# Illustrations of ISM and CoDA for 24HAC data analysis

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## Descriptive table of the data

Table 1: Descriptive statistics of the sample

| | Overall |
|---|---|
| n | 1000 |
| sex = Female (%) | 586 (58.6) |
| Sit (hrs/day), mean (SD) | 10.12 (2.14) |
| Stand (hrs/day), mean (SD) | 3.87 (1.72) |
| Step (hrs/day), mean (SD) | 1.42 (0.82) |
| Sleep (hrs/day), mean (SD) | 8.59 (1.27) |

## ISM approach

### Linear ISM

Consider an example where each minute of the 24-hour day is classified into one of four activities: sleeping, sitting, standing, and stepping. The ISM is formulated by including the total activity and all but one of the activity variables – the activity you will explore displacing – in the model. For example, with a continuous health outcome an ISM that leaves out the time stepping can be formulated, as below:

$$E(Y) = \beta_0 + \beta_1 Sit + \beta_2 Stand + \beta_3 Step + \beta_4 Ttoal + \gamma Sex$$

where $E(Y)$ abbreviates the conditional mean of the health outcome given the time allocation variables (Sit, Stand, Sleep, Total measured on the same unit, e.g., hours in a 24-hour day), and the covariate sex. When *Total* is exactly a constant 24 hours/day for every subject, like in this hypothetical example, only one of the intercept or the Total terms can be included in the model.

Four linear ISMs adjusted for sex were fit to the data, with each of the four activities omitted from the model one at a time.

Table 2: Isotemporal Substitution of Activities, per 10-Minute/Day Increase

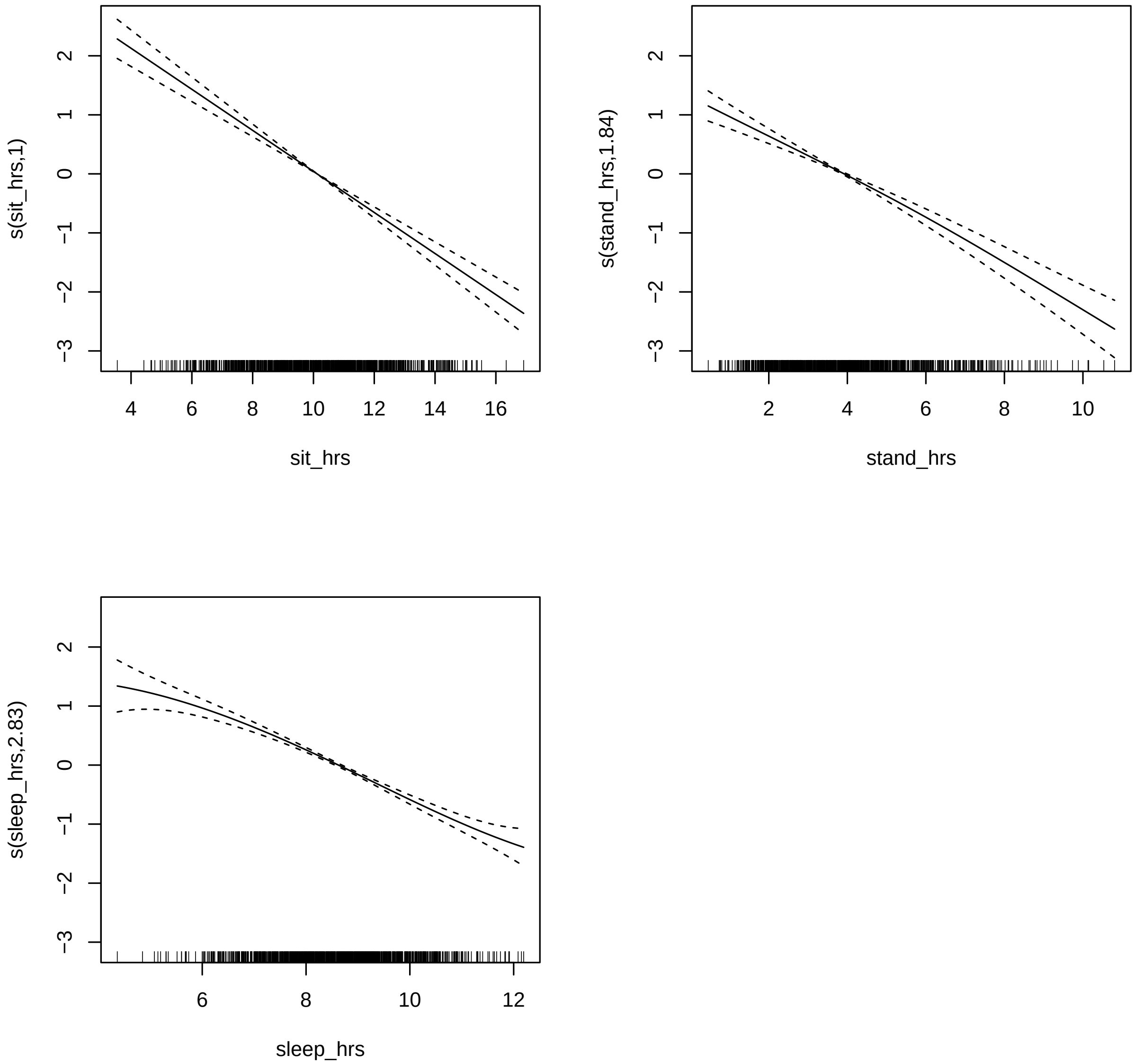
| | **Model with sit dropped** | | **Model with stand dropped** | | **Model with step dropped** | | **Model with sleep dropped** | |
|---|---|---|---|---|---|---|---|---|
| | Beta (95% C.I.) | p-value | Beta (95% C.I.) | p-value | Beta (95% C.I.) | p-value | Beta (95% C.I.) | p-value |
| Sit (hours) | Dropped | Dropped | 0.01 [-0.02, 0.03] | 0.656 | -0.35 [-0.40, -0.30] | <0.001 | 0.05 [0.02, 0.08] | 0.001 |
| Stand (hours) | -0.01 [-0.03, 0.02] | 0.656 | Dropped | Dropped | -0.35 [-0.42, -0.29] | <0.001 | 0.04 [0.01, 0.08] | 0.010 |
| Step (hours) | 0.35 [0.30, 0.40] | <0.001 | 0.35 [0.29, 0.42] | <0.001 | Dropped | Dropped | 0.40 [0.35, 0.45] | <0.001 |
| Sleep (hours) | -0.05 [-0.08, -0.02] | 0.001 | -0.04 [-0.08, -0.01] | 0.010 | -0.40 [-0.45, -0.35] | <0.001 | Dropped | Dropped |

The associations of 1-hr time reallocations between any two types of activity are summarized in Table 2. For example, the ISM model with Step dropped suggested that reallocating 1 hr/day from sitting, standing, or sleeping to stepping was associated with 0.35 [0.30, 0.40], 0.35 [0.29, 0.42], and 0.40 [0.35, 0.45] units higher mean (95% CI) outcome, respectively.

### Nonlinear ISM

A more flexible ISM could be fit with each activity term modeled by a penalized spline function, while keeping the total activity as a linear term. The slope of a spline term represents the instantaneous effect of increasing a small amount of time in the activity the spline term corresponds to, while decreasing the same small amount of time in the activity that is left out from the model. The optimal trade-off between smoothness and goodness of fit can be determined by either performing cross validation or minimizing the generalized cross validation (GCV) criteria. The significance of the association for each behavior in the nonlinear ISM model can be tested via a Wald like test [Wood SN 2006]. The nonlinear ISM analysis can be done in R with package “mgcv”.

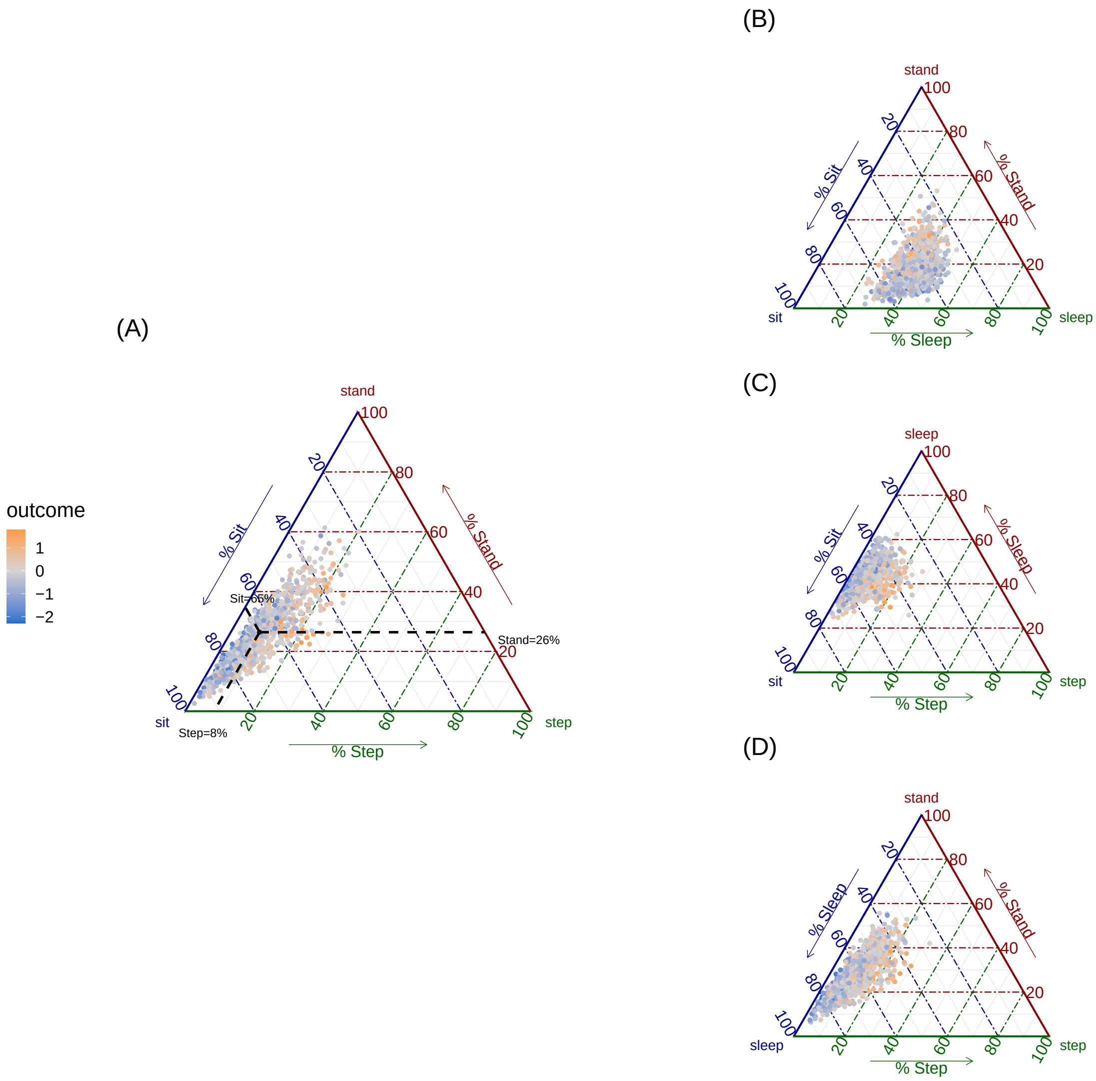
For example, we fit a nonlinear ISM dropping step time adjusted for sex. The estimated smoothing terms for sit, stand, step are shown below. At the mean composition, all the smoothing terms equal to 0 because of the identifiablility constrain. We observed that increasing time in each of the behavior was associated with worse mean outcome, and in the main range of the data, the associations were approximately linear. This shows consistent results to that obtained from linear ISMs, and indicates that linear ISM would fit the data equally well and could be a better option for this hypothetical data because of its parsimony.


s(sit_hrs,1)
sit_hrs
s(stand_hrs,1.84)
stand_hrs
s(sleep_hrs,2.83)
sleep_hrs

# CoDA

Visualizations and compositional descriptive statistics of 24HAC can be helpful, before fitting any models. The Figure below displays 24HAC compositions of sit, stand, step, and sleep for the fake data using ternary diagrams, a common tool to visualize composition with 3 parts. Since the 24HAC of interest here consists of four activity behaviors, we plotted four ternary diagrams (A-D below), with each graph representing a sub-composition of three activity behaviors. From the figure below, we can see how sub-compositions are distributed and possibly associated with the outcome.

## Simplex plot




## Comparisons of compositional means by groups

The compositional mean is a common descriptive statistics to describe central tendency of compositional data. It is defined as the vector of geometric means of each behavior, rescaled to sum to 1. Since the components of a composition are inter-correlated, it is not sensible to calculate the variance of a single component. In stead, a variation matrix for the log-ratio is used to describe the interdependence between every pair of behaviors i.e. each element of that matrix is the variance of log-ratio between two components. An off-diagonal value close to 0 means the two parts are highly proportional in the observed data. Both compositional mean and variation matrix can be easily coded in R.

For inferential analysis such as hypothesis testing and regressions, CoDA relies on the isometric log-ratio (ilr) transformation, which transforms each D-part composition to a unique D-1 vector on a new coordinate system where each new coordinate is a log-ratio which falls along the real line. For example, a possible transformation is as follows:

$$z_1 = \sqrt{\frac{3}{4}} ln(\frac{Sit}{(Stand \times Step \times Sleep)^{\frac{1}{3}}})$$

$$z2 = \sqrt{\frac{2}{3}} ln(\frac{Stand}{(Step \times Sleep)^{\frac{1}{2}}})$$

$$z_3 = \sqrt{\frac{1}{2}} ln\frac{Step}{Sleep}$$

In R, we used the function *pivotCoord()* from the package *robCompositions* for this transformation.

With transformed data ($z_1$, $z_2$, and $z_3$) and normality assumptions, we performed James multivariate analysis of variance with unequal variances to test the difference in the compositional mean between sex. The James test was available in the R pacage *Compositional*.

The table below presents the compositional means in the overall sample and the groups defined by sex. P-value = 0.348 indicating insufficient evidence to reject the null hypothesis that the compositional means are equal between males and females.

Table 3: Compositional mean in subgroups

| | N (%) | Sit | Stand | Step | Sleep | P-value |
|---|---|---|---|---|---|---|
| Overall | 1000 (100 %) | 10.3 ( 42.8%) | 3.6 ( 15.2%) | 1.3 ( 5.2%) | 8.8 ( 36.8%) | |
| Sex | | | | | | 0.348 |
| Male | 414 ( 41.4 %) | 10.23 ( 42.6 %) | 3.73 ( 15.5 %) | 1.27 ( 5.3 %) | 8.78 ( 36.6 %) | |
| Female | 586 ( 58.6 %) | 10.3 ( 42.9 %) | 3.58 ( 14.9 %) | 1.25 ( 5.2 %) | 8.87 ( 36.9 %) | |

*Note:*
p-value from multivariate analysis of variance on the isometric log-transformed time use variables without assuming equal variance across subgroups.

### CoDA regressions and interpretations

Next, we applied CoDA to estimate a type of time reallocation i.e. increasing time in one activity while simultaneously proportionally decreasing time in the other activities. To achieve this, it is convenient to create four sets of ilr-coordinates with each behavior in turn being singled out as the numerator in the pivot coordinate $z_1$. Four linear regression models were fit with the continuous outcome, and with the resulting ilr-coordinates ($z_1$, $z_2$, $z_3$) as predictors. Each regression model is adjusted for sex.

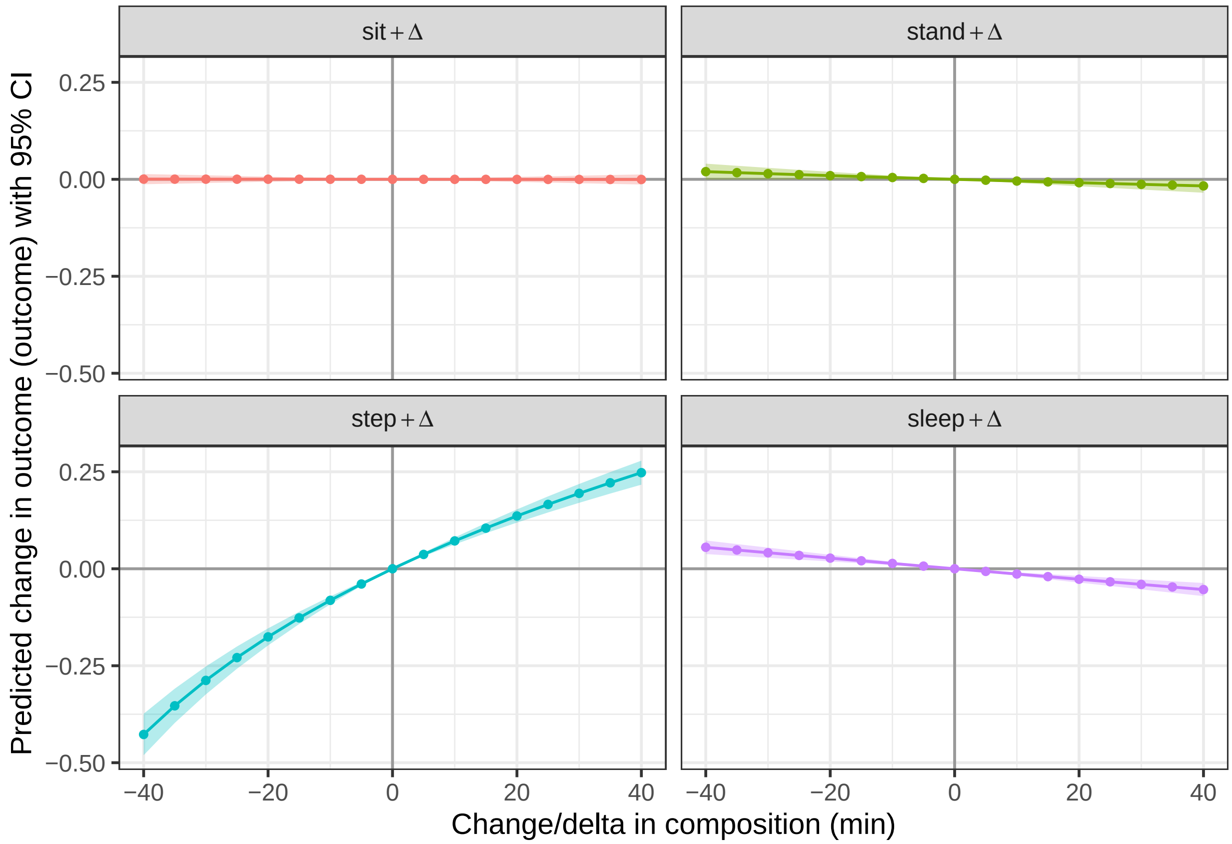
Table below summarized regression coefficient estimates $\hat{\beta}_1$ for the four CoDA pivot coordinates, each of which quantifies the effect of increasing time in one behavior by a factor while simultaneously decreasing time in the other behaviors by another factor. To make meaningful interpretation of those $\hat{\beta}_1$ estimates, we need to consider a referent composition in order to inform what magnitude difference in $z_1$ is a meaningful difference. Suppose the compositional mean calculated over the entire sample is chosen as the referent composition, and we are interested in the effect of increasing step by a factor of $1 + r$. Then, all the other components should simultaneously be decreased by another factor $1 - s$ to maintain $z_2$ and $z_3$ constant. Some derivations show that the difference in the mean outcome for such a time reallocation equals to $\hat{\beta}_1\sqrt{\frac{3}{4}}log(\frac{1+r}{1-r})$. We created a R function *comp_contrast* that can output the difference between any two compositions in terms of ilr-coordinates (using *pivotCoord* function from the package *robCompositions*). More specifically, with a fitted CoDA regression model with $z_1$, $z_2$, $z_3$ (from pivotCoord) as predictors and a given referent composition say $c_1$, to estimate the effect of a specific time reallocation, we only need to know the composition $c_2$ after such time reallocation, and enter $c_1$ and $c_2$ into the *comp_contrast* function, we can obtain the difference between the two compositions in terms of $z_1$, $z_2$, and $z_3$. The estimated effect of such time reallocation is a linear combination of $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$ with coefficients equal to that difference. Note that CoDA regression results should be the same regardless of the form of ilr-transformations i.e. which set of ilr-coordinates is used.

Table 4: Regression of pivot coordinates against the outcome. Analysis controlled for sex. Remaining = remaining behaviors

| | Estimate (95% C.I.) | P-value |
|---|---|---|
| Sit vs Remaining | -0.00 [-0.14, 0.13] | 0.949 |
| Stand vs Remaining | -0.10 [-0.20, 0.00] | 0.059 |
| Step vs Remaining | 0.63 [0.55, 0.71] | <0.001 |
| Sleep vs Remaining | -0.53 [-0.69, -0.36] | <0.001 |

Based on the results in Table, we can see increasing time in step while proportionally decreasing time in the other activities is associated with higher mean outcome. In contrast, increasing time in sleep while proportionally decreasing time in the other activities is associated with lower mean outcome. More specifically, increasing 30 minsin step while proportionally decreasing time in the other behaviors is associated with 0.19 [0.17, 0.22] increase in the outcome. Increasing 30 mins in sleep while proportionally decreasing time in the other behaviors is associated with a decrease in mean outcome of 0.04 [0.03, 0.05].

The results can be visualized by using the package *codaredistlm* available on Github: github.com/tystan/codaredistlm. Please see the blow plots created by using the function *pred_df* and *plot_delta_comp*.


sit + Δ
stand + Δ
step + Δ
sleep + Δ
Predicted change in outcome (outcome) with 95% CI
0.25
0.00
−0.25
−0.50
−40
−20
0
20
40
Change/delta in composition (min)

We can use the same functions to estimate and visualize the effect of time-reallocation between any pair of behaviors, e.g. reallocate time only between step and sit. See the plot below.