THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH4010 Functional Analysis 2021-22 Term 1
Homework 5
Deadline: 2021-11-18 Thursday

Notice:
- All the assignments must be submitted before the deadline.
- Each assignment should include your name and student ID number.

1. Let (x_n) be a sequence in an inner product space. Show that the conditions $\|x_n\| \to \|x\|$ and $\langle x_n, x\rangle \to \langle x, x\rangle$ imply $x_n \to x$.

2. Show that
$$X = \left\{x = (x_n) \in \ell^2 \colon \sum_{n=1}^{\infty} \frac{x_n}{n} = 0\right\}$$
is a closed subspace of ℓ^2.

3. (a) Prove that for every two subspaces X_1 and X_2 of a Hilbert space,
$$(X_1 + X_2)^\perp = X_1^\perp \cap X_2^\perp.$$

   (b) Prove that for every two closed subspaces X_1 and X_2 of a Hilbert space,
$$(X_1 \cap X_2)^\perp = \overline{X_1^\perp + X_2^\perp}.$$

— THE END —