CSCI 5350
Advanced Topics in Game Theory

http://course.cse.cuhk.edu.hk/~csci5350/

1st Term, 2020/2021

Teacher: Ho-fung Leung
Time: T4-5, W3

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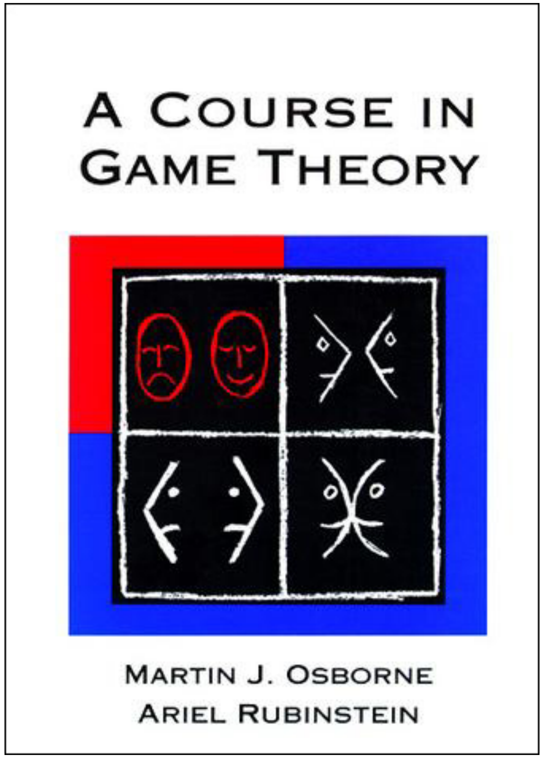
Text Book



Osborne, M. J. and Rubinstein, A., 1994. *A Course in Game Theory*. MIT Press.

Tentative Schedule:

Week	Topics	Osborne & Rubinstein	Remarks
1	Nash Equilibrium Zero-Sum Games	2.1 – 2.5	
2	Bayesian Games	2.6	
3	Mixed Strategies, Correlated Equilibrium and Evolutionary	3.1 – 3.4	Assignment 1
4	Extensive Games with Perfect Information	6.1	
5	Subgame Perfect Equilibrium	6.2 – 6.5	
6	Folk's Theorems	8.1 – 8.10	
7	Implementation Theory	10.1 – 10.5	
8	Extensive games with imperfect information	11.1	Assignment 2
9	Framing Effects	11.2 – 11.5	
10	Sequential Equilibrium	12.1 – 12.2	
11	Perfect Bayesian Equilibrium, Trembling Hand Perfect Equilibrium	12.3 – 12.5	
12	The Core	13.1 – 13.6	
13	Stable Sets and Shapley Value	14.1 – 14.2, 14.4	Assignment 3

Assessment Scheme:

Assignments: 45% (15% each)
Examination: 55%

- All assignments are written assignments.
- There is no programming assignment.
- There will be three assignments to be posted in week 3, week 8 and week 13.
- The final examination lasts for 2 hours.

The Prisoner's Dilemma

		Player 2	
		Cooperate	*Defect*
Player 1	*Cooperate*	freedom freedom	freedom & *award* imprisonment
	Defect	imprisonment freedom & *award*	remission remission

If you are one of them, what will **YOU** do?

Strategic Games

Prisoner's Dilemma is a typical instance of **strategic games** (also known as '**normal form games**').

In general, in a strategic game:

- There is a finite set N of players.
- Player i has a set A_i of available actions.
- An **outcome** is an action profile: player 1 plays a_1, player 2 plays a_2, player 3 plays a_3, and so on.
- For player i, some outcomes are better than ('$\succsim_i$') other outcomes. (Each player thinks differently.)

		Player 2	
		Cooperate	*Defect*
Player 1	*Cooperate*	freedom freedom	freedom & *award* imprisonment
	Defect	imprisonment freedom & *award*	remission remission

$$N = \{1,2\}. \quad A_1 = \{C, D\}. \quad A_2 = \{C, D\}.$$

$$A = \{(C, C), (C, D), (D, C), (D, D)\}.$$

$$(D, C) \gtrsim_1 (C, C) \gtrsim_1 (D, D) \gtrsim_1 (C, D)$$

$$(C, D) \gtrsim_2 (C, C) \gtrsim_2 (D, D) \gtrsim_2 (D, C)$$

Strategic Games

DEFINITION. A **strategic game** $\langle N, (A_i), (\succsim_i) \rangle$ consists of

- a finite set N (the set of **players**)
- for each player $i \in N$ a nonempty set A_i (the set of **actions** available to player i)
- for each player $i \in N$ a preference relation $\succsim_i$ on $A = A_1 \times A_2 \times \ldots \times A_n = \times_{j \in N} A_j$ (the **preference relation** of player i on the set of **action profiles**).

Payoff Functions

In most cases the preference relation $\succsim_i$ can be represented by a **payoff function** (or *utility function*)

$$u_i : A \to \mathbb{R}$$

such that

$$u_i(a) \geq u_i(b) \text{ if and only if } a \succsim_i b.$$

In these cases, we can denote that game by $\langle N, (A_i), (u_i) \rangle$ rather than $\langle N, (A_i), (\succsim_i) \rangle$.

Player 2

		Cooperate	*Defect*
Player 1	*Cooperate*	3, 3	0, 5
	Defect	5, 0	1, 1

$$A = \{(C, C), (C, D), (D, C), (D, D)\}.$$

$$u_1((D, C)) = 5. \qquad u_1((C, C)) = 3.$$

$$u_1((D, D)) = 1. \qquad u_1((C, D)) = 0.$$

$$(D, C) \ \gtrsim_1 \ (C, C) \ \gtrsim_1 \ (D, D) \ \gtrsim_1 \ (C, C).$$

$$5 \quad \geq \quad 3 \quad \geq \quad 1 \quad \geq \quad 0$$

Strategic games are also known as **bimatrix games**.

Nash Equilibrium

$$(a_1^*, a_2^*, \dots, a_{k-1}^*, a_k^*, a_{k+1}^*, \dots, a_n^*) \in A$$

A **Nash equilibrium** is an action profile,

in which

every player thinks that he is playing the best strategy with respect to this profile.

NOTATIONS

$$a = (a_1, a_2, \dots, a_{k-1}, a_k, a_{k+1}, \dots, a_n)$$

$$a_{-k} = (a_1, a_2, \dots, a_{k-1}, \qquad a_{k+1}, \dots, a_n)$$

$$(a_{-k}, x) = (a_1, a_2, \dots, a_{k-1}, x, a_{k+1}, \dots, a_n)$$
$$(a_{-k}, a_k) = (a_1, a_2, \dots, a_{k-1}, a_k, a_{k+1}, \dots, a_n)$$
$$(a_{-k}, a_k) = a$$

Nash Equilibrium

$$(a_1^*, a_2^*, \dots, a_{k-1}^*, a_k^*, a_{k+1}^*, \dots, a_n^*) \in A$$

In general, player k's best strategy depends on other players' strategies. Assume a_k^* is the best strategy player k plays <u>if other players do not change their strategies</u>, that is, $(a_{-k}^*, a_k^*) \succsim_k (a_{-k}^*, a_k)$ for all $a_k \in A_k$.

If <u>every</u> player thinks this way, this action profile is in equilibrium, called the **Nash equilibrium**.

Nash Equilibrium

DEFINITION. A **Nash equilibrium of a strategic game** $\langle N, (A_i), (\succsim_i) \rangle$ is a profile $a^* \in A$ of actions with the property that for every player $i \in N$ we have

$$(a^*_{-i}, a^*_i) \succsim_i (a^*_{-i}, a_i) \text{ for all } a_i \in A_i.$$

Best Responses

We consider Nash equilibrium in terms of players' *best responses*. Consider

$$a_{-k} = (a_1, a_2, \dots, a_{k-1}, \quad a_{k+1}, \dots, a_n)$$

Player k's *best strategies* are

$$\{a_k \in A_k : (a_{-k}, a_k) \succsim_k (a_{-k}, a_k') \text{ for all } a_k' \in A_k\}.$$

Player k's ***best-response function*** B_k:

$$B_k(a_{-k}) = \{a_k \in A_k : (a_{-k}, a_k) \succsim_k (a_{-k}, a_k') \text{ for all } a_k' \in A_k\}.$$

Consider a Nash equilibrium

$$(a_1^*, a_2^*, \dots, a_n^*) \in A.$$

By definition, each player is playing his best response. Therefore, we have

$$a_i^* \in B_i(a_{-i}^*) \text{ for all } i \in N.$$

Example: The Prisoner's Dilemma

		Player 2	
		Cooperate	*Defect*
Player 1	*Cooperate*	3, 3	0, 5
	Defect	5, 0	1, 1

Where are the Nash Equilibria?

Example: Battle of the Sexes

(Also known as 'Bach or Stravinsky')

		Wife	
		Boxing	*Opera*
Husband	*Boxing*	2, 1	0, 0
	Opera	0, 0	1, 2

Where are the Nash Equilibria?

Example: Coordination Game

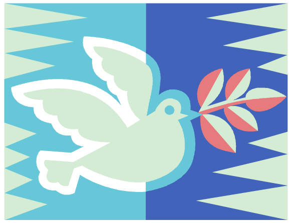
(Also known as 'Mozart or Mahler')

		Player 2	
		Square	*Circle*
Player 1	*Square*	2, 2	0, 0
	Circle	0, 0	1, 1

Where are the Nash Equilibria?

Example: Hawk-Dove Game

(Also known as 'Chicken Game')



		Bird 2	
		Dove	*Hawk*
Bird 1	*Dove*	3, 3	1, 4
	Hawk	4, 1	0, 0

Where are the Nash Equilibria?

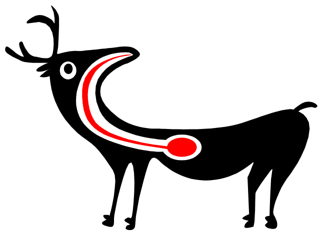
Example: Matching Pennies

Player 2

		Head	*Tail*
Player 1	*Head*	1, –1	–1, 1
	Tail	–1, 1	1, –1

Where are the Nash Equilibria?

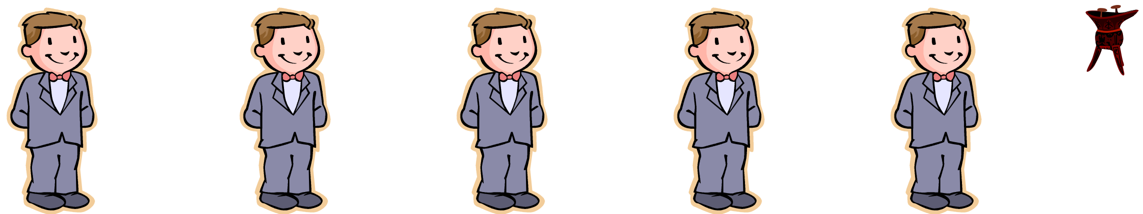
Example: Stag Hunt



		Player 2	
		Stag	*Hare*
Player 1	*Stag*	5, 5	0, 2
	Hare	2, 0	1, 1

Where are the Nash Equilibria?

Class Discussion: First-Price Sealed-Bid Auctions

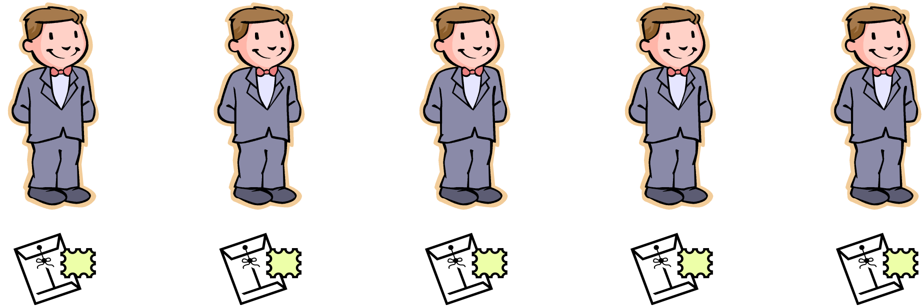
Player 

Valuations (v_i) \$10m > \$7m > \$6m > \$3m > \$1m > 0

Player with the lowest index wins if more than one player submits the highest bid.

Class Discussion

Player



Valuations $v_1 > v_2 > v_3 > v_4 > v_5 > 0$

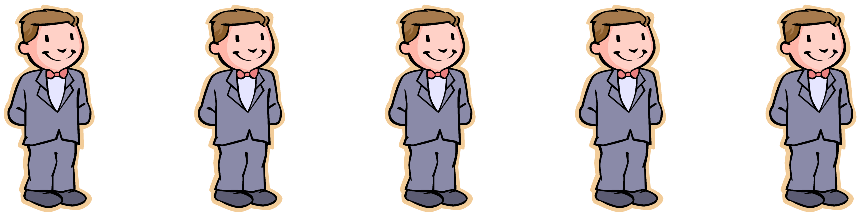
Submitted Bid $b_1 \quad b_2 \quad b_3 \quad b_4 \quad b_5$

$$b_i \in A_i = [0, \infty).$$

$$u_i(b_{-i}, b_i) = \begin{cases} v_i - b_i & \text{Player } i \text{ wins} \\ 0 & \text{Otherwise} \end{cases}.$$

Q: What are the Nash equilibria?

Player



Valuations (v_i) $v_1 > v_2 > v_3 > v_4 > v_5 > 0$

Submitted Bid b_1 b_2 b_3 b_4 b_5

Check whether the profile $(b_1, b_2, \ldots, b_n)$ is a Nash equilibrium:

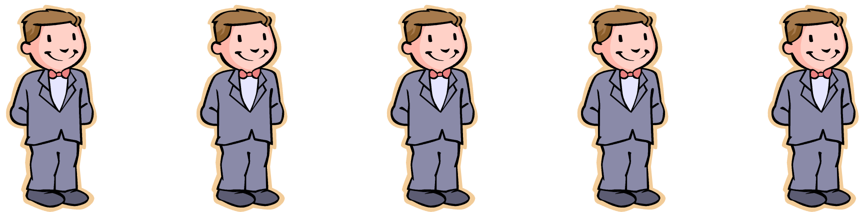
- $b_1 \in [v_2, v_1]$,
- $b_j \leq b_1$ for all $j \neq 1$, and
- $b_j = b_1$ for some $j \neq 1$.

(Comment: b_1 has to be the highest.)

Are there any other Nash equilibria?

Class Discussion: <u>Second</u>-Price Sealed-Bid Auctions

Player



Valuations (v_i)	v_1	$>$	v_2	$>$	v_3	$>$	v_4	$>$	v_5	$>$	0
Submitted Bid	b_1		b_2		b_3		b_4		b_5		

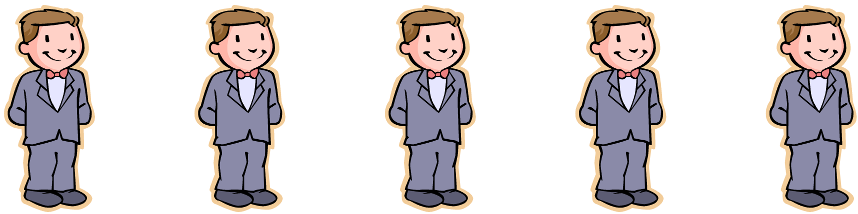
$$A_i = [0, \infty). \quad u_i(b_{-i}, b_i) = \begin{cases} v_i - b_j & \text{Player } i \text{ wins} \\ 0 & \text{Otherwise} \end{cases}.$$

Player i wins if b_i is the highest bid.

(Note: b_j is the highest of the other players' bids).

Q: What are the Nash equilibria?

Player



Valuations (v_i)	v_1	>	v_2	>	v_3	>	v_4	>	v_5	>	0
Submitted Bid	b_1		b_2		b_3		b_4		b_5		

Hint:

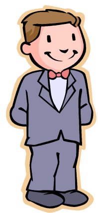
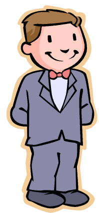
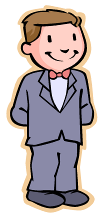
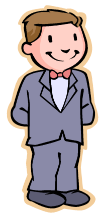
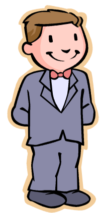
Consider $(v_1, \dots, v_n)$, that is, $b_i = v_i$.

- $\max\limits_{j \neq i} b_j \geq v_i$?
- $\max\limits_{j \neq i} b_j < v_i$?

Q: Is it better for $b_i \neq v_i$?

!**Q**: Is there a Nash equilibrium in which player 3 wins the item?

There is a Nash equilibrium in which player 3 wins:

Player											
Valuations (v_i)	10	>	7	>	6	>	3	>	1	>	0
Submitted Bid	5		0		11		0		0		

In general, any outcomes that satisfy the following conditions are Nash equilibria in which player j wins:

- $b_1 < v_j$.
- $b_j > v_1$.
- $b_i = 0$ for $i \notin \{1, j\}$.

Symmetric Games

A strategic game $\langle\{1,2\},(A_i),(u_i)\rangle$ is **symmetric** if $A_1 = A_2$, and $(a_1,a_2) \gtrsim_1 (b_1,b_2)$ if and only if $(a_2,a_1) \gtrsim_2 (b_2,b_1)$ if for all $a \in A$ and $b \in A$.

		Bird 2	
		Dove	*Hawk*
Bird 1	*Dove*	3, 3	1, 4
	Hawk	4, 1	0, 0

$$(H,D) \gtrsim_1 (D,D) \ \leftrightarrow\ (D,H) \gtrsim_2 (D,D)$$

$$(D,H) \gtrsim_1 (H,H) \ \leftrightarrow\ (H,D) \gtrsim_2 (H,H)$$

Is there a symmetric N.E. (a^*,a^*)?

Symmetric Games

A strategic game $\langle\{1,2\},(A_i),(u_i)\rangle$ is **symmetric** if $A_1 = A_2$, and $(a_1, a_2) \succsim_1 (b_1, b_2)$ if and only if $(a_2, a_1) \succsim_2 (b_2, b_1)$ if for all $a \in A$ and $b \in A$.

	L	M
T	$1, 1$	$0, 0$
B	$0, 0$	$0, 0$

Is there a symmetric N.E. (a^*, a^*)?

Symmetric Games

A strategic game $\langle\{1,2\},(A_i),(u_i)\rangle$ is **symmetric** if $A_1 = A_2$, and $(a_1, a_2) \succsim_1 (b_1, b_2)$ if and only if $(a_2, a_1) \succsim_2 (b_2, b_1)$ if for all $a \in A$ and $b \in A$.

	Left	*Right*
Left	1, 1	0, 0
Right	0, 0	1, 1

Is there a symmetric N.E. (a^*, a^*)?

Symmetric Games

A strategic game $\langle\{1,2\},(A_i),(u_i)\rangle$ is **symmetric** if $A_1 = A_2$, and $(a_1, a_2) \succsim_1 (b_1, b_2)$ if and only if $(a_2, a_1) \succsim_2 (b_2, b_1)$ if for all $a \in A$ and $b \in A$.

	X	Y
X	0, 0	1, 1
Y	1, 1	0, 0

Is there a symmetric N.E. (a^*, a^*)?

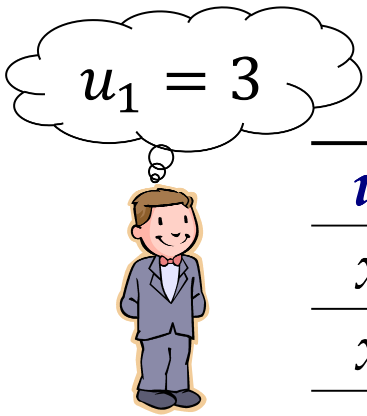
Strictly Competitive Games

DEFINITION. A strategic game $\langle\{1,2\},(A_i),(\succsim_i)\rangle$ is **strictly competitive** if for any $a \in A$ and $b \in A$ we have $a \succsim_1 b$ if and only if $b \succsim_2 a$.

A strictly competitive game is sometimes called *zerosum* when the two players' payoffs can be represented by u_1 and u_2 with $u_1 + u_2 = 0$.

Maxminimisation

A player *maxminimises* if he chooses an action on the assumption that his opponent always hurts him as much as possible.



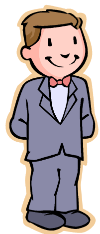
u_1	y_1	y_2	y_3
x_1	**–10**	8	0
x_2	4	7	**3**
x_3	**0**	1	2

This action is called a ***maxminimiser*** for player 1.

Maxminimiser Actions

DEFINITION. Let $\langle \{1,2\}, (A_i), (u_i) \rangle$ be a strictly competitive strategic game. The action $x^* \in A_1$ is a **maxminimiser for player 1** if

$$\min_{y \in A_2} u_1(x^*, y) \geq \min_{y \in A_2} u_1(x, y) \text{ for all } x \in A_1.$$

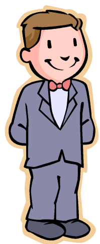


u_1	y_1	y_2	y_3
x_1	**–10**	8	0
$x^*=x_2$	4	7	**3**
x_3	**0**	1	2

Maxminimiser Actions

A maxminimiser maximises the payoff that the player can *guarantee*. A maxminimiser for player 1 solves the problem $\max_{x \in A_1} \min_{y \in A_2} u_1(x, y)$, that is,

$$x^* = \arg\max_{x \in A_1} \min_{y \in A_2} u_1(x, y)$$



u_1	y_1	y_2	y_3
x_1	**-10**	8	0
x^*	4	7	**3**
x_3	**0**	1	2

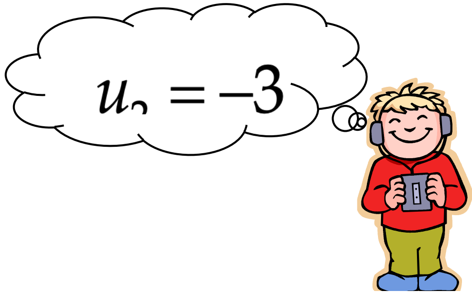
In the following, without loss of generality, we assume that

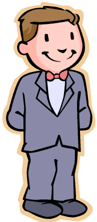
$$u_2 = -u_1.$$

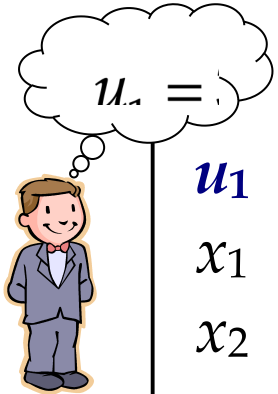
.

What if Player 2 also performs maxminimisation?

$$y^* = \arg\max_{y \in A_2} \min_{x \in A_1} u_2(x, y)$$

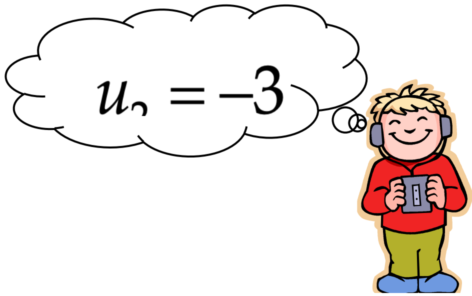


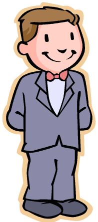




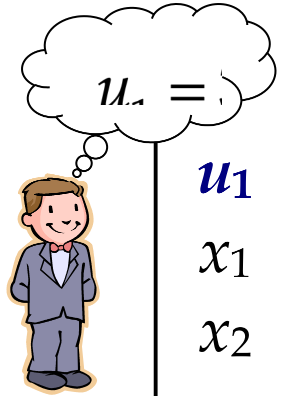
u_2	y_1	y_2	y^*
x_1	10	**–8**	0
x_2	**–4**	–7	**–3**
x_3	0	–1	–2

u_1	y_1	y_2	y^*
x_1	–10	**8**	0
x_2	**4**	7	**3**
x_3	0	1	2





u_2	y_1	y_2	y^*
x_1	10	**–8**	0
x_2	**–4**	–7	**–3**
x_3	0	–1	–2



u_1	y_1	y_2	y^*
x_1	–10	**8**	0
x_2	**4**	7	**3**
x_3	0	1	2

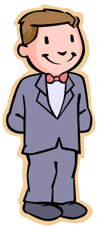
The maxminimisation of player 2's payoff is equivalent to the minmaximisation of player 1's payoff.

$$\max_{y \in A_2} \min_{x \in A_1} u_2(x, y) = -\min_{y \in A_2} \max_{x \in A_1} u_1(x, y)$$
$$(\textit{Note}:\ u_2(x, y) = -u_1(x, y))$$

Equivalence of Maxminimisation of u_2 and Minmaximisation of u_1

LEMMA. Let $\langle\{1,2\},(A_i),(u_i)\rangle$ be a strictly competitive strategic game. Then $\max_{y\in A_2}\min_{x\in A_1} u_2(x,y) = -\min_{y\in A_2}\max_{x\in A_1} u_1(x,y)$. Further, $y \in A_2$ solves the problem $\max_{y\in A_2}\min_{x\in A_1} u_2(x,y)$ if and only if it solves the problem $\min_{y\in A_2}\max_{x\in A_1} u_1(x,y)$.

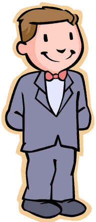
Nash Equilibrium and Maxminimisation



u_2	y_1	y_2	y^*
x_1	10	**–8**	0
x_2	**–4**	–7	**–3**
x_3	0	–1	–2

u_1	y_1	y_2	y^*
x_1	–10	**8**	0
x_2	**4**	7	**3**
x_3	0	1	2

In a Nash equilibrium (x^*, y^*), both players are maxminimising.



u_2	y_1	y_2	y^*
x_1	10	**–8**	0
x_2	**–4**	–7	**–3**
x_3	0	–1	–2

u_1	y_1	y_2	y^*
x_1	–10	**8**	0
x_2	**4**	7	**3**
x_3	0	1	2

Consider a Nash equilibrium (x^*, y^*).

- First, $u_2(x^*, y^*) \geq u_2(x^*, y)$ for all $y \in A_2$, or, $u_1(x^*, y^*) \leq u_1(x^*, y)$ for all $y \in A_2$.

  Hence $u_1(x^*, y^*) = \min_{y \in A_2} u_1(x^*, y) \leq \max_{x \in A_1} \min_{y \in A_2} u_1(x, y)$.

- Similarly, $u_1(x^*, y^*) \geq u_1(x, y^*)$ for all $x \in A_1$.

  Hence $u_1(x^*, y^*) \geq \min_{y \in A_2} u_1(x, y)$ for all $x \in A_1$.

  Hence $u_1(x^*, y^*) \geq \max_{x \in A_1} \min_{y \in A_2} u_1(x, y)$ for all $x \in A_1$.

$$\therefore u_1(x^*, y^*) = \max_{x \in A_1} \min_{y \in A_2} u_1(x, y).$$

In Strictly Competitive Games, Nash Equilibrium Implies Maxminimisation

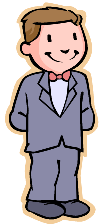
PROPOSITION. Let $G = \langle \{1,2\}, (A_i), (u_i) \rangle$ be a **strictly competitive** strategic game.

If (x^*, y^*) is a Nash equilibrium of G then x^* is a maxminimiser for player 1 and y^* is a maxminimiser for player 2.

This is an important result to remember:

In a **strictly competitive** strategic game, if (x^*, y^*) is a Nash equilibrium, then both players are maxminimising in (x^*, y^*)!

And we now go to the next important result…



u_2	y_1	y_2	y^*
x_1	10	**–8**	0
x_2	**–4**	–7	**–3**
x_3	0	–1	–2

u_1	y_1	y_2	y^*
x_1	–10	**8**	0
x_2	**4**	7	3
x_3	0	1	2

Now we know that in a Nash equilibrium (x^*, y^*),

$$u_1(x^*, y^*) = \max_{x \in A_1} \min_{y \in A_2} u_1(x, y), \text{ and}$$

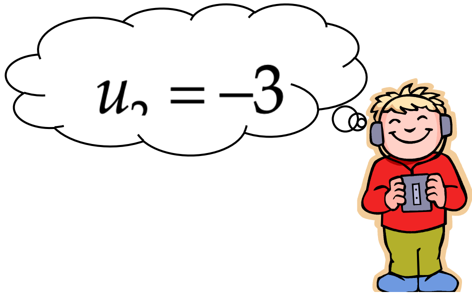
$$u_2(x^*, y^*) = \max_{y \in A_2} \min_{x \in A_1} u_2(x, y),$$

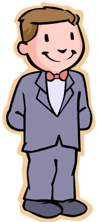
$$u_1(x^*, y^*) = -u_2(x^*, y^*)$$

$$u_1(x^*, y^*) = -\max_{y \in A_2} \min_{x \in A_1} u_2(x, y) = \min_{y \in A_2} \max_{x \in A_1} u_1(x, y).$$

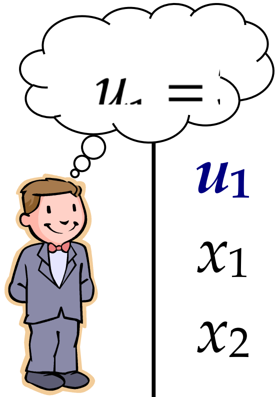
Therefore, $\max_{x \in A_1} \min_{y \in A_2} u_1(x, y) = \min_{y \in A_2} \max_{x \in A_1} u_1(x, y)$.

Value of Strictly Competitive Games





u_2	y_1	y_2	y^*
x_1	10	**–8**	0
x_2	**–4**	–7	**–3**
x_3	0	–1	–2



u_1	y_1	y_2	y^*
x_1	–10	**8**	0
x_2	**4**	7	**3**
x_3	0	1	2

$$u_1(x^*, y^*) = \max_{x \in A_1} \min_{y \in A_2} u_1(x, y) = 3$$

$$u_2(x^*, y^*) = \max_{y \in A_2} \min_{x \in A_1} u_2(x, y) = -3$$

Uniqueness of Payoff in Nash Equilibrium

PROPOSITION. Let $G = \langle \{1,2\}, (A_i), (u_i) \rangle$ be a **strictly competitive** strategic game.

If (x^*, y^*) is a Nash equilibrium of G then $\max_x \min_y u_1(x,y) = \min_y \max_x u_1(x,y) = u_1(x^*, y^*) = v^*$ and thus all Nash equilibria of G yield the same payoff v^* (called the '**value** of the game').

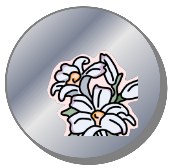
Class Discussion

IN THE GAME MATCHING PENNIES, WE HAVE

$$\max_x \min_y u_1(x, y) = ?$$

$$\min_y \max_x u_1(x, y) = ?$$

They are not equal?



		Player 2	
		Head	*Tail*
Player 1	*Head*	1, –1	–1, 1
	Tail	–1, 1	1, –1

Class Discussion

IN THE GAME OF BATTLE OF THE SEXES, WHICH IS NOT STRICTLY COMPETITIVE, WE DO *NOT* HAVE

$$\max_{x \in A_1} \min_{y \in A_2} u_1(x, y) = \min_{y \in A_2} \max_{x \in A_1} u_1(x, y)$$

		Wife	
		Boxing	*Opera*
Husband	*Boxing*	2, 1	0, 0
	Opera	0, 0	1, 2

This is an important result to remember:

In a **strictly competitive** strategic game, all Nash equilibria yield the same payoff v^*!

And we now go to the last important result…

Given a strictly competitive game, let $\max_x \min_y u_1(x,y) = \min_y \max_x u_1(x,y) = v^*$, hence $\max_y \min_x u_2(x,y) = -\min_y \max_x u_1(x,y) = -v^*$.

Let x^* and y^* be maxminimisers. Hence $u_1(x^*,y) \geq v^*$ for all $y \in A_2$, and $u_2(x,y^*) \geq -v^*$ for all $x \in A_1$. Let $y = y^*$, we obtain $u_1(x^*,y^*) = v^*$; and let $x = x^*$, we obtain $u_2(x^*,y^*) = -v^*$.

We conclude that (x^*,y^*) is a Nash equilibrium of G.

Uniqueness of Maxminimised Payoff Implies Nash Equilibria Being Maxminimisation

PROPOSITION. Let $G = \langle \{1,2\}, (A_i), (u_i) \rangle$ be a **strictly competitive** strategic game.

If $\max_x \min_y u_1(x,y) = \min_y \max_x u_1(x,y)$, x^* is a maxminimiser for player 1, and y^* is a maxminimiser for player 2, then (x^*, y^*) is a Nash equilibrium of G.

- If (x^*, y^*) is a Nash equilibrium of G then x^* is a maxminimiser for player 1 and y^* is a maxminimiser for player 2.
- If $\max_x \min_y u_1(x, y) = \min_y \max_x u_1(x, y)$, x^* is a maxminimiser for player 1, and y^* is a maxminimiser for player 2, then (x^*, y^*) is a Nash equilibrium of G.

This is an important result to remember:

In a **strictly competitive** strategic game, the Nash equilibria are *interchangeable*: if (x, y) and (x', y') are equilibria, then so are (x, y') and (x', y).

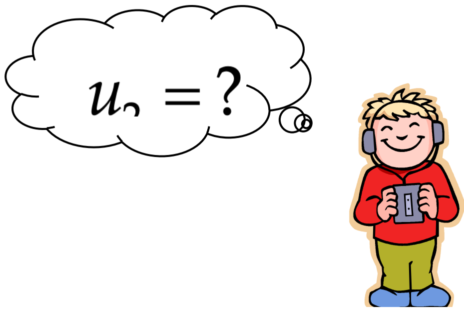
PROPOSITION. Let $G = \langle \{1,2\}, (A_i), (u_i) \rangle$ be a **strictly competitive** strategic game.

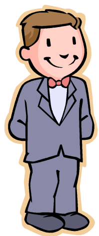
a. If (x^*, y^*) is a Nash equilibrium of G then x^* is a maxminimiser for player 1 and y^* is a maxminimiser for player 2.

b. If (x^*, y^*) is a Nash equilibrium of G then $\max_x \min_y u_1(x,y) = \min_y \max_x u_1(x,y) = u_1(x^*, y^*) = v^*$ and thus all Nash equilibria of G yield the same payoff v^* (called the '**value** of the game').

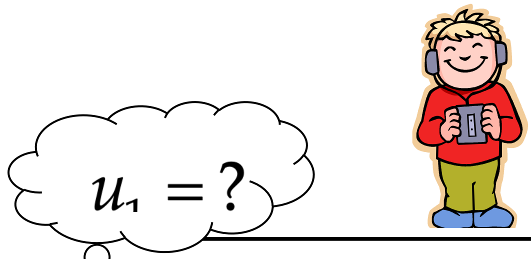
c. If $\max_x \min_y u_1(x,y) = \min_y \max_x u_1(x,y)$, x^* is a maxminimiser for player 1, and y^* is a maxminimiser for player 2, then (x^*, y^*) is a Nash equilibrium of G.

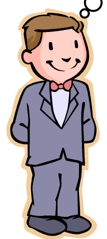
Class Discussion





u_2	y_1	y_2	y_3
x_1	10	–8	0
x_2	–4	–7	–3
x_3	8	–3	–9





u_1	y_1	y_2	y_3
x_1	–10	8	0
x_2	4	7	3
x_3	8	3	9

$$\max_x \min_y u_1(x, y) = ?$$

$$\min_y \max_x u_1(x, y) = ?$$