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1.1 1.1 We know that both $\{a\}$ and $\{a,b\}$ are subset of $\{a,b\}$, thus $\{a,b\},\{a\} \in \mathcal{P}(\{a,b\})$, it means that $\{\{a,b\},\{a\}\} \subseteq \mathcal{P}(\{a,b\})$ which implies $\{\{a,b\},\{a\}\} \in \mathcal{P}(\mathcal{P}(\{a,b\}))$.

we have $a,b \in \{a,b\}$, but $(a,b) = \{\{a\},\{a,b\}\}$ which means that there is some $C \in (a,b)$ such tha $a,b \in C$, thus $a,b \in \bigcup(a,b)$.

if $a,b \in A$ then $\{a,b\}$ and $\{a\}$ both are subset of A, thus $\{a,b\},\{a\} \in \mathcal{P}(A)$, again it implies that $\{\{a,b\},\{a\}\} \subseteq \mathcal{P}(A)$, thus $(a,b) = \{\{a,b\},\{a\}\} \in \mathcal{P}(\mathcal{P}(A))$.

1.2 1.2 if a and b exist, then by axiom of pairing and powerset $T = \mathcal{P}(\mathcal{P}(\{a,b\}))$ exist and by previous exercise $(a,b) \in T$. because $(a,b,c) = ((a,b),c)$ by previous argument we have $(a,b,c) \in \mathcal{P}(\mathcal{P}(\{(a,b),c\}))$ which clearly exist.

1.3 if $(a,b) = (b,a)$, it follows from Theorem 1.2 that $a = b$ and $b = a$, so $a = b$.

1.4 if $(a,b,c) = (a',b',c')$ then $((a,b),c) = ((a',b'),c')$, by Theorem 1.2 we have (*) $(a,b) = (a',b')$ and $c = c'$, but again by Theorem 1.2 and (*) we have $a = a'$ and $b = b'$.

1.5 Let $a = \emptyset$, $b = \{a\}$ and $c = \{b\}$, then if $((a,b),c) = (a,(b,c))$ we get $(a,b) = a = \emptyset = \{\{a\},\{a,b\}\}$ which is a contradiction.

1.6 We first prove that:

(1) $a = c$ or $d = \square$.

(2) $b = d$ or $c = \triangle$.

To prove (1): $\{\{a,\square\},\{b,\triangle\}\} = \{\{c,\square\},\{d,\triangle\}\}$ implies either $(\bullet)$ $\{a,\square\} = \{c,\square\}$ or $(\star)$ $\{a,\square\} = \{d,\triangle\}$, if $(\bullet)$ then either $a = c$ or $a = \square$, if first we are done, if the second then $\{a,\square\} = \{\square\} = \{c,\square\}$ which means $a = \square = c$, thus in both case $a = c$. if $(\star)$ then either $a = d$ or $a = \triangle$, if first then $\{a,\square\} = \{a,\triangle\}$ which implies $\triangle = \square$, contradiction, so we have $a = \triangle$, then $\{\triangle,\square\} = \{d,\triangle\}$ which implies $d = \square$. so we have either $a = c$ or $d = \square$.

To prove (2):

We also have (*) $\{b,\triangle\} = \{c,\square\}$ or (**) $\{b,\triangle\} = \{d,\triangle\}$, if (*) then either $b = c$ or $b = \square$, if first then $\{b,\triangle\} = \{b,\square\}$ which implies a contradiction: $\triangle = \square$, therefore the second case only remains which implies $c = \triangle$. if (**) then either $b = d$ or $b = \square$, if first we are done, if the second then $\{\square,\triangle\} = \{d,\triangle\}$ which implies $b = \square = d$, so in both case we have $b = d$. so we have either (2) $b = d$ or $c = \triangle$.

So we have (1) and (2), assume that $b = d$ from (2), now consider (1), if first case then we are done. if the second then $b = d = \square$, therefore $\{\{a,\square\},\{\square,\triangle\}\} = \{\{c,\square\},\{\square,\triangle\}\}$ which implies $a = c$.

Assume the second case of (2), then by first case of (1) we have $a = c = \triangle$, therefore $\{\{\triangle,\square\},\{b,\triangle\}\} = \{\{\triangle,\square\},\{d,\triangle\}\}$ which implies $b = d$.

Now consider the second case of (1), then we have $d = \square$ and $c = \triangle$ then $\{\{a,\square\},\{b,\triangle\}\} = \{\{\triangle,\square\},\{\square,\triangle\}\} = \{\{\square,\triangle\}\}$, then $a = \triangle = c$ and $b = \square = d$, we are done.