Simultaneous Grasping of Multiple Objects

Yosef Ovcharik and Amir Shapiro

***Abstract*—Here goes the abstract**
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I. Introduction

What's the motivation, what exists and what do I do. What is the contribution of the work done. Paper structure

Idea

AMAZON RC

Home assisted robotics

Who done what

What we show in the paper

The remainder is structured as follows

A. Problem Statement

Given.

Problem.

Restrictions / Assumptions, etc.

B. Related work

[1]- Stable transport assemblies

[2], [3], [4]

While methods for 3 finger fixturing are presented [5], [6], [7], [8]

Existence of grasps [9], [10]

Quality measures [11]

Configuration space [12]

Optimal grasps [13]

Machine learning optimal grasps [14]

Pushing of objects - object-edge interaction [15], [16]

[3] present methods to rearrange objects into desired configuration.

[17] presents graphical methods for multi-contact friction cone generalizations.

Convex hulls [18], [19]

Further planning shown by [20]

[21] presents descriptions of motion of points of contact of two contacting objects.

[22] Shows method for constructing force closure grasps.

II. Methods

Formal definitions, verbal description

A. Definitions

bla bla bla grasp map, force closure form closure

Ovcharik and Shapiro are with the Dept. of ME, Ben Gurion University, Israel, ovcharik@post.bgu.ac.il, ashapiro@bgu.ac.il

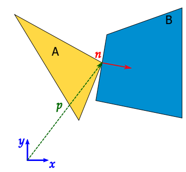


Figure 1: Contact description example.

B. Inter-object contact description

For the purpose of polygon arrangement evaluation, every contact is assumed to be performed by polygon A acting on the polygon B. The following description of a contact is adapted:

$$C = \{p, \hat{n}, id(P_A), id(P_B)\} \tag{1}$$

where $id(P)$ stands for polygon number/identification entry, p is a contact point location and $\hat{n}$ is the contact normal direction inward the polygon B. An example of inter-object contact description shown on figure 1. The contact is saved as: $C = \{p, \hat{n}, A, B\}$,where A, B are the names of the polygons, p is the contact location vector in selected coordinate frame and $\hat{n}$ is the inward normal direction of the second polygon at the contact location.

C. Solution existence for combination of internal and external contacts

Given a set of polygonal objects, where some of the objects are in contact one with another. Each object has at least 4 contacts: some of them are fingers and some are inter-object contacts. The equilibrium equation for each object is written in form of total wrench:

$$G\bar{f} = \bar{0} \tag{2}$$

where: $\bar{f} = \begin{bmatrix} f_{f1} & f_{f2} & f_{p12} & f_{p13} & \dots & f_{pMN} \end{bmatrix}^T$ is a set of forces between fingers and bodies, and between bodies and other bodies in contact. Grasp map matrix G can be obtained by as shows [23]. The matrix will be block diagonal with some cross-block columns which correspond to body-to-body contacts.

Lemma 1. *Given an edge of a polygonal object and a frictionless point contact, the generalized (superposition) wrench for all possible locations of contact along the edge can be expressed as 2 dimensional cone (2 vectors) in vertical plane of the configuration space (the plane's normal direction will be in $f_x f_y$ plane).*

Proof: All possible contacts on the edge will have the same direction and hence same f_x, f_y components. But for all these possible contacts the torques will be different. The positions of the contact points one the edge have limits, and at those limits will define the marginal wrenches that will form a two dimensional cone in a wrench space. ∎

Proposition 2. *4 frictionless contacts acting on a 2D object in distinct points along object's boundary may constitute a force closure equilibrium grasp.*

Proof: Caratheodory's theorem [24] shows that a N-dimensional space can be positively spanned by N+1 vectors. For two-dimensional task space, there are 3 degrees of freedom (x, y, θ) and hence both the configuration space and the wrench space are three dimensional ($\mathbb{R}^3$). It has been elaborately shown by [25] and proven by [26] that at least four wrenches are required to immobilize 2D objects. Equilibrium grasp is achieved under condition that the origin of the wrench space lies inside of the convex hull of these wrenches. ∎

Definition 3. A contact between two objects is called stable if for small perturbations in contact's location (movement of one object in respect to another) and applying same set of forces that kept two objects in equilibrium, the contact will either slip to the previous location or stay in the new location. Given 2 objects A, B located at $[x_A, y_A, x_B, y_B]$, set of forces F acting on the objects and a contact $C = \{p, \hat{n}, A, B\}$ and small perturbation ε:

$$\forall \varepsilon \quad C = \{p + \varepsilon, \hat{n}, A, B\}, \tag{3}$$

and applying same set of forces F, new location of the objects will stay in the vicinity of the previous locations: $\|[x_{new}, y_{new}] - [x_{old}, y_{old}]\| \leq \varepsilon$.

Proposition 4. *A set of N polygonal objects in $\mathbb{R}^2$ in stable contact one with another can be immobilized by at most $3N$ fingers assuming first order non-frictional contacts.*

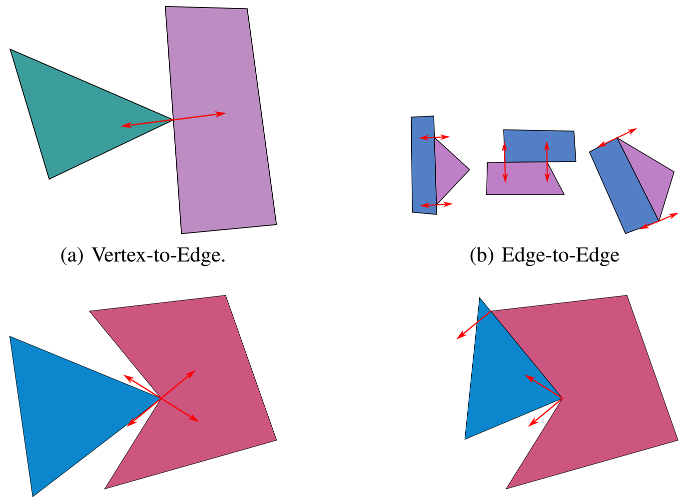
Proof: Given connected set of objects, for each object at least one contact exist. For an object in $\mathbb{R}^2$ to be first order immobilized minimum 4 constraints required. Knowing that at least one constraint already exists because the set is connected, for each object 3 external constrains are needed, yielding $3N$ fingers for the whole set. ∎

Proposition 5. *A set of N polygonal objects in $\mathbb{R}^2$ in contact one with another can be immobilized by at least 4 fingers assuming first order contacts and exceptional resulting shape.*

Proof: Assuming optimal configuration of the objects where each of them is immobilized relative to others by form closure conditions and possibly external constraints (EC) the total amount of external constraints for the set immobilization is $\max(4, |EC|)$. Example: jigsaw puzzle which once assembled can be immobilized by 4 fingers. ∎

Proposition 6. *Given a polygonal object and 3 parallel frictionless contacts on the object boundary (same edge or different edges). It is impossible to construct and first order immobilizing grasp with 1 additional contact.*

Proof: 4 contacts of a frictionless grasp have to not to intersect in one point, as showed. 3 parallel contacts are "intersecting" at infinity. Moreover, 4-th contact alone cannot span all moments around that intersection point. ∎



(a) Vertex-to-Edge. (b) Edge-to-Edge (c) Vertex-to-Inner-Vertex (d) Vertex-to-Inner-Vertex with contiguous edge

Figure 2: Contact type examples

D. Classification of inter-object contacts

The examination of given polygon configuration can be subdivided to interaction of each couple of polygons. The polygons can be disconnected - no points belong to 2 polygons simultaneously, or connected - one ore more points belong to 2 or more polygons. The connection between the polygons can be represented by a contact description. A stable contact between 2 polygons can exist in several variants:

1) Vertex to edge: is the discrete contact where a vertex of the first polygon (p_1) is coincident with an edge of the second polygon (p_2). The contact normal direction is defined by the edge the of the second polygon. The set of contacts for this variant consists of a single contact: $C = \{c_1\}$. Contact restrictions can be described as following: velocity of the contact point on the boundary of p_1 relative to the velocity of the contact point on the boundary of p_2 (denoted v_{12}) cannot have component in direction $\hat{n}_1^2$ – polygons cannot penetrate. This can be expressed as follows: $v_{12} \cdot \hat{n}_1^2 \leq 0$. Contact is maintained when the component of the relative velocity in the direction if inward normal of first polygon is 0. Since $\hat{n}_i^2 = -\hat{n}_i^1$, $-v_{12} \cdot \hat{n}_1^2 \leq 0$. Therefore, the contact is maintained when relative velocity of contact points of each polygon are perpendicular to the edge normal: $v_{12} \cdot \hat{n}_1^2 = 0$.

2) Edge to edge : the contact is a continuous contact between two bodies along a continuous segment. Since the edges are finite, there exist at least 2 vertices belonging to either one or two polygons that belong to the contact segment. In a way analogous to distributed load concentrated in a point, the continuous distributed contact can be concentrated in 2 different points. Two different vertices are selected to be such points. Since the vertices lie along the same edge, the contact normal directions are the same in this case. The set of contacts for this variant consists of two contacts: $C = \{c_1, c_2\}$, $c_i = \{r_i, \hat{n}\}$, where r_i stands for the locations of the contacts and $\hat{n}$ stands for the normal direction of the edge.

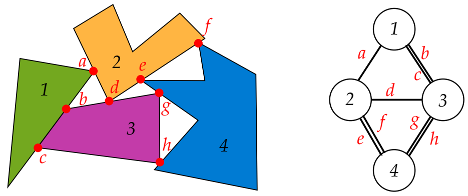


Figure 3: Set of objects and a corresponding graph.

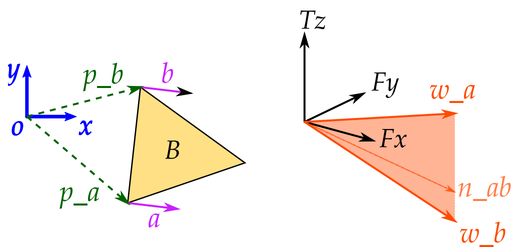


Figure 4: Edge with inner normal (left), Edge Generalized Wrench (right).

3) Vertex-inner-Vertex: is another type of discrete contact with multiple point-normal parameters. Polygon 1 vertex is in contact with 2 edges of p_2. In this case, since the movement of polygon 1 relative to polygon 2 is restricted in 2 directions, 2 normal directions exist at same contact point. The set of contacts for this variant consists of two contacts: $C = \{c_1, c_2\}$, $c_i = \{r, \hat{n_i}\}$, where$r$ stands for contact location and $\hat{n}$ stands for the normal directions of the edges.

Other variants of contact descriptions are variations of the mentioned above. Examples: edge and inner vertex, vertex and vertex, edge and edge. Outer vertex to outer vertex contact assumed to be unstable and is discarded in this work. One polygon can contact several others in distinct points, apart from a case where several vertices are coincident.

E. Graphs depicting the interaction between objects

The interaction between several objects can be qualitatively described by a graph where nodes describe objects, and edges describe contact existence between the objects. For the purposes of multi-object grasp evaluation graphs allow search of connected sub-sets and their correspondent contacts.

F. Edge Generalized Wrench

EGW is a graphical way of describing possible contact wrenches for a given linear edge of some 2D object. The evaluation of possible contact positions on an edged can be simplified by introducing an edge generalized wrench:

Definition 7. Given a polygon's edge e and a reference point o, the set of all possible wrenches w_e can can be defined by a two-dimensional convex cone in a vertical plane of a wrench space.

Denote the edge's normal by $\hat{n}$ and edge's ends by $\vec{e}_1, \vec{e}_2$ relative to o, the edge is parameterized by parameter $s \in [0, 1]$ the wrench of a contact along the edge can vary between:

$$(1-s)\begin{bmatrix}\hat{n} \\ \vec{e}_1 \times \hat{n}\end{bmatrix} + s\begin{bmatrix}\hat{n} \\ \vec{e}_2 \times \hat{n}\end{bmatrix}, \qquad s \in [0,1] \tag{4}$$

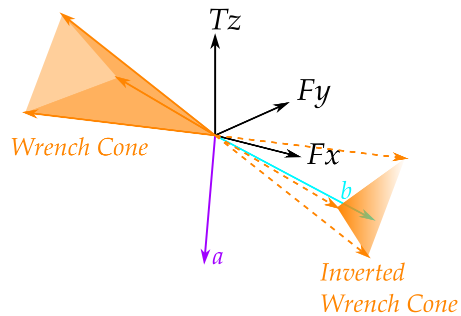


Figure 5: Inverted Cone, with vector a inside and vector b outside.

which is exactly a linear combination of two marginal wrenches. In a geometrical representation, it is a planar angle contained in a vertical plane (plane's normal direction has no torsional component) in the wrench space. Illustration shows an example of a generalized edge wrench. An edge of polygonal object B has inward normal direction $\hat{n}_{ab}$. Two marginal contacts a, b are shown in violet, with locations defined by vectors p_a, p_b respectively. Wrenches induced by these contacts are calculated by:

$$w_a = \begin{bmatrix}\hat{n}_a \\ \vec{p}_a \times \hat{n}_a\end{bmatrix}, \; w_b = \begin{bmatrix}\hat{n}_b \\ \vec{p}_b \times \hat{n}_b\end{bmatrix}. \tag{5}$$

As can be seen from the FIGURE , contact a induces positive torque about the selected reference point, while contact b induces negative torque. Hence, the 3-rd components of a wrench have opposite signs and are located on different sides of the xy plane. The expression for the edge generalized wrench can be rewritten to a form of:

$$w_e(s) = (1-s)\begin{bmatrix}\hat{n} \\ \vec{e}_1 \times \hat{n}\end{bmatrix} + s\begin{bmatrix}\hat{n} \\ \vec{e}_2 \times \hat{n}\end{bmatrix} = \dots, \qquad s \in [0,1]$$

$$= \begin{bmatrix} n_x \\ n_y \\ (x_1 \cdot n_y - y_1 \cdot n_x) + s\left(n_y(x_2 - x_1) + n_x(y_1 - y_2)\right)\end{bmatrix} \tag{6}$$

We can note that only the torque component of the wrench vector changes along the edge, and hence the edge generalized wrench is strictly "vertical" in the wrench space, and that it is linear along the parameter$s$.

G. Inverted cone test

As stated above, a convex hull of the contact wrenches should contain the origin of a wrench space. Given several contacts, and corresponding wrenches in the wrench space that do not contain the origin, a simple test can show whether additional vector will make total convex hull to contain the origin or not.

Proposition 8. *Given a set of 3 contacts, whose wrenches do not span the wrench space (the origin of the wrench space is not inside the convex hull of these wrenches) while not being in the same plane and additional contact, the contact can complete the force closure grasp of the object if the contact's* *wrench lies within the inverted cone of the wrenches of given contacts .*

Proof: Each couple of wrenches along with the origin point form a plane, Equilibrium condition for a grasp is that the total wrench for all contacts equals to zero:

$$\sum_{i=1}^{4} G_i f_i = 0 \tag{7}$$

Where G_i is $\begin{bmatrix} n_i \\ p_{c_i} \times n_i \end{bmatrix}$ for each contact. Rewriting the (7):

$$G_4 f_4 = -\sum_{i=1}^{3} G_i f_i \tag{8}$$

Given the fact that G columns are wrench space vectors, and f are scalar values, this allows the representation of a 4th wrench space vector as a linear combination of other wrench vectors, which is exactly the cone formed by these wrenches. If the 4th vector lies outside the cone then it cannot be represented as a linear combination of other wrenches and hence will not allow equilibrium grasp. ■

H. Maximizing the Inscribed Sphere

One of the proposed quality measures is a volume/radius of the sphere inscribed in the convex hull of grasp map basis vectors. Maximizing the sphere radius can be done by increasing the minimal of the distances between the origin end the faces of the convex hull. Since the hull is convex, some of the faces will be tangent to the sphere, and in no case a edge will be tangent. In cases where several contacts act on the object, it is always possible to construct a *convex* hull and thus eliminate redundant wrench vectors.

Proposition 9. *Given a set of k-1 contacts, and their corresponding wrench vectors, convex cone and it's inverse cone; for a generalized wrench that has a segment in the interior of the inverse convex cone an optimal k-th contact can be found by examining distances of the convex hull faces from the origin.*

Proof: As seen in 8, the wrench w_k has to lie inside of the Anti-Cone. If w_k lies on a cone boundary then it and 2 vectors that form that boundary (denoted by w_1, w_2), will form a surface that contains the origin of the wrench space. To maximize the distance of said surface from the origin we first parameterize the wrench vector end point location segment by using the (6). For each face of the convex hull, the distance of the face from the origin can be calculated by projecting a point on the plane on the plane's normal. We define a *central axis* of their convex cone as:

$$a = \frac{\sum_{i=1}^{k-1} w_i}{\left|\sum_{i=1}^{k-1} w_i\right|}$$

Assuming that the *k-1* wrenches are sorted in counter-clockwise direction about the cone axis, meaning that

$$a \cdot (w_i \times w_{i+1}) > 0 \; \forall i \in [1 : k-1],$$

$$a \cdot (w_{k-1} \times w_1) > 0.$$

The equation for the normal direction of a plane formed by w_i, w_{i+1}, w_k is:

$$n_p = (w_i - w_k) \times (w_{i+1} - w_k).$$

If the w_k lies inside the inverted convex cone of first*k-1* contacts, then the normal direction will point from the plane towards the origin. Consequently, the distance of the plane from the origin can be expressed as:

$$d_i = -w_k \cdot (w_i - w_k) \times (w_{i+1} - w_k),$$

note the "–" sign to obtain positive distance[1]. Substituting the expression for wrench vectors (shown in Appendix) leaves the distance of plane i (contacts *i,i+1* and parameter s along an edge) to be a function of that parameter only:

$$d_i = f(s),$$

Ultimately, distance from every face can be expressed as a function of edge parameter$s$ and thus a compound function of minimal distance can be built.

$$d = \min\{d_i\} \;\; i \in [1,].$$

Since projection is a linear transformation, and EGW endpoints segment being parameterized uniformly, the distance function will be linearly dependent on on s parameter. Meaning that the distance function will be either constant or monotonically increasing/decreasing along the change in$s$. Solution can be found by examining*k-1* 1st order polynomials and finding the maximized minimal value. Since the functions are 1st order polynomials, there either will be 1 such solution or infinite number of solutions (in case where there is a plane parallel to the EGW endpoints segment and hence the distance is constant). ■

I. Algorithm for objects rearrangement

1) Description:
2) Completeness:
3) Complexity:

J. Algorithm for complementing fingers placement

1) Description:
2) Completeness:
3) Complexity:

K. Multi object grasp evaluation and grasp securing

Once every given object is immobilized by other objects and fingers, the constellation still might be not fully immobilized. Example: figure 6 shows 3 objects: black object is immobilized by 4 contacts from red and blue objects while red and and blue are immobilized by two additional fingers each one. The contacts that act on blue and black objects (shown as red arrows) intersect at same point and hence do not immobilize these two objects together – namely, these two objects can rotate together about the contact intersection point. Total immobilization can

[1]MATLAB has polyshapes stored with vertices in clockwise direction, hence no minus sign in the code.

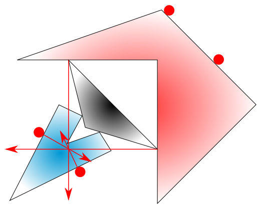


Figure 6: 2 of 3 objects are not immobilized.

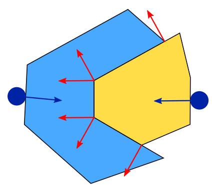


Figure 7: 2 objects set is not immobilized.

be achieved by ensuring that every possible subset of objects is immobilized. This requires extraction of all possible connected subsets from the given set object and evaluation whether they are or are not immobilized. Each constellation ought to have at least 4 external contacts. If there are less - additional contacts required to immobilize it. If the amount of external contacts is higher than 4, assessment of intersection of contacts external to the subset can show whether the subset is immobilized or not. If the contacts intersect at one point, contact positions of added fingers can be altered in a allowed regions that were found for that contact.

III. Results

A. Simulations

Show several examples of the algorithm performance:

- Two objects held by minimal amount of fingers
- Several objects held by minimum fingers.
- 2 objects distant, brought back and immobilized
- Several objects distant, brought to contact and then immobilized
- Several objects clustered in sets and then moved together
- Computation time / memory for these simulations

B. Experiments

Experiments were conducted to physically test the performance of the algorithms.

IV. Conclusions

In this work we -introduced, -to-exploit? -model-presented, -algorithms-proposed, -validated

In future we plan to -build, -assess, –more-complex, -online

V. AppendiXes

Appendi

Appe

Appendix

Accompanying code with MATLAB simulations is presented at https://github.com/yossioo/MSc-Research

Video complementing the submissions available at https://YouTube

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