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2.1 Assume that X is transitive, ket $a \in X$, then $a \subseteq X$ which means $a \in \mathcal{P}(X)$, thus $X \subset \mathcal{P}(X)$. Conversely suppose $X \subseteq \mathcal{P}(X)$, let $a \in X$ then $a \in \mathcal{P}(X)$, which means $a \subseteq X$, thus X is transitive.

2.2 Assume X is transitive, by previous case we get $X \subseteq \mathcal{P}(X)$, which implies $\bigcup X \subseteq \bigcup \mathcal{P}(X) = X$. For the converse, suppose $\bigcup X \subseteq X$, let $a \in X$, then $a \subseteq \bigcup X$, thus $a \subseteq X$. X is transitive.

2.3 (a) (b).

2.4 (a) (b) (e).

2.5 Suppose that every $X \in S$ is transitive, let $x \in \bigcup S$, then $x \in C$ for some $C \in S$, but S is transitive, thus $C \subseteq S$ which implies $x \in S$, therefore $x \subseteq \bigcup S$. We proved for any $x \in \bigcup S$, $x \subseteq \bigcup S$, thus $\bigcup S$ is transitive.

2.6 Assume that α is a natural number, then it is finite and every subset of it is finite and has a greatest element. For the converse, assume that α is an ordinal and every subset of it has the greatest element, It means that $\alpha < \omega$, (since otherwise $\omega \leq \alpha$, which means $\omega \subset \alpha$ and has no greatest element, a contradiction.), thus it is a natural number.

2.7 Assume that $supX$ is a successor ordinal, i.e. $supX = \alpha + 1$, then $\alpha \in supX$, which means for some $\beta \in X$, $\alpha \in \beta$, since X has no greatest element, there is some $\beta' \in X$ such that $\beta \in \beta'$, it means that $\alpha < \beta < \beta' \leq \alpha + 1$, it means that there is a ordinal between α and its successor, which is a contradiction.

2.8 Consider some $\alpha \in X$, then $\bigcap X \subseteq \alpha$, since $(\alpha, \in)$ is a well-ordering on $\in$, it is a well-ordering on its subset $\bigcap X$. We just show that it is transitive: let $x \in \bigcap X$, then $x \in \alpha$ for every $\alpha \in X$, since α is transitive, we get $x \subseteq \alpha$, so for any $\alpha \in X$ we have if $y \in x$ then $y \in \alpha$, thus $y \in \bigcap X$ and $x \subseteq \bigcap X$, this shows $\bigcap X$ is transitive, hence is an ordinal. it is the least element, since $\bigcap X \subseteq \alpha$ for every $\alpha \in X$.