

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MATH4010 Functional Analysis 2022-23 Term 1**  
Homework 1  
Deadline: 2022-09-22 Thursday

Notice:

- All the assignments must be submitted before the deadline.
- Each assignment should include your name and student ID number.

1. Let  $B(S)$  be the vector space of all bounded  $\mathbf{F}$ -valued functions on a nonempty set  $S$ . Define

$$\|x\| = \sup\{|x(t)| : t \in S\}.$$

Prove that  $(B(S), \|\cdot\|)$  is a complete normed space (cf. [Textbook, Theorem 3.5]).

2. Show that for  $1 \leq p < \infty$ ,

$$c_{00} \subset \ell_p \subset c_0 \subset c \subset \ell_\infty$$

and all inclusions are proper. (Please refer to [Textbook, Section 3.3 & 3.4] for the definitions of above spaces.)

Also show that

- (a)  $c_{00}$  is dense in the space  $c_0$ ,
- (b)  $c_{00}$  is dense in  $\ell_p$ ,
- (c)  $c_{00}$  is not dense in  $c$ ,
- (d)  $c_{00}$  is not dense in  $\ell_\infty$ ,

in the topology defined by the sup-norm,  $\|\cdot\|_\infty$ .

**Note.** The textbook, *Ovchinnikov, Functional analysis, Springer*, can be downloaded from the library web.

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