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SIMULTANEOUS GRASPING OF MULTIPLE OBJECTS

A thesis submitted in partial fulfillment of
the requirements for the degree of Master of Science

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Abstract
Something
General idea is to find an amount of missing finger contacts for each object and each set of objects by examining the relationship between them.

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Acknowledgments

I am very grateful to my arms for being by my side, to my legs for carrying me, and to my fingers - I can always count on them.

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Nomenclature

μ coefficient of frfiction

f_c contact force vector .... [N]

F_i force component in i direction .... [N]

f_i^j i-coordinate component of j contact force .... [N]

f_n normal force .... [N]

F_o wrench vector .... [N and Nmm]

f_t tangent force .... [N]

G grasp map matrix .... [unitless]

n_i i's contact inward normal direction .... [unitless]

p_{c_i} i's contact location in object's frame .... [m]

r contact location distance from origin .... [mm]

r_c distance between coordinate systems origins .... [mm]

T_i torque component in i direction .... [Nmm]

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1. Introduction

1.1. Motivation [1 page]

Robotics become a vast field of research and applications,

- Technologies
- Robotic manipulators
- Grasping
- Automation
- Home Assistance
- Defense applications
- Remote robotics (space etc.)

AMAZON ROBOTICS CHALLENGE

** ***The idea can be expanded for various tasks, such as home assistant robot that brings several items in one run. This should go to Motivation.***

1.2. Research objectives [.5-1 page]

While vast amount of research is done on grasping in different forms (manipulators, fixture, first\second order, machine learning approach, visual assessment, restraint analysis etc.) little work was done on grasping/manipulation of several objects an once.

The main objective of the research is to present a solution for immobilization of several objects simultaneously, while using minimal amount of actuators (fingers). That can be achieved by analyzing the spatial relationship of objects and the contacts between them.

The research can be divided to sub-objectives:

Classifying the types of contacts between bodies.

Assessment of additional constraints needed to immobilize the set of objects

Deriving an optimal way to arrange the objects in order to minimize the amount of external constraints needed for the immobilization.

1.3. Research contribution and innovation [.5-1 page]

The research yielded solid and reproducible results of multiple object immobilization that can be integrated in various robotics systems.

The methods described are novel solutions for the problems yet to arise.

The research can be further extended to manipulation of multiple objects, interaction with soft objects / fingers and, of course, all of this in 3D space.

Some of the products of the research can be used for more intelligent manipulation in cases where variety of items exist and decisions as to selection of groups to be moved simultaneously have to be made. Example of such use: home assistance robotic system that performs a task of sorting and storage of groceries.

Should mention ICRA here, and the book that could be published.

1.4. Literature Review

Important contribution to the future development of robotics in general and grasping in particular belongs to Reuleaux [38], Landau and Lifshitz [21] for bringing the basics of mechanics and mechanism kinematics, which serve as foundation for every analytical approach of robotics study. Lozano-Perez [24] for the introduction of Configuration Space use which contributed in vast majority of fields. Use of the Configuration Space representation has been a major interest for the simultaneous multi-body manipulation, yet because of high computation cost and lack of explicit analytical formulations was not fully incorporated in this research.

Ball [4] introduces an essential ideas of screw theory which allows use of wrench and wrench space concepts, tools that take major part in the contemporary robotics. Murray et al. [32] presents concise and yet thorough survey on appliance of the mathematical tools for the description of manipulations. With general tools for mechanisms and manipulators in particular, interaction of such manipulators with the environment can be formulated using Coulomb's friction, categorizing contacts for frictional, frictionless, hard or soft contacts Murray et al. [32]. Rimon and Burdick [40] proposed bounds on number of frictionless fingers for 2D polygonal object immobilization.

Given broad mathematical basics, grasping becomes of particular interest with Mishra et al. [29] showing existence of positive grasps, and allowing following grasp synthesis development: Bunis et al. [14], Borst et al. [10]. Derivations would not be possible without defining form and force closure by Asada and Kitagawa [3]Rimon and Burdick [39]. Nguyen [33] proposes a viable methods for constructing force closure grasps. More methods on grasp construction include synthesizing grasps evolved from predefined configurations Pollard [36], mimicking human behavior in grasping tasks: using predefined shapes and modes for closure grasps are planned with numerical computations of trajectories intersections Wren and Fisher [48].

Contact modeling Xydas and Kao [51]

Rimon and Burdick [41] propose a novel idea of mobility index based on second order curvature of bodies in contact, intended for multi-finger grasping.

Grasp evaluation takes a long way with Mishra and Silver [30] discussing the stability of grasps, Ferrari and Canny [17] introducing quality measures based on wrenches, Roa and Suárez [42] surveying several methods of grasp quality measure, while Lin et al. [23] extend the measures for complaint grasps.

Grasp generation can be addressed with different approach, with randomized grasp generation Borst et al. [9] where grasp candidates are generated, tested for force closure and evaluated according to desired quality measures.

Integrating methods in real life applications usually require use of vision systems as shows Corke [15] and often extended to use of machine learning for vision based grasping: Quillen et al. [37], Bezak et al. [7], Le et al. [22].

The list would not be full without mentioning Siciliano and Khatib [46] providing an overview and composition of concepts and techniques in grasp synthesis, evaluation and optimization.

Envelope grasping introduced by Harada and Kaneko [19] uses similar ideas fro constructing enveloping grasps, which induce multiple contacts between polygonal and polyhedral objects and chained fingers – not dissimilar to interaction of couple of polygonal objects

Shapiro et al. [45] present an assessment of passive forces in grasping fingers developed reacting to external perturbations acting on the

Sintov et al. [47] show a method for generation of common robotic grasps for set of objects. The objects are grasp individually by the same end-effector. This could be extended to simultaneous grasping in case of planar objects stacked one above another or for cases where spatial objects have planar (frictional) grasp.

Stability of frictionless assemblies is studied by Mattikalli et al. [28]. The paper deals with orientations of assemblies for immobilization under gravity.

Fixture of single part with 4 contacts in lattice points Brost and Goldberg [13]

Most methods for grasp construction are searching for force closure grasps, some of them for optimal grasps. Usually the complexity of the computations depends on the number of edges / faces of the object to be grasped.

Multiple objects grasping Pajarinen and Kyrki [35], Zeng et al. [52]

Polygons interaction is reviewed by Brost [12], who describes algorithm for construction of equilibrium configurations for two polygonal objects.

* **** Put more info on CONFIGURATION SPACE approach to grasping***

Multiple robot cooperation to move multiple objects was presented by Donald et al. [16]

A research with some common ideas is done by Bernheisel and Lynch [6] who describe methods to determine stable pushing motions for planar stack of polygonal parts as alternative to grasping and carrying the whole assembly.

Minimal Fixturing of Frictionless Assemblies: Complexity and Algorithms Baraff et al. [5] * **** WHAT?????? It's quite similar!!!***

Assembly fixturing Romney [43] present methods for assembly sequence generation and fixture determination which is more oriented on CAD assemblies planning and design and uses assembly-by-disassembly strategy. * ***This is similar as well***

Main difference of the presented work relative to the similar papers shown above is generation of optimal grasps while maximizing the grasp quality measure and online algorithms for optimal configuration construction. * ***Elaborate here?***

Various methods for assembly manipulations by pushing developed in past Akella and Mason [1], Bernheisel and Lynch [6], Lynch [26, 27]

in past by Akella and Mason [1], Bernheisel and Lynch [6], Lynch [26, 27]. The most suitable solution for this case seems to be the one proposed by Anders et al. [2], and in presence of suitable gripper the final step will overcome uncertainties should they rise.

For the purposes of simulations and experiments, objects are rearranged manually into desired configuration.

1.5. Preliminary Background

Basically this part should explain to the reader all the basics that are known prior this work. Everything that is out of the scope of the B.Sc in mechanical engineering should be addressed and explained.

Starting with introduction to grasping, contact models and grasp matrix, follow with for closure and form closure, grasp planning (fingers number, conditions and grasps). Restraint analysis (form closure). Need to describe: workspace, closure, contacts, twist and wrench duality.

Notation in robotics makes a wide use in vectors and matrices, along with coordinate frames and transformations. A lot of concepts are drawn from linear algebra.

Manipulator forces, relations between joint forces/torques and end-effector forces, singularities, dynamics of manipulators etc. is out of the scope of this work.

1.5.1. Motion and Forces

Traditionally, motion of rigid bodies can be described by utilizing a concept knows as twist: the description of both linear and angular velocities of a rigid body. Murray et al. [32] shows that any rigid body motion can be described using a twist, namely that every motion of a rigid body can be described by a rotation of a body about an axis and translation of that body in direction parallel to that axis – known as *screw* motion. The mapping between twist and screws can be done using matrix exponential.

$$V = \begin{bmatrix} \upsilon \\ \omega \end{bmatrix} \in \mathbb{R}^6 \qquad \upsilon \in \mathbb{R}^3, \omega \in \mathbb{R}^3 \tag{1.1}$$

A generalized force, acting on the body is described as a *wrench* – force/moment pair, which consists from a linear component and an angular component acting on a point.

$$F = \begin{bmatrix} f \\ \tau \end{bmatrix} \in \mathbb{R}^6 \qquad f \in \mathbb{R}^3,\, \tau \in \mathbb{R}^3 \tag{1.2}$$

Combination of wrench and twist define power: the dot product of twist and wrench yield instantaneous power. Reciprocal wrenches and twist are combinations that have instantaneous zero power: $F \cdot V = 0$. Both concepts can be described as screws, and this way threat them in same way. The notion allows simple analysis of kinematics of mechanisms in general, and, more importantly, grasping in particular.

In grasping, the wrenches applied to the object act as a set of constraints, and twists are possible motions of the object. If acting wrenches allow no reciprocal twists, i.e. there is no twist that for every wrench the dot product will be 0, then the object is immobilized.

1.5.2. Grasping and Immobilization

Contacts

Basic concept in grasping is a contact between a finger and an object. Contact describes the mapping of forces applied by fingers at some point on the object boundary to resultant wrenches in object coordinate frame. A set of properties should be defined for a contact:

- Contact point location
- Contact type
- Forces and torques applied by the contact

Contact point location is usually defined in object-attached coordinate frame. It is convenient to define the origin of this coordinate frame at object's center of mass, and to set contact coordinate frame where *z-axis* is normal to the object surface, pointing inside of the object, this way positive z value means pressure applied by a finger. Term pressure here means that finger can exert only pressing force and not pulling, as opposed to suction cups. Figure 1.1 illustrates coordinate frames: O – object coordinate frame, C – contact coordinate frame together with vector r_c which defines the location of contact point in object coordinate frame. Contact type would describe the interaction between the surface of the object and the surface of the finger. Common cases are: normal force only, in cases where friction coefficient is low; normal and tangent forces, in cases where friction is significant; normal and tangent forces along with normal torque (torsional friction). Typical examples for given contact types are: pressing a pen tip against a glass – normal force only; pressing a pencil tip against rubber - can provide normal and tangent forces; pressing a hand against a paper sheet laid on a table: friction coefficient between a sheet and a table is considerably lower than this between the hand and the sheet, this way we could move the sheet aside and rotate it about axis normal to the table surface. Figure 1.3 shows modeling of different contact types, with descriptions presented below:

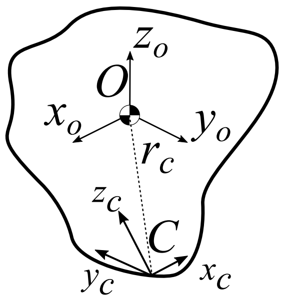


Figure 1.1.: Coordinate frames.

Frictionless point contact is a simplest type of contact which can apply forces only in the direction of the surface inward normal.

Point contact with friction is a more realistic contact which can apply both normal and tangent forces on an object. The modeling of a friction between a finger and an object surface could be simplified by assuming that the finger can exert small tangent forces, linearly depended with a normal force. This would produce a *friction cone* as shown in Figure 1.2. Normal force and tangent force written as f_n and f_t respectively. Tangent force depends on normal force. and thus can be described mathematically by inequality:

$$f_t \leq f_n \cdot \mu \tag{1.3}$$

whereμ stands for coefficient of friction between a finger and a surface. For practical applications, friction cone is approximated by finite set of vectors lying on cone's lateral surface. In 2 dimensional space it takes a form of an isosceles triangle with a vertex between two equal edges located at the contact point, and the bisector lies along the normal direction. The half angle of the aforementioned vertex is defined by $\alpha = \arctan(\mu)$.

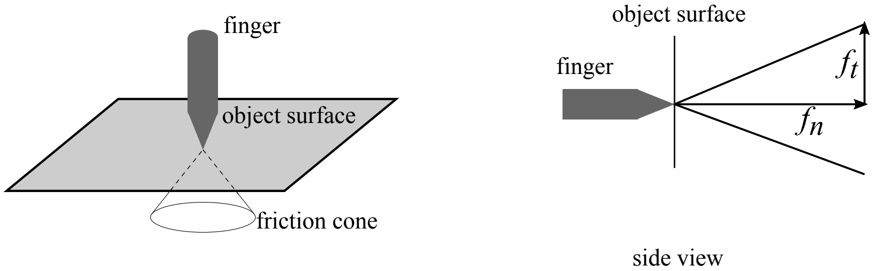


Figure 1.2.: Friction cone.

Soft finger is the most "capable" contact, able to apply both normal and tangent forces, and a torque about the contact normal as well. In $\mathbb{R}^2$ the concept is not used since rotations only possible around 1 axis.

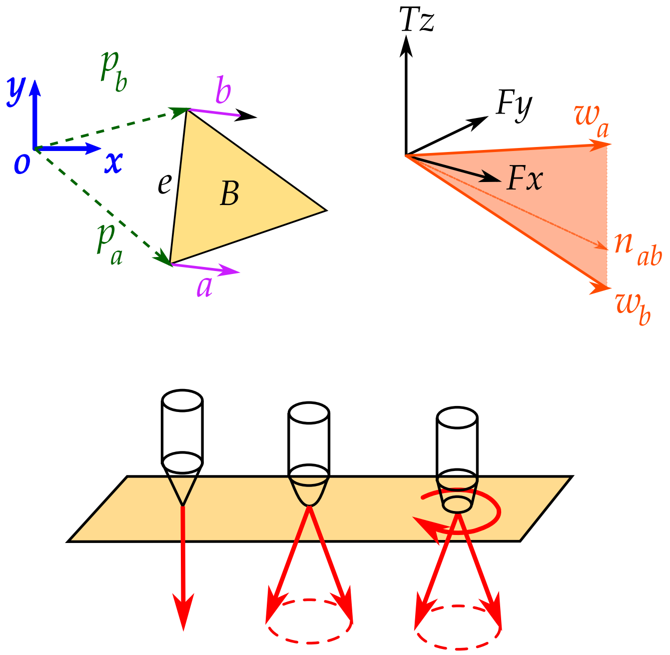


Figure 1.3.: Contact types

Grasp and Grasp Matrix

A grasp is a collection of contacts (types and locations) that can apply forces and torques on a body. Contact forces applied to the object will result in a resulting force and torque - wrench. A map from contact forces at contact location to wrenches that are induced by these contacts is called a Grasp Map. Derivation of the matrix is not shown here, reader can refer to Murray et al. [32] for more detailed explanation.

Example 1. Grasp map derived by Murray et al. [32] p.222 is presented below in Figure 1.4: a planar rectangular object held by two fingers which apply forces in the plane. Resulting wrench on the object from one finger can be described as following:

$$F_o = \begin{bmatrix} f_o \\ \tau_o \end{bmatrix} = \begin{bmatrix} R_c & 0 \\ \begin{bmatrix} -p_y & p_x \end{bmatrix} R_c & 1 \end{bmatrix} \cdot \begin{bmatrix} f_c \\ \tau_c \end{bmatrix} \tag{1.4}$$

whereR_c represents a transformation matrix from the contact reference coordinate frame to the body coordinate frame. Grasp map for this example is given by:

$$G = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ r & 0 & r & 0 \end{bmatrix}. \tag{1.5}$$

Although the derivation of the grasp matrix is not shown here, reader can intuitively see that locations of contacts and orientation of contact coordinate frames are reflected in this representation. Using grasp map, total wrench on the object could be defined as:

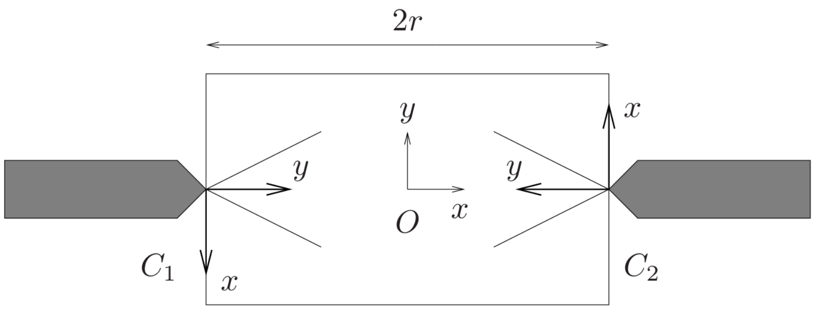


Figure 1.4.: Planar grasping.

$$F_o = G \cdot f_c = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ r & 0 & r & 0 \end{bmatrix} \cdot \begin{bmatrix} f_x^1 \\ f_y^1 \\ f_x^2 \\ f_y^2 \end{bmatrix}, \tag{1.6}$$

where F_o denotes a total wrench (force and torque combined) applied to the object, G is the grasp map as shown in Equation 1.5 and f_c is a vector of contact forces (normal and tangent forces applied by each and every finger). Expanding the matrix-vector product yields total wrench that can be confirmed by visual observation of Figure 1.4:

$$\begin{bmatrix} F_x \\ F_y \\ T_z \end{bmatrix} = \begin{bmatrix} f_y^1 - f_y^2 \\ -f_x^1 + f_x^2 \\ r \cdot f_x^1 + r \cdot f_x^2 \end{bmatrix}. \tag{1.7}$$

The space that is spanned by all contacts is called the Grasp Wrench Space (GWS) and characterizes the ability of the grasp to balance disturbance forces. A convex hull of vectors from grasp matrix rows with torque values normalized with some reference distance (radius of a sphere with same volume as the grasped object, for example) can be further used to analyze the grasp quality.

Grasp properties:

Given a grasp one can inquire what are the properties of that grasp, how good is it and whether it is suitable for desired task:

Equilibrium grasp is a grasp where fingers can exert forces that will keep the grasped object in the equilibrium state.

Manipulability defines whether arbitrary motions of a grasp object can be generated by applying forces by the fingers.

Immobilization grasp is called immobilizing if applied contacts prevent any motion of the object (translation or rotation).

Grasp redundancy the grasp can consist of number of fingers, while not every one of them is necessary (e.g. for immobilization, or for equilibrium grasp).

Grasp stability concept was introduced by Montana [31]. Bicchi and Kumar [8] summarize that the grasp stability depends on the local geometry of the grasp body. One of the definitions of stability is based on the potential function of the contacts forces, and the grasp said to be stable if for small perturbations from the equilibrium grasp the potential function has positive gradient, and unstable otherwise.

Force Closure A grasp is a force closure if the space of object wrenches is spanned by the set of finger forces and include the origin surroundings inside the convex hull of applied wrenches.

If a grasp can resist any external wrench applied to the object the grasp is called *force-closure*. One way to ensure that given grasp is force-closure is constructing *convex hull*. If convex hull contains the origin then the grasp is force closure.

Constructing of convex hull is done by using the columns of grasp map: each column represents a vector. Plotting these vectors and closing polygons between them will yield a convex hull shape (polyhedron). Normally tangent forces are result of friction and along with normal forces can be defined by cones, as shown above. For the matter of simplicity of the example, tangent forces are assumed to be independent of normal once. Hence, f_x assumed to be positive or negative, where f_y is positive only as shown in Figure 1.5 . Once convex hull is built,

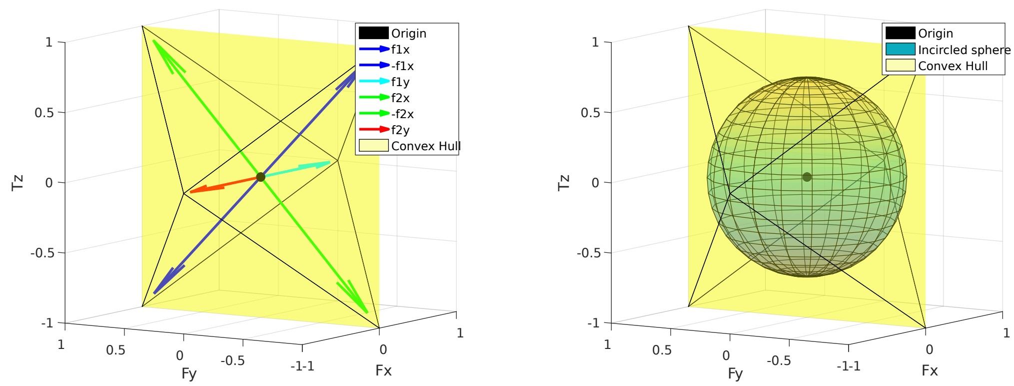


Figure 1.5.: Convex hull (left) and inscribed sphere (right).

quantitative assessment can be performed. One way of doing that is determining the largest sphere to inscribe in the hull with its center at origin, as shown in Figure 1.5, and measuring it's diameter. If vectors are normalized, the radius of the sphere can serve as ***grasp quality measure*** (dimensionless value). In case of 3-dimensional space and object convex hull is 6-dimensinoal: $F_x, F_y, F_z, T_x, T_y, T_z$ and it cannot be shown fully in 3-dimensinal domain, but projections such as F_x, F_y, F_z can be presented. Given the fact that vector describing inward direction of object boundary is typically normalized, it is easily seen that inscribed sphere radius will never by higher than 1. Example of force closure with 2 point contacts with friction is shown on right in Figure 1.6.

Form Closure A grasp is a form closure if it is a force closure and the fingers are frictionless contacts. The form closure is "geometrical" immobilization of an object, discarding friction forces etc. Example of form closure configurations shown in Figure 1.6 with blue frictionless fingers; left variant is first order form closure and middle one is second order form closure as introduced by Rimon and Burdick [40].

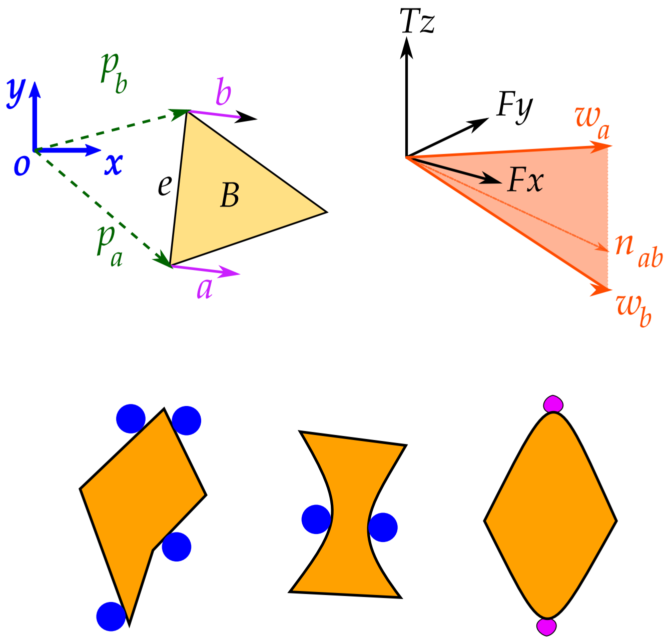


Figure 1.6.: Form closure and Force closure.

Quality Measures

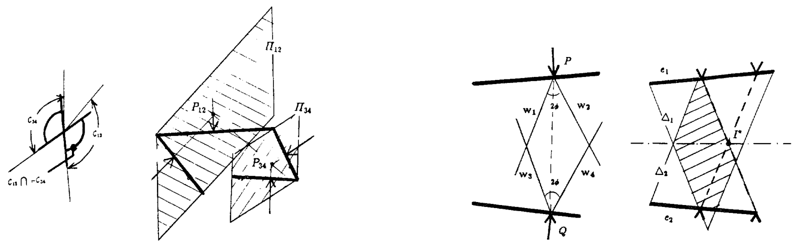
Several method proposed for calculation of Grasp Quality Measures (GQM) and they can be applicable for different cases. Some of them are focused on the mobility of the object that is grasped, others relate to the manipulator constraints.

Roa and Suárez [42] summarize common methods for deriving a grasp quality measures, both associated with contact points (such as Grasp Matrix singular values and wrench space ellipsoid volume) and with hand configuration (distance to singular configurations, manipulability ellipsoid, finger joints positions etc.). Ferrari and Canny [17] use radius of largest origin-centered hyper-sphere that fits in convex hull, this method is adopted for this research, and such sphere can be seen in Figure 1.5.

Grasp planning and construction

When planning a grasp for given object, one has to determine how many contacts and of what kinds are needed. Murray et al. [32] reminds that for spatial object, 4 friction contacts are sufficient for grasp. But, when dealing with robotic grippers, contact locations are limited due to gripper geometry, and forces gripper can apply are limited too. Given an object and a multi-fingered robot hand (along with robot itself) maximal wrenches an hand can exert can be calculated. Xiong et al. [50] shows extended grasp capability assessment, but for the purposes of this research it is safely to assume that the objects are light enough and the hand is robust enough to exert desired wrenches. Yet contact locations are limited because of the dimensions of finger and placement of the fingers on the palm (e.g. opposing finger vs symmetric pattern).

Variety of methods for grasp planning was presented during years, yet for the purposes of this research we use geometrical methods as proposed by Nguyen [33]. The paper shows that for construction of force closure grasp with 4 planar forces only, the geometric necessary and sufficient condition is that that no more than 2 force directions does not intersect in one point, and remaining 2 force directions result in moments with different signs about the aforementioned intersection point. Figure 1.7 shows images from the paper that summarize the idea of intersecting polygons formed by contact normal directions.



(a) Frictionless grasps on four edges.

(b) Grasps with friction on two edges.

Figure 1.7.: Grasp planning by Nguyen.

The methods uses geometric relations between object's edges and corresponding normal directions to construct polygons and determine edge regions suitable for finger placement. Method can be used both for frictional and frictionless contacts (only 2 fingers needed for frictional grasp). The resemblence between couple of contacts and frictional contact allows treatment of contact couples in a way similar to frictional contact.

Additional method that is used in a research is random grasp generation with evaluation of the resultant grasps, as proposed by Borst et al. [9]. Main idea is to generate random contact points along the object's boundary and evaluate the resultant grasp both for force closure and for given quality measure.

1.5.3. Graphs

Multiple objects grasping requires objects interaction classification. Some of the objects can be in contacts with other object(s) and these connections will be partially described by a graph: objects are represented as graph nodes and inter-object contacts serve as graph edges.

Graph is structure describing a set of objects and relations between them. Non-directed graphs alone are the utilized in this work. Adjacency matrix is a way to represent a finite graph. For non-directed graphs the adjacency matrix is symmetric. Matrix elements indicate whether a connection exists between 2 nodes, and also may quantify the connection.

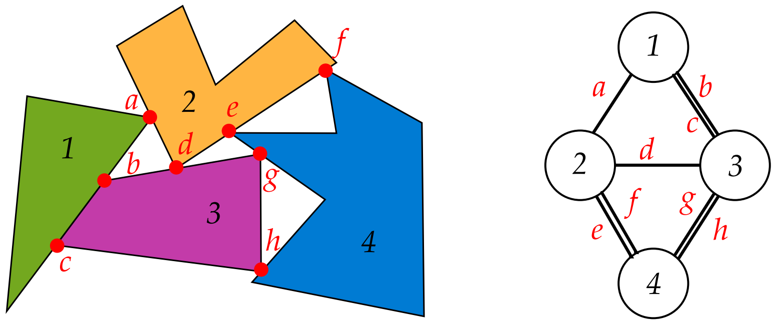


Figure 1.8.: Objects set and a corresponding graph.

Example 2. Four objects in Figure 1.8 where #3 touches ##1,2,4, and #2 touches ##2,3. Graph for the given object configuration is shown below. Adjacency matrix describing given graph is presented:

$$A = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 \\ 0 & 2 & 2 & 0 \end{bmatrix}.$$

1.6. Research objectives and assumptions

In our research we attempt to find an analytical solution for multi-fingered simultaneous grasping of multiple planar objects while minimizing the amount of fingers required for the grasp.

Formulation of the problem to be solved can be written as follows:

Given: A set of polygonal objects in 2D space.

Desired:

A Configuration of objects in space achieved by translation and rotation only (no reflections).

B Minimal set of finger locations required to immobilize all of the given objects.

This work focuses on planar space where objects represented as polygons with finite amount of edges. Knowledge of exact location and orientation of those polygons are assumed, along with the knowledge of the exact shape.

The polygons are assumed to be non-self-intersecting, while for practical purposes the null-thickness regions of self-intersection polygons can be infinitesimally thickened to form a non-self-intersection polygon.

In a given set of objects only connected configurations are of interest. The problem with more than one distinct group of connected polygons are easily divided to subproblems.

The work focuses on construction of equilibrium force closure grasps with frictionless fingers using first order geometry (i.e. form closure).

Finger contacts are point contacts and hence can act on one object at a time, two fingers cannot be placed at same location.

2. Proposed solution

In a common workspace polygonal objects can contact one another in several ways. Classification of the relations between the polygonal allows more thorough state analysis and further planning. After the classification is presented, an algorithm for deriving optimal object configuration and for finger placement given configuration of polygons are proposed, and finally an algorithm for configuration assessment is proposed.

Thus, the solution can be subdivided into 2 parts: determining the minimal amount and placement of fingers required to immobilize given set of contacting polygonal objects and a determination of desired configuration (rearranged set of objects) that will require minimal amount of finger to immobilize it. Both parts require examination of inter-object contacts and relations between objects, along with classification for further analysis.

2.1. Mathematical description

REWRITE THIS SECTION[1]

Given 2 polygons, p_1, p_2. The twist of polygon p_1 relative to p_2 can be defined as $\dot{x}_{12} = (v_{12}, \omega_{12})^T$. Polygons can maintain contact but not penetrate one another.

** ***Let IP denote the set of interaction points common to both polygons. Let $C = \{c_i\}$ be a set of contact descriptions between these polygons.***

Let $\hat{n}_i^j$ denote the inward normal direction of the boundary of polygon j at contact point i.

The contact between the polygons can be described using one or more contact points and corresponding contact normal directions.

2.1.1. Contact description

For the purpose of polygon arrangement evaluation, every contact is assumed to be performed by polygon A acting on the polygon B. The following description of a contact is adapted:

$$C = \{p, \hat{n}, id\,(P_A)\,, id\,(P_B)\} \tag{2.1}$$

where $id\,(P)$ stands for polygon number/identification entry, p is a contact point location and $\hat{n}$ is the contact normal direction inward the polygon B. An example of inter-object contact description shown on Figure 2.1. The contact is saved as:

[1] *Treat the contact point and not the whole polygon. Rethink what do I want to say here at all.*

$C = \{p, \hat{n}, A, B\}$,where A, B are the names of the polygons, p is the contact location vector in selected coordinate frame and $\hat{n}$ is the inward normal direction of the second polygon at the contact location.

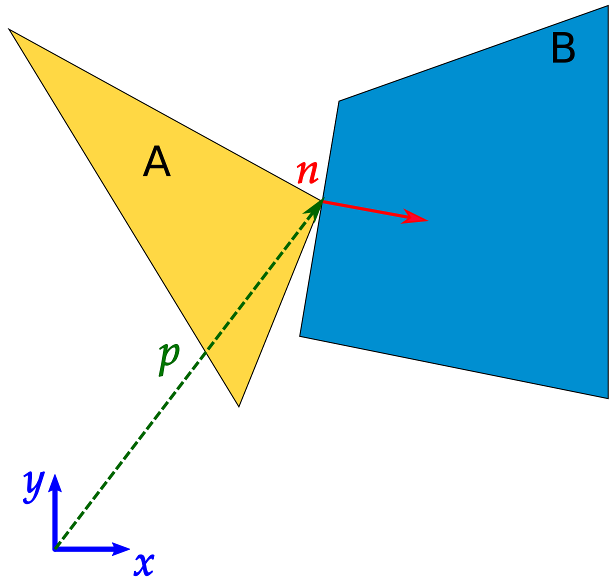


Figure 2.1.: Contact description example.

Extension of grasp stability on contact stability can be described as follows:

Definition 3. If two bodies are in contact and are held in equilibrium by external forces (e.g. fingers), if for small perturbation of contact position, the contact will stay in that new position or return to the original position than the contact is called ***stable***.

2.1.2. Relationship between polygons and polygon sets

The examination of given polygon configuration can be subdivided to interaction of each couple of polygons. The polygons can be disconnected - no points belong to 2 polygons simultaneously, or connected - one ore more points belong to 2 or more polygons. The connection between the polygons can be represented by a contact description. A stable contact between 2 polygons can exist in several variants:

** ***Make this more* math *style***

Vertex-Edge Vertex to edge contact is the discrete contact where a vertex of the first polygon (p_1) is coincident with an edge of the second polygon (p_2). The contact normal direction is defined by the edge the of the second polygon. The set of contacts for this variant consists of a single contact: $C = \{c_1\}$. Contact restrictions can be described as following: velocity of the contact point on the boundary of p_1 relative to the velocity of the contact point on the boundary of p_2 (denoted v_{12}) cannot have component in direction $\hat{n}_1^2$ - polygons cannot penetrate.

This can be expressed as follows: $v_{12} \cdot \hat{n}_1^2 \leq 0$. Contact is maintained when the component of the relative velocity in the direction if inward normal of first polygon is 0. Since $\hat{n}_i^2 = -\hat{n}_i^1$, $-v_{12} \cdot \hat{n}_1^2 \leq 0$. Therefore, the contact is maintained when relative velocity of contact points of each polygon are perpendicular to the edge normal.

$$v_{12} \cdot \hat{n}_1^2 = 0 \tag{2.2}$$

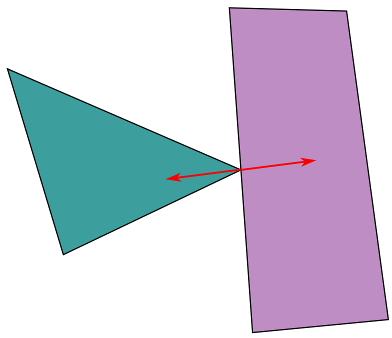


Figure 2.2.: Vertex-to-Edge contact example.

Edge-Edge Edge to edge contact is a continuous contact between two bodies along a continuous segment. Since the edges are finite, there exist at least 2 vertices belonging to either one or two polygons that belong to the contact segment. In a way analogous to distributed load concentrated in a point, the continuous distributed contact can be concentrated in 2 different points. Two different vertices are selected to be such points. Since the vertices lie along the same edge, the contact normal directions are the same in this case. The set of contacts for this variant consists of two contacts: $C = \{c_1, c_2\}$, $c_i = \{r_i, \hat{n}\}$, where r_i stands for contacts' location and $\hat{n}$ stands for the normal direction of the edge. Graphically this can be depicted as follows: Polygon 1, polygon 2, contact line, contact points. Cases with 2 points, case with 3 vertices, case with 4 vertices in contact.

Vertex-inner-Vertex is another type of discrete contact with multiple point-normal parameters. Polygon 1 vertex is in contact with 2 edges of p_2. In this case, since the movement of polygon 1 relative to polygon 2 is restricted in 2 directions, 2 normal directions exist at same contact point. The set of contacts for this variant consists of two contacts: $C = \{c_1, c_2\}$, $c_i = \{r, \hat{n_i}\}$, where$r$ stands for contact location and $\hat{n}$ stands for the normal directions of the edges.

Other variants of contact descriptions are variations of the mentioned above. Examples: edge and inner vertex, vertex and vertex, edge and edge. Outer vertex to outer vertex contact assumed to be unstable and is discarded in this work. One polygon can contact several others in distinct points, apart from a case where several vertices are coincident as shown in Figure 2.5, in this case,

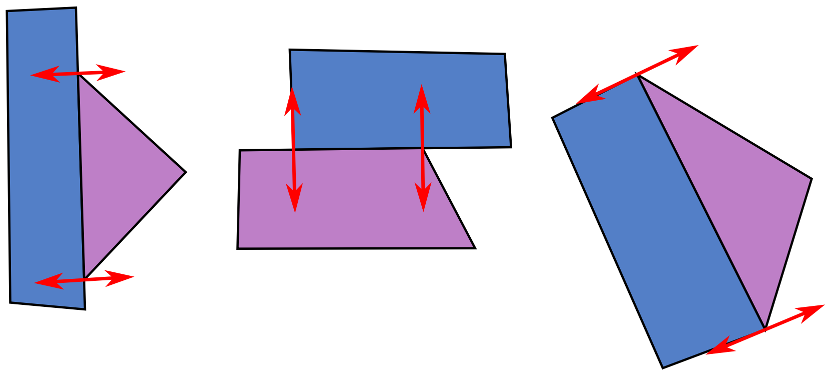


Figure 2.3.: Edge-to-Edge contact examples.

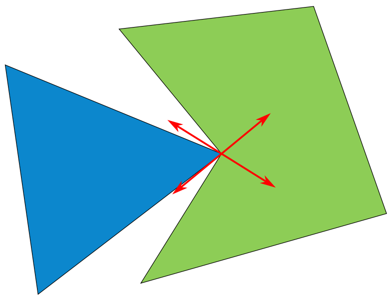


Figure 2.4.: Vertex-to-Inner-Vertex contact example.

2.1.3. Solution existence for combination of internal and external contacts

Given a set of polygonal objects, where some of the objects are in contact one with another. Each object has at least 4 contacts: some of them are fingers and some are inter-body contacts. The equilibrium equation for each object is written in form of total wrench:

$$G\bar{f} = 0 \tag{2.3}$$

where: $\bar{f} = \begin{bmatrix} f_{f1} & f_{f2} & f_{p12} & f_{p13} & \dots & f_{pMN} \end{bmatrix}^T$ is a set of forces between fingers and bodies, and between bodies and other bodies in contact. G matrix rows mul-

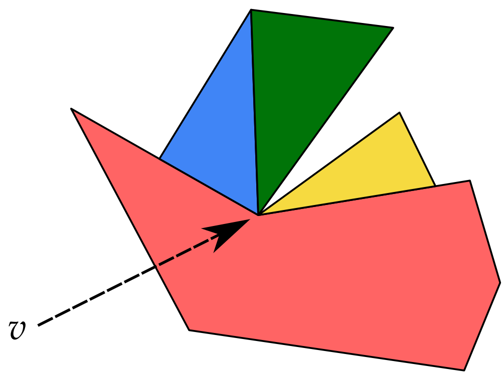


Figure 2.5.: Multi-object vertex v.

tiplied by the set of forces produce equations for $\begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ for each body. The matrix will be block diagonal with some cross-block columns which correspond to body-to-body contacts.

Lemma 4. *Given an edge of a polygonal object and a frictionless point contact, the generalized (superposition) wrench for all possible locations of contact along the edge can be expressed as 2 dimensional cone (2 vectors) in vertical plane of the configuration space (the plane's normal direction will be in$f_x f_y$ plane).*

Proof. All possible contacts on the edge will have the same direction and hence samef_x, f_y components. But for all these possible contacts the wrenches will be different. The positions of the contact points one the edge have limits, and at those limits will define the marginal moments that the contact can apply. These two marginal vectors will form a two dimensional cone in a wrench space. □

Proposition 5. *4 point fingers acting on a 2D object in distinct points along object's boundary may constitute a force closure equilibrium grasp.*

Proof. Caratheodory's theorem shows that a N-dimensional space can be positively span by N+1 vectors. For two-dimensional task space, there are 3 degrees of freedom (x, y, θ) and hence both the configuration space and the wrench space are three dimensional ($\mathbb{R}^3$). Four distinct contacts can be represented be 4 vectors in the wrench space and span it. Because of the configuration/wrench space duality, fully spanned wrench space means fully constricted configuration space: the object will have no freedom of movement. The convex hull of these vectors spans the origin as well, and thus it is an equilibrium grasp. □

Proposition 6. *A set of N polygonal objects in $\mathbb{R}^2$ in stable contact one with another can be immobilized by at most $3N$ fingers assuming first order non-frictional contacts.*

Proof. Given connected set of objects, for each object at least one contact exist. For an object in $\mathbb{R}^2$ to be first order immobilized minimum 4 constraints required. Knowing that at least one constraint already exists because the set is connected, for each object 3 external constrains are needed, yielding $3N$ fingers for the whole set. □

Proposition 7. *A set of N polygonal objects in $\mathbb{R}^2$ in contact one with another can be immobilized by at least 4 fingers assuming first order contacts and exceptional resulting shape.*

Proof. Assuming optimal configuration of the objects where each of them is immobilized relative to others by form closure conditions and possibly external constraints (EC) the total amount of external constraints for the set immobilization is $\max(4, |EC|)$. Example: jigsaw puzzle which once assembled can be immobilized by 4 fingers. □

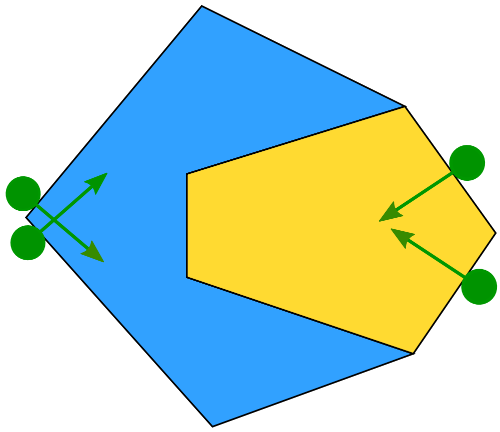


Figure 2.6.: Immobilization by minimum 4 fingers.

Proposition 8. *Given a polygonal object and 3 parallel frictionless contacts on the object boundary (same edge or different edges). It is impossible to construct and first order immobilizing grasp with 1 additional contact.*

Proof. Four contacts of a frictionless grasp have to not to intersect in one point, as showed Nguyen [33]. Three parallel contacts are "intersecting" at infinity. Moreover, 4-th contact alone cannot span all moments around that intersection point. □

Theorem 9. *For a given set of polygonal objects in contact, and some amount of external fingers which results in total minimum 4 distinct equilibrium contacts for each object, a solution for the finger contact forces exists that will maintain the equilibrium of the objects.*

Proof. Four contacts acting on each object (whether from inter-object contacts or from external contacts), maintaining equilibrium have a grasp map of form:

$$G_{f[m:n]} = \begin{bmatrix} c_m & \cdots & c_n \\ s_m & \cdots & s_n \\ \mathbf{x}_m \times \begin{bmatrix} c_m \\ s_m \end{bmatrix} & \cdots & \mathbf{x}_n \times \begin{bmatrix} c_n \\ s_n \end{bmatrix} \end{bmatrix}, \tag{2.4}$$

where $\mathbf{x}_i$ is a point of the contact location, and $\begin{bmatrix} c_i & s_i \end{bmatrix}^T = \begin{bmatrix} \cos\theta_i & \sin\theta_i \end{bmatrix}^T$ with θ_i being the direction of the object inward normal at $\mathbf{x}_i$. Since the contacts are distinct (located at different points), 3-rd row of the grasp map is not linear combination of first 2. An example of contrary for 2 contacts, 3-rd row is a combination of first two only if both contacts are at same location:

$$G_{f[m,n]} = \begin{bmatrix} c_m & c_n \\ s_m & s_n \\ \mathbf{x}_m \times \begin{bmatrix} c_m \\ s_m \end{bmatrix} & \mathbf{x}_n \times \begin{bmatrix} c_n \\ s_n \end{bmatrix} \end{bmatrix} = \begin{bmatrix} c_m & c_n \\ s_m & s_n \\ x_m s_m - y_m c_m & x_n s_n - y_n c_n \end{bmatrix} \tag{2.5}$$

$$\to \mathbf{x}_m = \mathbf{x}_n,$$

Building a global grasp map for the whole object set can be rearranged to have diagonal blocks – corresponding to the external contacts for each object and off diagonal blocks that represent the inter-object contact components. Since every diagonal block is linearly independent, the global grasp matrix rows will be linearly independent and hence there will be a (non-unique) solution. □

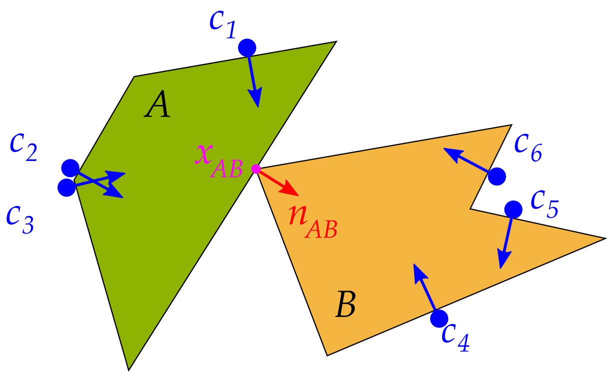


Figure 2.7.: Solution existence example.

Example 10. Two bodies in contact at one point. Contact forces shown in red, finger forces shown in blue. Each force f_i acts at point $\mathbf{x}_i$ located at body boundary, with direction θ_i which vector is $\begin{bmatrix}\cos\theta_i\\ \sin\theta_i\end{bmatrix} = \begin{bmatrix}c_i\\ s_i\end{bmatrix}$. The inter-body contact force f_{AB} selected to act in direction $\begin{bmatrix}c_{AB}\\ s_{AB}\end{bmatrix}$ at point $\mathbf{x}_{AB}$. The objects are immobilized by 3 fingers, according to the Proposition 6.

$$\begin{bmatrix} f_{Ax}\\ f_{Ay}\\ \tau_A\\ f_{Bx}\\ f_{By}\\ \tau_B \end{bmatrix} = \begin{bmatrix} G_{f[1:3]} & \left\{\begin{matrix} -c_{AB}\\ -s_{AB}\\ -\mathbf{x}_{AB}\times\begin{bmatrix}c_{AB}\\ s_{AB}\end{bmatrix}\end{matrix}\right\} & 0 \\ 0 & \left\{\begin{matrix} c_{AB}\\ s_{AB}\\ \mathbf{x}_{AB}\times\begin{bmatrix}c_{AB}\\ s_{AB}\end{bmatrix}\end{matrix}\right\} & G_{f[4:6]} \end{bmatrix} \begin{bmatrix} f_1\\ f_2\\ f_3\\ f_{AB}\\ f_4\\ f_5\\ f_6 \end{bmatrix} =$$

$$= \begin{bmatrix} G_{f[1:3]} & 0 & \left\{\begin{matrix} -c_{AB}\\ -s_{AB}\\ -\mathbf{x}_{AB}\times\begin{bmatrix}c_{AB}\\ s_{AB}\end{bmatrix}\end{matrix}\right\} \\ 0 & G_{f[4:6]} & \left\{\begin{matrix} c_{AB}\\ s_{AB}\\ \mathbf{x}_{AB}\times\begin{bmatrix}c_{AB}\\ s_{AB}\end{bmatrix}\end{matrix}\right\} \end{bmatrix} \begin{bmatrix} f_1\\ f_2\\ f_3\\ f_4\\ f_5\\ f_6\\ f_{AB} \end{bmatrix} = 0 \qquad (2.6)$$

As seen in the Equation 2.6 above, the grasp matrix for the given set of objects consists of a diagonal blocks and off diagonal columns. A system will be in equilibrium if a non-trivial set of forces exist that solves the equation.

** ***Consider rewriting the example as theorem.***

2.1.4. Edge Generalized Wrench

The evaluation of possible contact positions on an edged can be simplified by introducing an Edge Generalized Wrench (EGW):

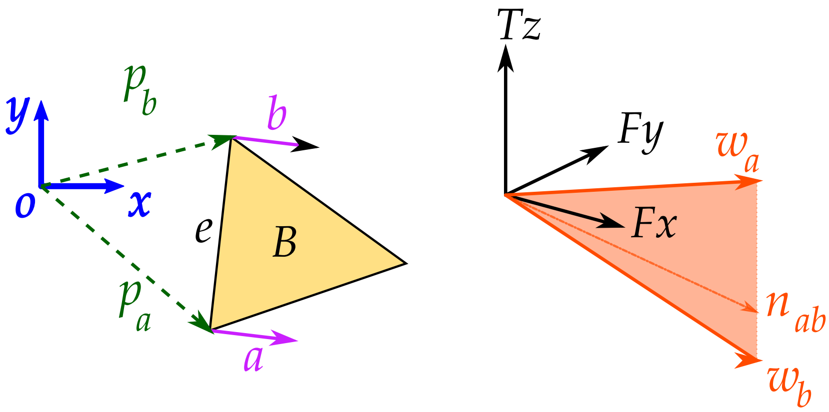


Figure 2.8.: Edge Generalized Wrench.

Lemma 11. *Given an edge of a polygonal object and a frictionless point contact, the generalized (superposition) wrench for all possible locations of contact along the edge can be expressed as 2 dimensional cone (2 vectors) in vertical plane of the configuration space (the plane's normal direction will be in* $f_x f_y$ *plane).*

Proof. All possible contacts on the edge will have the same direction and hence same f_x, f_y components. But for all these possible contacts the wrenches will be different. The positions of the contact points one the edge have limits, and at those limits will define the marginal moments that the contact can apply. These two marginal vectors will form a two dimensional cone in a wrench space. Denote the edge's normal by $\hat{n}$ and edge's ends by $\vec{e}_1, \vec{e}_2$ relative to o, the edge is parameterized by parameter $s \in [0, 1]$ the wrench of a contact along the edge can vary between:

$$(1-s)\begin{bmatrix}\hat{n}\\ \vec{e}_1 \times \hat{n}\end{bmatrix} + s\begin{bmatrix}\hat{n}\\ \vec{e}_2 \times \hat{n}\end{bmatrix}, \qquad s \in [0,1] \tag{2.7}$$

which is exactly a linear combination of two marginal wrenches. In a geometrical representation, it is a planar angle contained in a vertical plane (plane's normal direction has no torsional component) in the wrench space. The expression for the edge generalized wrench can be rewritten to a form of:

$$
\begin{aligned}
w_e(s) = (1-s)\begin{bmatrix}\hat{n}\\ \vec{e}_1\times\hat{n}\end{bmatrix} + s\begin{bmatrix}\hat{n}\\ \vec{e}_2\times\hat{n}\end{bmatrix} &= \begin{bmatrix}\hat{n}\\ ((1-s)\cdot\vec{e}_1 + s\cdot\vec{e}_2)\times\hat{n}\end{bmatrix} = \\
= \begin{bmatrix} n_x \\ n_y \\ (x_1\cdot n_y - y_1\cdot n_x) + s\left(n_y\left(x_2-x_1\right) + n_x\left(y_1-y_2\right)\right)\end{bmatrix} & \qquad s\in[0,1]\,.
\end{aligned} \tag{2.8}
$$

We can note that only the torque component of the wrench vector changes along the edge, and hence the edge generalized wrench is strictly“vertical” in the wrench space, and that it is linear along the parameter$s$. □

Illustration in Figure 2.8 shows an example of a generalized edge wrench. An edge of polygonal object B has inward normal direction $\hat{n}_{ab}$. Two marginal contacts a, b are shown in violet, with locations defined by vectors p_a, p_b respectively. Wrenches induced by these contacts are calculated by:

$$
w_a = \begin{bmatrix}\hat{n}_a\\ \vec{p}_a\times\hat{n}_a\end{bmatrix},\ w_b = \begin{bmatrix}\hat{n}_b\\ \vec{p}_b\times\hat{n}_b\end{bmatrix}.
$$

As can be seen from the Figure 2.8, contact a induces positive torque about the selected reference point, while contact b induces negative torque. Hence, the 3-rd components of a wrench have opposite signs and are located on different sides of the xy-plane.

2.1.5. Inverted cone test

As stated above, a convex hull of the contact wrenches should contain the origin of a wrench space. Given several contacts, and corresponding wrenches in the wrench space that do not contain the origin, a simple test can show whether additional vector will make total convex hull to contain the origin or not.

Proposition 12. *Given a set of 3 contacts, whose wrenches do not span the wrench space (the origin of the wrench space is not inside the convex hull of these wrenches) while not being in the same plane and additional contact, the contact can complete the force closure grasp of the object if the contact's wrench lies within the inverted cone of the given contacts' wrenches.*

Proof. Each couple of wrenches along with the origin point form a plane, Equilibrium condition for a grasp is that the total wrench for all contacts equals to zero:

$$
\sum_{i=1}^{4} G_i f_i = 0 \tag{2.9}
$$

Where G_i is $\begin{bmatrix} n_i \\ p_{c_i}\times n_i\end{bmatrix}$ for each contact. Rewriting the Equation 2.9:

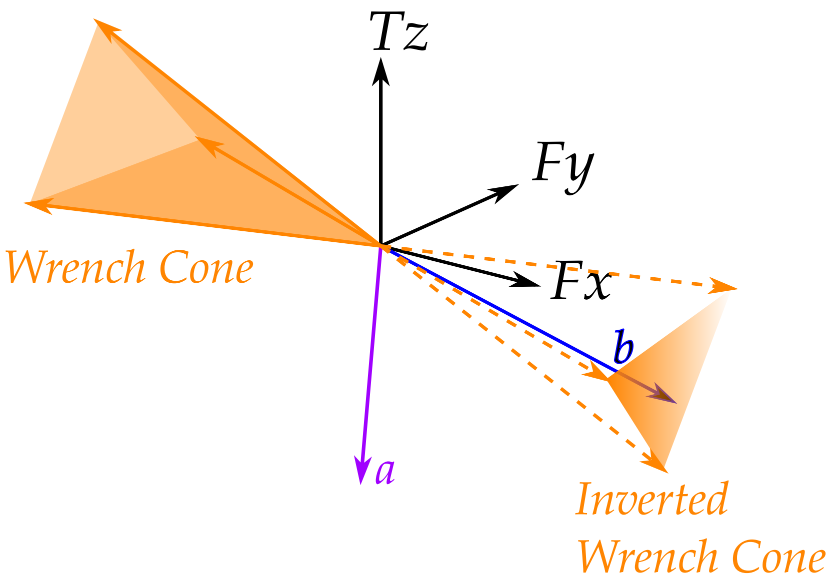


Figure 2.9.: Inverted Cone, with vector a inside and vector b outside.

$$G_4 f_4 = -\sum_{i=1}^{3} G_i f_i \tag{2.10}$$

Given the fact that G columns are wrench space vectors, and f are scalar values, this allows the representation of a 4-th wrench space vector as a linear combination of other wrench vectors, which is exactly the cone formed by these wrenches. If the 4-th vector lies outside the cone then it cannot be represented as a linear combination of other wrenches and hence will not allow equilibrium grasp. □

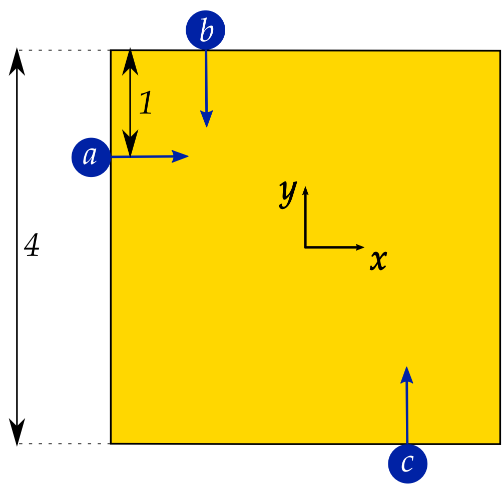


Figure 2.10.: A square object with 3 contacts.

Example 13. A square object with 3 contacts. Each contact is located 1 unit length from an edge. The contacts are described by the direction and the position as follows:

$$C_a = \left\{ n_a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, p_a = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$$
$$C_b = \left\{ n_b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, p_b = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$$
$$C_c = \left\{ n_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, p_c = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$$

Corresponding wrenches are:

$$w_a = \begin{bmatrix} \hat{n}_a \\ \vec{p}_a \times \hat{n}_a \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$w_b = \begin{bmatrix} \hat{n}_b \\ \vec{p}_b \times \hat{n}_b \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$w_c = \begin{bmatrix} \hat{n}_c \\ \vec{p}_c \times \hat{n}_c \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Convex cone of these wrenches along with it's inverse are shown in Figure 2.11. The distance of some point from 3 faces of the inverted cone is calculated by

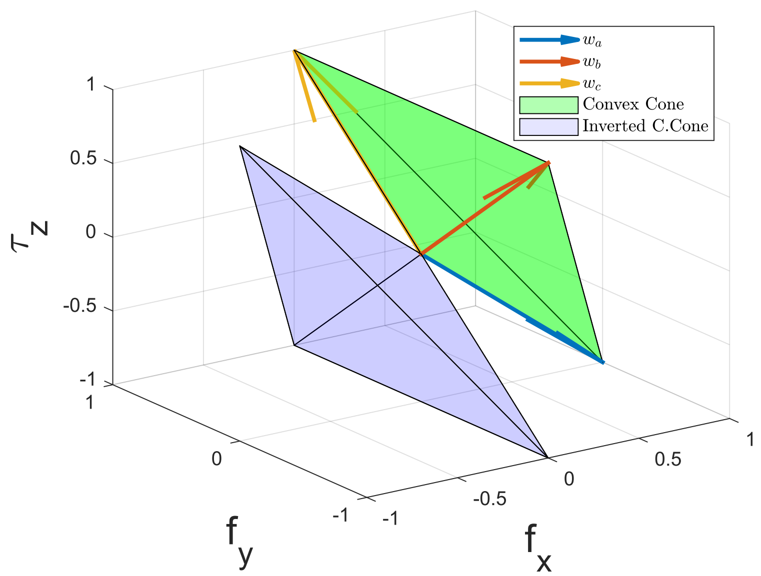


Figure 2.11.: Contact wrenches, convex cone and inverted convex cone.

projecting the point onto the face normal. Face normal directions are:

$$\hat{n}_{ab} = \frac{w_a \times w_b}{|w_a \times w_b|} = \frac{\begin{bmatrix}1\\0\\-1\end{bmatrix} \times \begin{bmatrix}0\\-1\\1\end{bmatrix}}{\left|\begin{bmatrix}1\\0\\-1\end{bmatrix} \times \begin{bmatrix}0\\-1\\1\end{bmatrix}\right|} = -\frac{\sqrt{3}}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\hat{n}_{bc} = \frac{w_b \times w_c}{|w_b \times w_c|} = \frac{\begin{bmatrix}0\\-1\\1\end{bmatrix} \times \begin{bmatrix}0\\1\\1\end{bmatrix}}{\left|\begin{bmatrix}0\\-1\\1\end{bmatrix} \times \begin{bmatrix}0\\1\\1\end{bmatrix}\right|} = \begin{bmatrix}-1\\0\\0\end{bmatrix}$$

$$\hat{n}_{ca} = \frac{w_c \times w_a}{|w_c \times w_a|} = \frac{\begin{bmatrix}0\\1\\1\end{bmatrix} \times \begin{bmatrix}1\\0\\-1\end{bmatrix}}{\left|\begin{bmatrix}0\\1\\1\end{bmatrix} \times \begin{bmatrix}1\\0\\-1\end{bmatrix}\right|} = \frac{\sqrt{3}}{3}\begin{bmatrix}-1\\1\\-1\end{bmatrix}$$

Building generalized edge wrenches for each edge: As can be seen from Fig-

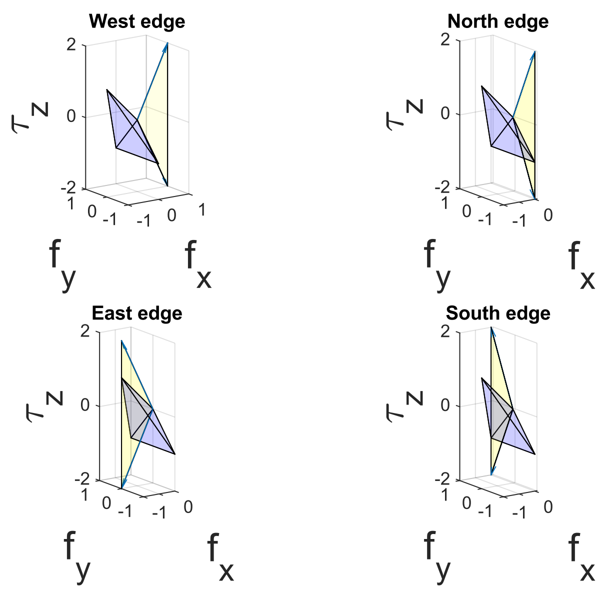


Figure 2.12.: Edge Generalized wrenches for 4 edges of the object.

ure 2.12, only the eastern edge wrench has the common intersection with the inverted cone. Visual examination of the object confirms intuitively that placement of a finger on the eastern edge only could immobilized the object. It can be seen that only a wrench vector from the eastern edge can complete the convex cone of 3 given contacts' wrenches and form a convex hull which contains the origin in it's interior. The idea is elaborated in Section 2.4.

2.1.6. Maximizing the Inscribed Sphere

One of the proposed quality measures is a volume/radius of the sphere inscribed in the convex hull of grasp map basis vectors. Maximizing the sphere radius can be done by increasing the minimal of the distances between the origin end the faces of the convex hull. Since the hull is convex, some of the faces will be tangent to the sphere, and in no case a edge will be tangent. In cases where several contacts act on the object, it is always possible to construct a *convex* hull and thus eliminate redundant wrench vectors.

Proposition 14. *Given a set of k-1 contacts, and their corresponding wrench vectors, convex cone and it's inverse cone; for a generalized wrench that has a segment in the interior of the inverse convex cone an optimal k-th contact can be found by examining distances of the convex hull faces from the origin.*

Proof. As seen in Proposition 12, the wrench w_k has to lie inside of the Anti-Cone. If w_k lies on a cone boundary then it and 2 vectors that form that boundary (denoted by w_1, w_2), will form a surface that contains the origin of the wrench space. To maximize the distance of said surface from the origin we first parameterize the wrench vector end point location segment by using the Equation 2.8. For each face of the convex hull, the distance of the face from the origin can be calculated by projecting a point on the plane on the plane's normal. We define a *central axis* of their convex cone as:

$$a = \frac{\sum_{i=1}^{k-1} w_i}{\left|\sum_{i=1}^{k-1} w_i\right|}$$

Assuming that the *k-1* wrenches are sorted in counter-clock-wise direction about the cone axis, meaning that

$$a \cdot (w_i \times w_{i+1}) > 0 \; \forall i \in [1 : k-1] \, ,$$

$$a \cdot (w_{k-1} \times w_1) > 0.$$

The equation for the normal direction of a plane formed by w_i, w_{i+1}, w_k is:

$$n_p = (w_i - w_k) \times (w_{i+1} - w_k) \, .$$

If the w_k lies inside the inverted convex cone of first *k-1* contacts, then the normal direction will point from the plane towards the origin. Consequently, the distance of the plane from the origin can be expressed as:

$$d_i = -w_k \cdot (w_i - w_k) \times (w_{i+1} - w_k) \, ,$$

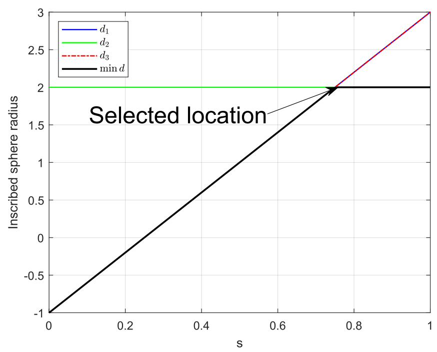
note the "–" sign to obtain positive distance[2]. Substituting the expression for wrench vectors leaves the distance of plane i (contacts i,i+1 and parameter s along an edge) to be a function of that parameter only:

$$d_i = f\left(s\right),$$

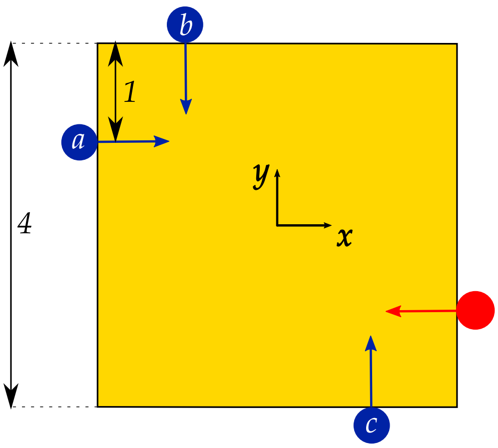
Ultimately, distance from every face can be expressed as a function of edge parameter s and thus a compound function of minimal distance can be built.

$$d = \min\left\{d_i\right\}\ \ i \in [1,].$$

Since projection is a linear transformation, and EGW endpoints segment being parameterized uniformly, the distance function will be linearly dependent on on s parameter. Meaning that the distance function will be either constant or monotonically increasing/decreasing along the change in s. Solution can be found by examining *k-1* 1st order polynomials and finding the maximized minimal value. Since the functions are 1st order polynomials, there either will be 1 such solution or infinite number of solutions (in case where there is a plane parallel to the EGW endpoints segment and hence the distance is constant). □



(a) Distance from convex cone faces.



(b) Object with added contact (red)

Figure 2.13.: Selecting the contact based on Inscribed Sphere Radius.

Elaborating on the Proposition 12, Figure 2.13 presents distances from convex hull faces as function of the parameter s. In this case, due to the symmetry of the object and the contact locations, d_1 and d_3 are equal, while d_2 is constant. In this case one of the distances is constant, and it is minimal at some part of the domain. Contact point selected to be furthest from the edge. Additional examples provided in Appendix.

[2]MATLAB polyshapes are stored with vertices in clockwise direction, hence no minus sign in the code.

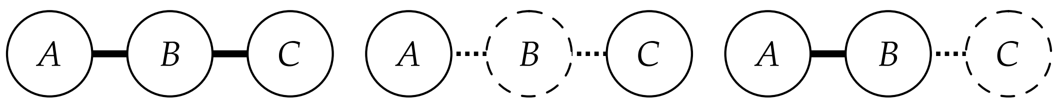
2.1.7. Connected graph components search

When a object arrangement is given (or obtained) and desired amount of external constraints is found by the proposed solution algorithms, there is no guarantee that there is no subset of objects which is not immobilized. This leads to a need of grasp assessment for subsets of the given arrangement. Each connected subset should be tested and ensured to be first order immobilized. Representing the objects arrangement as a connected graph, one can find all connected subgraphs. Different methods for subset extraction can be implemented, including filtering of power sets, connectivity analysis etc. as show Hartuv and Shamir [20], Gouda and Hassaan [18], Luxburg [25], Bron and Kerbosch [11]. For the purposes of this research, power set filtering is used.

Given a connected graph describing the objects configuration, where nodes are objects and edges are inter-object contacts, we iterate over the power set of graph's nodes and choose connected sets from the power set. Algorithm 2.1 describes a simple (yet computationally expensive) way to find connected subsets, where first a power set describing possible object combinations is constructed, and then each combination is evaluated to be connected. The evaluation part is non elaborated, it can be performed simply by evaluating off-diagonal components of the adjacency matrix of the graph describing the subset or by calculating *N-th* power of the adjacency matrix of size $N \times N$.

Algorithm 2.1 Connected subsets extraction from power set.

procedure ConnSubS(S, L) *// Power set S; linked list of L*
  for all set $s \in S$ **do**
    if isConnected(s) **then**
      $L = L + \{s\}$
    end if
  end for
  sort L ascending by number of elements in set
end procedure



(a) Full graph (b) B node removed no connected subgraphs (c) C node removed AB subgraph is connected

Figure 2.14.: Graph, connected and disconnected subgraphs

2.1.8. Grasp Quality Measure for a set of objects

We use simple GQM based on Ferrari and Canny [17], namely a radius of a origin-centered sphere inscribed in the convexand for any given contacts set and a reference point we can compute positive grasp quality measure if these contacts span the origin of the wrench space.

Algorithm 2.2 Grasp Quality Measure from contact set

function FINDGQM(p, C) *// reference point p; Contacts C*
  $W = \{w_i\,(p, c_i)\} \quad \forall c_i \in C$
  CH - Convex Hull of W
  if $\vec{0} \in CH$ **then**
    GQM = radius of origin-centered sphere inscribed in the CH.
  / The subset is indeed immobilized by the external contacts. */*
  else
    $GQM = 0$
  end if
  return GQM
end function

Assessment of the Grasp Quality Measure can be performed for given subset. For the purpose of this work, GQM for a set of objects computed as it was a single rigid object. Additional methods can be explored, such as minimal GQM of some object, inscribed hyper-sphere in the convex hull of external finger contacts in global $\mathbb{R}^{3\times N}$ wrench space of N objects etc. Note that Grasp Quality Measure for single object and for constellation of several object may have different trends at given finger configuration: the grasp may be optimal for the object, but for the constellation it will yield not the highest value.

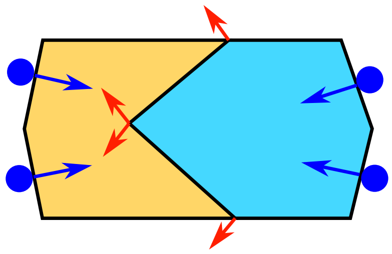
Example 15. 2 objects in contact. each object is immobilized individually, and the whole constellation is immobilized as well. Examining the grasps by computing GQMs shows that individual GQMs are high (maximal values) but total GQM (due to external contact vectors only) is low due to the directions of the external fingers contacts.

Figure 2.15 shows two polygonal objects immobilized by 4 external contacts. External contact direction vectors have relatively low y-components, and hence limit the GQM inscribed sphere radius to be at most as large as the$y$-component of the wrench vectors. The individual convex hulls are significantly larger the the convex hull for the both objects together.

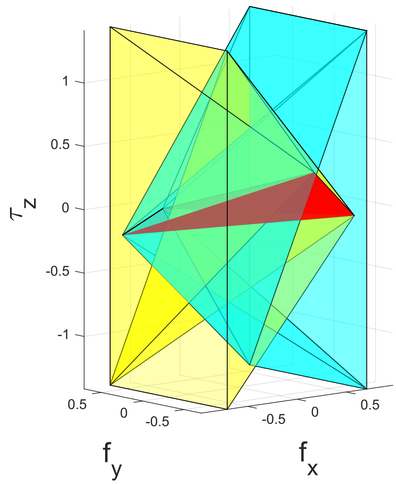
2.2. Determining new configuration for objects

2.2.1. Description

Given an unordered set of objects, each object is scanned and classified: number of edges, concave vertices, lengths of the edges. Scanned and classified objects allow assessment of different constellations for future grasping. Based on the algorithms and inter-object contacts classification as presented above, several strategies for object rearrangement can be implemented.



(a) Immobilized objects.



(b) Convex Hull for each object and for the set (red).

Figure 2.15.: GQMs differences.

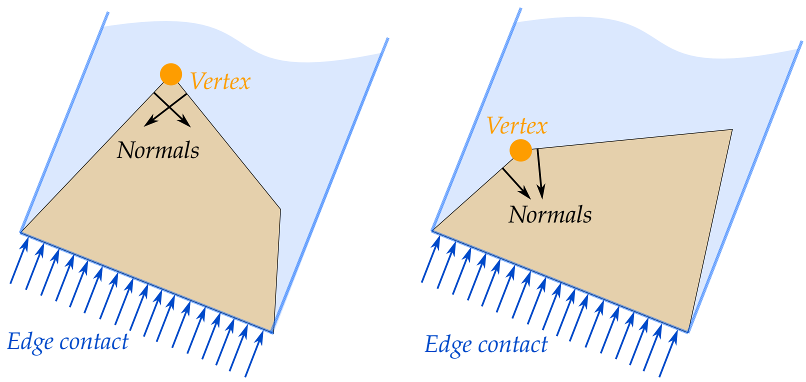
The approach for objects rearrangement is to stack objects in configuration that will require minimal amount of fingers for each object, maximizing number of inter-object effective contacts. Objects in the set in the new configuration can be described by a tree, where the root is some selected object, and all other objects described as children in a way similar to URDF style [44].

First of all, we need to know how many polygons with concave vertices exist, and how many exist with vertex angles less than 90°. Additionally, how many angles are smaller than concave angles if any.

Recapping the inter object contact types along with the additional contacts needed for the immobilization, we assume that the parent object is immobilized and inducing contact forces on a child object, which has to be immobilized by additional fingers. In the following paragraphs, term "fixation" refers to the contacts of the parent object acting on the child object.

Edge-to-edge fixation provides two parallel contacts, their convex cone can be seen as infinite "corridor", formed by the contact region swept along the contacts' normal direction. The interaction can be considered useful in cases where some other 2 different edges provide normal directions that are crossing in the parallel contacts' "corridor". Simple example would be a case with a vertex inside of the "corridor", where normal directions of 2 adjacent edges induce opposite wrenches about the edge with two contacts. This requires at least 2 additional external contacts for each object to complete the fixture.

One way of finding such configuration is checking whether there exists an object with a vertex (concave or convex) that is projected on the interior of some non-adjacent edge (base edge), the edge can be used as a base of edge-to-edge contact, and the vertex's adjacent edges will serve as locations for 2 additional fingers. The grasp will be feasible if normal directions from the adjacent edges and from the base edge will positively span $\mathbb{R}^2$. Example of feasible and non feasible grasps are shown in Figure 2.16.

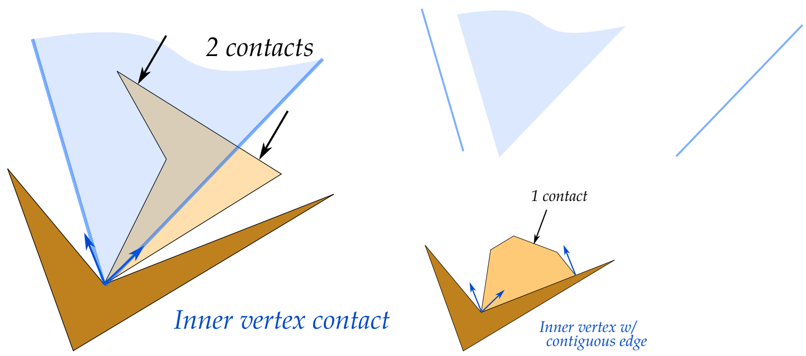


(a) Spanning vectors (b) Not spanning vectors

Figure 2.16.: Edge infinite projection and vertex with adjacent normal vectors.

Vertex-to-inner-vertex fixation provides 2 edges normal directions, which yield a convex cone with distinct vertex (unlike two parallel contacts which can be considered as convex cone with a vertex at infinity).

This fixation allows use of single edge to complete the fixture, given that the edge's inner normal direction is a negative combination of the given contacts' directions, and the edge allows two distinct contacts with opposing wrenches about the vertex (i.e. vertex is projected on the inner of the edge segment), as shown in Figure 2.17a. In case no such edge exist, two different edges provide normal directions which form a convex cone that mutually overlap(have a common region) the given convex cone (or their anti-cones), and the line connecting the vertices of the cones is in the interior of the overlapping region.



(a) (b)

Figure 2.17.: Inner vertex immobilization.

Vertex-to-inner-vertex with contiguous edge fixation contributes 3 effective contacts and needs only one external finger contact in the optimal case. For this there should exist an edge which projection along it's normal direction has 2-contact vertex inside it's interior and a there exists an intersecting segment of that projection and the 3rd normal direction that lies inside the convex cone of first 2 normals. Example of such configuration is shown in Figure 2.17b. If no such edge exists, 2 external contacts are required to immobilize this grasp by a method presented above

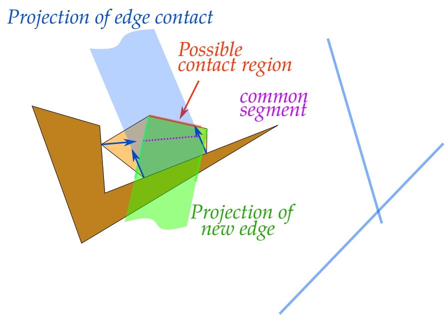
Vertex-to-inner-vertex with 2 contiguous edges fixation provides a optimal configuration which requires sole external contact for the immobilization of the object. Since objects are polygonal objects with non intersection edges and non-zero thickness, there will always exist an edge with a normal direction that will complete the form closure.

Vertex-to-inner-vertex with a contiguous edge and a contact point fixation is similar to the previous one except for the normal direction, and hence in certain cases can be self immobilizing.

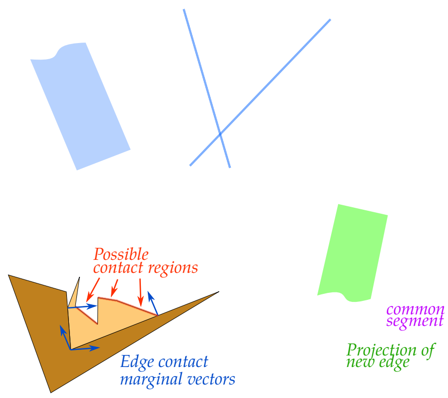
Vertex-to-inner-vertex with additional contact points is equivalent to the vertex-to-inner-vertex with contiguous edges formations. It provides same amount of contacts, but the normal directions can differ.

Contiguous edge with vertex point contact fixation forms a case with 3 contacts, which allows fixation with one additional finger under certain conditions: there should exist an edge, which projections common segment with the point contact's normal has a part lying inside the edge contact's infinite projection and also the new edge, point and given edge normal directions should span $\mathbb{R}^2$. Figure 2.18a shows an example of possible contact locations for this fixation.

Example in Figure 2.18b describes 2 contiguous edges, which can be seen as sub case for the described fixation and any contact with normal direction which is negative linear combination of the normals of 2 edges will immobilize the object.



(a) Contiguous edge with point contact.



(b) 2 Contiguous edges.

Figure 2.18.: Example of preferred edge contacts.

Given all these considerations, presented set of objects can be arranged to require the minimal amount of external fingers for each object and hence minimal amount of fingers in total.

The order of the stacking is suitable for the one-by-one object manipulation. Possible inter-object fixations can be ordered by amount and restrictions on additional contacts that are to be applied. **??** describes the order of preferable inter-object relations, based on amount of external fingers needed for the immobilization of child object relative to the parent.

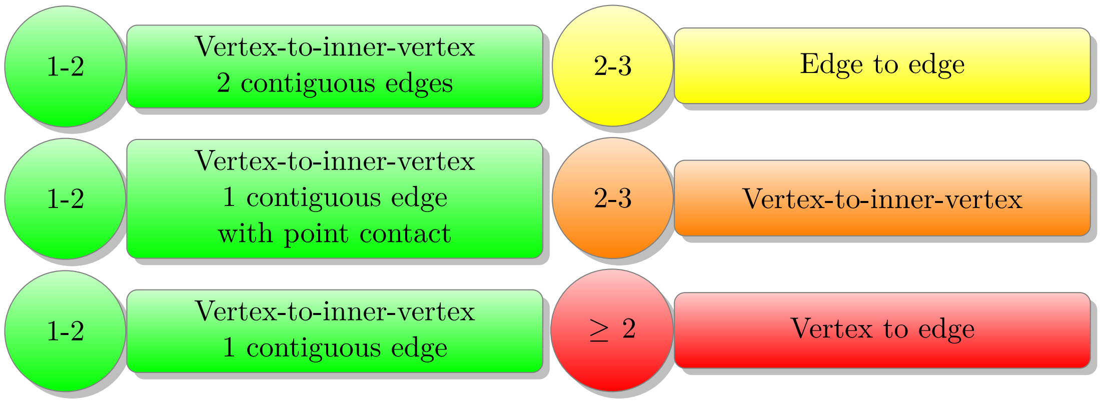


Figure 2.19.: Object placement priority.

Basically, last option (vertex to edge) will never be selected intentionally since if there is an option to place an vertex on an edge, there also is an option to place edge on edge. Vertex-to-inner vertex option also exists but for stability it is preferred to rotate the child object so it will have more parent contacts - vertex-to-inner-vertex with a contiguous edge.

2.2.2. Algorithms

First step is to define a root object, which will serve as base for further object stacking. This is done either by selecting a polygon with concave vertex, or stacking together two objects to form a concavity. Algorithm 2.3 describes the selection of root object.

When root object is determined, further stacking can be done. If there are objects with vertex angles less than the concavity angle, they are stacked together inside the concavity, possibly forming multiple concavities. If no such object exists, an object that will form a new concavity is stacked inside the root concavity. Example of such object is shown in Figure 2.18a; beige object stacked inside of a root brown object form 2 new concavities (potentially they could be wider than the root concavity).

Algorithm 2.4 iterates over objects in the given set, and stacks them one by one. Stacked objects are saved in a connected list, along with relative location; in order that allows physical stacking in case the configuration is accepted.

If at some step a potential configuration with no concavities is obtained, in case no more objects needed to be stacked it is accepted, but if some other objects left – other configurations can be considered to evade potential situation of edge-to-edge interaction. After the desired configuration is obtained, next step will be finding inter-object contacts that will be used for determination of required finger contact locations.

Algorithm 2.3 Defining the root object

```
procedure FindRoot(P)    // Set of polygons P = {p_i}
    if ∃ 1+ concave vertices then
        Root object = p_i with widest concave vertex
    else
        if ∃ 2 vertices with inner angles ≥ 90° then
            Root = p with greatest angle
            Root.Child = p with second greatest angle
            Align vertices to conctact, lean on longest adjacent edge.
        else
            Root = p with longest edge
            Root.Child = p with second longest edge
            Align longest edges to be collinear, vertex on child's longest edge coincident with midpoint of parent's longest edge.
        end if
    end if
end procedure
```

2.3. Contacts search

Given a set of polygonal objects, first step is to find the contacts between the objects. The problem is subdivided into assessment of polygon pairs. The polygon interactions assessed by simple algorithm that checks whether vertices of one polygon lie on edges or on vertices of another one. It is possible to restrict regions on edges due to adjacent objects, but at this point it is assumed that fingers are infinitesimally small and thus can fit in any desired location.

The Algorithm 2.5 is used to perform an inspection of contact points for given 2 polygons. The algorithm finds contacts between 2 polygons, returning a set that contains all contacts both the location and direction of the contact and the polygons IDs (numbers) that the contact relates to.

Once all contacts between all polygons are determined, remaining needed fingers placement can be done for each polygon – given a polygon, and a set of contacts acting on that polygon, a remaining amount of fingers needed can be determined, presented methods rely on a fact that 4 frictionless contacts is a minimum for immobilization of a two dimensional object.

2.4. Fingers placement

Given a configuration of objects in contact (connected group: each object has at least one contact), the configuration is parameterized by contact types. Each inter-object interaction is classified according to the variants defined above in subsection 2.1.2. When all contacts are found, each object is checked and an amount of additional contacts for first order form closure is derived. For every object required missing fingers are determined.

Algorithm 2.4 Objects stacking

```
procedure StackObjects(P, L)    // Polygons P = {p_i}, List L
    C = list of concavities
    initialize C with the root concavity
    L = {v} sorted list of polygon vertices
    for all v ∈ L do
        if ∠(v) ≤ ∠(c_i ∈ C) then
            Stack the child object in the c_i, vertex to innter vertex, edge aligned preferrably to a edge with non equal length
            move L.v to T.v and save child location and orientation
        else
            Find smallest concavity that allows contiguous edge with vertex contact interaction, stack object there.
        end if
        C = new formed concavities    // Upd. concavities list
    end for
end procedure
```

Several methods for single object finger placement were developed in past (presented in section 1.4), some of them can adopt predefined locations of part of fingers, among them Wu [49]. Our method is based on wrench space vectors evaluation and convex hull construction.

The global finger placement process is divided to several cases of polygon contact combinations. Cases of existing 3 and 2 inter-object contacts are addressed. When possible, contact locations selected to yield highest grasp quality measure, namely origin centered inscribed sphere radius.

2.4.1. Finding 1 finger contact given 3+ contacts

Given 3 contacts on an object, the viable regions for the 4th finger can be determined by evaluating EGW for each edge and assessing whether it is possible to achieve a force closure grasp by placing a finger on that edge. When existing contacts are represented by wrench space vectors, it is possible to perform assessment of the convex cone and convex hull. For a convex hull of four wrenches to span the origin of the wrench space, any 3 of the vectors should be linearly independent (not lying in one plane) and the 4th vector should be a negative combination of first 3, as shown in subsection 2.1.5. If there exists such wrench, corresponding contact location can be selected to be a complementary finger contact. Since on every edge could exist variety of possible contacts, simple method presented in subsection 2.1.6 is used to select the most promising contact location. The algorithm is presented below:

The algorithm gives a solution for cases where wrench space vectors of 3 given contacts are linearly independent. This method is complete and guaranteed to find a solution if there exist one. It is computationally inexpensive – $O(n)$, it iterates over all edges at most one time. If one of the contacts is a combination

Algorithm 2.5 Contacts between 2 polygons

procedure CONTACTSFROMTWOPOLYGONS(P_1, P_2)
    for all p_i in vertices of P_1 **do**
        if $p_i \in$ boundary (P_2) **then**
            if p_i is a vertex of P_2 **then**
                if p_i is inner vertex of P_2 **then**
                    Add 2 contacts at p_i, normals of adjacent edges of P_2
                else if p_i is inner vertex of P_1 **then**
                    Add 2 contacts at p_i, normals of adjacent edges of P_1
                end if
            else
                Add a contact at p_i with normal of P_2 edge
            end if
        end if
    end for
    for all p_i in vertices of P_2 **do**
        if $p_i \in$ boundary (P_1) **then**
            Add a contact at p_i with normal of P_1 edge
        end if
    end for
end procedure

of other two (or 3 vectors are parallel) – 2 marginal vectors that form the convex cone that includes the 3rd one should be chosen and passed to Algorithm 2.7.

Using the algorithm for cases of more than 3 contacts is practically the same, it provides solution for Example: Figure 2.20 shows 2 bodies in contact, 4 distinct contact forces act on each body. Adding one finger to each object will immobilized

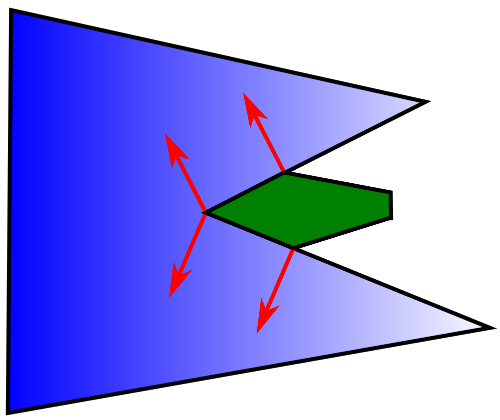


Figure 2.20.: 4 non-immobilizing contacts.

it but the whole configuration will no be immobilized having only 2 external contacts. Assessment of such cases is discussed in section 2.5.

Algorithm 2.6 Finding 1 finger given 3 effective contacts.

```
procedure FindLastFinger(P, C)    // polygon P, existing contacts C
    AntiCone = −1 · Cone ({w_{C_i}}, i = 1, .., |C|)
    F = finger with GQM = 0
  /* Form a cone of given contacts */
    for all e_i ∈ P.edges do
        EGW = EdgeGeneralizedWrench(e_i)
        CS = CommonSegment(GW, AntiCone)
        if CS ≠ ∅ then
            w_optimal = argmax_w GQM(Cone (C) , GW)
            GQM_optimal = max_w GQM(Cone (C) , GW)
            allowed region = backpropagation of CS to the edge.
            p = location from wrench w_optimal
            if GQM_optimal > F.GQM then
                F = {p, n̂_{e_i}, P, e_i, GQM_optimal}
  /* If this finger position is better – save it. */
            end if
        end if
    end for
end procedure
```

2.4.2. Finding 2 fingers for 2 effective contacts

** *$O(nlog(n))$ **or** $O(n^2)$?* Given 2 contacts, or more contacts that form a flat wrench cone, additional contacts can be found be examining wrench space vectors. For a case with some degenerate contacts, 2 marginal vectors are selected. Wrench space vectors and the origin form a plane (from now on“the plane”). Once such plane is determined, possible complementary contacts can be tested. First, simple one-edge solutions are proposed and evaluated as describes Algorithm 2.7. For each edge a corresponding generalized wrench is built and tested to be on both sides of the plane. If such edge found, two marginal wrenches are tested to get grasp quality measure, which is saved along with contact locations. Optimal locations along with optimal GQM are calculated numerically

If no such edge found, or if higher quality measures are desired, variants of 2 edge combinations are selected to check whether they complete the convex hull of wrench vectors. Similar numerical calculation of GQM is performed for 2 contacts along 2 different edges. Pseudocode is presented in Algorithm 2.8.

2.5. Multi object grasp evaluation and securing

Once every given object is immobilized by other objects and fingers, the constellation still might be not fully immobilized. Figure 2.21 shows three objects: black object is immobilized by four inter-object contacts from red and blue objects while red and and blue are immobilized by two additional fingers each one.

Algorithm 2.7 Finding 2 fingers on same edge given 2 effective contacts .

procedure FIND2FINGERS(P, C) *// polygon P, existing contacts C*
  $w_{c1}, w_{c2} = \text{WrenchesOf}\,(c_1, c_2)$
  $\hat{n}_{w_{c1,c2}} = w_{c1} \times w_{c2}$
  $F_{1,2} =$ fingers combination with $GQM = 0$
  for $e_i \in P$.edges **do**
    if span $\{\hat{n}_{c_1}, \hat{n}_{c_2}, \hat{n}_{e_i}\} = \mathbb{R}^2$ **then** *// Normal vectors span force space*
      $GW = GeneralizedWrench(e_i)$
      $w_a = \min\limits_{w}(GW)$
      $w_b = \max\limits_{w}(GW)$
      if $\left(w_a \cdot \hat{n}_{w_{c1,c2}}\right) \cdot \left(w_b \cdot \hat{n}_{w_{c1,c2}}\right) < 0$ **then**
        $w^*_{1,2} = \operatorname*{argmax}\limits_{w_{1,2}} GQM(Cone\,(C)\,, GW)$
        $GQM^*_{1,2} = \max\limits_{w_{1,2}} GQM(Cone\,(C)\,, GW)$
        if $GQM^*_{1,2} > F_{1,2}.GQM$ **then**
          $F_{1,2} \twoheadleftarrow \left\{\texttt{locationsFromWrenches}\left(w^*_{1,2}\right), \hat{n}_{e_i}, P, e_i, GQM\right\}$
        end if
      end if
    end if
  end for
end procedure

The contacts that act on blue and black objects (shown as red arrows) intersect at same point and hence do not immobilize these two objects together – namely, these two objects can rotate together about the intersection point of the contacts external to these objects.

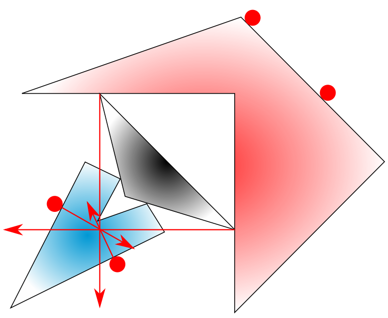


Figure 2.21.: 2 of 3 objects are not immobilized.

Total immobilization can be achieved by ensuring that every possible subset of objects is immobilized. This requires extraction of all possible connected subsets from the given set object and evaluation whether they are or are not immobilized. If the subset is not immobilized - existing fingers are moved if possible, and if not (or in case of less than 4 external contacts) – additional fingers are added.

If the amount of external contacts is higher than 4, assessment of intersection of contacts external to the subset can show whether the subset is immobilized or

Algorithm 2.8 Finding 2 fingers on different edges given 2 effective contacts.

procedure FIND2FINGERSDIFFEDGE(P, C)
  $F_{1,2}$ = fingers combination with $GQM = 0$
  for $e_a, e_b \in P$.edgePairs **do**
    if $\texttt{span}\left\{\hat{n}_{C_1}, \hat{n}_{C_2}, \hat{n}_{e_a}, \hat{n}_{e_b}\right\} = \mathbb{R}^2$ **then**
      $GW_a = GeneralizedWrench(e_a)$
      $GW_b = GeneralizedWrench(e_b)$
      $w^*_{1,2} = \underset{w_{1,2}}{\operatorname{argmax}}\, GQM(Cone\,(C)\,, GW_a, GW_b)$
      $GQM^*_{1,2} = \max_{w_{1,2}} GQM(Cone\,(C)\,, GW_a, GW_b)$
      if $GQM^*_{1,2} > F_{1,2}.GQM$ **then**
        $p^*_{1,2} = \texttt{locationsFromWrenches}\left(w^*_{1,2}\right)$
        $F_{1,2} \twoheadleftarrow \left\{p^*_{1,2}, \hat{n}_{e_a}, \hat{n}_{e_b}, P, e_a, e_b, GQM^*_{1,2}\right\}$
      end if
    end if
  end for
end procedure

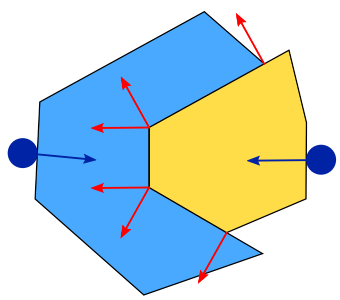


Figure 2.22.: Non immobilized set of 2 objects.

not. If the contacts intersect at one point, contact positions of added fingers can be altered in a allowed regions that were found for that contact.

Given set of polygons, $P = \{p_1, \ldots p_K\}$, set of inter-object contacts $C = \{c_1, \ldots c_L\}$ and set of selected finger contacts $F = \{f_i, \ldots f_M\}$. Recapping that each contact is defined by the object it is acting on, among other parameters. Let $\acute{P} = \{p_1, \ldots p_n\}$ $N < K$ denote a subset of polygons selected for evaluation. Set of the contacts used for the evaluation can be defined as follows: inter-object contacts that are acting on one of the objects but not applied by another object and set of finger contacts that are acting on one of the objects in the subset. Mathematical description of the subsets of inter-object and finger contacts can be written as follows:

$$\acute{C} = \left\{c_i \in C \mid c_i.id\,(P_A) \in \acute{P} \text{ and } c_i.id\,(P_B) \notin \acute{P}\right\} \tag{2.11}$$

$$\acute{F} \quad = \quad \left\{f_i \in F \mid f_i.id\,(P) \in \acute{P}\right\} \tag{2.12}$$

Using the notation, we assume that the subset is immobilized internally (all subsets of the subset are immobilized if the algorithm evaluates subsets by an increasing number of polygons) and hence we can treat it like a solid object. As was introduced above, to immobilize an object by first order frictionless grasp, four contacts have to positively span the origin of a wrench space and three of them cannot intersect in one point (Nguyen [34]). This can be extended to more than four contacts:

Theorem 16. *Object in $\mathbb{R}^2$ cannot be immobilized by N contacts if more than $N-2$ contacts intersect at one point.*

Proof. Assuming $N-1$ contacts intersect at point p, it leaves 1 contact with direction that is not passing p. Contacts which intersect at the p cannot induce torques about the point, and the remaining contact cannot span both positive and negative torques about p. Therefore, the object can rotate freely around p in direction opposite to the torque induced by last contact. $\square$

Therefore, assessing the subset $\acute{P}$ and set of contacts external to it $\check{C} = \acute{C} \bigcup \acute{F}$, we can determine whether this contacts set immobilize the objects subset. Algorithm 2.9 shows a pseudo code for grasp assessment and securing in case that the grasp is not immobilized. Since in previous steps finger positions are found along with allowed regions along the edge, some of the contacts external to subset can be moved and ensure immobilization of the grasp.

Algorithm 2.9 Grasp assessment and securing

procedure EnsureGrasp($\check{C}$) *// Contacts $\check{C}$*
  if $\left|\check{C}\right| < 4$ **then**
    Add fingers according to **??**
  else *// There are 4 or more contacts*
  / Find a subset of $\check{C}$ where all contacts intersect at one point. Select that point to be a reference point. */*
    $p \in c_i\ \forall c_i \in \acute{\check{C}}\ :\ \max_{\acute{\check{C}}} \left|\acute{\check{C}}\ \{c_i\}\right|$
    if FindGQM $\left(p, \check{C}\right) = 0$ **then** *// The grasp is not immobilizing*
      if $f \in \acute{F} \in \check{C}$ **then** *// We have a finger that can be moved.*
        Find new allowed region for the finger to complete the grasp
        if succeded **then return**
        end if
      end if
      Add fingers according to **??**
    end if
  end if
end procedure

With all connected subsets (combinations of objects in touch) found by Algorithm 2.1 and the subset grasp assessment algorithm, all configurations can be iterated over and ensured that selected fingers complete the grasp of the rearranged set.

2.6. Objects rearrangement

Though object manipulation and movement of assemblies is not in the scope of the work, the solution takes into account the need for the desired arrangement to be feasible, and hence the output of the stacking algorithm is arranged in a way that will allow consecutive stacking of objects in the presented order. This eliminates the need of multi-object simultaneous manipulation. That being said, solutions for moving object assemblies in plane were proposed in past by Akella and Mason [1], Bernheisel and Lynch [6], Lynch [26, 27]. The most suitable solution for this case seems to be the one proposed by Anders et al. [2], and in presence of suitable gripper the final step will overcome uncertainties should they rise.

For the purposes of simulations and experiments, objects are rearranged manually into desired configuration.

3. Results

The chapter presents the results of the algorithms performance.* ***Duh!***

3.1. Simulations

Geometrical simulations were performed in MATLAB environment. The simulations included assessment of polygonal objects, rearrangement algorithm evaluations and finger placement algorithms execution. The obtained results are tested for consistency, and mathematically assessed for the completeness of the solution. Several methods are compared and comparison results are presented.

3.1.1. Software

MATLAB computing environment was used for the purpose of geometrical simulations. Specialized graphical user interface (GUI) programs were created for evaluation and presentation of the algorithms[1].

3.1.2. Setup

V-REP environment was used for physical simulation of the performance of the algorithms. The software incorporated several common physics engines which allow testing of different scenarios of computer simulations.

Delta-robot model of && was used to perform as the main manipulator, custom end-effector was added to the model to perform grasping of desired object sets.

The end-effector consists of 10 individually actuated fingers, which can move in the"palm" plane freely and have limited movement in Z direction. Depending on the found solution, the desired number of fingers can be extended to execute the grasp.

Control of the robot is done in closed loop by internal means of the V-REP software, while the control of the algorithm steps is done in the specialized GUI. The simulation is divided into several steps:

1. Obtaining environment information:
    a) Number of objects
    b) Positions
    c) Shapes

[1]The programs are available at https://github.com/yossioo/MSc-Research.

2. Obtaining the desired constellation variants
   a) variants are shown in preview pane along with quality measures
   b) desired variant can be selected for the execution
3. Rearranging the objects
   a) objects are moved by pushing algorithms to obtain the desired configuration
4. Immobilizing the objects
   a) Required amount of fingers are selected and extended to perform the grasp
   b) manipulator moves above the objects
   c) Manipulator and the end-effector are lowered down to constrain the constellation
5. Immobilization tests
   a) moving in plane
   b) moving to vertical plane and rotating

Executions of the steps can be controlled from the UI.

3.1.3. Results

A simple case of 3 equilateral triangles presented in the *SIMULATIONS. The designed configuration and designed grasp are presented above. 3D printed end-effector is presented on the FIGURE. Due to*

A case of 6 equilateral triangles
A case of 3 squares
A case of 4 squares
A case of 3 hexagons
Cases of 3-4-5-... irregular triangles.
A case of 3-4-5 octagons?

3.2. Experiments

Bring a 3D printed set of objects and designed gripper to the defense. Include the results in the Appendix, and in the presentation

The algorithms were tested in experimental setups. Robotic manipulator with 6DOF was equipped with an end-effector designed according to the algorithm's output. The robot is controlled by a laptop with Robot Operating System (ROS) hosted in Ubuntu. Several polygonal objects were selected to be stacked, and after the verification by the algorithm a custom-made end-effector with stationary fingers was manufactured by 3D printing. The end-effector was tested and confirmed to immobilize the desired set of objects both in simulation and in experiments.

Due to the real world imperfections, the fingers of the designed end-effectors are made with chamfers, this allowed grasping of the arranged set of items despite small perturbations of item locations and robotic arm performance.

3.2.1. Experimental Setup

For planar problems in this researchFor several examined cases of polygon sets,

3.2.2. Experimental Results

A simple case of 3 equilateral triangles presented in the SIMULATIONS. The designed configuration and designed grasp are presented above. 3D printed end-effector is presented on the FIGURE. Fingers - 1 joint finger - touches 2 triangles.

A case of 6 equilateral triangles

A case of 3 hexagons

Cases of 3-4-5-... irregular triangles.

4. Discussion

Written before Conclusions

Limitations ****This should be in discussion***
What can I write here?

Limitations of the finger placement? Finger sizes, proximity to the vertex – dealing with uncertainty. While rearranging:

Limitation of the multi-object evaluation? Object placement?

Quality measure contradictions: what's best for one polygon not necessary is best for polygon constellation.

5. Conclusions

Written before the Introduction

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Appendix

A. Algorithms

This section presents more elaborate representation of the algorithms presented in the paper.

B. Software

Source code is available at https://github.com/yossioo/MSc-Research.

Additional videos are available at YouTube channel: &&&

The software used for the research is:

MATLAB r2018a

V-REP 3.5.0 Educational Version

ROS Kinetic on Ubuntu 16.04

C. Examples of calculated grasps

3 fingers - 1 found

D. Examples of rearranged object sets

E. Examples of finger combinations found for sets

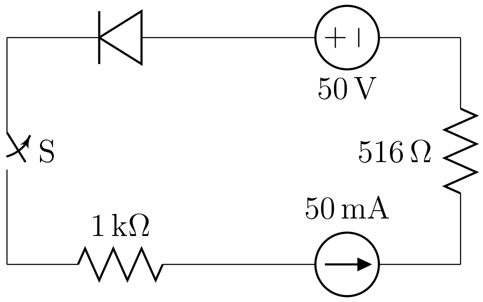
F. Simulations

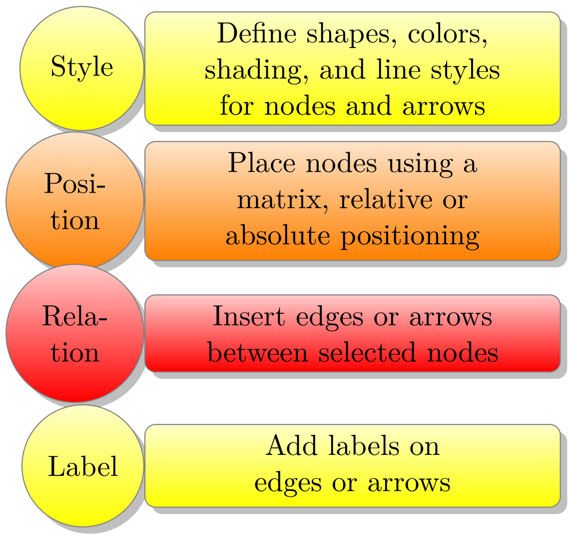
V-REP

G. Experiments

Real robot

H. Junk





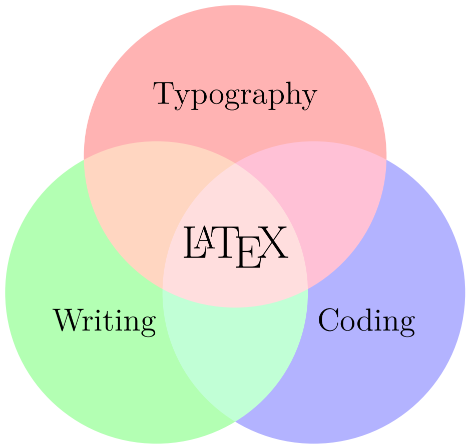
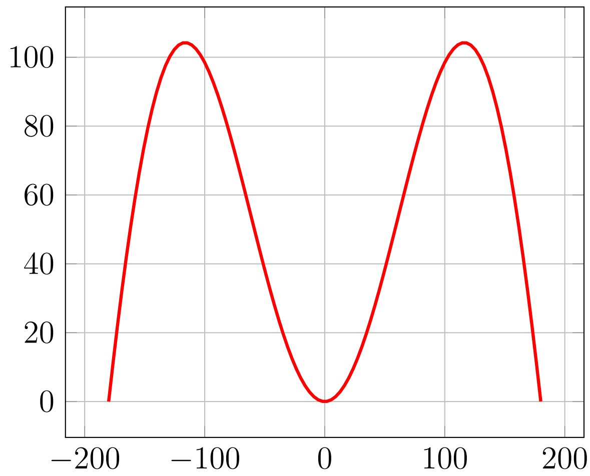


Figure H.1.: 222



X example

$$
\begin{tikzcd}
\text{Some text here} \arrow[dr] & \cos(\theta) & e \\
c \arrow[ur] \arrow[rr, "Over", "Under"'] & & f
\end{tikzcd}
$$