CMPSC 465 | Data Structures & Algorithms
Spring 2021 | Paul Medvedev and Chunhao Wang

HW 10

Due April 22nd, 10:00 pm

Instructions: You may work in groups of up to three people to solve the homework. You must write your own solutions and explicitly acknowledge up everyone whom you have worked with or who has given you any significant ideas about the HW solutions. You may also use books or online resources to help solve homework problems. All consulted references must be acknowledged.

You are encouraged to solve the problem sets on your own using only the textbook and lecture notes as a reference. This will give you the best chance of doing well on the exams. Relying too much on the help of group members or on online resources will hinder your performance on the exams.

Late HWs will be accepted until 11:59pm with a 20% penalty. HWs not submitted by 11:59pm will receive 0. There will be no exceptions to this policy, as we post the solutions soon after the deadline. However, you will be able to drop the three lowest HW grades.

For the full policy on HW assignments, please consult the syllabus.

1. **(0 pts.) Acknowledgements.** The assignment will receive a 0 if this question is not answered.

   (a) If you worked in a group, list the members of the group. Otherwise, write "I did not work in a group."

   (b) If you received significant ideas about the HW solutions from anyone not in your group, list their names here. Otherwise, write "I did not consult without anyone my group members".

   (c) List any resources besides the course material that you consulted in order to solve the material. If you did not consult anything, write "I did not consult any non-class materials."

2. **(10 pts.) Unit-time task scheduling**

   Recall the unit-time task scheduling problem covered in the class. Let $S = \{a_1, \ldots, a_n\}$ be a set of n unit-time tasks, i.e., each task takes a unit time to complete. Let $d_1, \ldots, d_n$ be the corresponding deadlines for the tasks and $w_1, \ldots, w_n$ be the corresponding penalties if you don't complete task a_i by d_i. Note that $1 \le d_i \le n$, and $w_i > 0$ for all i. The goal is to find a schedule (i.e., a permutation of tasks) that minimized the penalties incurred.

   Recall that we can model this problem as a matroid maximum independent subset problem. Consider the matroid $M = (S, \mathcal{I})$, where $S = \{a_1, \ldots, a_n\}$ and

   $$\mathcal{I} = \{A \subseteq S, \text{ s.t. there exists a way to schedule the tasks in } A \text{ so that no task is late}\}.$$

   Finding the maximum independent subset of M is equivalent to finding the optimal schedule (as shown in the class).

   An important step in the greedy algorithm for the maximum independent subset problem is to check whether $A \cup \{x\} \in \mathcal{I}$ for $x \in S$. Show that for all $x \in S$, checking whether $A \cup \{x\} \in \mathcal{I}$ can be done in $O(n)$ time.

   You may find the following lemma useful:

Lemma. For $t = 0, 1, \ldots, n$, let $N_t(A)$ denote the number of tasks in A whose deadline is t or earlier. Note that $N_0(A) = 0$ for any set A. Then, the set A is independent if and only if for *all* $t = 0, 1, \ldots, n$, we have $N_t(A) \leq t$.

3. (12 pts.) Destroying missiles

Enemy missiles are arriving over the course of n seconds; in the i-the second, x_i missiles arrive. Based on remote sensing data, you know the sequence $x_1, x_2, \ldots, x_n$ in advance. You are in charge of an electromagnetic pulse (EMP), which can destroy some of the missiles as they arrive. The power of EMP depends on how long it has been allowed to charge up. More precisely, there is a function f so that if j seconds have passed since the EMP was last used, then it is capable of destroying up to $f(j)$ missiles. So, if the EMP is being used in the k-th second and it has been j seconds since it was previously used, then it destroys $\min\{x_k, f(j)\}$ missiles in the k-th second. After this use, it will be completely drained. We assume that the EMP starts off completely drained, so if it used for the first time in the j-th second, then it is capable of destroying up to $f(j)$ missiles. Your goal is to choose the points in time at which the EMP is going to activated so as to destroy as many as missiles as possible.

Give an efficient algorithm that takes the data on missile arrivals $x_1, \ldots, x_n$, and the recharging function f, and returns the maximum number of missiles that can be destroyed by a sequence of EMP activations. Analyze the running time of your algorithm.

4. (18 pts.) Card game

A dealer produces a sequence $s_1, \ldots, s_n$ of cards, face up, where each card s_i has value v_i. Then two players take turns picking a card from the sequence, but can only pick the first or the last card of the (remaining) sequence. The goal is to collect cards of largest total value. (For example, you can think of the cards as bills of different denominations.) Assume n is even.

(a) Consider a greedy approach for the first player: always picking up the available card of larger value. Give a sequence of cards such that is not optimal for the first player. Explain what the greedy approach will give you and what a better solution is.

(b) Give an $O(n^2)$ algorithm to compute an optimal strategy for the first player. Given the initial sequence, your algorithm should precompute in $O(n^2)$ time some information, and then the first player should be able to make each move optimally in $O(1)$ time by looking up the precomputed information.

5. (5 pts.) SRTE (Bonus credit) Please go to `srte.psu.edu` to submit your SRTE feedback (Student Rating of Teaching Effectiveness) which is available after April 19, 2021. This is not required, but if 90% of you have submitted your evaluations, *all* of you will get this bonus credit. I appreciate your feedback!