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3.1 We must prove that the set $\{x : x \in A \text{ } and \text{ } x \notin B\}$ exist. Let $P(x, A, B)$ be the property "$x \in A \text{ } and \text{ } x \notin B$", $P(x, A, B)$ implies $x \in A$, because A exist, we have $\{x : x \in A \text{ } and \text{ } x \notin B\} = \{x \in A : x \in A \text{ } and \text{ } x \notin B\} = \{x \in A : x \notin B\}$, this set clearly exist by the axiom of comprehension.

3.2 Weak Axiom of Existence implies that some set exist, call one of them A and let $P(x)$ be the property "$x \neq x$", by axiom of comprehension the set $X = \{x \in A : x \neq x\}$ exist, it has no element because no object satisfy the property $P(x)$.

3.3 (a) Suppose that V is set of all sets, by Comprehension $X = \{x \in V : x \notin x\}$ exist. Because V is set of all sets, clearly $X \in V$. Now suppose that $X \in X$ then $X \notin X$ by definition, a contradiction. suppose $X \notin X$, then $X \in X$ again by definition.

(b) Assume the contrary, there is a set A that any $x \in A$. then $A = V$ is set of all sets, by previous exercise there is no V.

3.4 By axiom of pairing the set $\{A, B\}$ exist and union axiom implies the existence of $\bigcup\{A, B\}$, let $P(x, A, B) = (x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B)$ by comprehension there is a set that its elements satisfy $P(x, A, B)$ and $x \in \bigcup\{A, B\}$.

3.5 3.5(a) by axiom of pairing there is $\{A, B\}$ and $\{C\}$. again by pairing $\{\{A, B\}, \{C\}\}$. by axiom of union there is $X = \bigcup\{\{A, B\}, \{C\}\}$. Now $x \in X$ iff $x \in \{A, B\}$ or $x \in \{C\}$ iff $x = A$ or $x = B$ or $x = C$.

(b) Take $\{C, D\}$ instead of $\{C\}$ in the previous exercise.

3.6 Assume that $\mathcal{P}(X) \subseteq X$, Now let $Y = \{x \in X : x \notin x\}$, clearly $Y \subseteq X$, so $Y \in \mathcal{P}(X)$, thus $Y \in X$. also we have either $Y \in Y$ or $Y \notin Y$. if first, $Y \notin Y$, if th second $Y \in Y$, thus $Y \in Y$ iff $Y \notin Y$, a contradiction.

3.7 Let $P(x, A, B)$ be the property "$x = A \vee x = B$", apply axiom of comprehension to C, we get the set $X \subseteq C$ such that $x \in X$ iff $x = A$ or $x = B$, so $X = \{A, B\}$.

Let $P'(x, S)$ be the property "$\exists A(A \in S \wedge X \in A)$", apply axiom of comprehension to U, we get the set Y such that $x \in Y$ iff for some $A \in S$ we have $x \in A$, thus $Y = \bigcup S$.

Let $P'(x, S)$ be the property "$x \subseteq S$", apply axiom of comprehension to P, we get the set Z such that $x \in Z$ iff $x \subseteq S$, thus $Y = \mathcal{P}(S)$.