1 Homework 9

Name: Kangdong Yuan

1.1 problem1

a).I did not work in a group.
b).I did not consult without anyone my group members
c).I did not consult any non-class materials.

1.2 problem2

We prove by contradiction.
Given e^* is the heaviest edge in some cycle C of undirected graph G.
We assume the claim is Flase, so T is a minimal spanning forest and $e^* \in T$. Let $S,\ V-S$ be the two connected components in $T \setminus \{e\}$. Since C is a cycle, there are two edges in C that cut $S,\ V-S$. We define another edge is r. The e^* has more cost than edge r, which is $w(e) > w(r)$. So, the new minimal spanning forest with edge r cost less than minimal spanning forest with edge e^*. It is a contradiction to the minimal cost principle of minimal spanning forest.
So, e^* cannot in minimal spanning forest, which cannot appear in any MST of G.

1.3 problem3

a). $f_a > f_b\ and\ f_a > f_c$, so $f_a = 10, f_b = 5, f_c = 5$
b). This encoding is not possible, because the code for a is (0), is a prefix of the code for c is (00).
c). This encoding is not optimal, because the $\{0, 10, 11\}$ cost less space. And, the Huffman tree of this encoding is not complete, which is not optimal.

1.4 problem4

we can construct Huffman tree to find prefix-free encoding of minimal total cost. As a common convention, bit '0' represents following the left child and bit '1' represents following the right child.

First, using a priority queue Q to store all the words as key and $f_i * c_i$ as value. In Q, lowest value is given highest priority.

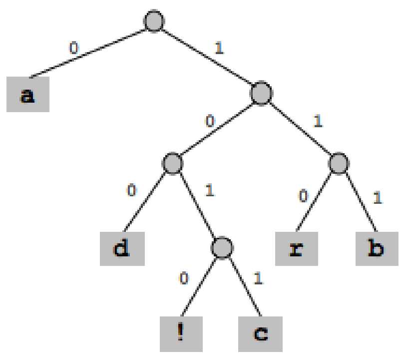
1. Create a leaf node for each symbol and add it to the Q.
2. While there is more than one node in the queue:

    Remove the two nodes from Q (lowest $f_i * c_i$)

    Create a new internal node with these two nodes as children and with value $(f_i * c_i)$ equal to the sum of the two nodes' value $(f_i * c_i)$.

    Add the new node to the queue.
3. The remaining node is the root node and the tree is complete.

Then the prefix-free encoding of each word is path of each node. for example



char	encoding
a	0
b	111
c	1011
d	100
r	110
!	1010

Time complexity: $O(nlogn)$, because each iteration requires $O(logn)$ time to determine the lowest frequencies and insert the new word in priority queue. There are $O(n)$ iterations.