolved to not be a capture move and white C4 is resolved to be a capture move. So Black D4 is captured by White.

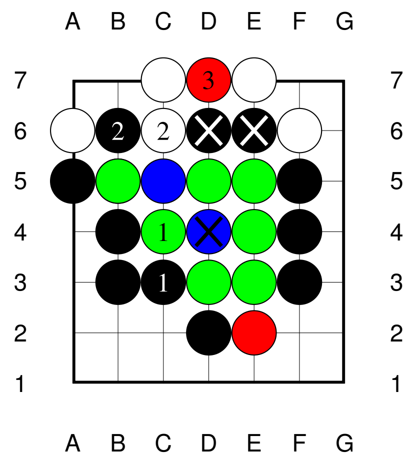


2.3.1. *Resolution.* The state reduces as depicted on the right.

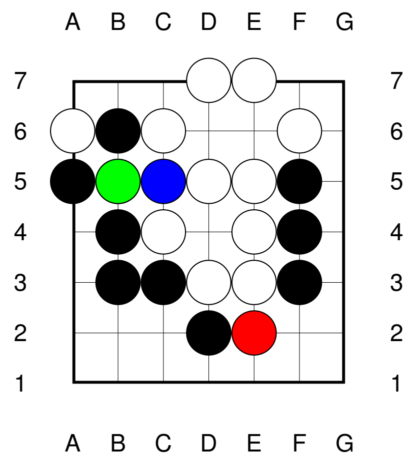
Reduced state.



In the game state on the right D7 is a capture move for White, hence by stage one of rule 3 the entire solid green group on the right is resolved to white. Thus to be causally consistent, black C3 is resolved not to be a capture move and so white C4 is resolved to be a capture move, and black's D4 is now captured. However there are no further causal consequences, the ambiguous state of C5, B5 is consistent with the new situation situation on the board.



Reduced state on move 3.



2.4. **Reduction of nested entanglement and the third rule of S-Go.** The above then gives us the idea how to proceed in general. Although the following may seem a little complicated, the essence is very simple: the complete third rule just tells us that we must work out the causal consequences of applying the first stage of rule 3, to get consistent game states. In practice this would be handled by a computer and not the players, even though experienced players will likely know what exactly is going to happen in the reduction.

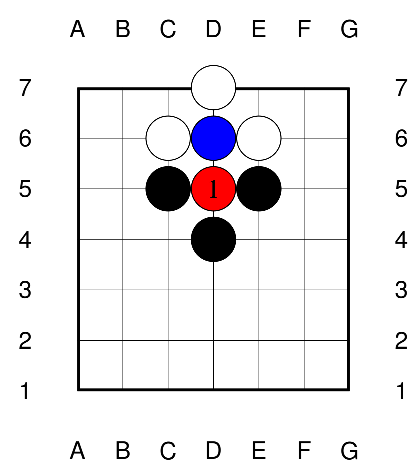
The third rule of S-Go. This is the rule of *entanglement reduction*. We first give *stage 1* of this rule. Again from the point of view of Black player, this states the following. Let $G'_{1,i}$ be a solid group of blue stones. It is the i'th solid component of a uniquely determined group G'_1 of blue stones entangled with some G_1 by a dual capture move (p_1, p'_1), or an ambiguous save by Black move. In the latter case G_1 consists of a single red stone. The next turn stones in $G'_{1,i}$ are unambiguously (that is so that the second rule does not get involved in the capture) used in capturing a group G'' non intersecting G_1, all the stones in G'_1 are resolved to be black, (in particular alive, on this turn). In case (p_1, p'_1) was a dual capture the entangled partner group G_1 is removed together with G'' as Black's prisoner, furthermore if $p_1 = p_1'$ then the red stone at p resolves to Black. If (p_1, p_1') was an ambiguous save then G_1 which is a single red stone resolves to black. Similarly in the case of ambiguous save by White, but from the point of view of Black, let G_1 be a group of green stones, entangled with a red stone R via move (p_1, p_1'). Then next turn Black uses R for a unambiguous capture move of a group G'' non-intersecting G_1. The stones in G_1 are resolved black prisoners, and R is resolved black.

We then descend to the second stage. As p_1 is a capture move, the collection of all the stones G used in this capture move may have contained some ambiguous solid blue groups $\{G_{2,i}\}_{i=1}^{i=k}$, or red stones $\{R_i\}_{i=1}^{i=m}$ then apply stage 1 of rule 3 to each $G_{2,i}$, R_i.

Continuing to descend this way, we reduce the nested entanglement. If we have a computer S-Go simulation then the resolution of nested entanglement can be automatic and seamless. We emphasize that the status of dead or alive (captured or not) is by rule always assumed to propagate to the entire component of the solid green-blue group, while in a solid group of red stones, some may resolve to white or black, while others stay red.

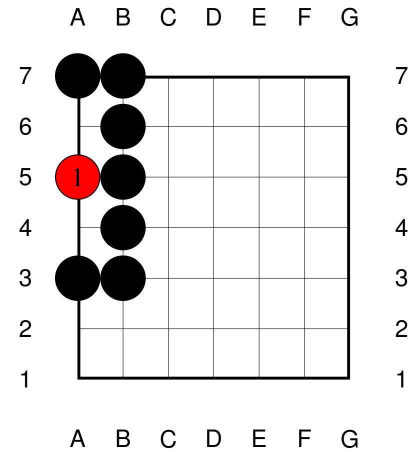
2.5. **Ko situation.** The Ko rule is no longer meaningful and so is removed.

On the right on turn 1, we get an entangled state but there is nothing special about it and the game can proceed normally. In classical Go if White had the turn initiative we would get a symmetric situation where Black could recapture and so on. In classical Go this is solved by adding the Ko rule which forbids immediate recapture.



2.6. **Life and death S-Go style.** Life and death for the most part works exactly like in Go, here is an example.

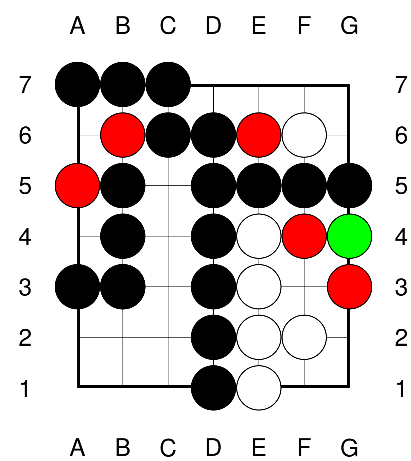
The Black group is alive, that is impossible to kill.



3. End of the game

As in Go, S-Go game ends after both players pass. At which point the winner is decided as in Go by territory and prisoner count. Our convention is that Red stones are never removed as prisoners. Territory can be determined by ambiguous stones, as in the example below.

On the right we have an example of an end game position. Both players pass. Just from the board, White has 4 points, Black has 18 points.



A tie is now possible although compared to chess it might be much less likely, (on a typical 19×19 go board).

4. A theoretic connection of Go with S-Go

We now describe a certain formal connection of classical Go with S-go. First we give some definitions. These definitions are somewhat primitive, treating "games" on a similar footing as deterministic automata, but they work well to illustrate the essential idea.

Definition 1. *A* **deterministic game** *G with 2 players P_1, P_2 or Black, White consists of:*

- *A set: $S(G)$, called the set of game states of G. For us $S(G) = P \times I$, with P set of "positions", and $I = \{0,1\}$, so that in game state $s = (p,i)$, P_i has the initiative.*
- *Nonempty subsets $W = W(G), B = B(G), D(G) \subset S(G)$, with empty pairwise intersection, which are understood as states where White wins, Black wins or both players draw, respectively.*
- *A distinguished state $q_0 = q_0(G) \in S(G)$, understood as the state in which the game is initialized.*
- *A finite set $M(G) = M_1(G) \times M_2(G)$ of moves which may be undertaken by the players. Thus an element of $M(G)$ is an ordered pair which tells us which moves P_i make on some abstract turn.*
- *A partially defined game evolution function:*

$$E : S(G) \times M(G) \to S(G).$$

  Thus E says how the game state evolves after both players have performed their move. We say that a move $m = (m_1, m_2) \in M(G)$ is **allowed** *in game state s if $E(s,m)$ is defined. We assume for convenience that if s is in B, W or D, then $E(s,m) = s$ for every m so that such an s is frozen, and every move is allowed.*

Definition 2. *A* **game sequence** *is sequence of states*

$$\{s_k = E(q_0, m_1, \ldots, m_k) = E(E(q_0, m_1, \ldots, m_{k-1}), m_k)\}$$

for $\{m_i\}$ a sequence of moves. And where the definition of $E(q_0, m_1, \ldots, m_k)$ is expanded recursively. It is assumed that for all k, m_k is allowed in state $E(q_0, m_1, \ldots, m_{k-1})$). We say that a game sequence $\{s_k\}_{k=1}^m$ is **wining for Black, White, or is Drawing** *if $s_m \in B$, $s_m \in W$, or $s_m \in D$, respectively.*

Definition 3. *A game is* **simultaneous***, if at each turn all players can move without knowledge of the other's move. In this case E must satisfy the following: if $E(p,0,m_1,m_2)$ is defined then so is $E(p,1,m_1,m_2)$, moreover*

$$E(p,1,m_1,m_2) = E(p,0,m_1,m_2)$$

and vice versa. If $E(s,m_1,m_2)$ is defined then for all $m \in M_2(G)$ $E(s,m_1,m)$ is defined. Similarly if $E(s,m_1,m_2)$ is defined then for all $m \in M_1(G)$ $E(s,m,m_2)$ is defined. We take this property of E as the definition of a simultaneous game.

Note that if G is simultaneous by definition above it merely means that its state evolution rules allow it to be played as a simultaneous game, but it may also be played with White and Black taking turns, and knowing the other's move. In other words our definition of a game does not prescribe how the players elect their moves.

We now need to introduce morphisms of games.

Definition 4. *A morphism $f : G_1 \to G_2$ of two games consists of functions:*

$$f_S : S(G_1) \to S(G_2),\ f_M : M_1(G) \to M_2(G),$$

which satisfy:

$$E_2(f_S(s), f_M(m)) = E_2(E_1(s,m)),$$

for E_i the game evolution functions of G_i, respectively. Moreover f_S takes $W(G_1), B(G_1)$ into $W(G_2), B(G_2)$ respectively and $q_0(G_1)$ to $q_0(G_2)$.

It is easy to see that the collection of games forms a (large) category $\mathcal{G}$, with morphisms as above. Let us denote by $\mathcal{SG}$ the full-subcategory formed by simultaneous games.

Definition 5. *For a pair of games G_1, G_2, we say that G_2* **refines** *G_1 if there is a function: $f : M(G_2) \to M(G_1)$ s.t a game sequence $\{q_0(G_2), q_1, \ldots, q_m\}$ is winning for Black, White, or is drawing, only if the induced game sequence in G_2:*

$$\{q_0(G_2), q_1', \ldots, q_m')\}$$

is winning for Black, White or is drawing, respectively. Here

$$q_k' = E_2(q_0(G_2), f(m_1), \ldots, f(m_k)))$$

if $q_k = E_1(q_0(G_1), m_1, \ldots, m_k))$. And there is an implicit condition in this definition that for all k q_k' are defined, if q_k are defined.

For the second part of the following we need to slightly modify classical *Go*. Let us denote by $\overline{Go}$ the classical *Go*, but with Komi, suicide and Ko rules removed. Instead of Ko rule, to avoid infinite loops and have a playable game, we can add a tie by agreement rule like in Chess, but this is not necessary for what follows.

Theorem 1. *There is a natural and non-trivial functor:*

$$Sym : \mathcal{G} \to \mathcal{SG},$$

which is a retraction so is identity on $\mathcal{SG} \subset \mathcal{G}$. Moreover if $\overline{Go}$ is as above, and SGo denotes the game S-Go described above then SGo refines $Sym(\overline{Go})$.

Proof. We define the functor:

$$Sym : \mathcal{G} \to \mathcal{SG},$$

as follows. For a game G, $G' = Sym(G)$ has the same moves as G. The set of states $S(G')$ is defined to be the set of non-empty subsets of $S(G)$, with initial state $q_0' = \{q_0\}$. The sets $W(G'), B(G')$ are defined to be elements of $S(G')$, which as sets have nonempty intersection with $W(G)$, respectively $B(G)$. The evolution function:

$$E' : S(G') \times M(G') \to S(G'),$$

of G' is defined to satisfy $E'(\{s = (p,i)\}, m) = \{E(p,i,m)\}$ if $E(p,0,m) = E(p,1,m)$ in particular E is defined on both sides. Otherwise if one of $E(p,0,m)$ or $E(p,1,m)$ is undefined or if they do not coincide set

$$E'(\{s = (p,i)\}, m) = \{(\pi_S(E(p,1,m)), i), (\pi_S(E(p,0,m)), i)\},$$

for $\pi_S : S(G) \to P$ the projection. If both $E(p,i,m)$ are undefined then E' is not defined on (s,m), so that m is not an allowed move in this state. Then set

$$E'(\{s_1, \ldots, s_k\}, m) = \bigcup_{1 \le i \le k} E'(\{s_i\}, m),$$

where the union on the right is over all s_i s.t. $E'(\{s_i\}, m)$ is defined. It is then immediate from construction that $Sym(G)$ is a simultaneous game. Moreover the construction is clearly functorial in G so that Sym is a functor.

To verify the last assertion of the theorem let

$$i : M(SGo) \to M(Sym(\overline{Go}))$$

be the natural identification map, (the set of abstractly possible moves is the same in both games). Note that on states the evolution in *SGo* is similar to $Sym(\overline{Go})$, in that it is a sum over certain histories or game sequences in $\overline{Go}$, except in *SGo* we deterministically eliminate some of these possible histories. In other words the state $E_{SGo}(q_0, m_1, \ldots, m_k)$ of *SGo* can be naturally understood as a subset of the state $E_{Sym(\overline{Go})}(q_0', m_1, \ldots, m_k)$ understood as a set. The assertion is then obvious. $\square$

So if every game G has a simultaneazation or symmetrization $Sym(G)$, what is so special about S-Go? Well as apparent from construction $Sym(G)$ can be extraordinarily complex even for a simple game G, it is then a bit of miracle that in the case of Go there is a refinement - our S-Go, which is rather simple and can actually be played. We propose that in the case of Chess there is no such simple refinement, at least it is very difficult to imagine one. The issue in Chess aside from practical concerns of how to denote a black knight and say white queen in the same position on the board, is that interactions of pieces are non-local, and for this reason a naive version of our reduction mechanism does not seem to work.

4.1. **Other thoughts.** A theorem of Zermelo [4] says roughly that in every game with Black and White taking turns, (and knowing each other's move), either Black or White has a winning strategy or both have a draw strategy. Obviously Black and White cannot have a winning strategy in S-Go played simultaneously, as that would be absurd, but what about strategy to draw? As we point out in Section 2.6 life and death works similarly to Go in S-Go, so this is not totally unlikely. More generally:

Question 1. *Can we put conditions on G such that $Sym(G)$ has a strategy to draw?*

Some conditions must be necessary as rock-paper-scissors is a simultaneous game, and has no drawing strategy.

5. Concluding remarks

As we see S-Go solves the problem of time asymmetry in Go, and introduces even more variety and complexity into game states. My feeling is that simultaneity of movement, and hence partially imperfect information makes the game more intuitive and natural, since in real world military or economic conflicts actions by parties are often simultaneous for all practical purposes because of "fog of war".

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