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4.2 (a) Left to right, assume $A \subseteq B(*)$, and let $x \in A \cap B$, which means that $x \in A$ and $x \in B$, we can conclude $x \in A$, thus $A \cap B \subseteq A(**)$. to prove the other direction, let $x \in A$, by assumption (*) we get $x \in B$, we can conclude $x \in A$ and $x \in B$, which means that $x \in A \cap B$, so we have $A \subseteq A \cap B$, so by this and (**) we have $A = A \cap B$.

Right to left, suppose $A \cap B = A(*)$, let $x \in A$, by (*) $x \in B$, so we have $A \subseteq B$.

Second part, $x \in A \cup B$ iff $x \in B$, it means that there is nothing in A such that is not in B, thus $A - B = \emptyset$.

(b) Left to right, suppose $A \subseteq B \cap C$, let $x \in A$, by previous assumption we have $x \in B \cap C$, which implies that $x \in B$ and $x \in C$, so we have $A \subseteq B$ and $A \subseteq C$.

Right to left, suppose $A \subseteq B$ and $A \subseteq C$, let $x \in A$, by two previous assumtion we have both $x \in B$ and $x \in C$ which implies that $x \in B \cap C$, thus we have $A \subseteq B \cap C$.

(c) Suppose $B \cup C \subseteq A$, let $x \in B$, we can get also $x \in B \cup C$, by previous assumption we conclude that $x \in A$, thus $B \subseteq A$. by similar argument we can show $C \subseteq A$.

(d) $x \in A - B$ iff $x \in A \wedge \neg(x \in B)$ iff $x \in A \wedge \neg(x \in B) \vee (x \in B \wedge \neg(x \in B))$ iff $(x \in A \vee x \in B) \wedge \neg(x \in B)$ iff $x \in (A \cup B) - B$ iff $(x \in A \wedge \neg(x \in B)) \vee (x \in A \wedge \neg(x \in A))$ iff $x \in A \wedge (\neg(x \in A) \vee \neg(x \in B))$ iff $x \in A - (A \cap B)$.

(e) $x \in A \cap B$ iff $x \in A \wedge x \in B$ iff $(x \in A \wedge x \in B) \vee (x \in A \wedge \neg(x \in A))$ iff $(x \in A \wedge x \in B) \vee (x \in A \wedge \neg(x \in A))$ iff $x \in A \wedge (x \in B \vee \neg(x \in A))$ iff $x \in A \wedge \neg(\neg(x \in B) \wedge (x \in A))$ iff $x \in A \wedge \neg(x \in A - B))$ iff $x \in A - (A - B)$.

(f) $x \in A - (B - C)$ iff $x \in A \wedge \neg(x \in B - C)$ iff $x \in A \wedge \neg(x \in B \wedge \neg(x \in C))$ iff $x \in A \wedge (\neg(x \in B) \vee (x \in C))$ iff $(x \in A \wedge \neg(x \in B)) \vee (x \in A \wedge x \in C)$ iff $x \in A - B \vee x \in A \cap C$ iff $x \in (A - B) \cup (A \cap C)$.

(g) $(A-B)\cup(B-A)=\emptyset$ iff both (*) $A-B=\emptyset$ and $B-A=\emptyset$, by (a) we get (*) iff $A\subseteq B$ and $B\subseteq A$ iff $A=B$.

4.4 Suppose it exist, then $A'\cup A$ is equal to universal set which does not exist.

4.5 (a) let $x\in A\cap\bigcup S$, then $x\in A$ and $x\in C$ for some $C\in S$, it means that $x\in A\cap C$, clearly $A\cap C\in P(A)$ so $A\cap C\in T_1$ by definition, thus $x\in\bigcup T_1$. (Note that if we take $A\cap C=C$, then we can say that for some $C\in T_1$ we have $x\in C$). Now let $x\in\bigcup T_1$, then there is some $Y\in T_1$ such that $x\in Y$, but by definition of T_1 we know that $Y=A\cap X$ for some $X\in S$, it means that $x\in\bigcup S$ and $x\in A$, thus $x\in A\cap\bigcup S$.

(b) Let $x\in A-\bigcup S$, we have $x\in A-\bigcup S$ iff $x\in A$ and $x\notin X$ for any $X\in S$. it equally means that (*) $x\in A-X$ for every $X\in S$. we know that any set in the form of $A-X$ such that $X\in S$ is in T_2, thus (*) means that we have $x\in\bigcap T_2$.

$x\in A-\bigcap S$ iff $x\in A$ and $x\notin C$ for some $C\in S$ iff $x\in A-C$ for some $C\in S$, because any set in the form of $A-X$ such that $X\in S$ is in T_2 we have some $x\in\bigcap T_2$.

4.6 if S is not empty, then there is some $C\in S$, by Axiom Schema of Comprehension the set $\{x\in C:(\forall X)(X\in S\to x\in X)\}$ exist. if it is empty, then we can not apply the axiom of comprehension.