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3.1 Fix some $n \in N$, We prove the claim $P(m)$="$n < m \to f_n \prec f_m$" for all $k \leq m$ such that $k = n+1$, $P(k)$ holds since $n < n+1 \to f_n \prec f_{n+1}$ holds by assumption (has a true consequence). Now assume that for an m, $k \leq m$, $P(m)$ holds, thus $k \leq m$ and $n < m \to f_n \prec f_m$. assume that $n < m+1$ then either $n < m$ or $n = m$, if $n < m$ then by induction hypothesis $f_n \prec f_m$, since $f_m \prec f_{m+1}$ is true by assumption, we get $f_n \prec f_{m+1}$ by transitivity of $\prec$. if $m = n$ then trivially $f_n \prec f_{n+1} = f_{m+1}$ holds, so we proved if $P(m)$ then $n < m+1 \to f_n \prec f_{m+1}$ which is $P(m+1)$, thus $P(m)$ holds for all $k = n+1 \leq m$, since n was arbitrary it holds for all $m, n \in N$.

3.2 Let $g : A \times N \to A$ be the function that $g(x, n)$ is the successor of x. Let u be the $\prec$-least element of A, by recursion theorem there is a function $f : N \to A$ such that $f_0 = u$ and $f_{n+1} = g(f_n, n) =$ successor of f_n, the function is total since by (a) every element of A has a successor. if p is successor of q then $q \prec p$, thus we have $f_n \prec f_{n+1}$. by previus exercise for every $m < n$ we have $f_n \prec f_m$ thus f is one-to-one. To prove it is onto, assume that there is some $a \in A$ such for no $n \in N, f(n) = a$ (not in $ranf$), let a be the least of them, clearly $a \neq u$ since $f_0 = u$, thus by (c) a is successor of some $q \in A$, because a was the least element that is not in range of f, and because $q \prec a$, q must be in range of f, thus for some $k \in N, f_k = q$, but then $f_{k+1} = g(f_k, k)$= successor of $f_k = q$ which is a, thus $a \in ranf$, a contradiction.