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1.4 (a) for every $(a, b) \in A \times B$, let $f((a, b)) = (b, a)$.
(b) $f(((a, b), c)) = (a, (b, c))$.
(c) Since $B \neq \emptyset$ there is some $b \in B$, let $f(a) = (a, b)$ for every $a \in A$.

1.5 for every $s \in S$ let $f(s) = \{s\}$, clearly $f(s) \in \mathcal{P}(S)$ and for every $s \in S$ there is a unique $\{s\}$, thus f is one-to-one.

1.6 We need to show there is a one-to-one mapping $f : A \to A^S$. if $A = \emptyset$ then $A^S = \emptyset$ and this case is trivial, so assume that it is non-empty, for every $a \in A$, let $f(a) = h_a$ such that $h : S \to A$ is a function such that for every $s \in S$, $h_a(s) = a$, clearly there is just one function for each $a \in A$, therefore f is one-to-one.

1.7 Like previous exercise for empty A the proof is trivial, assume that it is non-emty, so there is some $a \in A$. for every $f \in A^S$ define $F(f) = f'$ such that $f' \in A^T$, $f'|S = f$ and for every $t \in T - S, f'(t) = a$, clearly $F : A^S \to A^T$, to prove that it is one-to-one assume $F(f) = F(g)$, then there are two function $f', g' \in A^T$ such that $f' = g'$ and $f'|S = f$ and $g'|S = g$, it means that $g = f$.

1.8 Since $2 \leq |S|$ there are at least two distinct element $a, b \in S$. define F as follows: for every $t \in T$ let $f_t \in S^T$ such that $f_t(t) = a$, for every $t \neq x \in T$, $f_t(x) = b$, clearly this is function in A^T. To prove it is one-to-one, assume that $F(t) = F(t')$, then $f_t = f_{t'}$, it means that $f_t(t) = f_{t'}(t)$, but $f_t(t) = a$, therefore $f_{t'}(t) = a$ but the only value for which $f_{t'}(x)$ is equal to a is when $x = t'$, from this we get $t = t'$.