CMPSC 465 Data Structures & Algorithms
Spring 2021 Paul Medvedev and Chunhao Wang

HW 8

Due March 25, 10:00 pm

Instructions: You may work in groups of up to three people to solve the homework. You must write your own solutions and explicitly acknowledge up everyone whom you have worked with or who has given you any significant ideas about the HW solutions. You may also use books or online resources to help solve homework problems. All consulted references must be acknowledged.

You are encouraged to solve the problem sets on your own using only the textbook and lecture notes as a reference. This will give you the best chance of doing well on the exams. Relying too much on the help of group members or on online resources will hinder your performance on the exams.

Late HWs will be accepted until 11:59pm with a 20% penalty. HWs not submitted by 11:59pm will receive 0. There will be no exceptions to this policy, as we post the solutions soon after the deadline. However, you will be able to drop the three lowest HW grades.

For the full policy on HW assignments, please consult the syllabus.

1. **(0 pts.) Acknowledgements.** The assignment will receive a 0 if this question is not answered.
   (a) If you worked in a group, list the members of the group. Otherwise, write "I did not work in a group."
   (b) If you received significant ideas about the HW solutions from anyone not in your group, list their names here. Otherwise, write "I did not consult without anyone my group members".
   (c) List any resources besides the course material that you consulted in order to solve the material. If you did not consult anything, write "I did not consult any non-class materials."

2. **(10 pts.) Connectivity detection** Given a connected and undirected graph $G = (V,E)$. Design an $O(|V|)$ time algorithm that outputs "yes" if there is an edge you can remove from G while still leaving G connected, and outputs "no" otherwise.

3. **(12 pts.) Shortest path and MST**

   Let $G = (V,E)$ be an undirected graph with edge weights $w_e \geq 1$. Suppose that you have computed a minimum spanning tree of G, and that you have also computed a shortest path from a vertex $s \in V$ to another vertex $t \in V$. Now, each edge weight is decreased by 1, i.e., the new weights are $w'_e := w_e - 1$.
   (a) Does the MST change? Give an example where it changes or prove it cannot change.
   (b) Does the shortest path change? Give an example where it changes or prove it cannot change.

4. **(12 pts.) MST and cut property**

   Let $T = (V,E_T)$ be an MST of graph $G = (V,E)$. Given a connected subgraph $H = (V_H,E_H)$ of G, i.e., $V_H \subseteq V$ and $E_H \subseteq E$, show that $T \cap H := (V \cap V_H, E_T \cap E_H)$ is contained in some MST of H.

   (Hint. Recall the cut property: let A be part of some MST of $G = (V,E)$ and let $(S,V-S)$ be a cut that respects A. If e is the lightest edge across this cut, then $A \cup \{e\}$ is part of some MST.)