TKU211202
Linear Algebra
Lesson 10: Linear Transformation

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I. The Idea of a Linear Transformation

The Idea of a Linear Transformation: Rules

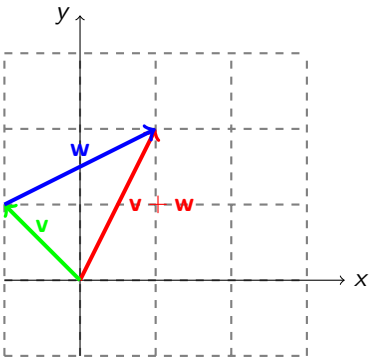
- A linear transformation T takes vectors v to vectors $T(v) \rightarrow$ Linear map
- A transformation T follows the same idea as a function.
- Linearity requires:
  - Rule #1 - additivity / operation of addition

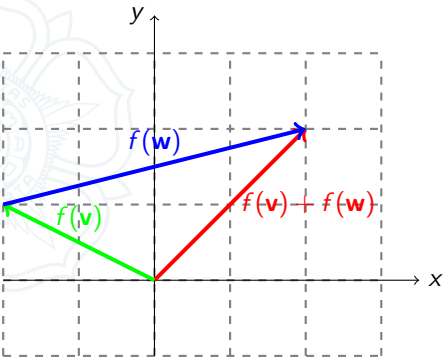
$$T(\mathbf{v} + \mathbf{w}) = T(\mathbf{v}) + T(\mathbf{w})$$

  - Rule #2 - operation of scalar multiplication

$$T(c\mathbf{v} + d\mathbf{w}) = cT(\mathbf{v}) + dT(\mathbf{w})$$

The Idea of a Linear Transformation: Rules



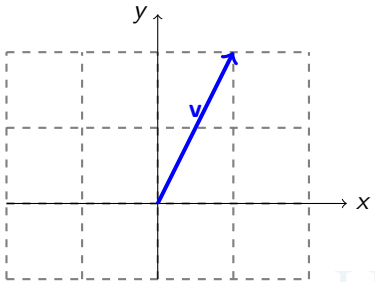


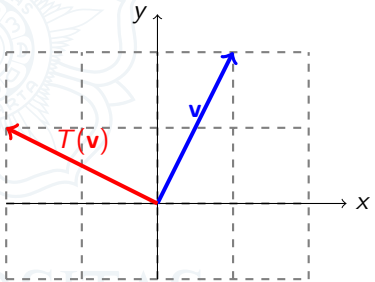
$$\underbrace{\begin{bmatrix}-1\\1\end{bmatrix}}_{\mathbf{v}} + \underbrace{\begin{bmatrix}2\\1\end{bmatrix}}_{\mathbf{w}} = \underbrace{\begin{bmatrix}1\\2\end{bmatrix}}_{\mathbf{v}+\mathbf{w}}$$

$$f(x, y) = (2x, y)$$

The Idea of a Linear Transformation: Example in $\mathbf{R}^2$

- In two-dimensional space $\mathbf{R}^2$, linear maps are described by 2 x 2 matrices (called A).
- Example #1 - rotation by 90 degrees counterclockwise





$$\underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}_{\mathbf{v}} = \underbrace{\begin{bmatrix} -2 \\ 1 \end{bmatrix}}_{T(\mathbf{v})}$$