THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH4010 Functional Analysis 2022-23 Term 1

Homework 7

Deadline: 2022-11-03 Thursday

Notice:

- All the assignments must be submitted before the deadline.
- Each assignment should include your name and student ID number.

1. Let (x_n) be a sequence in an inner product space. Show that the conditions $\|x_n\| \to \|x\|$ and $\langle x_n, x\rangle \to \langle x, x\rangle$ imply $x_n \to x$.

2. Prove that in a complex (resp. real) inner product space, $x \perp y$ if and only if

$$\|x + \lambda y\| = \|x - \lambda y\|$$

for all scalars $\lambda \in \mathbb{C}$ (resp. $\mathbb{R}$).

3. (a) Prove that for every two subspaces X_1 and X_2 of a Hilbert space,

$$(X_1 + X_2)^\perp = X_1^\perp \cap X_2^\perp.$$

(b) Prove that for every two closed subspaces X_1 and X_2 of a Hilbert space,

$$(X_1 \cap X_2)^\perp = \overline{X_1^\perp + X_2^\perp}.$$

— THE END —