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- 4.1 (a) transitive.  
(b) reflexive, transitive.  
(c) symmetric.  
(d)  $\subseteq$ : reflexive, transitive.  $\subset$ : transitive.  
(e) reflexive, transitive, symmetric.  
(f) symmetric and transitive.

$E$  is reflexive. Let  $aEb$ , then for some  $a, b \in A$  we have  $f(a) = f(b)$  but also  $f(b) = f(a)$ , thus  $bEa$ , therefore  $E$  is symmetric. Suppose that  $aEb$  and  $bEc$ , then we get  $f(a) = f(b)$  and  $f(b) = f(c)$ , since  $f$  is a function we have  $f(a) = f(c)$ , thus  $aEc$ ,  $E$  is transitive.

(b) We define  $\phi : A/E \rightarrow B$  such that  $\phi([a]_E) = f(a)$  for every  $[a]_E \in A/E$ , if  $[a]_E = [a']_E$  then  $aEa'$ , by definition we get  $f(a) = f(a')$  which means that  $\phi([a]_E) = \phi([a']_E)$ .

(c) for every  $a \in A$  we have  $\phi \circ j(x) = \phi(j(a)) = \phi([a]_E) = f(a)$ , because  $f$  and  $j$  have the same domain we can conclude  $\phi \circ j = f$ .

4.2 Because for every  $(r, \gamma) \in P$  we have  $r = r$  and  $\gamma - \gamma = 0 = 2\pi \times 0$  which 0 is an integer multiple of  $2\pi$ , we get  $(r, \gamma) \sim (r, \gamma)$ . Now let  $(r, \gamma) \sim (r', \gamma')$ , then  $r = r'$  and  $\gamma - \gamma' = 2\pi k$  is an integer, because  $\gamma' - \gamma = -(\gamma - \gamma') = 2\pi(-k)$  is also an integer, together with symmetricity of  $\sim$  we get  $r' = r$ , so we conclude that  $(r', \gamma') \sim (r, \gamma)$ , thus  $\sim$  is symmetric.

Let  $(r, \gamma) \sim (r', \gamma')$  and  $(r', \gamma') \sim (r'', \gamma'')$ , by transitivity of identity we simply get  $r = r''$ , also  $\gamma - \gamma' = 2\pi k$  and  $\gamma' - \gamma'' = 2\pi k'$  such that  $k, k'$  are both integer, but then  $\gamma - \gamma'' = (\gamma - \gamma') + (\gamma' - \gamma'') = 2\pi k + 2\pi k' = 2\pi(k + k')$  clearly is an integer, thus  $(r, \gamma) \sim (r'', \gamma'')$ .

Consider  $(r, \gamma)$ , then there is some  $(r, \gamma')$  such that  $\gamma - \gamma' = 2\pi k$  and is an integer. then  $\gamma' = \gamma - 2\pi k$ , we argue that for some integer  $k'$  we have  $0 \leq \gamma - 2\pi k' \leq 2\pi$ , if there is no such  $k'$  that satisfies last inequality then we also do not have  $-\gamma \leq -2\pi k' \leq 2\pi - \gamma$  and also  $\gamma - 2\pi \leq 2\pi k' \leq \gamma$ , dividing by  $2\pi$  yields that there is no  $\gamma/2\pi - 1 \leq k' \leq \gamma/2\pi$ , but it contradicts the fact tht for any real number  $X$  there is an integer  $X - 1 \leq k' \leq X$ , so we can take  $(r, \gamma - 2\pi k')$ .