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Optimization of a plate-fin heat exchanger design through an improved multi-objective teaching-learning based optimization (MO-ITLBO) algorithm

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ABSTRACT

Teaching-learning-based optimization (TLBO) is a recently developed heuristic algorithm based on the natural phenomenon of teaching-learning process. In the present work, multi-objective improved teaching-learning-based optimization (MO-ITLBO) algorithm is introduced and applied for the multi-objective optimization of plate-fin heat exchangers. The basic TLBO algorithm is improved to enhance its exploration and exploitation capacities by introducing the concept of number of teachers, adaptive teaching factor, tutorial training and self-motivated learning. The MO-ITLBO algorithm uses a grid-based approach to adaptively assess the non-dominated solutions maintained in an external archive. Minimizing total annual cost and the total weight of heat exchanger as well as minimization of total pressure drop and maximization of heat exchanger effectiveness for specific heat duty requirement are considered as objective functions. Two application examples are presented to demonstrate the effectiveness and accuracy of the proposed algorithm.

Keywords: Multi-objective optimization; Plate-fin heat exchanger; Teaching-learning-based optimization (TLBO) algorithm; Improved teaching-learning-based optimization (ITLBO); Meta-heuristics; Imperialist competitive algorithm (ICA)

1. Introduction

Finding the global optimum value(s) of the problem involving more than one objective with conflicting nature arises in many scientific applications. The problem of optimization involving more than one objective function with conflicting nature is known as multi-objective optimization (MOO) problem. Multi-objective optimization has been defined as finding a vector of decision variables while optimizing several objectives simultaneously with a given set of constraints. Unlike single objective optimization, MOO solutions are in such a way that the performance of each objective cannot be improved without sacrificing the performance of another one. Hence, the solution of MOO problem is always a trade-off between the objectives involve in the problem. Moreover, the obtained result in multi-objective optimization is a set of solutions since objective functions are conflicting in nature (Zhou et al., 2011).

The computational effort required to solve the MOO problems are quite considerable. Moreover, many of these problems cannot be solved analytically and consequently they have to be addressed by numerical algorithms. Recently several authors have proposed different evolutionary and swarm intelligence based MOO algorithms to solve these types of problems. Dynamical Multi-Objective Evolutionary Algorithm (Liu et al., 2009), Multiple Trajectory Search (Tseng and Chen, 2009), Multi-Objective Evolutionary Programming (Qu and Suganthan, 2009), NSGA-II (Deb et al., 2002), Local Search Based Evolutionary Multi-Objective Optimization Algorithm (Sindhya et al., 2009), Multi objective Biogeography-Based Optimization (Silva et al., 2012), etc. are some of the

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Nomenclature

A	heat exchanger surface area (m^2)
A_{cf}	annual co-efficient factor
A_{ff}	free flow area (m^2)
C	heat capacity rate (W/K)
C_A	cost per unit area ($/$m^2$)
C_p	specific heat (J/kg K)
C_{in}	initial cost ($)
C_{op}	operating cost ($)
C_r	heat capacity ratio = C_{min}/C_{max}
d_h	hydraulic diameter (m)
f	Fanning friction factor
G	mass flux velocity (kg/m^2 s)
h	convective heat transfer co-efficient (W/m^2 K)
H	height of fin (m)
j	Colburn factor
l_f	lance length of the fin (m)
L	heat exchanger length (m)
m	mass flow rate (kg/s)
n	fin frequency (m^{-1})
n_1	exponent of non-linear increase with area increase
N_h	number of hot side layer
NTU	number of transfer units
Pr	Prandtl number
ΔP	pressure drop (kPa)
Re	Reynolds number
roi	rate of interest
fs	fin spacing (m)
t	fin thickness (m)
T	temperature (°C)
TAC	total annual cost
T_F	teaching factor
U	overall heat transfer co-efficient (W/m^2 K)
z_1	depreciation time

Greek letters

ε	effectiveness
μ	viscosity (N s/m^2)
ρ	density (kg/m^3)
η	compressor efficiency
ζ	electricity price ($/MW h)
τ	hour of operation

Subscripts

h	hot
c	cold
max	maximum
min	minimum

evolutionary MOO algorithms that aimed to obtain approximate Pareto front for multi-objective problems. Similarly, PSO-based multi-objective optimization with dynamic population size and adaptive local archives (Coello et al., 2004), Dynamic Multiple Swarms in Multi-Objective Particle Swarm Optimization (Yen and Leong, 2009), Autonomous bee colony optimization for multi-objective function (Zeng et al., 2010), Particle swarm inspired evolutionary algorithm (PS-EA) for multi-objective optimization problem (Srinivasan and Seow, 2003), Interactive Particle Swarm Optimization (Agrawal et al., 2008), Multi-objective artificial bee colony algorithm (Akbari et al., 2012), etc. are some of the swarm intelligence algorithms which efficiently solved the multi-objective problems.

The evolutionary and swarm intelligence based algorithms are probabilistic algorithms and required common controlling parameters like population size and number of generations. Besides the common control parameters, different algorithms require their own algorithm-specific control parameters. For example, genetic algorithm (GA) is evolutionary optimization techniques and works on the Darvin's theory of evolution and the survival of the fittest. GA requires mutation rate and crossover rate (which is algorithm-specific control parameters) for its working. Particle swarm optimization (PSO) is swarm intelligence based optimization techniques. PSO mimics the social behavior of a flock of migrating birds to find the destination. PSO requires inertia weight, social and cognitive parameters (which is algorithm-specific control parameters) for its working. Similarly, artificial bee colony (ABC) algorithm requires number of employed bees, number of onlooker bees and limit for its working. The proper tuning of the algorithm-specific parameters is very important factor for the efficient working of the evolutionary and swarm intelligence based algorithms. The improper tuning of algorithm-specific parameters either increases the computational effort or yields the local optimal solution. Considering this fact, recently Rao et al. (2011, 2012) and Rao and Patel (2012) introduced the teaching-learning based optimization (TLBO) algorithm which does not require any algorithm-specific parameters. TLBO requires only common control parameters like population size and number of generations for its working. Thus, TLBO algorithm avoid all the difficulties associated with the proper tuning of the algorithm specific parameters which in turn increases the exploration and exploitation capacity of the TLBO algorithm and this feature of the TLBO algorithm made it better than the other optimization algorithms.

In the present work, an attempt is made to investigate the optimum geometric parameters of cross flow plate-fin heat exchanger using the multi-objective improved teaching-learning based optimization (MO-ITLBO) algorithm. The main objectives of this work are (i) single objective optimization of the influential parameters of cross flow plate-fin heat exchanger (ii) multi-objective optimization of the influential parameters of cross flow plate-fin heat exchanger considering conflicting objectives simultaneously and (iii) to demonstrate the effectiveness of TLBO and MO-ITLBO algorithms for optimization of cross flow plate-fin heat exchanger. In this paper, ability of TLBO and MO-ITLBO algorithms is demonstrated using application examples.

The remainder of this paper is organized as follows. Section 2 describes the previous work on plate-fin heat exchanger optimization. Section 3 describes the thermal modeling and objective function formulation of plate-fin heat exchanger. Section 4 briefly describes the basic TLBO algorithm. Section 5 explains the proposed MO-ITLBO algorithm. Section 6 presents application examples. Finally, the conclusion of the present work is presented in section 7.

2. Previous work on plate-fin heat exchanger (PFHE) optimization

Plate fin heat exchangers are widely used in different phases of industry such as automobile, cryogenics, aerospace, chemical and petrochemical processes because of their high compactness and relatively good heat transfer efficiency. The design of a PFHE is a complex task based on trial and error process in which geometrical and operational parameters are selected to satisfy specified requirements such as heat duty, pressure drop and outlet temperature.

Previously different authors used different optimization techniques for the single as well as multi-objective optimization of PFHE. In addition to using traditional mathematical methods (Reneaume et al., 2000; Reneaume and Niclout, 2003), simulated annealing (Reneaume and Niclout, 2001) and artificial neural network (Jia and Sunden, 2003) many researchers have successfully employed evolutionary and swarm intelligence based computation in design optimization of PFHE. Mishra et al. (2004) and Mishra and Das (2009) used GA for optimal design of plate-fin heat exchangers. The authors had considered minimization of total annual cost and total thermo-economic cost. Najafi et al. (2011) carried out multi-objective optimization of PFHE considering total heat transfer rate and total annual cost of the heat exchanger simultaneously using multi-objective GA. Xie et al. (2008) used GA for optimal design of PFHE. The authors had considered minimization of total annual cost as an objective function.

Sanaye and Hajabdollahi (2009) carried out thermo-economic multi-objective optimization of PFHE using non-dominated sorting GA. The authors had considered maximization of effectiveness and minimization of total cost of the heat exchanger simultaneously. Peng et al. (2010) used PSO to optimize the structure dimensions of PFHE. The authors had considered total weight and total annual cost minimization of the heat exchanger as an objective function for given constrained conditions. Rao and Patel (2010) applied PSO for the thermodynamic optimization of PFHE. The authors had considered minimization of entropy generation units as an objective function. Peng and Ling (2008) used neural networks cooperated with genetic algorithm for optimal design of PFHE. The authors had considered minimization of weight and minimization of total annual cost as objectives. Yousefi et al. (2012a,b) used imperialist competitive algorithm (ICA) for the optimization of PFHE. The authors had considered minimization of total weight and total annual cost of the heat exchanger as objective functions. Yousefi et al. (2013) used improved harmony search algorithm for the single objective optimization of PFHE. Ahmadi et al. (2011) used multi-objective GA for cost and entropy generation minimization of a cross flow PFHE. Yousefi et al. (2012a,b) applied genetic algorithm hybrid with particle swarm optimization for design optimization of a plate-fin heat exchanger. Several other investigators (Najafi and Najafi, 2010; Ozkol and Komurgoz, 2005; Amin and Ali, 2013; Rao and Patel, 2013a,b) used different optimization strategies to optimize the heat exchanger design.

3. Thermal modeling of PFHE

In the present work a cross-flow plate fin heat exchanger with offset strip fin is considered for the optimization. Fig. 1 shows the schematic diagram of the considered PFHE geometry. For the cross flow heat exchanger with both fluids unmixed, effectiveness (ε) is given by Incropera and DeWitt (1998) as,

$$\varepsilon = 1 - \exp\left[\left(\frac{1}{C_r}\right) NTU^{0.23}\left[\exp(-C_r\, NTU^{0.78}) - 1\right]\right] \quad (1)$$

where number of transfer units (NTU) and heat capacity ratio (C_r) is given as,

$$\frac{1}{NTU} = \frac{C_{\min}}{UA} = C_{\min}\left[\frac{1}{(hA)_h} + \frac{1}{(hA)_c}\right] \quad (2)$$

where subscript h and c indicate hot side fluid and cold side fluid respectively.

The free flow area (A_{ff} in m^2) for the plate fin heat exchanger geometry is given by,

$$A_{ff,h} = (H_h - t_h)(1 - n_h t_h) L_c N_h \quad (3)$$

$$A_{ff,c} = (H_c - t_c)(1 - n_c t_c) L_h N_c \quad (4)$$

where L_c and L_h indicate cold side and hot side flow length. N_h and N_c indicate number of hot side and cold side layer respectively.

Heat transfer areas (A in m^2) for the two sides are obtained by,

$$A_h = L_h L_c N_h[1 + 2n_h(H_h - t_h)] \quad (5)$$

$$A_c = L_h L_c N_c[1 + 2n_c(H_c - t_c)] \quad (6)$$

So, total heat transfer area of the considered heat exchanger are formulated as,

$$A = A_h + A_c = L_h L_c[N_h(1 + 2n_h(H_h - t_h)) + N_c(1 + 2n_c(H_c - t_c))] \quad (7)$$

The Colburn factor (j), Fanning factor (f) and heat transfer co-efficient for offset strip fin is given by Manglik and Bergles (1995) as,

$$j = 0.6522(Re)^{-0.5403}(\alpha)^{-0.1541}(\delta)^{0.1499}(\gamma)^{-0.0678} \left[1 + 5.269 \times 10^{-5}(Re)^{1.34}(\alpha)^{0.504}(\delta)^{0.456}(\gamma)^{-1.055}\right]^{0.1} \quad (8)$$

$$f = 9.6243(Re)^{-0.7422}(\alpha)^{-0.1856}(\delta)^{0.3053}(\gamma)^{-0.2659} \left[1 + 7.669 \times 10^{-8}(Re)^{4.429}(\alpha)^{0.920}(\delta)^{3.767}(\gamma)^{0.236}\right]^{0.1} \quad (9)$$

$$h = j\, G\, C_p (Pr)^{-0.667} \quad (10)$$

where α, δ and γ are dimensionless parameters and given by,

$$Re = \frac{Gd_h}{\mu}, \quad \alpha = \frac{fs}{H - t}, \quad \delta = \frac{t}{l_f}, \quad \gamma = \frac{t}{fs}, \quad fs = \left(\frac{1}{n} - t\right) \quad (11)$$

The above equations of Colburn factor (j) and Fanning factor (f) are valid for $120 < Re < 10^4$, $0.134 < \alpha < 0.997$, $0.012 < \delta < 0.048$ and $0.041 < \gamma < 0.121$.

For the given fin geometry, hydraulic diameter (d_h) is computed by,

$$d_h = \frac{4 fs l_f (H - t)}{2(fs l_f + (H - t) l_f + (H - t)t) + t fs} \quad (12)$$

Frictional pressure drop for the two fluid streams are given by Shah and Sekulic (2003) as,

$$\Delta P_h = \frac{2 f_h L_h G_h^2}{\rho_h d_{h,h}} \quad (13)$$

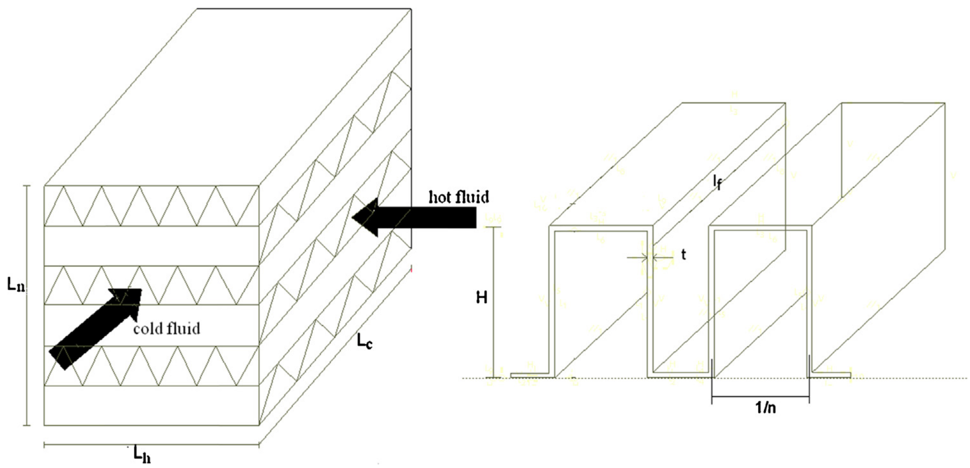

Fig. 1 – Schematic diagram of cross flow plate-fin heat exchanger (PFHE) and offset-strip fin.

Table 1 – Economic parameters of application example 1.

Cost per unit area (C_A), \$/m^2	90
Hours of operation (τ), h	5000
Electricity price (ζ), \$/MW h	20
Compressor efficiency (η)	0.6
Non-linear exponent (n_1)	0.6
Depreciation time (z_1), year	10
Rate of interest (roi)	0.1

$$\Delta P_c = \frac{2 f_c L_c G_c^2}{\rho_c d_{h,c}} \tag{14}$$

Total annual cost (TAC) of a PFHE can be seen as the combination of two major costs: the initial cost (C_{in}) and the operating cost (C_{op}). For the consistency of the comparison both the cost are calculated using the same cost function of the previous work of Yousefi et al. (2012a,b). The initial cost of a heat exchanger is estimated from the following correlation

$$C_{in} = A_{cf} C_A A^{n_1} \tag{15}$$

where C_A and n_1 are cost per unit surface area and the exponent of non-linear increase with area increase respectively. A_{cf} is the annual coefficient factor and calculated by,

$$A_{cf} = \frac{roi}{1-(1+roi)^{-z_1}} \tag{16}$$

where roi and z_1 represent interest rate and depreciation time respectively.

The operating cost is governed by the pumping power required for driving the hot and cold fluids through the exchanger and calculated by,

$$C_{op} = \left[\zeta\tau\frac{\Delta P\, m}{\eta\, \rho}\right]_h + \left[\zeta\tau\frac{\Delta P\, m}{\eta\, \rho}\right]_c \tag{17}$$

All the parameters required for cost evaluations in the present work are presented in Table 1.

4. Teaching-learning-based optimization (TLBO) algorithm

Teaching-learning is an important process where every individual tries to learn something from other individual to improve himself/herself. Rao et al. (2011, 2012) and Rao and Patel (2012) proposed an algorithm known as teaching-learning based optimization (TLBO) which simulates the traditional teaching-learning phenomenon of class room. The algorithm simulates two fundamental modes of learning: (i) through teacher and (ii) interacting with the other learners. TLBO is a population based algorithm where a group of students (i.e. learner) is considered as population and the different subjects offered to the learners is analogous with the different design variables of the optimization problem. The grades of a learner in each subject represent a possible solution to the optimization problem (value of design variables) and the mean result of a learner considering all subjects corresponds to the quality of the associated solution (fitness value). The best solution in the entire population is considered as the teacher.

At the first step, the TLBO generates a randomly distributed initial population $p_{initial}$ of n solutions, where n denotes the size of population. Each solution X^k (k = 1, 2, ..., n) is a m-dimensional vector where m is the number of optimization parameters (design variables). After initialization, the population of the solutions is subjected to repeated cycles, i = 1, 2, ..., g, of the teacher phase and learner phase. Working of the TLBO algorithm is explained below with the teacher phase and learner phase.

4.1. Teacher phase

This phase of the algorithm simulate the learning of the students (i.e. learners) through teacher. During this phase a teacher convey knowledge among the learners and put efforts to increase the mean result of the class. Suppose there are 'm' number of subjects (i.e. design variables) offered to 'n' number of learners (i.e. population size, k = 1, 2, ..., n). At any sequential teaching-learning cycle i, $M_{j,i}$ is the mean result of the learners in a particular subject 'j' (j = 1, 2, ..., m). Since a teacher is the most experienced and knowledge person on a subject, so the best learner in the entire population is considered as a teacher in the algorithm. Let $X^b_{j,i}$, is the grades of the best learner and $f(X^b)$ is the result of the best learner considering all the subjects, who is identified as a teacher for that cycle. Teacher will put maximum effort to increases the knowledge level of the whole class, but learners will gain knowledge according to the quality of teaching delivered by a teacher and the quality of learners present in the class. Considering this fact the difference between the grade of the teacher and mean grade of the learners in each subject is expressed as,

$$Difference\_Mean_{j,i} = r_i(X^b_{j,i} - T_F M_{j,i}) \tag{18}$$

where $X^b_{j,i}$ is the grade of the teacher (i.e. best learner) in subject j. T_F is the teaching factor. Value of T_F can be either 1 or 2 which is again a heuristic step and so it is decided randomly with equal probability as $T_F = round\ [1 + rand\ (0, 1)\ \{2 - 1\}]$. r_i is the random number in the range [0,1].

Based on the $Difference\_Mean_{j,i}$, the existing solution 'k' is updated in the teacher phase according to the following expression.

$$X'^k_{j,i} = X^k_{j,i} + Difference\_Mean_{j,i} \tag{19}$$

where $X'^k_{j,i}$ is the updated value of $X^k_{j,i}$. Accept $X'^k_{j,i}$ if it gives better function value otherwise keep the previous solution.

4.2. *Learner phase*

This phase of the algorithm simulate the learning of the students (i.e. learners) through interaction among themselves. The students can also gain knowledge by discussing and interacting with the other students. The learning phenomenon of this phase is expressed below.

Randomly select two learners p and q such that $f(X^p) \neq f(X^q)$ (where, $f(X^p)$ and $f(X^q)$ are the updated result of the learner p and q at the end of teacher phase)

$$X''^p_{j,i} = X'^p_{j,i} + r_i(X'^p_{j,i} - X'^q_{j,i}), \quad \text{If} \quad f(X_p) < f(X_q) \tag{20}$$

$$X''^p_{j,i} = X'^p_{j,i} + r_i(X'^q_{j,i} - X'^p_{j,i}), \quad \text{If} \quad f(X_q) < f(X_p) \tag{21}$$

(Above equations is for minimization problem, reverse is true for maximization problem)where $X''^p_{j,i}$ is the updated value of $X'^p_{j,i}$. Accept $X''^p_{j,I}$ if it gives a better function value.

5. Multi-objective ITLBO (MO-ITLBO) algorithm

The proposed MO-ITLBO algorithm is the improved version of basic TLBO algorithm. In the basic TLBO algorithm, the result of the learners is improved either by a teacher (through class room teaching) or by interacting with other learners. However, in the traditional teaching-learning environment the students also learn during the tutorials hours by discussing with their fellow classmates or even by discussing with the teacher. Sometimes the students are self-motivated and try to learn the things by self-learning. Furthermore, the teaching factor in the basic TLBO algorithm is either 2 or 1 which reflects two extreme circumstances where the learner learns either everything or nothing from the teacher. During the course of optimization, this situation results in a slower convergence rate of optimization algorithm. So considering this fact, to enhance the exploration and exploitation capacity, some modifications have been introduced in the basic TLBO algorithm.

The basic TLBO algorithm has been already modified by Rao and Patel (2012, 2013a,b) to improve its performance and apply it to the optimization of thermal systems. In the present work the previous modifications are further enhanced to improve the performance of the algorithm.

5.1. *Number of teachers*

Population sorting is an important concept used in evolutionary algorithms to avoid the premature convergence. In the basic TLBO algorithm, the population sorting mechanism is provided by introducing the multi teacher concept. As per this modification, entire population (i.e. students) is divided into groups based on their results. The best learner of each group acts as a teacher for that group and tries to increase the mean result of his/her group. If the level (i.e. result) of the individual in the group reaches up to the level of the teacher of that group then this individual is assigned to the next group (i.e. next better teacher). The mathematical explanation of this modification is given in Fig. 2.

5.2. *Adaptive teaching factor*

Another modification is related to the teaching factor (T_F) of the basic TLBO algorithm. The teaching factor decides the value of mean to be changed. In the basic TLBO, the decision of the teaching factor is a heuristic step and it can be either 1 or 2. This practice corresponds to the situation where learners learn nothing from the teacher or learn all the things from the teacher respectively. But in actual teaching-learning phenomenon this fraction is not always at its end state for learners but varies in-between also. The learners may learn in any proportion from the teacher. Considering this fact the teaching factor is modified as,

$$(T_F)_{s,i} = \left(\frac{f(X^k)}{T_s}\right)_i \quad \text{if} \quad T_s \neq 0 \tag{22}$$

$$(T_F)_i = 1 \quad \text{if} \quad T_s = 0 \tag{23}$$

where $f(X^k)$ is the result of any learner k associated with group 's' considering all the subjects at iteration i and T_s is the result of the teacher of the same group at same iteration i. Thus in ITLBO algorithm, teaching factor varies automatically during search depending upon the result of the learner and teacher. Thus, automatic tuning of T_F improves the performance of the algorithm.

5.3. *Learning through tutorial*

This modification is based on the fact that students can also learn by discussing with their fellow classmates or even with the teacher during the tutorial hours while solving the assigned tasks. So, in the ITLBO algorithm, the learner improved his/her result in the teacher phase through the class room teaching provided by the teacher along with the discussion with the fellow classmates or teacher during tutorial hours. Mathematically this modification can be modeled as:

$$X'^k_{j,i} = (X^k_{j,i} + Difference\_Mean_{j,i}) + r_i(X^h_{j,i} - X^k_{j,i}) \quad \text{if} \quad f(X^h) < f(X^k), \quad h \neq k \tag{24}$$

$$X'^k_{j,i} = (X^k_{j,i} + Difference\_Mean_{j,i}) + r_i(X^k_{j,i} - X^h_{j,i}) \quad \text{if} \quad f(X^k) < f(X^h), \quad h \neq k \tag{25}$$

where the first term in the right side indicates the class room learning and second term indicates learning through tutorial.

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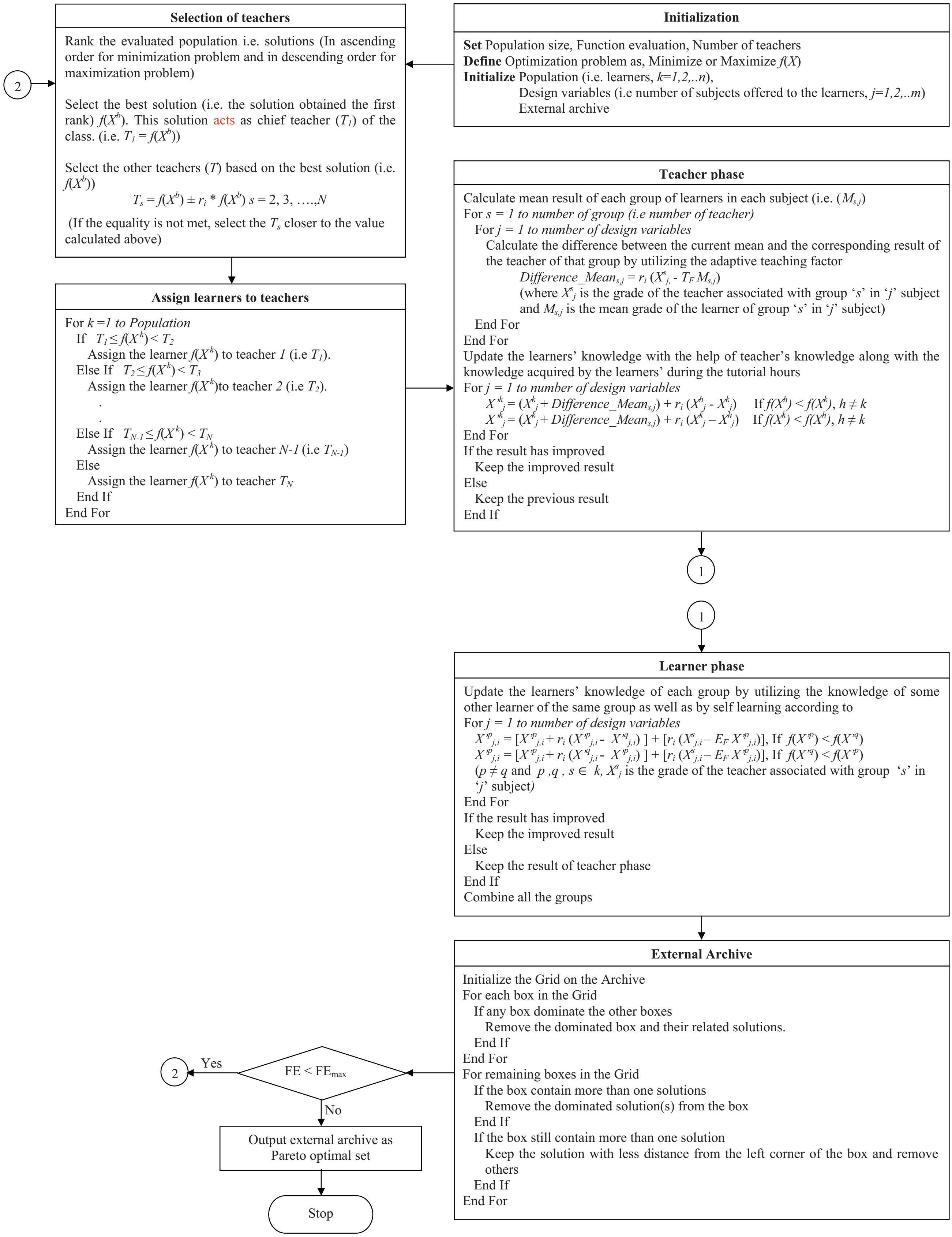

Fig. 2 – Schematic diagram of the MO-ITLBO algorithm.

5.4. *Self-motivated learning*

In the basic TLBO algorithm, the results of the students are improved either by learning from the teacher or by interacting with the other students. However, it is also possible that students are self-motivated and improve their knowledge by self-learning. Thus the self-learning aspect to improve the knowledge is considered in the ITLBO algorithm. Since the students learn without the aid of the teacher, we incorporate this search mechanism in the learner phase. Mathematically this modification can be modeled as:

$$X''^{p}_{j,i} = X'^{p}_{j,i} + [r_i(X'^{p}_{j,i} - X'^{q}_{j,i})] + [r_i(X^{s}_{j,i} - E_F X'^{p}_{j,i})],\quad \text{if } f(X'^{p}) < f(X'^{q}) \tag{26}$$

$$X''^{p}_{j,i} = X'^{p}_{j,i} + [r_i(X'^{q}_{j,i} - X'^{p}_{j,i})] + [r_i(X^{s}_{j,i} - E_F X'^{p}_{j,i})],\quad \text{if } f(X'^{q}) < f(X'^{p}) \tag{27}$$

($p \neq q$ and p, q, $s \in k$, X^{s}_{j} is the grade of the teacher associated with group 's' in 'j' subject)where E_F is the exploration factor and its value is decided randomly as:

$$E_F = Round \quad (1 + r_i) \tag{28}$$

The second term in the right side of Eqs. (26) and (27) indicate the learning by interacting with the other learner and third term indicates self-motivated learning.

5.5. External archive

The main objective of the external archive is to keep a historical record of the non-dominated vectors found along the search process. This algorithm uses a fixed size external archive to keep the best non-dominated solutions that it has found so far. In the proposed algorithm an ε-dominance method is used to maintain the archive (Deb et al., 2005). This method has been used widely in multi-objective optimization algorithms to manage the archive.

The archive is a space with dimension equal to the number of problem's objectives. The archive is empty at the beginning of the search. In ε-dominance method each dimension of the objective space is divided into segments whose width is ε, so that the objective space is divided into squares, cubes or hyper-cubes for two, three and more than three objectives respectively. If a box that holds the solution(s) can dominate other boxes then those boxes will be removed. Then each box is examined to check if only one non-dominated solution is present, while the dominated ones are eliminated. The proposed MO-ITLBO algorithm uses the grid based approach for the archiving process which was previously used by MOABC algorithm (Akbari et al., 2012). The schematic diagram of the MO-ITLBO algorithm is shown in Fig. 2.

Both the teacher phase and learner phase iterate cycle by cycle according to Fig. 2 till the termination criterion is satisfied. At the termination of the algorithm, the external archive found by the algorithm is returned as the output.

6. Application examples

In this section, the effectiveness of the proposed algorithm is assessed by analyzing two application examples. The first example was earlier analyzed using imperialist competitive algorithm by Yousefi et al. (2012a,b) while second example was attempted by Zarea et al. (2013).

6.1. Application example 1

A gas-to-air single pass cross-flow heat exchanger having heat duty of 1069.8 kW is needed to be designed and optimized for the minimum total annual cost and the minimum total weight separately as well as simultaneously. Maximum dimension of the exchanger is limited to 1 m × 1 m × 1.5 m respectively. The offset strip fin surface having same specifications are assumed on both the sides of heat exchanger. The heat exchanger material is aluminum with density of 2700 kg/m^3. Allowable pressure drops are 9.50 kPa and 8.00 kPa at hot side and cold side respectively. Table 2 presents the operating conditions of PFHE.

Table 2 – Operating parameters of application example 1.

Parameters	Hot side	Cold side
Mass flow rate (m), kg/s	1.66	2
Specific heat (Cp), J/kg K	1122	1073
Viscosity (μ), N s/m^2	4.01E−05	3.36E−05
Density (ρ), kg/m^3	0.6296	0.9638
Inlet temperature (T), °C	900	200
Prandtl number (Pr)	0.731	0.694

The fluid inlet temperatures and flow rates are considered as design specifications. While seven design variables such as hot side flow length (L_h), cold side flow length (L_c), fin height (H), fin thickness (t), fin frequency (n), lance length of fin (l_f) and number of hot side layers (N_h) are considered for the optimization problem.

In the present work, the proposed approach is used for multi-objective constrained minimization problem. The problem can generally be described as follows,

$$Minimise \quad f(x), \quad x = [x_1, x_2, \ldots, \quad x_k] \tag{29}$$

where constraints are stated as,

$$g_j(x) \le 0, \quad j = 1, \ldots, nc \tag{30}$$

$$x_{i,\min} \le x_i \le x_{i,\max}, \quad i = 1 \ldots, k \tag{31}$$

Here in the present work, the objective is to minimize the total weight and total annual cost of the heat exchanger. Moreover, to take in to account the effect of constraints violation during the optimization process a static penalty parameter is also added in the objective function. So, finally the objective function for the present work is represented as,

$$Minimize\, f(x) + \sum_{j=1}^{m} R1 \ (g_j(x))^2 \tag{32}$$

where R1 is the penalty parameter. The second term takes into account the effect of constraints violation. R1 comes in to picture when constraint violation takes place. Too low value of R1 during the course of optimization does not reflect the constraints violation and hence algorithm does not explore the different region of the search space but continue to exploit the current region though the solution is not feasible in that region. Similarly, too high value of R1 during the course of optimization results in arbitrary exploration of search space and hence more function evaluations are required to reach at optimum value. So, for any problem experiment are conducted by considering different values of R1 and based on the experiment results a proper value of R1 is decided.

The objective function is subjected to seven inequality constraints which are bound by lower and upper limits of the design variables.

$$0.1 \le L_h \le 1 \tag{33}$$

Table 3 – Comparison of the heat exchanger design geometries for minimum TAC consideration.

Design variables	ICA approach[a]	TLBO approach
Hot side flow length (L_h), m	0.83	0.8353
Cold side flow length (L_c), m	1	1
Fin height (H), mm	9.7	9.986
Fin thickness (t), mm	0.2	0.1921
Fin frequency (n), m^{-1}	228.2	205.2
Lance length of fin (l_f), mm	10	8.296
Number of hot side layer (N_h)	73	71
No flow length (L_n), m	1.5	1.5
Hot side pressure drop (ΔP_h), kPa	0.28	0.304
Cold side pressure drop (ΔP_c), kPa	0.31	0.33
Total weight (kg)	374.55	348.846
Initial cost ($)	713.2	679.91
Operating cost ($)	228.8	247.72
Total cost ($)	942	927.63
Function evaluations	12,000	8000
Computational time (s)	3.55	2.63

[a] Yousefi et al. (2012a,b).

$$0.1 \le L_c \le 1 \tag{34}$$

$$0.002 \le H \le 0.01 \tag{35}$$

$$100 \le n \le 1000 \tag{36}$$

$$0.0001 \le t \le 0.0002 \tag{37}$$

$$0.001 \le l_f \le 0.01 \tag{38}$$

$$1 \le N_h \le 200 \tag{39}$$

$$\Delta P_h \le 9.50 \quad \text{kPa} \tag{40}$$

$$\Delta P_c \le 8\,\text{kPa} \tag{41}$$

Now in the present work, the above design optimization problem is attempted using the proposed algorithm for single objective consideration as well as multi-objective consideration.

6.1.1. *Single objective consideration*

Initially, the single objective optimization of the PFHE is carried out to identify the effectiveness of the TLBO algorithm. After conducting number of trials, population size 25 and number of generations 400 is considered for the single objective optimization.

Table 3 shows the optimized parameters of the considered example obtained by using the TLBO approach for minimum total annual cost consideration and its comparison with the optimized parameters obtained by Yousefi et al. (2012a,b) using the ICA approach. Results show that reduction in lance length of the fin, fin frequency, fin thickness and number of hot side layer reduces the weight of the heat exchanger which in turn reduce the capital cost (i.e. initial cost) associated with the exchanger by 4.67% in the present approach. However, the increase hot side flow length and fin height increases the fluid pressure drop which in turn increase the operating cost associated with the exchanger. Overall, the combined effect of reduction in capital investment and increment in operating costs result in reduction of the total cost of about 1.53% using TLBO as compared to the ICA approach suggested by Yousefi et al. (2012a,b).

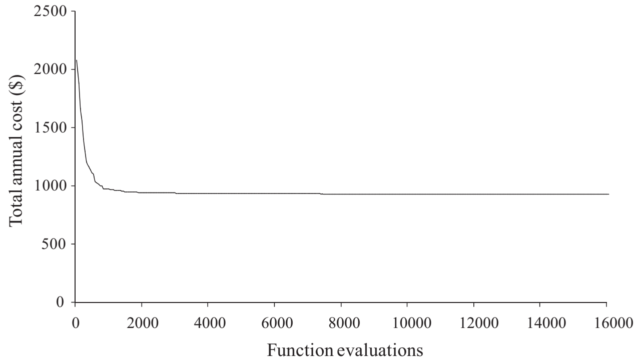

Fig. 3 – Convergence of ITLBO algorithms for minimum total annual cost consideration.

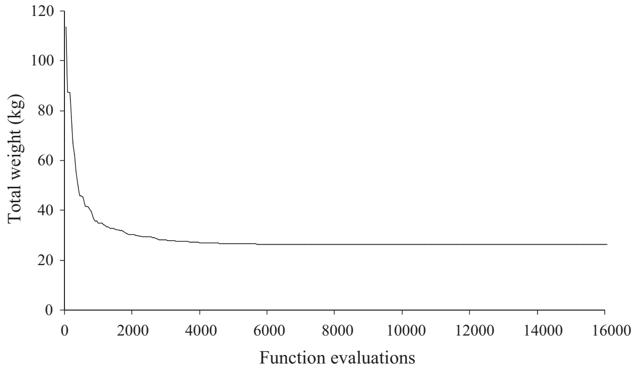

Fig. 4 – Convergence of ITLBO algorithms for minimum total weight consideration.

Table 4 shows the optimized parameters of the considered example obtained by using the TLBO approach for minimum total weight consideration. Results show that reduction in no flow length and number of hot side layer reduces the weight of the heat exchanger while larger fin height increases the weight of the heat exchanger. Overall, the combine effect of all these parameters reduces the weight of the heat exchanger by 2.28% in the present approach as compared to the ICA approach considered by Yousefi et al. (2012a,b).

Figs. 3 and 4 show the convergence of both the objective function using TLBO algorithms. It is observed from Figs. 3 and 4 that objective function converged within about 8000 and 6000 function evaluations for minimum total annual cost and minimum total weight consideration respectively while ICA requires 20,000 and 11,000 function evaluation (Yousefi et al., 2012a,b) for the convergence of the same objective functions respectively. Similarly, it is observed from Tables 3 and 4 that computational effort required for the TLBO algorithm is less as compared to previous approach suggested by Yousefi et al. (2012a,b).

6.1.2. *Multi-objective consideration*

The results of single objective optimization for minimum total annual cost and minimum total weight reveal that lower total annual cost accompanies the higher total weight and vice versa, which reflects the necessity of multi-objective optimization. Considering this fact MO-ITLBO algorithm is implemented for the simultaneous optimization of the both the objective functions. Here also population size 25 and number of generations 400 is set for the MO-ITLBO algorithm after

Table 4 – Comparison of the heat exchanger design geometries for minimum weight consideration.

Design variables	Preliminary design[a]	ICA approach[b]	TLBO approach
Hot side flow length (L_h), m	0.3	0.21	0.213
Cold side flow length (L_c), m	0.3	0.23	0.2241
Fin height (H), mm	2.49	6.8	6.815
Fin thickness (t), mm	0.102	0.1	0.1
Fin frequency (n), m^{-1}	782	1000	1000
Lance length of fin (l_f), mm	3.18	2.1	2.084
Number of hot side layer (N_h)	167	80	79
No flow length (L_n), m	1	1.178	1.164
Hot side pressure drop (ΔP_h), kPa	9.34	9.15	9.492
Cold side pressure drop (ΔP_c), kPa	6.9	7.96	7.959
Total cost ($)	6780.7	6987.9	7169.2
Total weight (kg)	64.63	26.73	26.12
Function evaluations	–	11,000	6000
Computational time (s)	–	3.29	2.37

[a] Shah and Sekulic (2003).
[b] Yousefi et al. (2012a,b).

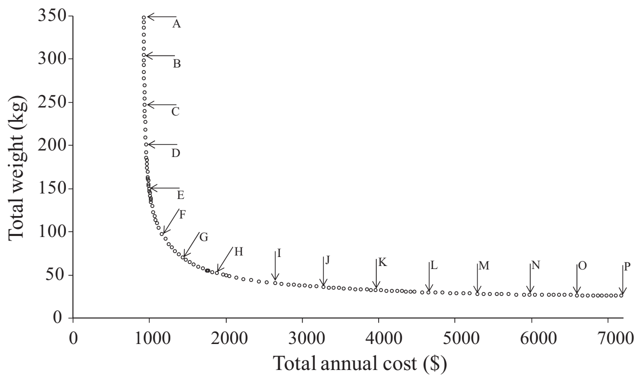

Fig. 5 – The distribution of Pareto-optimal points solutions for Example-1 using the MO-ITLBO algorithm.

conducting number of trials with different combination of population size and number of generations.

Fig. 5 represents the Pareto optimal curve obtained by using the MO-ITLBO algorithm for multi-objective optimization. As seen from Fig. 5, minimum total annual cost exists at design point A corresponding to 927.63$ where the total weight is 348.846 kg which is also validated by single objective optimization where total annual cost is only objective. Similarly, the total weight is lowest at design point P corresponding to 26.12 kg where the total annual cost is 26.12$. Specifications of sixteen sample design points A–P in Pareto optimal fronts are listed in Table 5. It is observed from Table 5 that reduction in hot side flow length, cold side flow length, fin height and lance length of fin are observed as we move from design point A to P on Pareto front. The combine effect of reduction of these parameters reduces the weight of the heat exchanger which in turn reduce the capital cost (i.e. initial cost) associated with the exchanger. However, the fin frequency as well as number of hot side layer increases as we move from design point A to P on Pareto front. Hence, the fluid pressure drop increases, which in turn increases the operating cost associated with the exchanger. Depending upon requirement, design corresponding to any combination of total cost and total weight will selected by end users from the proposed results.

Fig. 6(a)–(d) shows the convergence of the MO-ITLBO algorithm to the optimal Pareto front at the interval of 5000, 10,000, 15,000 and 20,000 function evaluations respectively. In Fig. 6(a)–(d), the feasible solutions obtained from the external archive at the end of corresponding function evaluations are plotted in two-dimensional objective spaces. It is observed from Fig. 6 that with the increase in the function evaluations the approximated solution points in the two dimensional objective space is also increased. Moreover, the distribution of the solution points becomes uniform as the number of function evaluations proceeds. Both these points indicate that the performance of the MO-ITLBO algorithm is continuously improved throughout the function evaluations.

6.2. *Application example 2*

To demonstrate the effectiveness of the proposed algorithm for multi-objective optimization of plate-fin heat exchanger, one more application example considered. Moreover, maximization of effectiveness and minimization of total pressure drop are taken as an objective function in the considered example to verify the sensitivity of proposed algorithm for objective functions.

A cross flow plate-fin heat exchanger having hot gas and air on both the sides of heat exchanger needs to be designed and optimized for maximization of effectiveness and minimization of total pressure drop simultaneously. Dimension of the exchanger is limited to 1 m × 1 m × 1 m. Mass flow rates of gas and air are 1.66 kg/s and 2 kg/s, respectively. Inlet temperature of gas and air are considered 900 K and 200 K, respectively. The gas and air inlet pressures are 160 kPa and 200 kPa absolute. Pressure drops are set to be limited to 9.50 kPa on gas side and 8.00 kPa on air side. The offset strip fin surface is used on the gas and air sides. The other design specifications, design variables and property values for both the fluids are available in the previous work of Zarea et al. (2013).

The MO-ITLBO algorithm is experimented with the population size 25 and number of generations 400 on the considered example. Fig. 7 represents the Pareto optimal curve obtained by using the MO-ITLBO algorithm for multi-objective optimization. The results for Pareto optimal curve shown in Fig. 7 clearly reveal the conflict between the two objectives, heat exchanger effectiveness and total pressure drop. Any parametric change that increases the effectiveness leads to an increase in the total pressure drop and vice versa which shows the need for multi-objective optimization.

It is observed from Fig. 7 that the maximum effectiveness exists at design point A (0.9277) where the total pressure

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Table 5 – Optimal output variables for A–P Pareto optimal front shown in Fig. 5.

Design point	L_h (m)	L_c (m)	H (mm)	t (mm)	n (m^{-1})	l_f (mm)	N_h	Total cost ($)	Total weight (kg)
A	0.8353	1	9.986	0.1921	205.2	8.296	71	927.63	348.846
B	0.7666	0.968	9.986	0.1738	226.74	8.932	71	930.41	304.823
C	0.6984	0.8865	9.986	0.145	271.78	8.92	71	940.24	247.727
D	0.6311	0.81	9.986	0.1265	311.5	10	71	955.91	201.022
E	0.5498	0.7046	9.986	0.1084	363.63	9.023	71	1000.70	149.758
F	0.4297	0.5479	9.986	0.1	474.71	8.333	71	1167.40	97.518
G	0.3473	0.4433	9.986	0.1	591.96	8.333	71	1433.70	70.661
H	0.286	0.3637	9.986	0.1	701.88	6.514	71	1883.40	52.133
I	0.2567	0.3155	9.986	0.1	701.88	2.182	71	2650.20	40.569
J	0.2279	0.2823	8.812	0.1	788.61	2.136	80	3281.04	36.082
K	0.2037	0.2616	7.957	0.1	867.19	2.09	88	3961.60	32.704
L	0.1923	0.2349	7.269	0.1	942.77	2.084	96	4651.10	30.094
M	0.1837	0.2204	6.816	0.1	1000	2.084	101	5293.02	28.273
N	0.191	0.2274	6.816	0.1	999.96	2.084	91	5976.10	27.347
O	0.2017	0.2272	6.816	0.1	1000	2.084	84	6594.10	26.652
P	0.213	0.2241	6.815	0.1	1000	2.084	79	7169.20	26.12

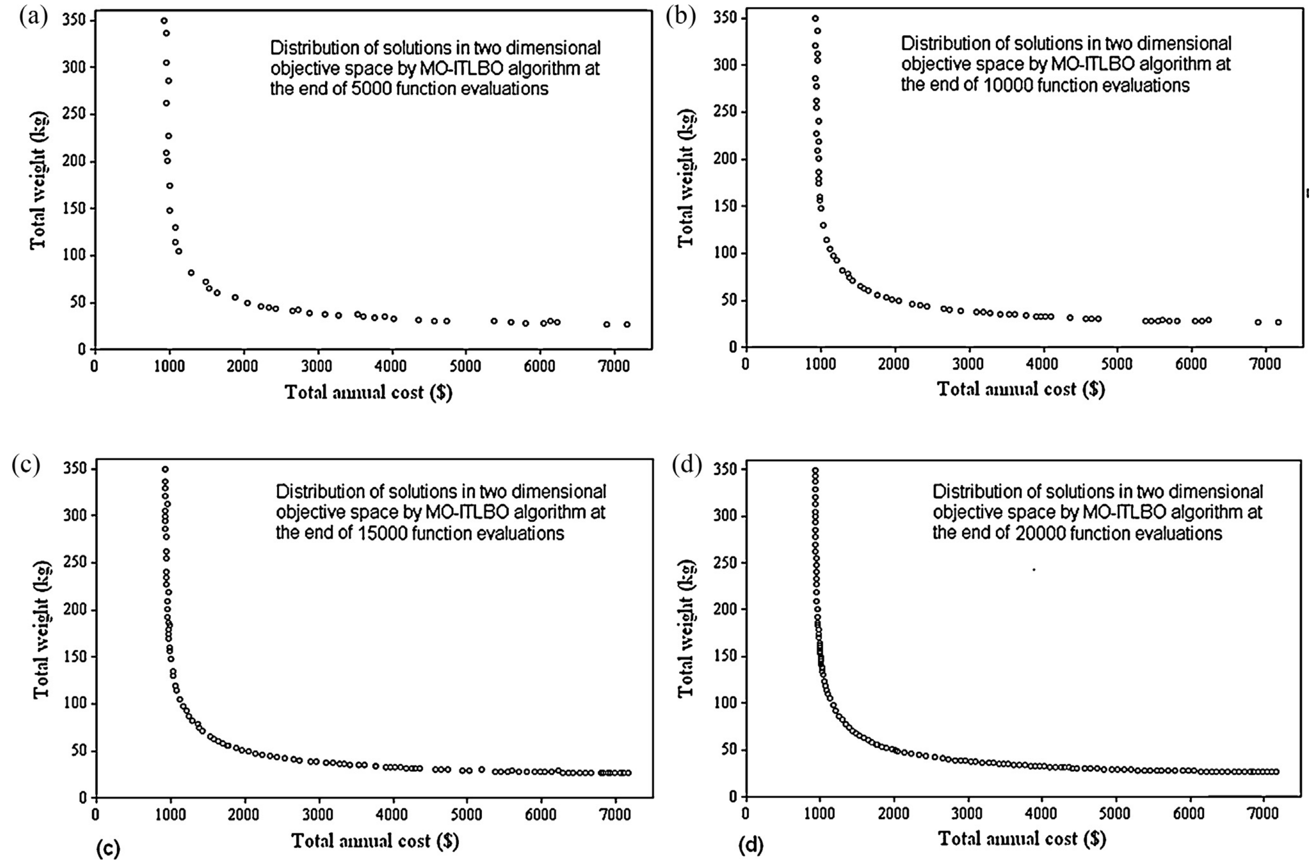

Fig. 6 – Convergence of MO-ITLBO algorithm (a) 5000 function evaluation, (b) 10,000 function evaluation, (c) 15,000 function evaluation and (d) 20,000 function evaluation.

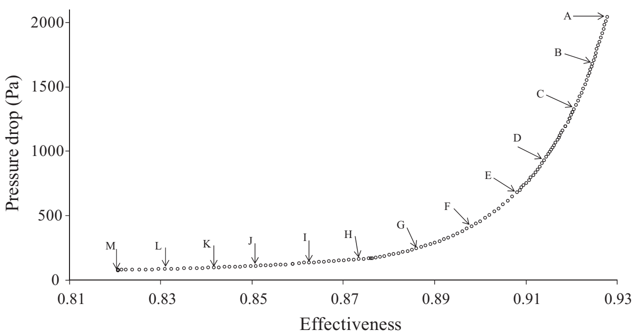

Fig. 7 – The distribution of Pareto-optimal points solutions for example 2 using the MO-ITLBO algorithm.

drop is highest (2045.68 Pa). On the other hand the minimum total pressure drop occurs at design point M (79.32 Pa) where the effectiveness has minimum (0.8205) value. Specifications of thirteen sample design points A–M in Pareto optimal fronts are listed in Table 6. It is observed from Fig. 7 and Table 6 that cold side flow length and lance length of fins are increased while fin frequency is reduced as we move from design point A to M. Higher cold flow length and lance length of fins increased the effectiveness as well as total pressure drop of heat exchanger. On the other hand reduction in fin frequency reduced the effectiveness as well as total pressure drop of the heat exchanger. The combine effect of these parameters reduces the effectiveness and total pressure drop of heat exchanger as we move from design point A to M.

Table 6 – Optimal output variables for A–M Pareto optimal front shown in Fig. 7.

Design point	L_h (m)	L_c (m)	H (mm)	t (mm)	n (m^{-1})	l_f (mm)	N_h	Pressure drop (Pa)	Effectiveness
A	1	0.7211	0.01	0.0002	648.83	0.0052	128	2045.68	0.9277
B	1	0.7542	0.01	0.0002	627.02	0.0071	127	1682.88	0.9245
C	1	0.7555	0.01	0.0002	562.31	0.0058	138	1330.94	0.9204
D	1	0.7649	0.01	0.0002	521.7	0.0086	147	935.93	0.9140
E	1	0.7376	0.01	0.0002	466.02	0.0082	167	685.29	0.9080
F	1	0.7394	0.01	0.0002	393.49	0.0093	192	421.39	0.8980
G	1	0.8558	0.01	0.0002	308.45	0.01	200	249.52	0.8860
H	1	1	0.01	0.0002	240.02	0.01	200	166.86	0.8734
I	1	1	0.01	0.0002	208.81	0.01	200	140.21	0.8624
J	1	1	0.01	0.0002	180.87	0.01	200	114.84	0.8507
K	1	1	0.01	0.0002	162.16	0.01	200	101.81	0.8415
L	0.9849	1	0.01	0.0002	145.02	0.01	200	90.50	0.8309
M	0.9783	1	0.01	0.0002	130.05	0.01	200	79.32	0.8205

Thus, for the considered objective functions, the design of the heat exchanger is more sensitive to fin frequency than other parameters.

7. Conclusion

In the present work, MO-ITLBO algorithm has been adapted to handle the MOO problem of PFHE. The MO-ITLBO algorithm used a fixed size archive to maintain the good solutions obtained in the every iteration and a grid-based approach to control the diversity over the external archive. The algorithm's ability is demonstrated using application examples of PFHE involving conflicting objectives. Seven geometric variables including the total length of the hot and cold side of the heat exchanger, fin height, fin frequency, lance length of the fin, fin thickness and the number of fin layers are considered as optimization parameters. The MO-ITLBO algorithm provides a set of optimal solutions each of which is a trade-off between the conflicting objectives. The principal advantage of this work is providing a wide range of optimal solutions which allows the user to choose the best design parameters regarding the application. The proposed algorithm can be easily customized to suit the optimization of other types of thermal systems involving large number of variables and objectives. These features boost up the applicability of the proposed algorithm for the thermal systems optimization.

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