Replication of Aiyagari(1994)

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The paper we replicated is "Uninsured Idiosyncratic Risk and Aggregate Saving" by S. Rao Aiyagari (Aiyagari, 1994). The paper was published in The Quaterly Journal of Economics, Vol. 109, No. 3 (Aug., 1994). Besides this document, you can also find the corresponding notebook here, which contains concrete steps of our replication.

1 Abstract

The paper modifies standard growth model to include precautionary saving motives and liquidity constraints.The paper also examines the impact of the introduction of a particular kind of uninsurable idiosyncratic risk on the aggregate saving rate; the importance of asset trading to individuals; and the relative inequality of the wealth and income distributions.

2 Introduction

The paper wants to provide an exposition of models whose aggregate behavior is the result of market interaction among a large number of agents subject to idiosyncratic shocks. Moreover, another goal of this paper is to use such a model to study the quantitative importance of individual risk for aggregate saving.

The paper mainly has five features: Endogenous heterogeneity, aggregation, infinite horizons, exogenous borrowing constraint, and general equilibrium. i.e. interest rate is endogenously determined since in a steady state equilibrium the capital per capita must equal the per capita asset holdings of consumers, and the interest rate must equal the net marginal product of capital.

3 Related Literature

The model in Aiyagari(1994) originates from Bewley model and a subsequent literature Zeldes (1989), Deaton (1991), Carroll (1992), and puts these kinds of models into a general equilibrium context. These models all share the same key components as mentioned in the previous part. And they are used to study the following topics:

- How much of observed wealth inequality does a particular choice of uninsurable idiosyncratic income uncertainty explain?
- In this model, what is the fraction of aggregate savings due to the precautionary motive?
- In this model, what are the re-distributional implications of various policies?

4 Model

4.1 Individual's Problem

$$\max E_0 \left(\sum_{t=0}^{\infty} \beta^t U(c_t) \right) \tag{1}$$

$$\text{s.t.} \tag{2}$$

$$c_t + a_{t+1} = wl_t + (1+r)a_t \tag{3}$$

$$c_t \geq 0 \tag{4}$$

$$a_t \geq -\phi \tag{5}$$

where ϕ (if positive) is the limit on borrowing; l_t is assumed to be i.i.d with bounded support given by $[l_{min}, l_{max}]$, with $l_{min} > 0$; w and r represent wage and interest rate respectively.

Let $\hat{a}_t \equiv a_t + \phi$ and $z_t \equiv wl_t + (1+r)\hat{a}_t - r\phi$, where z_t can be interpreted as total resources of the agent at date t respectively. Then the Bellman equation is as follows:

$$V(z_t, \phi, w, r) \equiv \max_{\hat{a}_{t+1}} \left(U(z_t - \hat{a}_{t+1}) + \beta \int V(z_{t+1}, \phi, w, r) \; dF(l_{t+1}) \right) \tag{6}$$

Consequently, Euler equation is:

$$U'(z_t - \hat{a}_{t+1}) = \beta(1+r) \int U'(z_{t+1} - \hat{a}_{t+2}) \; dF(l_{t+1}) \tag{7}$$

Solve the model, the decision rule can be written as:

$$\hat{a}_{t+1} = A(z_t, \phi, w, r) \tag{8}$$

And the law of transition would be:

$$z_{t+1} = wl_{t+1} + (1+r)A(z_t, \phi, w, r) - r\phi \tag{9}$$

4.2 Firm's Problem

$$\max F(K, L) - wL - rK \tag{10}$$

where K is the aggregate capital, L is the aggregate labor, $F(K, L)$ is the production function.

4.3 General Equilibrium

In the steady state, variables are time invariant and all markets are clear, i.e.,

- $F_K(K, L) = r + \delta$
- $F_L(K, L) = w$
- $\int l_i di = L$
- $\int a_i di = K$

5 Model Specification, Parameters and Computation

5.1 Model specification and parameters

We follow the parameters in Aiyagari(1994) for calibration. Parameters are listed in the table below.

Production function is Cobb Douglas production function with the capital share taken to be α

$$F(K, L) = K^{\alpha} L^{1-\alpha} \tag{11}$$

Utility function is CRRA utility function with the relative risk aversion coefficient μ.

Parameter	Description	Value
β	Time Preference Factor	0.96
δ	Depreciation Rate	0.08
α	Capital Share	0.36
ϕ	Borrowing Limit	0
μ	Risk Aversion Coefficient	$\{1, 3, 5\}$
ρ	Serial Correlation of Labor Shocks	$\{0, 0.3, 0.6, 0.9\}$
σ	Variance of Labor Shocks	$\{0.2, 0.4\}$

Table 1: Parameters for Calibration

Finally, labor endowment shocks follow an AR process.

$$\log(l_t) = \rho \log(l_{t-1}) + \sigma(1-\rho^2)^{\frac{1}{2}}\epsilon_t,\ \epsilon_t \sim N(0,1) \tag{12}$$

5.2 Computation

This notebook uses EconForge/Dolark toolkit to describe the results and reproduce the tables in the linked paper. And you can our application of this toolkit for this paper in the notebook we created.

6 Key Results

6.1 Aggregate Saving Rates

Table 1 shows the comparison between aggregate saving rates calculated by us and those in Aiyagari(1994). The results in Aiyagari(1994) are shown in section 7 of this document.

Our results are highly similar to but a bit different fom those in Aiyagari(1994). This is very likely because we use a different discrete-valued Markov Chain(MC) to approximate the AR process of individuals' idiosyncratic income shock. In our replication, three grid points (i.e. three MC states), which is set

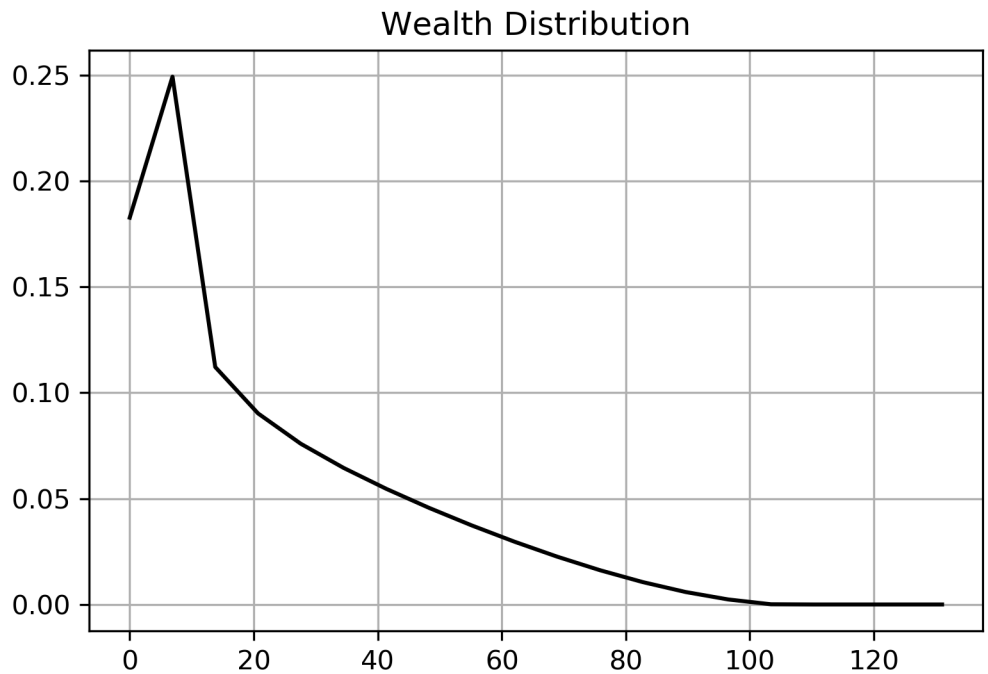
by default in `Dolo` and `Dolark`, are used to simulate the AR process, whereas this number is seven in Aiyagari(1994).

	Risk Averse Coefficient	Variance of Labor Shocks	Serial Correlation	Saving Rate	Saving Rate_Aiyagari
0	1	0.2	0	24.28	23.67
1	1	0.2	0.3	24.28	23.73
2	1	0.2	0.6	24.33	23.82
3	1	0.2	0.9	24.38	24.14
4	1	0.4	0	24.4	23.87
5	1	0.4	0.3	24.48	24.09
6	1	0.4	0.6	24.65	24.5
7	1	0.4	0.9	24.75	25.47
8	3	0.2	0	25.66	23.71
9	3	0.2	0.3	25.73	23.91
10	3	0.2	0.6	26.08	24.25
11	3	0.2	0.9	28.38	25.51
12	3	0.4	0	26.8	24.44
13	3	0.4	0.3	27.17	25.22
14	3	0.4	0.6	28.35	26.71
15	3	0.4	0.9	36.53	31
16	5	0.2	0	26.69	23.83
17	5	0.2	0.3	26.83	24.19
18	5	0.2	0.6	27.41	24.86
19	5	0.2	0.9	31.64	27.36
20	5	0.4	0	28.35	25.22
21	5	0.4	0.3	28.94	26.66
22	5	0.4	0.6	30.8	29.37
23	5	0.4	0.9	44.49	37.63

Table 2: Aggregate Saving Rate

6.2 Wealth Distribution

The following figure shows the wealth distribution, where the x-axis represents the level of wealth and the y-axis represents the share of population.



7 Appendix

According to the results in Aiyagari(1994), the differences between the saving rates with an without insurance are quite small for moderate and empirically plausible values of σ, ρ, and μ. However for high values of σ, ρ, and μ, the presence of idiosyncratic risk can raise the saving rate quite significantly.

The table below is extracted from Aiyagari(1994) and clearly shows this point.

TABLE II

A. Net return to capital in %/aggregate saving rate in % ($\sigma = 0.2$)			
$\rho\backslash\mu$	1	3	5
0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.14	3.2903/25.51	2.5260/27.36
B. Net return to capital in %/aggregate saving rate in % ($\sigma = 0.4$)			
$\rho\backslash\mu$	1	3	5
0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/31.00	−0.3456/37.63

References

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, *109*(3), 659–684.