

Prestige as a Driving Force in Cultural Transmission

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Abstract

Copying role-models can be an efficient method for acquiring knowledge. A common bias when choosing a role-model to copy is success bias: copying whoever appears more successful. This bias depends on the performance of the role-model alone, with no other factors. We propose an additional bias that may be prevalent in cultural transmission: influence bias, in which role-model choice is affected by the number of individuals that have already copied each potential role-model. We combine success and influence bias together to a “prestige bias” and analyze its effects on cultural-evolutionary dynamics using mathematical analysis and stochastic simulations. We find analytic approximations to our stochastic model, facilitating further mathematical analysis and reducing the computational complexity of simulations. We validate these approximations using simulations, and demonstrate their robustness to model assumptions. We also find approximations to the fixation probability and the fixation time of an invading advantageous cultural trait, in both a constant and changing environment, which resemble Kimura’s classical formulas. These approximations show that success bias effectively plays the part of natural selection, whereas influence bias effectively reduces the population size. It also accelerates the evolutionary dynamics, as can be expected in a *rich-getting-richer* process. Our model may provide a better description of human cultural transmission, especially in the last years where social networks are very popular. Further work is needed to test if this model could predict various phenomena in human cultural evolution when extended with the effects of selection and innovation.

Introduction

28 **Cultural transmission.** In cultural transmission, individuals transmit cultural traits (i.e., behavior, beliefs, norms) to one another, typically via learning and demonstrating (Cavalli-Sforza and Feldman, 30 1981). Examples for cultural traits in humans are behavioral patterns, such as personalities and habits, transmitted via both verbally and by observations (O'Brien et al., 2010). Although cultural 32 transmission is most common in humans, it is also observed in other animals such as chimpanzees (Horner et al., 2010; Kendal et al., 2015), dolphins and whales (Whitehead, 2017). In elephants, 34 McComb et al. (2001) showed that once a matriarch is removed from the group, the group's survival instincts are inferior and that "the possession of enhanced discriminatory abilities by the oldest 36 individual [matriarch] in a group can influence the social knowledge of the group as a whole." By playing audio recordings of African elephants, they showed that groups with a matriarch recognize 38 and react better to hostile or friendly calls than groups without a matriarch. Battesti et al. (2012) showed that choice of oviposition site in fruit flies is culturally transmitted: inexperienced flies that 40 spent some time with experienced flies chose the same type of oviposition site even without directly observing this behavior. How the information is transmitted is still an open question, but it has been 42 suggested that flies may use olfactory cues like other animals, such as rodents and bees.

Direction of transmission. Similar to genetic transmission, culturally transmitted traits can be trans- 44 mitted from parents to offspring, and their effects of can be physiological rather than behavioral. For example, parents can "teach" their children to be strong or tall, within some biological limits, by 46 instructing them to maintain a specific diet and engage in physical activity. Contrary to genetic transmission, cultural transmission can be non-vertical, that is, traits may be transmitted via social learning 48 from non-parental individuals, and even unrelated individuals such as teachers, leaders, media, or any stranger that interacts with the learning individual. Thus, cultural transmission may combine 50 vertical transmission, where parents transmit to their offspring; oblique transmission, where adults transmit traits to unrelated offspring; and horizontal transmission, where peers from the same age 52 cohort transmit to one another. Vertical transmission is also possible in the opposite direction: parents may copy traits from their offspring (Cavalli-Sforza and Feldman, 1981; Creanza et al., 2017).

54 **Transmission biases.** In social learning, transmission biases cause a trait to have a disproportionate probability to be transmitted compared to its frequency in the population. Although more common in 56 cultural transmission, transmission biases do occur in genetic transmission. For example, *wtf* genes in yeast bias their transmission to the gamete by secreting a long life-expectancy poison together with a 58 short life-expectancy antidote, so that a gamete without the gene will perish because the poison will outlive the antidote (Eickbush et al., 2019). Importantly, even when a trait is disfavored by natural 60 selection, it may still spread in a population due to transmission biases that are strong enough to overcome selection (Boyd and Richerson, 1988, Ch. 8 pg. 279). Cohen et al. (2021) show that 62 cooperative behavior can evolve in an individual due to horizontal transmission bias even when there is no benefit to it, or when it benefits its competitors.

64 **Success bias.** Rendell et al. (2010) have conducted a tournament between learning strategies. Each
strategy defines when individuals observe and copy from others, and when they engage in individual
66 learning, in which an individual learns a cultural trait on his own. The best strategies had a high
frequency of social learning relative to individual learning, even when the transmission error was
68 almost 50%. From these results we understand that all the winning strategies were mostly based on
success biased social learning, meaning it contributed more to the general success of the population
70 than individual learning. However, all winning strategies included individual learning to some degree,
implying that success-biased learning alone isn't the best way to advance, even when success is
72 measured accurately.

Evaluating success. Boyd and Richerson (1988, Ch. 5) suggest that the evaluation of success can
74 be divided into three groups: *direct bias*, *indirect bias* and *frequency-dependent bias*. A direct bias
occurs when a variation of a trait is more attractive than others, and is evaluated by *directly* testing the
76 variation of the trait. For example, an individual observing a Ping-Pong match can attempt both of the
observed paddle grips to determine which grip is better. An indirect bias occurs when an individual
78 uses the value of one trait to determine the attractiveness of another, so it *indirectly* evaluates the
attractiveness of the role-model. For example, an observer may copy the paddle grip of the Ping-Pong
80 player who scored more points in the match, thus indirectly evaluating the grip by the points scored. A
frequency-dependent bias occurs when an individual has a probability to copy a variant of the trait that
82 higher or lower than trait's frequency among demonstrators. For example, when an individual is 80%
likely to copy the common paddle grip even when only 60% of the population is using it, it is said to be
84 frequency-biased, or in this case, conformist. Frequency bias could be negative, i.e., non-conformist
bias. Conformity and non-conformity are well-known biases in cultural transmission (Molleman et al.,
86 2013), and its effect on cultural evolution have been studied with both models (Denton et al., 2020a,b)
and experiments (Aljadeff et al., 2020).

88 **Prestige.** Prestige means having a good reputation or high esteem, therefore does not directly signify
success (although it may imply it), making it an indirect bias. Both Boyd and Richerson (1988, Ch.
90 8) and Fogarty et al. (2017) suggest that prestige biases are probably more common in humans than
success biases. Boyd and Richerson (1988, Ch. 8) add that maladaptive traits may spread widely
92 in a population if indirect biases are strong enough. They suggest that such biases could lead to a
runaway process caused by a cultural equivalent of sexual selection (Andersson, 1994). On the other
94 hand, Henrich and Broesch (2011) suggest that prestige biases, over generations, can lead to cultural
adaptations, and that although prestige can lead to maladaptive traits spreading in the population, it
96 can also accelerate the spread of adaptive traits. Prestige is often mentioned in the cultural-evolution
literature, but seldom modeled.

98 **Influence bias.** Today, social media provides an easy way to estimate the influence individuals have
over others, and therefore may have an effect on decision making. Online social networks such as
100 *Facebook* or *Instagram* are known to affect the social influence of individuals (Anagnostopoulos
et al., 2008; Peng et al., 2018; Diga and Kelleher, 2009). For example, specific marketing practices
102 were invented following this realization (Lee et al., 2012), Here we propose an indirect bias that we

call *influence bias*, in which the choice of a role-model depends on the the choices made by other
 104 individuals that have already chosen a role-model. This bias depends on the state of a role-model
 rather than a trait, in contrast to frequency biases such as conformity, which depend on the frequency
 106 of a trait in the population or in a sample of role-models. We define a model for prestige bias that
 combines both success and influence biases, provide analytic approximations for this model, and
 108 analyze its dynamics.

Models and Methods

110 We begin with a continuous trait model with indirect bias suggested by Boyd and Richerson (1988),
 develop an extension with influence bias, and then develop a model with a dichotomous trait. We
 112 implemented our stochastic models and approximations, performed statistical analyses, and produced
 figures using Python (Van Rossum et al., 2007) with NumPy (Van Der Walt et al., 2011) and Matplotlib
 114 (Hunter, 2007).

Source code is available at <https://github.com/yoavram-lab/PrestigeBias>.

116 Continuous trait

We follow the model of Boyd and Richerson (1988), assuming only oblique transmission of the
 118 indicator trait and omitting the indirect trait in order to reduce model complexity. We consider a
 population of N individuals, described by a single indicator trait with a continuous value. Each
 120 generation, N naive individuals, or copiers, choose an individual from the previous generation, or
 role-models, from which they will copy their trait. Similar to a Wright–Fisher model, we assume non-
 122 overlapping generations such that the entire population is replaced in each generation. The population
 at time t can be described by $\mathbf{A}(t) = (A_1(t), \dots, A_N(t))$ where $A_i(t)$ is trait value of individual i at
 124 time t . We assume the initial population is drawn from a standard normal distribution, $\mathbf{A}(0) \sim N(0, 1)$
 . Cultural transmission is modeled by a function F such that

$$126 \quad A_i(t+1) = F_i(\mathbf{A}(t)) . \quad (1)$$

Success bias. Boyd and Richerson (1988, Ch.8, p.247-249) describe a blended transmission algorithm
 128 by defining F as a weighted average of the traits of all role-models,

$$F_i(\mathbf{A}) = \sum_{j=1}^N G_{i,j} \cdot A_j , \quad (2)$$

130 where $G_{i,j}$ is the success bias of role-model j in the eyes of copier i ,

$$G_{i,j} = \frac{\beta(A_{i,j})}{\sum_{k=1}^N \beta(A_{i,k})} , \quad (3)$$

132 $A_{i,j}$ is the absolute indicator trait value copier i estimates for role-model j with some error $e_i \sim N(0, \frac{1}{\eta^2})$,

134
$$A_{i,j} = A_j + e_i, \quad (4)$$

and $\beta(\cdot)$ is the bias function that quantifies the success bias of a role-model,

136
$$\beta(A_{i,j}) = b \cdot \exp\left(-\frac{(A_{i,j} - \hat{A})^2}{2J}\right), \quad (5)$$

with \hat{A} as the arbitrary optimal indicator value, and J and b as parameters that control the bias strength.

138 Therefore, $G_{i,j}$ is a relative success score that copier i assigns to role-model j , equivalent to relative fitness in evolutionary-genetic transmission models.

140 Boyd and Richerson (1988) note that the deterministic blended transmission algorithm they use has alternatives. We thus develop a similar stochastic model with transmission from a single random
142 role-model. Instead of eq. (2), we define the transmission function F as a random variable with its distribution given by

144
$$\Pr(F_i(\mathbf{A}) = A_j) = G_{i,j}, \quad (6)$$

such that $G_{i,j}$ gives the probability of copier i to choose to copy the trait of role-model j .

146 **Influence bias.** Here we introduce a new element to the model. We assume that in each generation, copiers choose their role-models one by one. We denote $K_{i,j}$ as the number of copiers that chose
148 role-model j after copier i chose a role-model. Thus, i out of N copiers already chose a role-model, $\sum_{j=1}^N K_{i,j} = i$, and there are $N - i$ copiers that have yet to choose a role-model. The stochastic process
150 of role-model choice,

$$\{\mathbf{K}_i = (K_{i,1}, \dots, K_{i,N})\}_{i=1}^N, \quad (7)$$

152 is described by the recurrence equation

$$K_{i,j} = K_{i-1,j} + S_{i,j}, \quad i, j = 1, 2, \dots, N, \quad (8)$$

154 where $S_{i,j} = 1$ if the i -th copier chose role-model j and 0 otherwise, and the initial state is $K_{0,j} = 0$.

156 Following eq. (6), the probability that the i -th copier chose role-model j is given by the prestige of role-model j in the eyes of copier i ,

158
$$\Pr(S_{i,j} = 1 \mid S_{1,j}, S_{2,j}, \dots, S_{i-1,j}) = G_{i,j}. \quad (9)$$

The prestige $G_{i,j}$ of role-model j in the eyes of copier i is determined by the estimated biased indicator
160 value $\beta(A_{i,j})$ and the number of copiers that chose role-model j before copier i , $K_{i-1,j}$, replacing eq. (3) of Boyd and Richerson (1988) with

162
$$G_{i,j} = \frac{\alpha_j \cdot \beta(A_{i,j}) + (1 - \alpha_j) \cdot K_{i-1,j}}{W_i}. \quad (10)$$

Here, the bias weight α_j is a characteristic of role-model j that determines the relative significance of success and influence in his prestige, the indicator value of role-model j estimates by copier i , $A_{i,j}$, remains as in eq. (4), and W_i is a normalizing factor that sums the numerator over all role-models ($1 \leq j \leq N$) to ensure $\sum_{j=1}^N G_{i,j} = 1$.

Dichotomous trait

To allow for mathematical analysis of the model, we introduce a simplified version where the indicator trait only has two phenotypes: the optimal phenotype \hat{A} and the sub-optimal phenotype A . All role-models with the same phenotype will contribute to the probability of that phenotype to be transmitted, and thus influence is determined by the number of copiers that have already chosen a role-model with either phenotype. Therefore, the probability of the i -th copier to copy phenotype A is

$$P_{i,A} = \frac{(N - X)\alpha'\beta(A) + K_{i,A}}{i - 1 + (N - X)\alpha'\beta(A) + X\alpha'\beta(\hat{A})} = \frac{(N - X)\alpha'\beta(A) + K_{i,A}}{i - 1 + (N - X)\alpha'\beta(A) + \alpha'X} \quad (11)$$

where X is the number of role-models with trait \hat{A} and $K_{i,A}$ is the number of copiers that already chose A when copier i chooses a role-model and α' is the odd ratio of the bias weight $\alpha' = \frac{\alpha}{1-\alpha}$.

We prove this equation later, based on the proof of the Dirichlet-Multinomial approximation.

Initially, we assume the population has a single individual with phenotype \hat{A} and $N - 1$ individuals with phenotype A . The rest of the details follow the continuous trait model.

Results

Approximations

Our model is defined by two nested stochastic processes. Change over multiple generations is described by the phenotype distribution at each generation, $\{\mathbf{A}(t)\}_t$. The transition from one generation to the next is described by the number of copiers each role-model has after i copiers have chosen a role-model, $\{\mathbf{K}_i\}_{i=1}^N$. We emphasize that the models are nested: $\mathbf{A}(t + 1)$ can be computed from $\mathbf{A}(t)$ by evaluating \mathbf{K}_N . However, the former requires iterating over eqs. (8) and (9). Thus, we sought to find an equivalent stochastic process that has the same joint distribution as \mathbf{K}_N . We found two such approximations, summarized here and explained in detail below:

1. The number of copiers of a specific role-model at each step, $K_{i,j}$, follows the *generalized binomial distribution* (Drezner and Farnum, 1993) and therefore,
 - (a) the expected number of copiers of role-model j equals its prestige in the eyes of the first copier, multiplied by the total number of copiers, that is, $E[K_{N,j}] = N \cdot G_{1,j}$ if trait estimation error is uniform for all copiers ($e = e_i$ for $i = 1, \dots, N$).
 - (b) the expected number of copiers of each role-model equals its relative biased indicator value, similar to the role of relative fitness in population-genetic models, that is, $E[K_{N,j}] =$

$\beta(A_j + e)/\bar{\beta}$ if the bias weight is uniform for all role-models ($\alpha = \alpha_j$ for $j = 1, \dots, N$),
 where $\bar{\beta} = 1/N \sum_{j=1}^N \beta(A_j + e)$ is the population mean estimated indicator value.

2. The role-model choice process, $\{\mathbf{K}_i\}_{i=1}^N$, is equivalent to a *Pólya urn* model if trait estimation error is uniform for all copiers ($e = e_i$ for all $i = 1, \dots, N$), meaning there's no meaning for the order of copiers. Therefore, the number of copiers of all role-models, $\mathbf{K}_i = (K_{i,1}, \dots, K_{i,N})$, follows a Dirichlet-Multinomial distribution, $\mathbf{K}_i \sim DM(N, \mathbf{G}_1)$, where $\mathbf{G}_1 = (G_{1,1}, \dots, G_{1,N})$ is the prestige vector of all role-models in the "eyes" of the first copier. Note that here $G_{i,j}$ is only a function of the indicator values A_j and the bias weights α_j , as the estimation error is uniform, meaning all copiers are identical.

Generalized binomial distribution

The generalized binomial distribution (GBD) emerges from a series of dependent Bernoulli trials (in contrast to the standard binomial distribution in which trials are independent) and is parameterized by $GBD(n, p, \theta)$ where n is the number of trials, p is the probability of success of the first trial, and θ is the correlation between trials ($\theta = 0$ gives the standard binomial distribution).

Result 1 (GBD approximation). *The number of copiers of role-model j after i copiers have chosen a role-model follows the GBD, $K_{i,j} \sim GBD(i, \alpha_j \cdot \beta(A_j + e))$ if $e_i = e$ for all role-models $i = 1, \dots, N$.*

Proof. We denote $Q_j(k, i) = P(K_{i,j} = k \mid K_{i-1,j})$ as the probability that exactly k out of i copiers choose role-model j given $K_{i-1,j}$ our of $i-1$ copiers chose role-model j . Using conditional probability and eq. (8),

$$Q_j(k, i) = P_j(S_{i,j} = 1 \mid k-1, i-1) \cdot Q_j(k-1, i-1) + P_j(S_{i,j} = 0 \mid k, i-1) \cdot Q_j(k, i-1), \quad (12)$$

where $S_{i,j} = 1$ when the i -th copier chooses role-model j . Equation (12) is equivalent to eq. (2.1) by Drezner and Farnum (1993), which completes the proof.

Corollary 1. $E[K_{N,j}] = N \cdot G_{1,j}$.

Proof. Drezner and Farnum (1993, eq. (2.3)) show that $E[k] = N \cdot Q_j(1, 1)$ (modified from their notation). $Q_j(1, 1)$ is the initial probability to choose role-model j , before any role-model choices are made. $Q_j(1, 1) = G_{1,j}$ by definition, which completes the proof.

Corollary 2. $E[K_{N,j}] = \alpha_j \cdot \beta(A_j + e) / \overline{\alpha \cdot \beta(A + e)}$, where the averaging in the denominator is over the role-models index, j .

Proof. The initial prestige of role-model j based on eq. (10) is

$$G_{1,j} = \frac{\alpha_j \cdot \beta(A_j + e)}{\sum_{m=1}^N \alpha_m \cdot \beta(A_m + e)}. \quad (13)$$

226 The denominator of eq. (13) can also be formulated as:

$$\sum_{m=1}^N \alpha_m \beta(A_m + e) = N \cdot \overline{\alpha \cdot \beta(A + e)}, \quad (14)$$

228 where $\overline{\alpha \beta(A + e)}$ is the mean value of $\alpha_m \cdot \beta(A_m + e)$ for all m . Using eq. (14) and **Corollary 1** we get,

$$230 \quad \mathbb{E}[K_{Nj}] = \alpha_j \cdot \beta(A_j + e) \left/ \overline{\alpha \cdot \beta(A + e)} \right., \quad (15)$$

completing the proof.

232 The special case where the bias weight is uniform for all role-models ($\alpha = \alpha_j$ for $j = 1, \dots, N$) is interesting, because we can evaluate the expected number of copiers using a linear equation

$$234 \quad \mathbb{E}[K_{Nj}] = N \cdot \frac{\alpha \cdot \beta(A_j + e)}{\sum_{m=1}^N \alpha \cdot \beta(A_m + e)} = \beta(A_j + e) \left/ \overline{\beta(A + e)} \right., \quad (16)$$

where the only variable is $A_j + e$, because $\overline{\beta(A + e)}$ is the mean of the distribution we draw the indicator values from, modified by some constant parameters of β . We can then denote $L = 1/\overline{\beta(A + e)}$ and write

$$238 \quad \mathbb{E}[K_{Nj}] = L \cdot \beta(A_j + e). \quad (17)$$

Dirichlet-Multinomial distribution

240 **Pólya urn model.** This stochastic process consists of N draws from an urn with an initial amount of colored balls of M colors. When a ball is drawn, it is then placed back in the urn together with an additional new ball of the same color. Let $\mathbf{U}_i = \{u_{i,1}, u_{i,2}, \dots, u_{i,M}\}$ where $u_{i,j}$ is the number of balls of the j -th color in the urn after i draws. Let $S_{i,j} = 1$ when drawing a j -colored ball on the i -th draw, and 0 otherwise. The probability that $S_{i,j} = 1$ given \mathbf{U}_{i-1} is

$$P(S_{i,j} = 1 \mid \mathbf{U}_{i-1}) = \frac{u_{i-1,j}}{\sum_{m=1}^M u_{i-1,m}} = \frac{o_j + w_{i-1,j}}{\sum_{m=1}^M o_m + w_{i-1,m}} = \frac{o_j + w_{i-1,j}}{i - 1 + \sum_{m=1}^M o_m}, \quad (18)$$

246 where o_j is the initial number of balls of the color j in the urn, and $w_{i,j}$ is the cumulative number of new balls that were added to the urn after i draws of the color j .

248

Result 2 (Pólya urn model). *The role-model choice process, $\{\mathbf{K}_i\}_{i=1}^N$, is equivalent to a Pólya urn model if both trait estimation error and bias weight are uniform in the population, $e = e_j$ and $\alpha = \alpha_j$ for all $j = 1, \dots, N$.*

252 *Proof.* Denote $\alpha' = \frac{\alpha}{1-\alpha}$ as the bias weight ratio, and $A'_j = A_j + e$. From eq. (10) and because $\sum_{j=1}^N K_{i,j} = i$, we have

$$254 \quad G_{i,j} = \frac{\alpha' \beta(A'_j) + K_{i-1,j}}{\sum_{m=1}^N \alpha' \beta(A'_m) + K_{i-1,m}} = \frac{\alpha' \beta(A'_j) + K_{i-1,j}}{i - 1 + \sum_{m=1}^N \alpha' \beta(A'_m)}. \quad (19)$$

Substituting $M = N$, $\alpha_j = \alpha' \beta(A'_j)$, and $w_{i,j} = K_{i,j}$ in eq. (18) gives eq. (19), thus completing the
256 proof.

Frigyik et al. (2010, section 2) prove that the proportion of different colored balls in a *Pólya urn*
258 *model* converges to the Dirichlet distribution as the number of draws approaches infinity, based on
the *Martingale Convergence Theorem* (Durrett, 1999). We therefore have an approximation for the
260 relative prestige each role-model has when evaluated by copiers. Thus, choosing the role-models for
all copiers is equivalent to drawing from a Multinomial distribution where the parameters are the
262 modified weights from a Dirichlet distribution and we have the following corollary.

264 **Corollary 3.** *The number of copiers of each role-model follows a Dirichlet-Multinomial distribution, $\mathbf{K}_i \sim DM(N, \mathbf{G}_1)$, under the conditions of Result 2.*

266 **Numerical validation.** To validate our analytical result (corollary 3) and test its sensitivity to the
assumptions ($e_i = e$ and $\alpha_i = \alpha$ for $i = 1, \dots, N$) we compare it to results of stochastic simulations of
268 the full model. First, we computed an observed distribution of the number of copiers from the average
empirical distribution of multiple simulations. We then compared this observed distribution with
270 the expected theoretical DM distribution using Pearson's chi-squared test of goodness-of-fit. Thus,
we can reject or accept the null hypothesis that simulation results are effectively drawn from a DM
272 distribution.

Examining variable values of the bias weight parameter α , the test p-value was always 1, meaning
274 that the null hypothesis cannot be rejected. For high α values (above 0.5), very few simulations are
needed to reach p-value of 1 (less than five simulations for $\alpha = 0.9$, and less than 20 for $\alpha = 0.7$) For
276 very low α values, which means very high influence weight, the number of simulations needed was
100. When testing our observed distributions against different distributions, for example the uniform
278 distribution (all role-models have exactly one copier), the p-value was 0, so that the null hypothesis
was rejected, as required.

280 Next, we examined the fixation probability and fixation time of an advantageous phenotype \hat{A} when
invading a population of phenotype A and compared results from the full model and the DM approx-
282 imation. We find that the number of simulations needed to sufficiently approximate our model with
the DM approximation is roughly 1,000 (Figure 1). Next, we examined the robustness of the DM
284 approximation to relaxing the approximation assumptions. First, we relaxed our assumption of no
estimation error e . Estimation error in the original model was drawn from a normal distribution, and
286 added to the trait value before evaluation of the bias ($A_{ij} = A_j + e_i$). When estimation error is applied,

we sample J_i for each copier i from a normal distribution with varying scale (variance). Even when the standard deviation is 0.1, the fixation probability and time is similar (fig. 2). We also relaxed our assumption of a uniform bias weight α (i.e., $\alpha_i = \alpha$). We allowed α to vary in the population, drawing α_j for each role-model j from a normal distribution such that $\alpha_j \sim N(X, X)$. We found again that results of the DM approximation are similar to those from stochastic simulations of the full model (fig. 3).

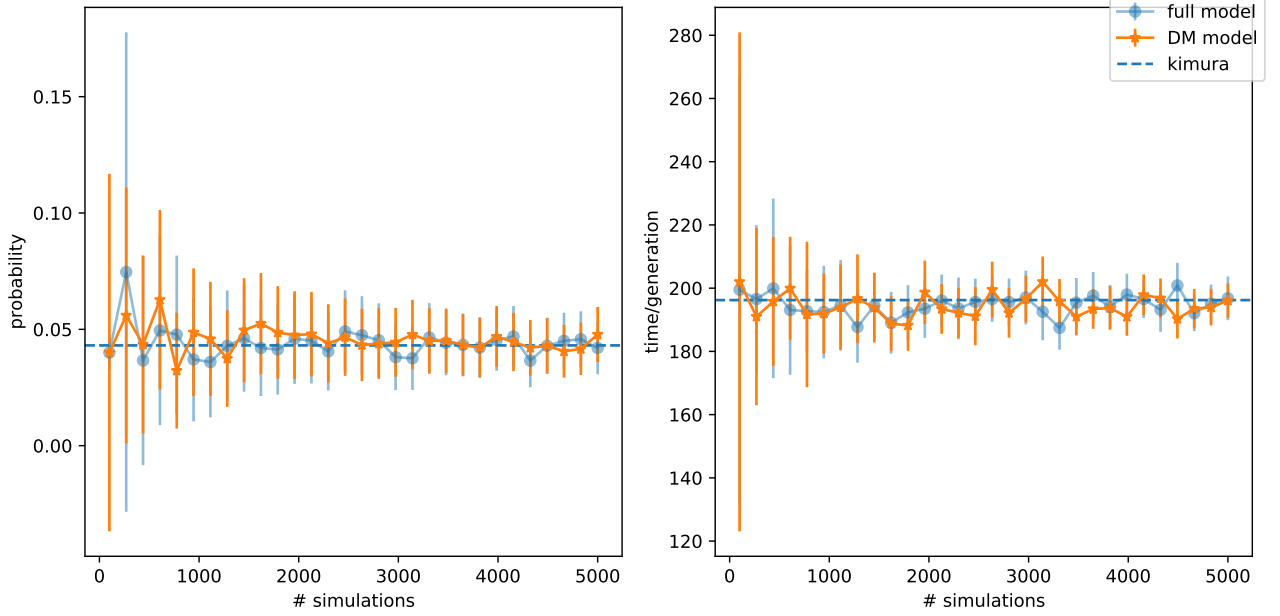


Figure 1: Number of simulations needed to get a good approximation. The approximation (orange) fits simulation results (blue) well when using 1,000 simulations for both models. Markers for average value across simulations, error bars for 95% confidence interval. Here, population size, $N = 1000$; bias weight, $\alpha = 0.5$; phenotype values, $\hat{A} = 1$, $A = 0.7$; success bias value, $\beta(A) = 0.956$.

Fixation probability and time

After finding that the DM distribution is a good approximation of the (within-generations) role-model choice process, we turn our attention to the (between-generations) evolutionary dynamics. We focus on the fixation probability and fixation time of an advantageous phenotype, similar to analyses in population-genetic models (Kimura, 1962; Kimura and Ohta, 1969; Otto and Whitlock, 2006).

We are mainly interested in the effect of the bias weight (α), which determines the relative effects of success and influence on prestige bias. For simplicity, we do not include estimation error in this analysis. As shown above, transmission in our model is approximately DM distributed with the parameters $\sim DM(\alpha'X, (N - X)\alpha'\beta(A))$ based on corollary 3 and eq. (19).

Result 3 (Effective selection coefficient). $1 - \beta(A)$ is equivalent to the selection coefficient s in the diffusion-equation approximation of the a classic Wright-Fisher model that approximate the fixation probability and fixation time of an advantageous allele.

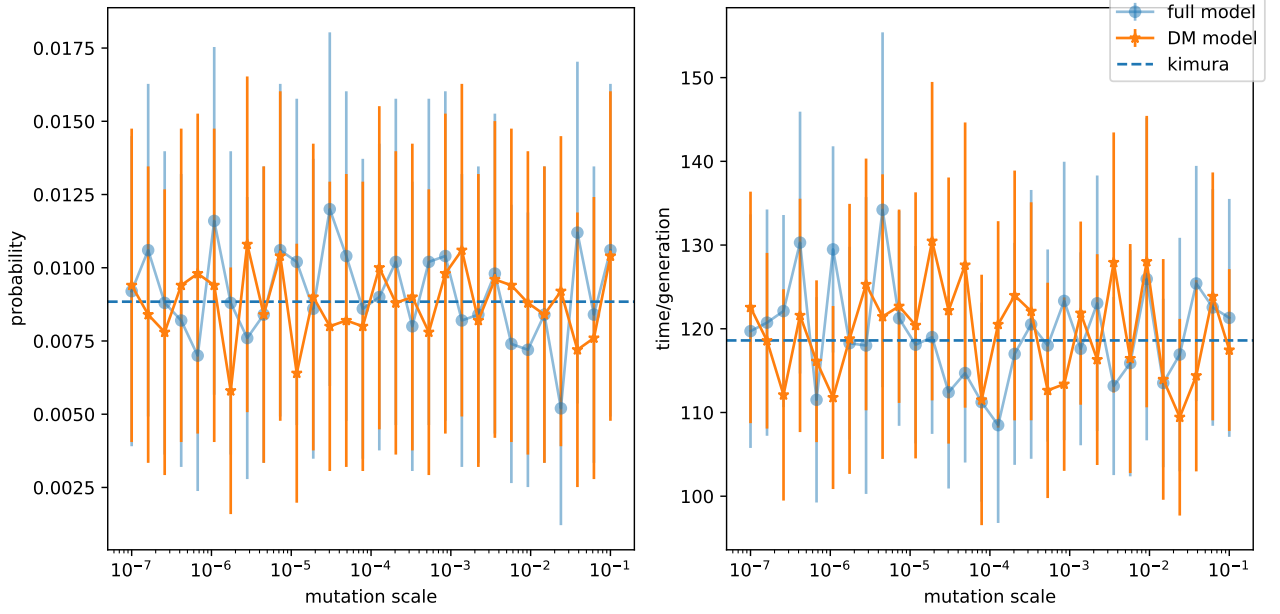


Figure 2: Robustness of DM approximations to inclusion of estimation error. Both the DM approximation (orange) and Kimura's equation (black line) fit the stochastic simulations (blue) well even with a high estimation error rate. Markers for average across simulations, error bars for 95% confidence intervals. 5,000 simulations per data point; population size, $N = 1000$; bias weight, $\alpha = 0.1$; phenotype values, $\hat{A} = 1, A = 0.7$; $J \sim N(1, x^2)$ where x is the estimation error scale in the x-axis.

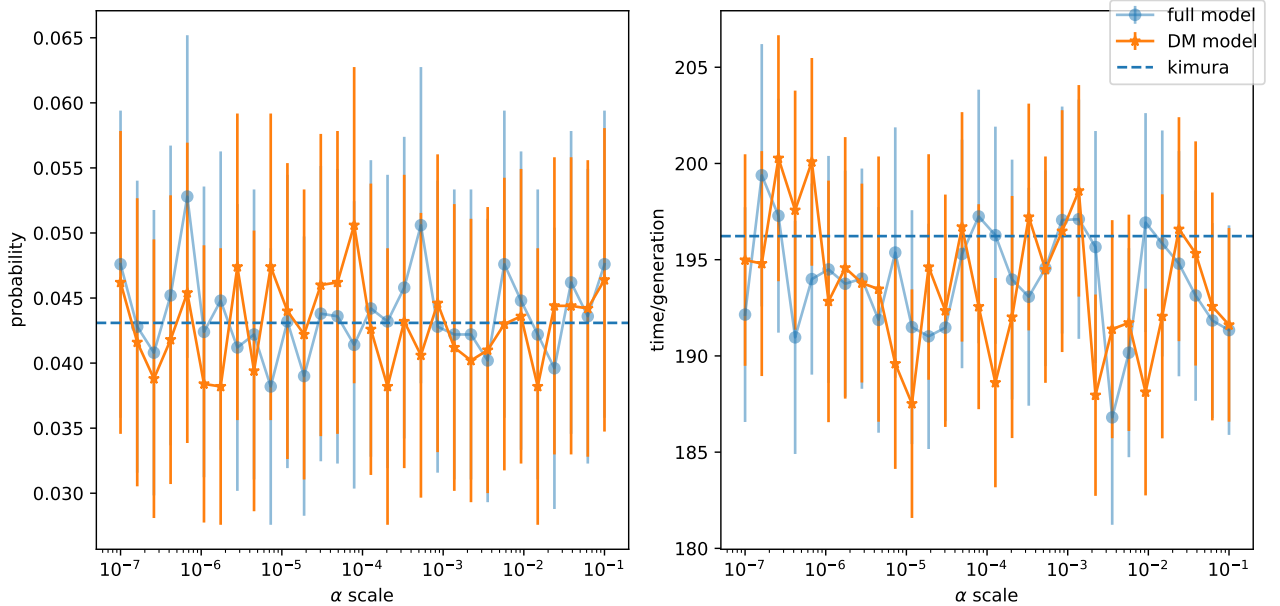


Figure 3: Robustness of DM approximations to variation in the bias weight α . High bias weight variance distances the approximation and the full model of generations to fixation from the Kimura's approximation, but not by much (confidence intervals still cover it). Error bars are 95% confidence intervals, and are less condensed (± 0.03 probability and ± 40 generations) 5000 simulations per data point, $N = 1000$, $\alpha \sim N(0.5, x^2)$, $\hat{A} = 1, A = 0.7$, $J = 1$, $\beta(A) = 0.956$.

Proof. Let x be the frequency of type \hat{A} in the population with N individuals. Let X be the number of
 306 individuals of type \hat{A} so $x = X/N$. X' is the number of individuals with \hat{A} in the next generation and
 x' their frequency. By definition $\beta(\hat{A}) = 1$, and for simplicity we'll denote $\beta(A) = \beta$ ($\beta < 1$).

308 The expected number of individuals of a DM distribution is:

$$E[X'] = N \frac{\alpha_1}{\alpha_1 + \alpha_2}, \quad (20)$$

310 where $\alpha_1 = \alpha'X$ and $\alpha_2 = \alpha'(N - X)\beta$, from eq. (11). We want to use frequencies instead of quantities
 to follow Durret's process so:

$$312 \quad E[x'] = E\left[\frac{X'}{N}\right] = \frac{1}{N}E[X'] \quad (21)$$

Putting it together we get:

$$314 \quad \begin{aligned} E[x'] &= \frac{1}{N}N \frac{\alpha'xN}{\alpha'xN + \alpha'N(1-x)\beta} \\ &= \frac{x}{x + (1-x)\beta} \end{aligned} \quad (22)$$

which is identical to the equation in the top of page 253, chap 7.2 in Durrett (2008). We therefore use
 316 the same approximation and say that:

$$\begin{aligned} E[x'] &= \frac{x}{x + (1-x)\beta} = \frac{x}{x + (1-x)(1-s)} = \\ &= x + x(1-x)s + o(s) \\ &= x + x(1-x)(1-\beta) + o(1-\beta) \end{aligned} \quad (23)$$

318 By definition, x is constant, so $E[x] = x$. We continue to calculate $E[x' - x]$:

$$E[x' - x] = E[x'] - E[x] = x(1-x)(1-\beta) + o(1-\beta) \quad (24)$$

320 where when substituting $1 - \beta$ with s , we get the same result as Durrett (2008) which is the desired
 result.

322 **Result 4** (Effective population size). $N_e = \alpha N + (1 - \alpha)$, where N_e is the effective population size of
 our binary model.

324 *Proof.* The variance of a DM distribution is:

$$V(X') = N \frac{\alpha_1}{\alpha_1 + \alpha_2} \left(1 - \frac{\alpha_1}{\alpha_1 + \alpha_2}\right) \left(\frac{N + \alpha_1 + \alpha_2}{1 + \alpha_1 + \alpha_2}\right) \quad (25)$$

326 And again, we want to use frequencies so:

$$V\left(\frac{X'}{N}\right) = \frac{1}{N^2}V(X') \quad (26)$$

328 Putting it together with our model's notations:

$$V(x') = \frac{1}{N^2} N \frac{x}{x + (1-x)\beta} \left(1 - \frac{x}{x + (1-x)\beta}\right) \left(\frac{N + \alpha' x N + \alpha' N(1-x)\beta}{1 + \alpha' x N + \alpha' N(1-x)\beta}\right) \quad (27)$$

330 Like Durrett, we'll use the zero order of the approximation when $\beta \approx 1$, so:

$$\frac{x}{x + (1-x)\beta} \approx x \quad (28)$$

332 and we also use $\beta \approx 1$ for the entire variance expression and get:

$$\begin{aligned} V(x') &\approx \frac{1}{N} x(1-x) \left(\frac{N + \alpha' x N + \alpha' N - \alpha' x N}{1 + \alpha' x N + \alpha' N - \alpha' x N}\right) \\ &= x(1-x) \left(\frac{1 + \alpha'}{1 + \alpha' N}\right) \end{aligned} \quad (29)$$

334 Again following Durrett we want to calculate:

$$V(x' - x) = V(x') - V(x) \approx x(1-x) \left(\frac{1 + \alpha'}{1 + \alpha' N}\right) \quad (30)$$

336 because x is a constant so $V(x) = 0$

In our model, α' is the odds ratio of a parameter we called "bias weight", denoted in our model as α ,

338 so:

$$\alpha' = \frac{\alpha}{1 - \alpha} \quad (31)$$

340 Combining eq. (30) and eq. (31) we get:

$$\begin{aligned} V(x' - x) &\approx x(1-x) \left(\frac{1 + \frac{\alpha}{1-\alpha}}{1 + \frac{\alpha}{1-\alpha} N}\right) \\ &= x(1-x) \left(\frac{\frac{1-\alpha+\alpha}{1-\alpha}}{\frac{1-\alpha+\alpha N}{1-\alpha}}\right) \\ &= x(1-x) \left(\frac{1}{1 - \alpha(1-N)}\right) \\ &= x(1-x) \left(\frac{1}{\alpha N + (1-\alpha)}\right) \\ &= x(1-x) \frac{1}{N_e} \end{aligned} \quad (32)$$

342

Using our substitute for a selection coefficient, $1 - \beta$, and the effective population size N_e , we can

344 estimate the fixation probability and time of our binary model.

The fixation probability derived from Kimura is therefore:

$$P_{fix} = \frac{1 - e^{-2(1-\beta)N_e x}}{1 - e^{-2(1-\beta)N_e}} \quad (33)$$

where x is the initial frequency of the advantageous phenotype \hat{A} .

The time to fixation can be estimated by:

$$T_{fix} = \frac{1 - P_{fix}}{1 - \beta} \int_0^x \frac{e^{2(1-\beta)\xi} - 1}{\xi(1-\xi)} d\xi + \frac{P_{fix}}{1 - \beta} \int_x^1 \frac{1 - e^{-2(1-\beta)(1-\xi)}}{\xi(1-\xi)} d\xi \quad (34)$$

where the integrals cannot be solved in closed form, so we can only estimate them numerically.

To validate our math we ran multiple simulations comparing our binary model with the classic Wright-Fisher model, using different α and β each time, and using the corresponding values of s and N_e for the WF simulations. In fig. 4 we changed α (and N_e accordingly) and used a constant β . In fig. 5 we changed β and used a constant α . In both cases we can see that the two models behave similarly, and both are approximated well by the Kimura's equations: eq. (33) and eq. (34).

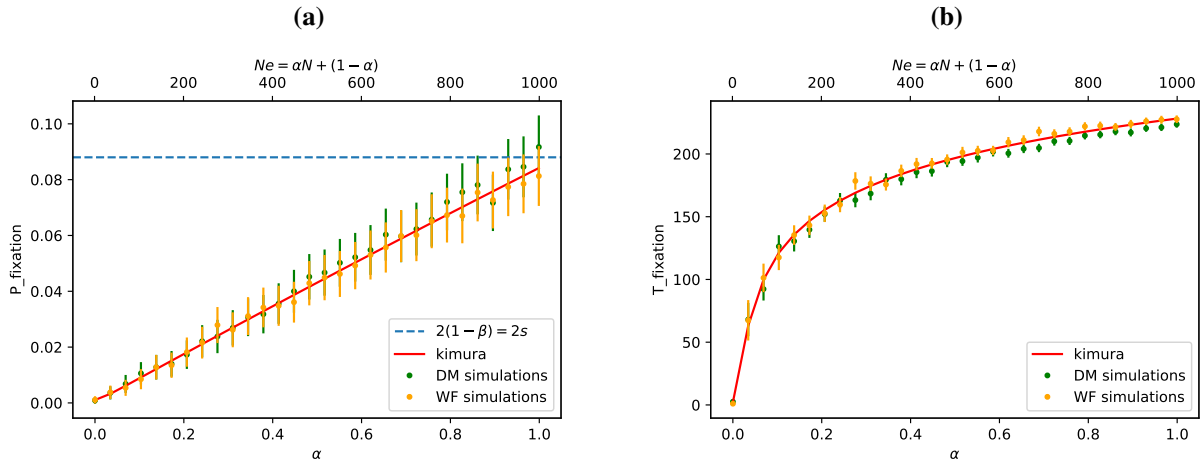


Figure 4: Comparison of the DM approximation and the WF model for different values of the effective population size. Error bars show mean and 95% confidence intervals. Effective population, $N_e = \alpha N + (1 - \alpha)$. 5, 000 simulations per data point; Population size, $N = 1, 000$; $\hat{A} = 1$, $A = 0.7$, $J = 1$, $1 - \beta = s = 0.044$.

Changing environment. After finding good estimations for our model in a constant environment, when the favorable trait is always \hat{A} , we want to find an estimation for our model in a changing environment.

For that we will find an expression for the expected and variance of the change in frequency between t generations. Let $s_t = N(1 - \beta_t)$, and $S_n = \sum_{i=1}^n s_i$, where β_t is $\beta(A)$ at time/generation t .

Proposition: $E[\frac{X_t}{N} - x] \simeq \frac{1}{N} S_t x(1 - x)$, $V(\frac{X_t}{N}) \simeq \frac{1}{N_e} t x(1 - x)$, where x is the initial frequency of the favorable/invading trait and X_t is the number of individuals with the trait at time t .

The proof is based on the proof of Ram et al. (2018), proving a similar scenario.

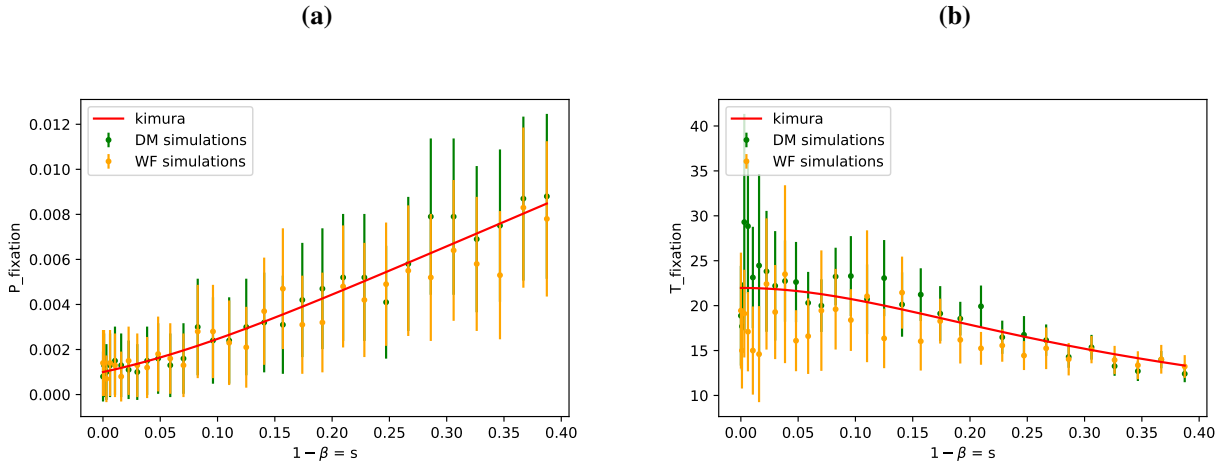


Figure 5: Comparison of the DM approximation and the WF model for different values of the selection coefficient, manifested as success bias in our model. Error bars are 95% confidence intervals. Effective population calculated by $N_e = \alpha N + (1 - \alpha)$. 5,000 simulations per data point, $N = 1,000$, $\hat{A} = 1$, $A = 0.7$, $J = 1$, $\alpha = 0.01$.

364 **Proof by induction:** From eq. (24) we know that

$$\begin{aligned} E \left[\frac{X_{t+1}}{N} - \frac{X_t}{N} \middle| X_t \right] &= \frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) (1 - \beta_{t+1}) \\ &= \frac{1}{N} \frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) s_{t+1} \end{aligned} \quad (35)$$

366 Also note that using the definition of $V(y) = E[y^2] - (E[y])^2$

$$\begin{aligned} E \left[\frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \right] &= E \left[\frac{X_t}{N} - \left(\frac{X_t}{N} \right)^2 \right] \\ &= E \left[\frac{X_t}{N} \right] - E \left[\left(\frac{X_t}{N} \right)^2 \right] \\ &= E \left[\frac{X_t}{N} \right] - V \left(\frac{X_t}{N} \right) - \left(E \left[\frac{X_t}{N} \right] \right)^2 \end{aligned} \quad (36)$$

368 we can now use the induction assumption of $V(\frac{X_t}{N})$ and get

$$E \left[\frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \right] \simeq E \left[\frac{X_t}{N} \right] \left(1 - E \left[\frac{X_t}{N} \right] \right) - \frac{1}{N_e} tx(1-x) \quad (37)$$

370 From eq. (35) we know that

$$\begin{aligned} E \left[\frac{X_{t+1}}{N} - \frac{X_t}{N} \right] &= \frac{1}{N} s_{t+1} E \left[\frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \right] \\ &\simeq \frac{1}{N} s_{t+1} \left(E \left[\frac{X_t}{N} \right] \left(1 - E \left[\frac{X_t}{N} \right] \right) - \frac{1}{N_e} tx(1-x) \right) \\ &\simeq \frac{1}{N} s_{t+1} \cdot E \left[\frac{X_t}{N} \right] \left(1 - E \left[\frac{X_t}{N} \right] \right) - \frac{1}{N_e N} s_{t+1} tx(1-x) \end{aligned} \quad (38)$$

372 Now we'll omit $O(\frac{1}{Ne \cdot N})$ and get

$$E \left[\frac{X_{t+1}}{N} - \frac{X_t}{N} \right] \simeq \frac{1}{N} s_{t+1} \cdot E \left[\frac{X_t}{N} \right] \left(1 - E \left[\frac{X_t}{N} \right] \right) \quad (39)$$

374 We'll now look at the induction assumption to see that

$$E \left[\frac{X_t}{N} - x \right] = E \left[\frac{X_t}{N} \right] - E[x] = E \left[\frac{X_t}{N} \right] - x, \quad (40)$$

376 so using the assumption we get

$$\begin{aligned} E \left[\frac{X_t}{N} \right] &\simeq \frac{1}{N} S_t x (1 - x) + x \\ 1 - E \left[\frac{X_t}{N} \right] &\simeq 1 - \frac{1}{N} S_t x (1 - x) + x \end{aligned} \quad (41)$$

378 we'll use both expressions in eq. (39) and get

$$\begin{aligned} E \left[\frac{X_{t+1}}{N} - \frac{X_t}{N} \right] &\simeq \frac{1}{N} s_{t+1} \left(\frac{1}{N} S_t x (1 - x) + x \right) \left(1 - \frac{1}{N} S_t x (1 - x) + x \right) \\ &\simeq \frac{1}{N} s_{t+1} \cdot x (1 - x) \end{aligned} \quad (42)$$

380 after again omitting $O(\frac{1}{N^2})$ parts of the equation. To conclude our proof, we see that

$$E \left[\frac{X_{t+1}}{N} - x \right] = E \left[\frac{X_{t+1}}{N} - \frac{X_t}{N} \right] + E \left[\frac{X_t}{N} - x \right] \quad (43)$$

382 so again using the induction assumption, together with eq. (42) we get

$$\begin{aligned} E \left[\frac{X_{t+1}}{N} - x \right] &\simeq \frac{1}{N} s_{t+1} \cdot x (1 - x) + \frac{1}{N} S_t \cdot x (1 - x) \\ &\simeq \frac{1}{N} x (1 - x) (S_t + s_{t+1}) \\ &\simeq \frac{1}{N} S_{t+1} x (1 - x) \end{aligned} \quad (44)$$

384 which proves the first part of our preposition.

For the second part, we'll use a property of variance:

$$386 \quad V \left(\frac{X_{t+1}}{N} \right) = E \left[V \left(\frac{X_{t+1}}{N} \middle| X_t \right) \right] + V \left(E \left[\frac{X_{t+1}}{N} \middle| X_t \right] \right) \quad (45)$$

using eq. (35) we see that:

$$\begin{aligned} E \left[\frac{X_{t+1}}{N} \middle| X_t \right] - E \left[\frac{X_t}{N} \middle| X_t \right] &= \frac{1}{N} s_{t+1} \frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \\ E \left[\frac{X_{t+1}}{N} \middle| X_t \right] &= \frac{X_t}{N} + \frac{1}{N} s_{t+1} \frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \end{aligned} \quad (46)$$

Using eq. (32) we get:

$$V \left(\frac{X_{t+1}}{N} \middle| X_t \right) = \frac{1}{N_e} \frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \quad (47)$$

and using the equation $y'(1 - y') \simeq y(1 - y)$ on the first part of eq. (45) we get:

$$E \left[V \left(\frac{X_{t+1}}{N} \middle| X_t \right) \right] = \frac{1}{N_e} E \left[\frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \right] \simeq \frac{1}{N_e} x(1 - x) \quad (48)$$

and moving on to simplify the second part of eq. (45) using eq. (46):

$$V \left(E \left[\frac{X_{t+1}}{N} \middle| X_t \right] \right) = V \left(\frac{X_t}{N} + \frac{1}{N} s_{t+1} \frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \right) \quad (49)$$

and now, because $\frac{X_t}{N}$ is a frequency, i.e $0 \leq \frac{X_t}{N} \leq 1$, we know that $V \left(\frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \right) \leq \frac{1}{4}$. We therefore see that:

$$V \left(\frac{1}{N} s_{t+1} \frac{X_t}{N} \left(1 - \frac{X_t}{N} \right) \right) \leq \frac{1}{4N^2} s_{t+1}^2 \quad (50)$$

and so it can be ignored. Combining our equations we get:

$$V \left(E \left[\frac{X_{t+1}}{N} \middle| X_t \right] \right) = V \left(\frac{X_t}{N} \right) + O \left(\frac{1}{N^2} \right) \simeq V \left(\frac{X_t}{N} \right) \quad (51)$$

Using the induction assumption and eq. (48):

$$V \left(\frac{X_{t+1}}{N} \right) \simeq \frac{1}{N_e} x(1 - x) + \frac{1}{N_e} tx(1 - x) \simeq \frac{1}{N_e} x(1 - x)(t + 1) \quad (52)$$

proving the second part of our preposition.

Following our proof, we can say that after many cycles, we can use a modified version of our fixation probability:

$$P_{fix} = \frac{1 - e^{-2 \frac{S_n}{n} N_e x}}{1 - e^{-2 \frac{S_n}{n} N_e}} \quad (53)$$

where $\frac{S_n}{n} = \frac{k-l}{k+l} (1 - \text{beta})$, $n = k + l$. Put into words, we use the average selection coefficient of a cycle $(k + l)$ as the selection coefficient in our original equation. In our proof we showed that the expected change in frequency and variance is only manifested in the selection coefficient S_n , and that we can use those modified equation as a base for Kimura's equation.

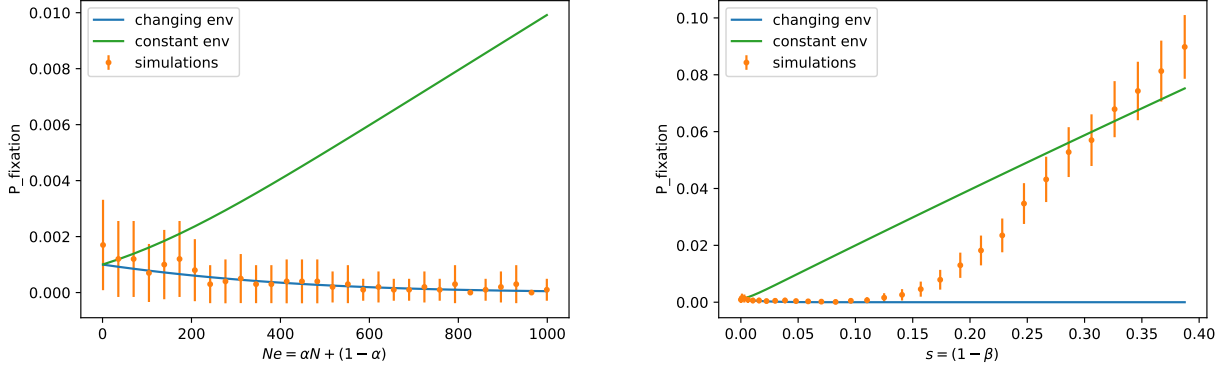


Figure 6: Model simulations compared with both the constant environment and the changing environment equations with different effective populations sizes and selection coefficients. Changing the effective population size doesn't affect the approximation, and it is condensed the mathematical expected values across all values. High values of success bias ($s > 0.1$) will distance the simulations from the changing environment expected values. Very high values ($s > 0.35$) will even deviate from the constant environment expected values. This is expected because Kimura's approximation are only viable for low selection coefficient values. 10,000 simulations per data point, $N = 1,000$, $\hat{A} = 1$, $A = 0.9$, $J = 1$, (a) success bias/selection coefficient is: $1 - \beta = s = 0.005$, (b) bias weight is: $\alpha = 0.1$.

410 We validate our results using simulations. We find that α does not have a significant effect on the approximations (fig. 6). However, β does.

412 We suspect that when β is too large, there won't be many cycles in the simulations. This might happen if either the population reaches a high frequency of the fitter phenotype after only a few cycles, or the fitter phenotype get extinct very quickly. For the very high β values (0.35 and above), the fixation probability exceeds even the constant environment approximation (which is the upper limit), but it's to be expected, because Kimura's equations are only appropriate for weak fitness (i.e., low selection coefficients, s).

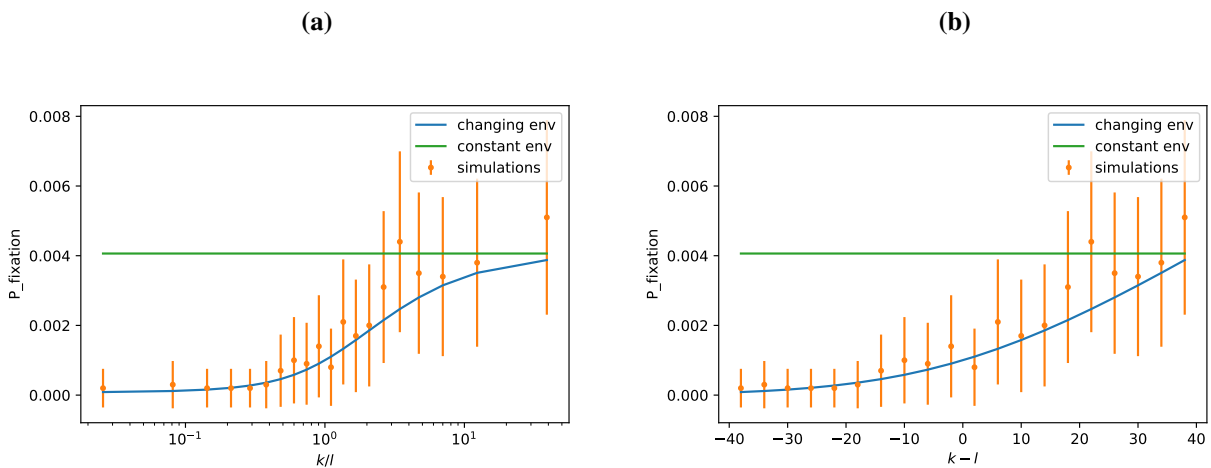


Figure 7: Model simulations compared with both the constant environment and the changing environment equations for different values of k, l , where k is the number of generations where the bias is advantageous, and l are the generations it is disadvantageous. When $k < l$ the approximation is good. When $k > l$, the approximation and the simulations are both very close to the constant environment approximation. We ran 10,000 simulations per data point, $N = 1,000$, $\hat{A} = 1$, $A = 0.8$, $J = 1$, $1 - \beta = s = 0.02$, $\alpha = 0.1$.

418 We also examined the effect of different choices of k and l while keeping a constant total cycle length,
420 $n = k + l = 40$. We found that larger k -to- l ratio increases the agreement between the changing
environment equation and the original estimation of the constant environment. When using higher
values of n , the agreement between the approximation and simulation results is weaker. This is
422 because our approximation is more precise when more cycles occur. When n is high, the cycles are
longer. When the cycles are long, the settings are similar to a constant environment equation, and
424 fixation/extinction is more likely to happen. This means there will be less cycles, which lowers our
approximation accuracy.

426 Discussion

Summary. During cultural transmission, cultural traits such as attitudes, values, beliefs, and behav-
428 ioral patterns are transmitted between individuals, for example via copying and social learning. Some
cultural traits or cultural role-models may be more likely to be copied due to transmission biases. A
430 common one is success bias, in which copiers are more likely to copy a successful role-model. Many
models assume that success can be correctly identified. However, it has been suggested (Fogarty et al.,
432 2017) that because estimating success is hard, a more common bias is *prestige bias*—a bias towards
role-models perceived to be successful by some other indicator.

434 Inspired by a model by Boyd and Richerson (1988), we developed a cultural-evolution model with
prestige bias, which included both success and influence biases, where the latter is a bias towards
436 role-models with many copiers. We found approximations for the role-model choice process: the
generalized binomial distribution that approximates the number of copiers of a single role-model,
438 and the Dirichlet-Multinomial distribution that approximates the distribution of copiers in the entire
population in a *rich-getting-richer* type of process. We then studied the effects of prestige bias on the
440 fixation probability and time of an invading advantageous cultural phenotype in both a constant and a
changing environment.

442 **Prestige in the literature.** According to King and Cowlishaw (2009), there are two main approaches
to group decision-making in nature: leadership and consensus. Leaders may lead the group when
444 traveling, decide on a nesting site, or choose foraging patches. They found that leadership is mostly
observed when there are a lot of connections in the social network of the group, and when there are
446 individuals that present leadership behavior. Leaders would usually be high-ranking members of the
group: elders, individuals with many kin relations, or individuals possessing other dominant traits.
448 When no individual possesses such traits, or when there are not enough connections and relations in the
social network, decisions are more likely to be made by consensus. Leaders may carry selection costs,
450 e.g. due to higher predation risk, poisoning when exploring new foraging patches. In some cases,
the leader may be the hungriest or the weakest individual, while the others would prefer to follow the
452 leader and minimizing their risks and costs. However, leadership may also carry selection benefits.
For example, given the route to the foraging site was successful, the leader and his closest followers
454 would gain most or the best of the food, whereas in consensus decision making, the food would be
shared more equally. Or, sexual selection may also provide a selection benefit advantage to leaders,

456 who might gain more sexual partners due to their leadership of the group. King and Cowlshaw (2009)
describe benefits for the closest associates of a dominant baboon, such as protection from predators.
458 In some species, like the females of *Eulemur fulvus rufus* (Red lemur), leaders may arise due to
nutritional needs, and not due to possessing superior traits (Erhart and Overdorff, 1999).

460 In humans, leadership also has its costs and benefits. Leaders can make decisions that would most
benefit them and their closest followers, while still maintaining group cohesion. However, wrong
462 decision making that harms the group could result in harm.

In modern society humans strive for the prestigious positions, as they may reap rewards greater than
464 the risk and costs to achieve them, or due to individual personality and pressure/education from the
family.

466 Van Vugt and Smith (2019) suggest a different view of leadership. They note that most discussions
assume there is one type of leadership, as seen above, and so they differ in their definitions. Van Vugt
468 and Smith (2019) suggest a way to solve said contradiction by defining two types of leaderships:
prestige-based and dominance-based. They present classical views of leaderships by Confucius and
470 Machiavelli. Confucius views leaders as role-models who exercise influence through possessing
superior knowledge, skills, and (outstanding) personal qualities. This description is very similar to
472 success bias in our model. By contrast, Machiavelli views leaders as rulers who exercise influence by
imposing costs through (the threat of) punishment and violence. They say that these two opposing
474 views are both partially supported by the available evidence but each one on its own offers an
incomplete view into the complex and dynamic processes of leadership. Our model does not reflect
476 these leadership styles, but several adjustments could be made in order to match it, such as assuming
there is a correlation between phenotype to leadership style. The emerging cultural-evolutionary
478 dynamics and their dependence on the costs and benefits are intriguing.

Henrich and Gil-White (2001) support said claim that there are two types of leadership, and also
480 define the two as prestige based and dominance based leadership types. By their definition, the latter
is defined by acquiring social status by using aggression, intimidation and violence. It is also more
482 common than prestige in non humans. Their definition of prestige is somewhat synonymous with
ours. According to their manuscript, prestige is composed both of estimation in the eyes of people
484 (our indicator/success trait) and commanding position in people's minds, i.e number of copiers people
think they have, which they define as *influence* (similar to our definition for influence). In their paper,
486 they show that prestige evolved from natural selection, as an efficient process to extract reproductive
benefit from the flow of socially transmitted information. Simply put, prestige is a natural step where
488 social learning exists, due to saving costs of individual learning. It could be interesting in the future to
expand our model using this idea: observing the copier trait of *evaluation*, rather than only observing
490 the evolution of the indicator trait copied. Henrich and Gil-White (2001) suggest that the most skilled
role-models will, on average, end up with most copiers. Their research, definitions and results, is
492 consistent with ours.

So far we presented the theory behind prestige, and it's appearance in nature. The following will show
494 the appearances of prestige biases in humans, and in recent times.

Chudek et al. (2012), for example, tested the existence of prestige in young children. Chudek et al.
496 (2012) report the first direct tests in children that suggest the existence of *prestige bias*, a tendency
to learn from individuals to whom others have preferentially attended, learned or deferred. Their
498 definition of prestige is similar to our *influence bias*, and brings concrete proof of its existence and
effects. Their study showed that the odds of 3-4 years-old children learning from an adult model to
500 whom bystanders had previously preferentially attended for 10 seconds were over twice those of their
learning from a model whom bystanders ignored. In addition to this first study, they also discovered
502 prestige effects are domain-sensitive. They saw that prestigious models were listened to by most when
demonstrating artifact-use, but not as much as when presenting food preferences. It lead Chudek et al.
504 (2012) to believe that when the trait is costly to learn individually, prestige will have a higher effect. It
would be interesting to include costs in our model to try and observe these effects and their dynamics
506 in the simulations of a larger population than this study.

Henrich and Broesch (2011) researched Fijian villages, looking for evidence of social learning biases
508 and their origins. They mention that:

evolutionary theorists propose that natural selection has favored the emergence of psy-
510 chological biases for learning from those individuals most likely to possess adaptive
information.

512 Their goal is to bridge from the laboratory to the field by examining if and how these biases emerge
in a small-scale society. During their research they found that:

514 Fijian villagers (ages 10 and up) are biased to learn from others perceived as more
successful/knowledgeable, both within and across domains (prestige effects).

516 Their research shows promising evidence for our prestige model, suggesting that copying from others
who are *perceived* as successful, rather than actually are successful. In their paper, they show that the
518 social networks representing copier-role-model relationships are centralized, suggesting:

520 This degree of centralization is consistent with the prediction that people substantially
share notions about who is a good cultural model (network centrality), but that individuals'
model selections are influenced by multiple factors.

522 We see here support for both our indicator trait and our influence bias in their data.

Aside from children's learning biases and small villages in a relatively primitive population, we can
524 see prestige in more advanced domains as well, like western medicine. Norredam and Album (2007)
present a specific and important effect of prestige - its significance for medical specialties and diseases.
526 They examined literature from 1950 to 2005 regarding the effects of prestige on medicinal practices.
They discovered that active, specialized, biomedical, and high-technological types of medicine on
528 organs in the upper part of the bodies of young and middle-aged people were accorded high levels of
prestige, while medicine with opposite characteristics had low levels of prestige. They have concluded

530 that such differences in prestige bear consequences for actual priority setting in healthcare systems. They discovered that surgery counts as the most prestigious specialty, while psychiatry is the less
532 prestigious. In addition, doctors tend to rank practices that require more time to master as more prestigious, while other procedures that are considered *easier* are less prestigious. Simply put, they
534 found that the advance in technology and research was in accordance with the prestige rankings. This means that there may be very important practices that are neglected due to the prestige bias.

536 As we seen so far, prestige can explain many behaviors and evolution of cultural traits. It is a tool to cheaply estimate and acquire knowledge, which helps an individual to survive and breed. However,
538 it is not always the case, and there could be negative repercussions to this bias, such as invasion of maladaptive traits.

540 Takahashi and Ihara (2019) mention that social learning not only takes the form of random copying of other individuals, but also involves learners' choice of what to learn and from whom to learn. They
542 suggest a best-of-K model where an individual samples k role-models and choose the one he deems most "successful". They mentioned that a previous mathematical analysis has shown that it may
544 sometimes result in maladaptive cultural evolution when the payoffs associated with cultural variants vary stochastically. In such a case, learners may be selectively disfavored and in the long run replaced
546 by unbiased learners, who simply copy someone chosen at random. They develop new mathematical models that are simpler and mathematically tractable. They found that best-of-k learning, unlike
548 unbiased learning, can facilitate the invasion of an on average inferior variant that sometimes gives a very high payoff. Our model, which includes influence bias, is consistent with that claim. When
550 a maladaptive trait is "piggybacking" a role-model with high influence, said trait could spread in the population, as mentioned. In addition, they show that best-of-k learning can be stable against invasion
552 by unbiased learning if social learning is sometimes combined with individual learning. Our model is based on copying based learning only, but it could be interesting to combine it with individual learning
554 and see how it affects the dynamics of the population.

We discussed prestige in depth, and provided several proofs for its existence in nature, humans, and
556 even medicine. We saw it could aid invasion of maladaptive traits, or neglect of important medicinal specialities. But, it can also accelerate reversal of harmful traditions. Harmful traditions can be child
558 marriage, open defecation, and domestic violence, to name a few. Efferson et al. (2020) suggest a mechanism called *spillover*. By their definition, a spillover is when an intervention affect a large
560 enough group in a target population, so that others not included in the intervention starts changing their behavior as well. In their research, they found that there are individuals who act as *agents*, who
562 are often looked upon, and therefore they are ideal targets for interventions. This is the same concept as our role-models, where a more prestigious individual will be copied more, therefore spreading his
564 trait wider in the population. Their research support therefore in our assumption that there are social biases, conformist influence specifically. They also suggest a way to use this phenomena to change
566 existing traditions in a population. It is very clear however, that just as it can be used to end harmful traditions, the same agents could be used for any negative way that comes to mind.

Up until now, we showed that cultural transmission is a process that manifests in many species, with emphasis on humans. We also displayed similarities between this process and genetic transmission, while presenting differences between them, specifically selective biases such as influence and prestige. We also presented examples of good and bad usages of such biases. All of these are mainly presented as a parallel process to the natural selection process in regards to physical anatomy, or at least have an indirect effect on it. Muthukrishna and Henrich (2016) offer a take on prestige as a factor of human physical evolution directly. They present a concept called *cultural brains* | brains that evolved primarily for the acquisition of adaptive knowledge. They build on the hypothesis of Dunbar (2009) that shows that larger, more complex brains can store and manage more information and in turn, this information can support the costs of a larger brain. Muthukrishna and Henrich (2016) built a model that predicts a strong relationship between brain size and group size, because group size also provides access to more adaptive knowledge. They later present their *cumulative cultural brain* hypothesis, an approach which proposes that human brains have evolved with an ability and proclivity for selective, high fidelity social learning. As part of this process, there are a variety of strategies and biases that have evolved to hone in on the most adaptive knowledge. These strategies and biases include direct and indirect cues of the popularity of cultural traits (e.g. success and prestige biases). In short, they suggest that some of the reasons for the extreme increase in brain size in humans, are the ability to "cheaply" acquire adaptive knowledge, i.e transmission biases, such as prestige.

Further work. Our model can be expanded in various ways: observing the effects of different bias functions, including errors when estimating influence or costs when copying from an influential role-model, and inclusion of multiple, rather than a single, optimal phenotype.

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