CALCULUS III, EXAM 1

Question 1.

(1) Find the partial derivatives a, b, c: $D_1 f(P) = a, D_2 f(P) = b, D_3 f(P) = b$, $P = (1, 0, 2)$ of $f(x, y, z) = z\sin(xy) + e^x z^2 y$.

(2) Find the gradient of the function f at P: $\operatorname{grad} f(P)$ (please express the answer in terms of a, b, c).

(3) Find the tangent plane to the surface $f(x, y, z) = 0$ at the point P (please express the answer in terms of a, b, c).

(4) Let $c(t)$ be a curve with $c(0) = P$, and $c'(0) = d$ what is the derivative at 0: $(f \circ c)'(0)$ in terms of a, b, c, d? (Hint: use Chain rule.)

Question 2. Let $f(x, y, z) = x^2 + 3y^2$.

(1) Find critical points in the interior of the closed domain D bounded by the sphere $x^2 + y^2 + z^2 = 1$.
(2) Find local max and min in the interior of D, i.e. in $x^2 + y^2 + z^2 < 1$.
(3) Find the critical points of f on the boundary of D. (Hint it may help to use Lagrange multipliers.)
(4) Find the absolute max and min of f on D. Justify your answer.

Question 3.

(1) Find the critical points for
$$f(x, y) = x^2 e^y - 6y^2.$$
(2) Find the Taylor quadratic form
$$q = f_{xx}(P)x^2 + 2f_{xy}(P)xy + f_{yy}(P)z^2,$$
at each critical point P.

(3) Determine whether the critical points of f are local maxima, local minima, or saddle points. (Justify your answer.)

Question 4.

(1) Let $F(x,y) = (ye^x + 2x, e^y)$ be a vector field on $\mathbb{R}^2$. Does it have a potential function? If so find it. Find the curve integral of F: $\int_C F$ for the curve
$$c(t) = (sin(t^3)e^t t, t^4 + 3t + 1),$$
for $0 \leq t \leq 1$.

(2) Same question for $F(x,y) = (x^2 \sin y, x)$ and the curve $c(t) = (t^2, t)$.

Question 5. Let $f(x, y) = x$. Find the double integral of f over the region D bounded by the curves $x^2 + y^2 = 1$, and $x = 0$. (So $x, y \in D$ satisfies $x \geq 0$ and $x^2 + y^2 \leq 1$).