RESEARCH STATEMENT

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Broadly I am working in differential geometry, particularly symplectic, dynamical systems, and algebraic topology. I also recently became interested in computer science. This research statement will mainly concern a chain of ideas around rigidity phenomena in locally conformally symplectic geometry as partly initiated in [22]. Locally conformally symplectic manifolds of l. c. s. manifolds for short, are an extremely natural generalization of both contact and symplectic manifolds. At the moment this subject is full of fundamental mysteries, and is relatively new.

1. Sky catastrophes, Reeb vector fields, and the no Reeb sky catastrophe conjecture

The original Seifert conjecture [24] asked if a non-singular vector field on S^3 must have a periodic orbit. In this formulation the answer was shown to be no for C^1 vector fields by Schweitzer [23], for C^2 vector fields by Harrison [12] and later for C^∞ vector fields by Kuperberg [19]. A C^1 volume preserving counter-example is given by Kuperberg in [18]. For a vector field X C^0 close to the Hopf vector field it was shown to hold by Seifert and later by Fuller [7] in his 1967 paper, using his Fuller index, which is a kind fixed point index for orbits. Part of the importance of the C^0 condition for Fuller is that it rules out "sky catastrophes" for an appropriate homotopy of non-singular vector fields connecting X to the Hopf vector field. The latter "sky catastrophes" are the last discovered kind of bifurcations originally constructed by Fuller himself [6]. He constructs a smooth family $\{X_t\}, t \in [0,1]$ of vector fields on a solid torus, for which there is a continuous (and isolated) family of $\{X_t\}$ periodic orbits $\{o_t\}$, with the period of o_t going to infinity as $t \mapsto 1$, and so that for $t = 1$ the orbit disappears. Let us give the following, a bit more general, but still incomplete definition here, a full definition (according to us) appears in [21].

Definition 1.1. *[Partial] A* **sky catastrophe** *for a smooth family* $\{X_t\}$, $t \in [0,1]$, *of vector fields on a manifold* M *is a continuous family of closed orbits* $\tau \mapsto o_{t_\tau}$, o_{t_τ} *is a non-constant periodic orbit of* X_{t_τ}, $\tau \in [0,\infty)$, *such that the period of* o_{t_τ} *unbounded from above.*

These sky catastrophes (and their more robust analogues called blue sky catastrophes) turned out to be common in many kinds of systems appearing in nature and have been studied on their own, see for instance Shilnikov-Turaev [25].

However since the time of Fuller's original papers it has not been understood if this the only thing that can go wrong. That is if without existence of a "sky catastrophe" in an appropriate general sense, the time 1 limit of a homotopy of smooth non-singular vector fields on S^{2n+1} starting at the Hopf vector field must have a periodic orbit. The difficulty in answering this is that although our orbits cannot "disappear into the sky", as there are infinitely many of them they may "cancel each other out", even if the Fuller index is "locally positive" - that is the index of isolated components in the orbit space is positive. In the C^0 nearby case this cancellation is prevented as orbits from isolated components of the orbit space may not interact. The reader may think of trying to make sense of the infinite sum

$$(5-1) + (5-1) + \ldots + (5-1) + \ldots.$$

While generally meaningless it has some meaning if we are not allowed to move the terms out of the parentheses. So one has to develop a version of Fuller's index which precludes such total cancellation in general. We do this in [21] and using this prove as a particular case:

Theorem 1. *Let $X = X_1$ be a smooth non-singular vector field on S^{2k+1} homotopic to the Hopf vector field $H = X_0$ through homotopy $\{X_t\}$ of smooth non-singular vector fields. Suppose that $\{X_t\}$ has no sky catastrophes then X has periodic orbits.*

Can we use Theorem 1 and its general analogues to show existence of orbits? The most promising case where this should be possible is for Reeb vector fields.

1.1. **Reeb vector fields and sky catastrophes.** As a first step we may ask if a homotopy of Reeb vector fields $\{X_t\}$ on a closed manifold is necessarily free of sky catastrophes. Our following elementary theorem puts a very strong restriction on the kinds of sky catastrophes that can happen, for general contact manifolds. It is likely, that if they exist, they must be pathological, and very hard to construct.

We note however that in the proof of [17, Theorem 1.19] a kind of partial Reeb plug is constructed, which is missing the matching condition, see for instance [19] for terminology of plugs, also see Kerman [16], Ginzburg [10], where plugs are utilized in Hamiltonian context. If one had a plug with all conditions, then it is simple to construct a sky catastrophe. For we may deform such a plug through partial plugs satisfying all conditions except the trapping condition (condition 3 in [19]) to a trivial plug. This deformation then readily gives a sky catastrophe corresponding to the trapped orbit. Without matching, this argument does not obviously work.

Theorem 2. [21] *Let $\{X_t\}$, $t \in [0, 1]$ be a smooth homotopy through Reeb vector fields on a contact manifold M. (Informally) let S denote the space of closed orbits of the family $\{X_t\}$, where period is allowed to vary. Then there is no (period) unbounded from above locally Lipschitz continuous path $p : [0, \infty) \to S$ whose composition with the projection $\pi : S \to [0, 1]$ has finite length, with π the projection to the time coordinate t.*

The above rules out for example Fuller's sky catastrophe that appears in [6], and described in the beginning of our paper. Inspired by this we may further conjecture:

Conjecture 1. *Let $\{X_t\}$, $t \in [0, 1]$ be a smooth homotopy through Reeb vector fields on a compact contact manifold M. Then there is a C^0 nearby smooth family $\{X'_t\}$, $t \in [0, 1]$, $X'_i = X_i$, $i = 0, 1$, such that $\{X'_t\}$ has no sky catastrophes.*

Given this conjecture we may readily apply general analogues in [21] of Theorem 1 to get applications to existence of Reeb orbits. Of course one exciting thing here is that it will be without so called "hard" elliptic pde techniques of pseudo-holomorphic curves. Instead the methods would just be conceptual methods of topology. It might be possible however that the proof of the conjecture above, itself may initially involve elliptic techniques. Not surprisingly given how powerful and far reaching these techniques have become. However whatever the method of proof, if we could verify this conjecture, we would obtain new qualitative dynamical-topological information about Reeb vector fields.

This is an ambitious problem, but we could also set some near term goals. For example we can at first restrict to homotopies $\{X_t\}$ of vector fields where the space of orbits S is well behaved, for example:

Question 1. Suppose S that is rectifiable in the sense of measure theory, does the conclusion of the Conjecture 1 hold in this case?

If S is rectifiable, then there are powerful additional tools at our disposal, most important of which is Sullivan's theory [27] which develops a connection between measure theory, and Hahn-Banach theorem with dynamics. Indeed under the rectifiable hypothesis it should be technically possible to just build on the proof of Theorem 2, and Sullivan's [27] to produce this special case of the conjecture, but it should still be interesting and challenging.

1.2. **Goal 1 of the proposal.** One of the principal goals of the proposal is to answer Question 1. This is possibly work in collaboration with Dennis Sullivan, of Stony Brook University, and City College of New York. We had a number of discussion on related topics.

2. Locally conformally symplectic geometry and rigidity

A locally conformally symplectic manifold or l. c. s. m. is a smooth $2n$-fold M with an l. c. s. structure: which is a non-degenerate 2-form ω, which is locally diffeomorphic to $f \cdot \omega_{st}$, for some (non-fixed) positive smooth function f, with ω_{st} the standard symplectic form on $\mathbb{R}^{2n}$. These were originally considered by Lee in [20], arising naturally as part of an abstract study of "a kind of even dimensional Riemannian geometry", and then further studied by a number of authors see for instance, [1] and [30]. This is a fascinating object, an l. c. s. m. admits all the interesting classical notions of a symplectic manifold, like Lagrangian submanifolds and Hamiltonian dynamics, while at the same time forming a much more flexible class. For example Eliashberg and Murphy show that if a closed almost complex $2n$-fold M has $H^1(M, \mathbb{R}) \neq 0$ then it admits a l. c. s. structure, [4], see also [2].

l. c. s. m.'s can also be understood to generalize contact manifolds. This works as follows. First we have a natural class of explicit examples of l. c. s. m.'s, obtained by starting with a symplectic cobordism (see [4]) of a closed contact manifold C to itself, arranging for the contact forms at the two ends of the cobordism to be proportional (which can always be done) and then gluing together the boundary components. As a particular case of this we get Banyaga's basic example.

Example 1 (Banyaga). Let (C, ξ) be a contact manifold with a contact form λ and take $M = C \times S^1$ with 2-form $\omega = d^{\alpha}\lambda := d\lambda - \alpha \wedge \lambda$, for α the pull-back of the volume form on S^1 to $C \times S^1$ under the projection.

Using above we may then faithfully embed the category of contact manifolds, and contactomorphism into the category of l. c. s. m.'s, and ***loose*** l. c. s. morphisms. These can be defined as diffeomorphisms $\phi : (M_1, \omega_1) \to (M_2, \omega_2)$ s.t. $\phi^*\omega_2 = f \cdot \omega_1$, for a positive function f.

Banyaga type l. c. s. m.'s give immediate examples of almost complex manifolds where the energy function is unbounded on the moduli spaces of fixed class pseudo-holomorphic curves, as well as where null-homologous J-holomorphic curves can be non-constant. We show in [22] that it is still possible to extract a variant of Gromov-Witten theory for l. c. s. m.'s. The story is closely analogous to that of the Fuller index in dynamical systems, which is concerned with certain rational counts of periodic orbits. In that case sky catastrophes prevent us from obtaining a completely well defined invariant, but Fuller constructs certain partial invariants which give dynamical information. In a very particular situation the relationship with the Fuller index becomes perfect as one of the results of [22] obtains the classical Fuller index for Reeb vector fields on a contact manifold C as a certain genus 1 Gromov-Witten invariant of the l. c. s. m. $C \times S^1$. The latter also gives a conceptual interpretation for why the Fuller index is rational, as it is reinterpreted as an (virtual) orbifold Euler number.

2.0.1. *Non-squeezing and rigidity.* Of course what we are really interested in is what kind of rigidity phenomenon can appear in l. c. s. geometry. As a first attempt what can be said about non-squeezing? Recall that one of the most fascinating early results in symplectic geometry is the so called Gromov non-squeezing theorem appearing in the seminal paper of Gromov [11]. The most well known formulation of this is that there does not exist a symplectic embedding $B_R \to D^2(r) \times \mathbb{R}^{2n-2}$ for $R > r$, with B_R the standard closed radius R ball in $\mathbb{R}^{2n}$ centered at 0. Gromov's non-squeezing is C^0 persistent in the following sense.

Theorem 3. *Given $R > r$, there is an $\epsilon > 0$ s.t. for any symplectic form ω' on $S^2 \times T^{2n-2}$ C^0 ϵ-close to a split symplectic form ω, which satisfies:*
$$\langle \omega, (A = [S^2] \otimes [pt]) \rangle = \pi r^2,$$
there is no symplectic embedding $\phi : B_R \hookrightarrow (S^2 \times T^{2n-2}, \omega')$.

On the other hand we have:

Theorem 4. *Given $R > r$ and every $\epsilon > 0$ there is a (necessarily by the theorem above non-closed) 2-form ω' on $S^2 \times T^{2n-2}$ C^0 close to a split symplectic form ω, satisfying $\langle \omega, A \rangle = \pi r^2$, and s.t. there is an embedding $\phi : B_R \hookrightarrow S^2 \times T^{2n-2}$, with $\phi^*\omega' = \omega_{st}$.*

Theorem 3 follows immediately by Gromov's argument in [11], while Theorem 4 is to be proved in a paper in preparation by the author. In what follows we outline an extension of Theorem 3 to l. c. s. manifolds.

One may think that recent work of Müller [26] may be related to the above. But there seems to be no obvious such relation as pull-backs by diffeomorphisms of nearby forms may not be nearby. Hence there is no way to go from nearby embeddings that we work with to ϵ-symplectic embeddings of Müller.

Definition 2.1. *Given a pair of* l. c. s. m.*'s (M_i, ω_i), $i = 0, 1$, we say that $f : M_1 \to M_2$ is a* **morphism***, if $f^*\omega_2 = \omega_1$. A morphism is called an* l. c. s. *embedding if it is injective.*

A pair (ω, J) for ω l. c. s. and J compatible will be called a compatible l. c. s. pair, or just a compatible pair, where there is no confusion. Note that the pair of hypersurfaces $\Sigma_1 = S^2 \times S^1 \times \{pt\} \subset S^2 \times T^2$, $\Sigma_2 = S^2 \times \{pt\} \times S^1 \subset S^2 \times T^2$ are naturally foliated by symplectic spheres, we denote by $T^{fol}\Sigma_i$ the sub-bundle of the tangent bundle consisting of vectors tangent to the foliation. The following theorem proved in my [22] says that it is impossible to have certain "nearby" l. c. s. embeddings, which means that we have a first rigidity phenomenon for l. c. s. structures. There is a small caveat here that in what follows we take the C^0 norm on the space of l. c. s. structures that is (likely) stronger then the natural C^0 norm (with respect to a metric) on the space of forms.

Theorem 5. *Let ω be a split symplectic form on $M = S^2 \times T^2$, and A as above with $\langle \omega, A \rangle = \pi r^2$. Let $R > r$, then there is an $\epsilon > 0$ s.t. if ω_1 is an* l. c. s. *on M C^0 ϵ-close to ω, then there is no* l. c. s. *embedding*

$$\phi : (B_R, \omega_{st}) \hookrightarrow (M, \omega_1),$$

s.t $\phi_* j$ preserves the bundles $T^{fol}\Sigma_i$, for j the standard almost complex structure.

We note that the image of the embedding ϕ would be of course a symplectic submanifold of (M, ω_1). However it could be highly distorted, so that it might be impossible to complete $\phi_*\omega_{st}$ to a symplectic form on M nearby to ω. We also note that it is certainly possible to have a nearby volume preserving as opposed to l. c. s. embedding which satisfies all other conditions. Take $\omega = \omega_1$, then if the symplectic form on T^2 has enough volume, we can find a volume preserving map $\phi : B_R \to M$ s.t. $\phi_* j$ preserves $T^{fol}\Sigma_i$. This is just the squeeze map, which as a map $\mathbb{C}^2 \to \mathbb{C}^2$ is $(z_1, z_2) \mapsto (\frac{z_1}{a}, a \cdot z_2)$. In fact we can just take any volume preserving map ϕ, which doesn't hit Σ_i.

2.1. **Goal 2.a.** Remove the condition in the theorem above that $\phi_* j$ preserves $T^{fol}\Sigma_i$. With this condition removed we obtain a simple and direct extension of the original Gromov non-squeezing to l. c. s. manifolds.

2.1.1. *Toward non-squeezing for loose morphisms.* In some ways loose morphisms of l. c. s. m.'s are more natural, particularly when we think about l. c. s. m.'s from the contact angle. So what about non-squeezing for loose morphisms as defined above? We can try a direct generalization of contact non-squeezing of Eliashberg-Polterovich [3], and Fraser in [5]. Specifically let $R^{2n} \times S^1$ be the pre-quantization space of R^{2n}, or in other words the contact manifold with the contact form $d\theta - \lambda$, for $\lambda = \frac{1}{2}(ydx - xdy)$. Let B_R now denote the open radius R ball in $\mathbb{R}^{2n}$.

Question 2. If $R \geq 1$ is there a compactly supported, loose endomorphism of the l. c. s. m. $\mathbb{R}^{2n} \times S^1 \times S^1$ which takes the closure of $U := B_R \times S^1 \times S^1$ into U?

2.2. **Goal 2.b.** Show that the answer to the above question is no. In this case we obtain an analogue of contact non-squeezing in l. c. s. geometry. We can prove the no answer, assuming the following l. c. s. analogue of the Weinstein conjecture.

2.3. **Holomorphic Weinstein conjecture.** In contact geometry rigidity phenomena circulate around existence phenomena of Reeb orbits. The most important conjecture concerning Reeb orbits is the Weinstein conjecture, which says that a closed contact manifold always has a Reeb orbit. For contact 3-folds this is now a very deep theorem of Taubes [29]. If l. c. s. m.'s are generalizations of contact manifolds, what is the analogue of this conjecture in l. c. s. geometry? To start we propose the following, which we we show in [22] to directly extend the Weinstein conjecture.

Conjecture 2. *Let* (ω, J)*, be a compatible pair of an almost complex structure and an* l. c. s. *form on* $M = C \times S^1$*, for* C *a threefold or* S^{2k+1}*. Suppose* ω *is of the form* $\omega = d\lambda + \lambda \wedge d\theta$ *for* λ *a 1 form on* M*. Then there is an elliptic, non-constant* J*-holomorphic curve in* M*.*

In [22] we prove this conjecture for ω C^∞ nearby to the Hopf l. c. s. structure on $S^{2k+1} \times S^1$, by exploiting the connection with Fuller index, to which we already alluded in our discussion of sky catastrophes. Moreover we show that either this conjecture holds for any l. c. s. structure ω homotopic to the Hopf l. c. s. structure, or there exist holomorphic sky catastrophes, which are analogues in world of pseudo-holomorphic curves of sky catastrophes of Definition 1.1. In particular this partially elaborates the connection of the rigidity story for l. c. s. manifolds and dynamical systems.

Of course it is natural to ask, other then curiosity what is the significance of the above conjecture? Well as we mentioned contact geometry rigidity is linked to existence of Reeb orbits, for example there are certain capacities called ECH capacities, and related constructions [14] that come from the machine of embedded contact homology of Huchings-Taubes, (and built using Reeb orbits). Recall that the embedded contact homology constructed by Hutchings is used by Taubes for the proof of the three dimensional Weinstein. There is an analogous story in the l. c. s. setting, and using it we will get new rigidity phenomena in l. c. s. geometry that we are looking for, in particular Question 2 on loose l. c. s. non-squeezing will be answered, given the conjecture above.

2.4. **Outline of the proof of Conjecture 2 when C is a 3-fold.** In this section we shall assume that the reader knows some basic language of symplectic field theory, Seiberg-Witten theory and embedded contact homology. When C is a 3-fold, and the Seiberg-Witten invariant of $M = C \times S^1$ is non-vanishing the path to the proof of Conjecture 2 is almost obvious. First we take a separating contact hypersurface Σ in M, and perform neck-stretching on the pair (ω, J) at Σ. In the end we brake up M into a pair of pieces M_i with cylindrical ends, with l. c. s. forms ω_i which are in fact globally conformally symplectic, at least when M_i are simply connected - for a general C we have to make additional assumptions on the Lee class of ω, and or the hypersurface Σ. (Lee class is a certain invariant of an l. c. s. form which vanishes when it is symplectic, see my [22] for instance.)

Globally conformally symplectic is as good as symplectic for the purpose of Gromov-Witten or SFT analysis, more specifically compactness works the same way. We may then consider the count (in the total homology class A determined by the spin-c structure induced by ω) of holomorphic buildings consisting of a pair of holomorphic curves with ECH index 0 in each part M_i with the same asymptotic constraints at the cylindrical end. This count is an invariant of the l. c. s. (M, ω), even though the count of J-holomorphic curves in M by itself is not a priori an invariant, and we claim that this invariant is the Seiberg-Witten invariant of M, for the spin-c structure determined by ω. This of course builds on the foundational work of Hutchings and Taubes, there are series of groundbreaking papers here, we give a very short list, [13], [15], [28], [29]. Luckily the technical work for this correspondence in a setup closely analogous to the above is worked out in the thesis of Chris Gerig [9], [8], which generalizes work of Taubes on GW-SW correspondence to general smooth 4-folds.

So if the Seiberg-Witten invariant of M is non-vanishing then a gluing argument gives an existence of a non-constant, class A, J-holomorphic curve in M. This may be enough for the applications that we have in mind (e.g. Question 2), however a more careful analysis should yield that this curve is actually an elliptic curve.

2.5. **Goal 3.** Prove holomorphic Weinstein conjecture for $M = C \times S^1$ with C a 3-fold, following the above outline.

3. An l. c. s.-homology theory

The above approach to existence of elliptic curves in $C \times S^1$ via Seiberg-Witten theory is very partial, we need C to be dimension 3, and we need topological restrictions on C, otherwise the Seiberg-Witten invariant may not even be defined. For general l. c. s. manifolds M we need to develop an analogue of contact homology, denoted by $CSH(M)$ for example. Indeed for the Banyaga l. c. s. structure on $M = C \times S^1$ with (C, λ) contact as in Example 1, for an appropriate almost complex structure J_λ, all J_λ-holomorphic tori are in one to one correspondence with Reeb orbits of (C, λ). They are just products of Reeb orbits by the S^1 factor of $M \times S^1$. But these Reeb tori as we call them have an additional structure: the form $d\lambda$ vanishes on them identically, we say that they are ***calibrated*** by $d\lambda$.

We first generalize the above to a Lichnerowitz exact l. c. s. structure ω on $M = C^{2n-1} \times S^1$, with C closed, i.e. $\omega = d\lambda + \lambda \wedge d\theta$, for λ a general 1-form on M, s.t. ω is non-degenerate. Let $\mathcal{V}$ denote the vanishing distribution of $d\lambda$. That is $v \in \mathcal{V}_p \subset T_pM$ iff $\omega(v, \cdot) = 0$. Then $\mathcal{V}$ is a 2-dimensional distribution: $\mathcal{V}_p$ has dimension at least 2 since $d\lambda$ cannot be symplectic since M is closed, and has dimension at most 2 since $d\lambda + \lambda \wedge d\theta$ is non-degenerate. Let ξ denote the co-vanishing distribution that is ξ_p is the ω-orthogonal complement to $\mathcal{V}_p$. We work with a class of ω-compatible complex structures J which preserve both ξ and $\mathcal{V}$. This extends the type of J used in symplectizations.

For J as above, and a closed J-holomorphic curve u in M, we have $u^* d\lambda = 0$, so again such a u is calibrated. We define l. c. s.-homology $CSH(M)$ over $\mathbb{Z}_2$ to have generators non-constant J-holomorphic elliptic curves u in M.

The instantons are J-holomorphic maps $u : S^1 \times \mathbb{R} \to M$ with $\int u^* d\lambda < \infty$.

Lemma 3.1. *An instanton is asymptotic at the ends to generators, that is asymptotically wraps around the generators.*

Proof. (Outline) In this case M fibers over S^1 with contact fibers, with contact distributions restrictions of ξ above. Let $(M_\theta, \lambda_\theta)$ denote the corresponding contact fibers. In this case analogously to the Banyaga example a non-constant elliptic curve in M must be foliated by $\{\lambda_\theta\}$-Reeb closed orbits, by the calibration condition. Now given an instanton u, at the ends $u^* d\lambda$ is asymptotically vanishing which means that u is asymptotically a "Reeb cylinder": a smooth s-family of $\lambda_{f(s)}$-Reeb orbits, for $s \in \mathbb{R}_+$ and $f(s) \in S^1$ for f determined by u. To finish the proof we need to show that given any Reeb cylinder as above, with finite energy, it must be a Reeb torus. Let u_s denote the slice of a Reeb cylinder u over $f(s) \in S^1$, that is u_s is a $\lambda_{f(s)}$-Reeb orbit. Let s_0 be fixed, and suppose that there is no $s > s_0$ with

$$f(s) = f(s_0) = \theta_0$$

such that $u_s = u_{s_0}$. Then by the finite energy condition we obtain a non-constant sequence $\{\gamma_n = u_{s_n}\}$ of λ_{θ_0}-Reeb orbits with bounded period, which must have a convergent subsequence $\{\gamma_{n_k}\}$ by Azrella-Ascolli. If we assume that λ_{θ_0} is Reeb non-degenerate then this sequence must eventually be constant and we are done. $\square$

3.1. **Goal 4.a.** Develop the homology theory $CSH(M)$.

3.2. **Goal 4.b.** Use $CSH(M)$ to prove l. c. s. non-squeezing, as in Goal 2.b.

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