COMPLEX VARIABLES, PARCIAL 2

Nombre:

Please justify all answers!

It is highly recommended to search for the simplest technique to evaluate the integral, using anti-derivatives, Cauchy theorems, residues, or any other tools.

Question 1. Find the contour integral of $f(z) = 4z^2 + e^z + sin(z)$ over the half circle $C(t) = e^{it}$, $0 \leq t \leq \pi$.

Question 2. Calculate the contour integral of $f(z) = \frac{1}{z^2}$ over the left half of the circle $C(t) = e^{it}$, $-\pi/2 \leq t \leq \pi/2$. Hint: there are a couple of ways to do this.

Question 3. Let C be the contour, which is the square bounded by $x = \pm 2$, $y = \pm 2$, oriented counterclockwise.

- Find the contour integral over C of $f(z) = \frac{e^z sin(z)}{(z^2+8)}$.
- Find the contour integral over C of $f(z) = \frac{e^z sin(z)}{z(z^2+8)}$.

Question 4.

- Starting with the power series of $\sin(z)$, find the Laurant series of $f(z) = \sin(z^3)/z^4$.
- Find the contour integral of f around the unit circle C centered at the origin.

Question 5. Let $C = \{|z| = 2\}$ be the circle oriented counterclockwise and $f(z) = \tan z$.

- Find the residues of f at each singularity inside (enclosed by) C.
- Find the contour integral of f over C.

Question 6. (Bonus question).

(1) Show that there is no biholomorphic map (analytic with analytic inverse) from the punctured disk $0 < |z| < 1$ to the disk $|z| < 1$. Hint: One way to do this is to use contour integrals.
(2) Why doesn't the Riemann mapping theorem apply?