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4.1 (a) transitive.
(b) reflexive, transitive.
(c) symmetric.
(d) $\subseteq$: reflexive, transitive. $\subset$: transitive.
(e) reflexive, transitive, symmetric.
(f) symmetric and transitive.

4.2 (a) for every $a \in A$, $f(a) = f(a)$ since f is a function, thus aEa and E is reflexive. Let aEb, then for some $a, b \in A$ we have $f(a) = f(b)$ but also $f(b) = f(a)$, thus bEa, therefore E is symmetric. Suppose that aEb and bEc, then we get $f(a) = f(b)$ and $f(b) = f(c)$, since f is a function we have $f(a) = f(c)$, thus aEc, E is transitive.

(b) We define $\phi : A/E \to B$ such that $\phi([a]_E) = f(a)$ for every $[a]_E \in A/E$, if $[a]_E = [a']_E$ then aEa', by definition we get $f(a) = f(a')$ which means that $\phi([a]_E) = \phi([a']_E)$.

(c) for every $a \in A$ we have $\phi \circ j(x) = \phi(j(a)) = \phi([a]_E)) = f(a)$, because f and j have the same domain we can conclude $\phi \circ j = f$.

4.3 Because for every $(r, \gamma) \in P$ we have $r = r$ and $\gamma - \gamma = 0 = 2\pi \times 0$ which 0 is an integer multiple of 2π, we get $(r, \gamma) \sim (r, \gamma)$. Now let $(r, \gamma) \sim (r', \gamma')$, then $r = r'$ and $\gamma - \gamma' = 2\pi k$ is an integer, because $\gamma' - \gamma = -(\gamma - \gamma') = 2\pi(-k)$ is also an integer, together with symmetricity of $=$ we get $r' = r$, so we conclude that $(r', \gamma') \sim (r, \gamma)$, thus $\sim$ is symmetric.

Let $(r, \gamma) \sim (r', \gamma')$ and $(r', \gamma') \sim (r'', \gamma'')$, by transitivity of identity we simply get $r = r''$, also $\gamma - \gamma' = 2\pi k$ and $\gamma' - \gamma'' = 2\pi k'$ such that k, k' are both integer, but then $\gamma - \gamma'' = (\gamma - \gamma') + (\gamma' - \gamma'') = 2\pi k + 2\pi k' = 2\pi(k + k')$ clearly is an integer , thus $(r, \gamma) \sim (r'', \gamma'')$.

Consider (r, γ), then there is some (r, γ') such that $\gamma - \gamma' = 2\pi k$ and is an integer. then $\gamma' = \gamma - 2\pi k$, we argue that for some integer k' we have $0 \leq \gamma - 2\pi k' \leq 2\pi$, if there is no such k' that satisfies last inequality then we also do not have $-\gamma \leq -2\pi k' \leq 2\pi - \gamma$ and also $\gamma - 2\pi \leq 2\pi k' \leq \gamma$, dividing by 2π yields that there is no $\gamma/2\pi - 1 \leq k' \leq \gamma/2\pi$, but it contradicts the fact tht for any real number X there is an integer $X - 1 \leq k' \leq X$, so we can take $(r, \gamma - 2\pi k')$.