THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH4010 Functional Analysis 2022-23 Term 1

Homework 4

Deadline: 2022-10-10 Monday

Notice:

- All the assignments must be submitted before the deadline.
- Each assignment should include your name and student ID number.

1. Prove that for every x in a normed space X, the following identity holds:

$$\|x\| = \sup\left\{\frac{|f(x)|}{\|f\|} : f \in X^*,\ f \neq 0\right\}.$$

2. Let $C[0,1]$ be the vector space of continuous functions on $[0,1]$. Define $\delta(x) = x(0)$ for $x \in C[0,1]$.

   (a) Show that δ is a bounded linear functional if $C[0,1]$ is endowed with the sup-norm. Find the norm of δ.

   (b) Show that δ is an unbounded linear functional if $C[0,1]$ is endowed with the norm

$$\|x\| = \int_0^1 |x(t)|\, dt.$$

— THE END —