About

These notes are from the Metric Geometry and Gerrymandering Group's (MGGG) workshop at Tufts University, August 7th-11th, 2017. For more information about the workshop itself, see `http://sites.tufts.edu/gerrymandr`.

As taking notes in LaTeX on-the-fly is not an easy task, I am sure this document is full of typos, sloppy notation, and small mathematical errors. If you find such an error, please send me an email at {`ianzach+notes[at]seas.upenn.edu`} *so I can correct it.*

MGGG Workshop at Tufts University **August 7th, 2017**

Situating Redistricting

Moon Duchin (Tufts) *Zach Schutzman*

Congressional Representation

Constitutionally mandated, allocations according to decennial Census.

There are issues:

> Census-taking isn't straightforward or perfect
>
> Apportionment isn't straightforward or perfect - how many reps should we have for each state?
>
> Drawing districts isn't straightforward or perfect - this is the topic of the week

Mathematically, we are interested in partitioning a population with attributes

We have a population of nodes, each with attributes

We want to partition the sets into blocks ("districts") and think about how the attribute patterns at the district level compare to that at the population model.

In practice, we also have geographic features to think about (S^2 embedding).

What are the goals?

We can think about proportionality - can we get the districts to "look like" the population at large?

We can gerrymander! - can we maximize/minimize the incidence of some attribute at the district level?

What are the constraints?

Districts must be (very nearly) equipopulous

Districts should be contiguous and non-punctured

Districts shouldn't be weirdly shaped (!)

Math v Politics

Any goal or constraint represents a mathematization of a normative ideal of politics

> Equal population - representational equality (one person - one vote)

Geographical division - bare majorities shouldn't dominate (appeal to the Law of Large Numbers - if people are assigned to a district randomly, a scant majority should make scant majorities in each district)

Shape - may indicate gerrymandering or some other extreme agenda

Proportionality - government should reflect the populace

Competitiveness - elections should be "fair"

Partisan favor - prevent government deadlock

The latter three of these are not encoded in the law.

How to Gerrymander

Packing and Cracking

Definition 1.1 ***Packing*** *is the act of creating a few districts with a strong majority of individuals with a certain attribute.*

Definition 1.2 ***Cracking*** *is the act of spreading out individuals with a certain attribute across several districts so as to make them a minority in those districts*

.

Together, Packing and Cracking looks like taking members of one group and making a few districts where they have a strong majority and many where they are a scant minority so as to minimize that group's representation at the district level.

Evaluating Shapes

Intuitively, any weird agenda should make weird looking districts.

At the legal level, districts are usually only stipulated to be "reasonably compact". What does that mean?

Mathematically, there are numerous definitions for compactness.

Compactness

Isoperimetry

Definition 1.3 ***Isoperimetry*** *is a measure of how close to being circular a region is. The Poslby-Popper score is* $0 \leq 400\frac{\pi A}{P^2} \leq 100$ *and is one way to measure this.*

This is weak because "perimeter" isn't really a thing. We have a Coastline Paradox effect at play.

Convexity

We can look at the convex hull of a district and see how far the district deviates from this. Also not great, because there are some very good reasons for nonconvexity.

Dispersion

Look at things like moment of inertia or how spread the district is. The failings of this are a little more subtle, well-detailed in the literature.

All of this is based on old mathematics.

Courts have discarded maps based on "weird" shapes, but there is no standard. This is the "Eyeball Test".

Race as Issue

People of Color tend to vote for Democrats. We have to think about the issue of race proxying for partisan allegiance.

Large cities tend to vote for Democrats. Since cities are populous and dense, we need to think carefully about how we divide cities. There are strong correlations between method of commuting and Presidential vote in 2016. Three of the top 40 largest cities voted for Trump in 2016 (OKC, Mesa, AZ, Colorado Springs). Even cities in red states go blue.

The IL-4 has two neighborhoods joined by a highway (zero population, of course). The northern chunk is a Puerto Rican neighborhood and the southern chunk is a Mexican neighborhood. This might look like an instance of packing, but it was actually done in order to give these two Latino populations the ability to pick their own representative.

Density and Splittability

Density of population creates "natural gerrymandering" (Chen and Rodden). How you draw the lines in and around cities decides how much packing and cracking you end up doing. In this sense, density and shape both contribute to how easy or hard it is to draw nice lines.

Thinking about District "Guts"

What abstraction should we be thinking about to capture the information we care about?

First, note that our data is discrete - we get block-level data from the census, and individual people are obviously discrete units.

We can think about Census units like vertices in a graph - but what are the edges?

> Adjacency - the spatially obvious thing to do
>
> Distance or travel time
>
> Commonalities - edge between blocks that "look similar"

Curvature as an Approach to Compactness

Graphs have shape, which reveals something about both isoperimetry and dispersion. What if you try to build your district out of a sheet of paper, with a face for each block. This shape will have (discrete) **curvature** which tells us something about the geometry of the distric.

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Partisan Gerrymandering

Steve Ansolabehere (Harvard) *Zach Schutzman*

How to Gerrymander (Revisited)

Suppose we have a square state where the Purple Party members all lives in a square in the center and the Yellow Party members all live in the surrounding region. You get hired to draw the districts (four of them) by the Yellow Party, how do you do it? Packing or cracking does the job here. If the Purples are a slight majority, draw one district that is all Purple, and the Yellow Party can win three districts. If the Purples are a minority, crack them across the four districts so that Yellow can win four.

Definition 2.4 *A **wasted vote** is a vote for the losing candidate or a vote for the winning candidate beyond the* $50\% + \epsilon$ *threshold for victory.*

At the base, packing puts "too many" people of one party into a district and "packing" puts too few, if we are thinking about how representative our districts are of the population. In our toy examples, we can think of drawing districts to waste as many Purple votes as possible. In the packing case, the Purple waste a lot of votes by winning unanimously in their district while the Yellow waste a smaller proportion of their votes in their winning districts, and waste none in the Purple district. In cracking, all of the Purple votes are wasted, while not all of the Yellow votes are.

How to Detect Gerrymandering (Revisited)

Definition 2.5 ***Distortion** is a quantification of how non-representative the legislature is of the voting populace.*

Definition 2.6 ***Partisan bias** is the difference between the proportion of the vote that a party wins and the number of seats that the party wins if the vote is split 50/50. An equivalent definition is that if one party earns* x *share of the vote statewide, they in half of the districts they earn more than* x *and half less than* x *share of the vote.*

For an example, think of FL, NC, or PA, where the Congressional vote is split fairly evenly, but Republicans won a majority of seats.

We can also think of *symmetry*, which considers how much one party wastes votes compared to the other.

Definition 2.7 *The **efficiency gap** is the ratio of the difference between the wasted votes for each party to the total votes.*

The efficiency gap concept gained traction in the current Wisconsin SCOTUS case. We have to ask whether this concept actually captures the notion of equal protection as enshrined in the Constitution and Voting

Rights Act. The case is notable because it was the first time a court found a violation of the 14th Amendment as a result of *partisan* (as opposed to racial or population-based) gerrymandering.

One thing that has been measured is *historical gerrymandering*. Before the 1960s (particularly in the South), partisan gerrymandering was bad. In the 1960s and 1970s, that declined to the point that by the 1990s, partisan bias was minimal at the national level. We see an uptick in the 2010s

Making Good Maps

We see evidence of distortion when all three branches of a state government are controlled by the same party. The REDMAP effort in 2010 led to Republicans taking control of many states, which contributed to the partisan bias increase this decade.

Every 10 years, Congressional districts are up for being redrawn. Since *Reynolds v Sims*, this process is really strict, as zero population deviation is tolerated at the Congressional district level. This process is long and slow in buildup, but districts are drawn quickly, requiring lots of government bureaucracy, then they get the data, then they only have a few months to actually draw the lines.

Public mapping has been a powerful and important change in recent years. Now that data and GIS is available widely, public mapping will only become more important.

AZ and CA have removed the power of redistricting from their legislatures. CA used an independent commission with members not permitted to be themselves or relatives of State employees. While this sounds crazy, it did a a really good job making fair maps. In AZ, they had three Democrats, three Republicans, and one Independent member who was targeted by politicians and court cases, although she eventually succeeded. ("Fairness" here refers to partisan bias and efficiency metrics).

The VRA and its interpretation of the 14th Amendment present an "equal treatment" idea of representation, which may come into play when SCOTUS hears arguments later this year.

Going to Court

Compactness is an important tool, because it is a mathematical tool which is easy to understand, easy to understand, and easy to interpret. NC-12 got thrown out partially because of arguments of it being the least compact district in the US, and one of the least compact in history.

Courts care about equal treatment. Expert witnesses are employees of the court, and cannot be seen to be taking sides.

Historians have (unfortunately) been largely absent through these cases. Historians are good at evaluating how (un)equal treatment and intentionality has impacted people.

We also need to think about consequences. Stranding minorities isn't illegal, but it has profound implications on outcomes and is an important argument in a courtroom setting as potential evidence of gerrymandering.

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Voting Rights Litigation

Kristen Clarke (LCCR) *Zach Schutzman*

The Voting Rights Act

Yesterday was the 52nd anniversary of the VRA (1965). The VRA is an important part of the context and history of the issues we'll talk about this week.

The VRA is one of the most important piece of civil rights legislation. It followed the incidents at the march from Selma to Montgomery and directly targeted racial discrimination in voting, banning literacy tests, grandfather clauses, and other tools of disenfranchisement. Certain states (mostly the South, but also NY and CA) were also required to get federal pre-clearance for passing voting legislation.

Section 2 of the VRA protects minority voters' equal opportunity to elect a representative of their choice, i.e. create "minority-majority" districts.

In 2006, Congress reauthorized the VRA for 25 years, including Sec. 5, by a wide majority. Things like packing and cracking of minorities and canceling a community election to prevent African-Americans from running for town council and mayor are examples of things struck down under the authority of Sec. 5.

Groups opposed to policies which attempt to correct historical wrongs, such as the VRA or race-conscious admissions policies, challenged the VRA. In 2009, SCOTUS (*Austin Municipal Util. Dist. No. 1 v Holder*), questioned, but did not strike down, Sec. 5. In *Shelby County v Holder* (2013) made a direct Constitutional challenge to Sec. 5. SCOTUS found that the coverage formula, used to determine which states were subject to Sec. 5, was unconstitutional, striking it down. This verdict opened the floodgates for a lot of the voter suppression we are seeing today, such as ID requirements.

Experts found that 600,000 people were disenfranchised the day this verdict passed, largely poor (disproportionately minorities). Costs of getting an ID are $20+, which is significant for people living below the poverty line. North Carolina cut early voting, eliminated pre-registration for teenagers, eliminated same-day registration, and made absentee voting more difficult, and this law would likely have been blocked by Sec. 5. A Court of Appeals found that this law discriminated against minorities with near "surgical precision" after examining how the State used data like Black voters huge participation in early voting to inform how it made its restrictions.

Today's Congress is fairly unproductive, and it does not look to be a fruitful avenue for protecting voting rights. Progress is being made case-by-case in court challenges. Unfortunately, in order to make a court case, you need evidence that the discrimination is occurring, which entails having to live under these repressive laws.

In Texas, 25% of Blacks, compared to 8% of Whites, do not have an ID valid for voting. As an example, concealed carry permits are valid (held disproportionately by White men, but student IDs do not).

This doesn't just affect Congressional elections, but also state- and municipal-level elections which also use redistricting procedures. Over 8,000 jurisdictions will engage in redistricting for the first time without Sec. 5.

Fears of vote fraud are being used to justify implementing restrictive laws. Claims that undocumented

Americans are voting illegally are an unfounded but powerful tool used to create support for these laws. Recently, the Election Integrity Commission (chaired by Kansas Secy. of State Kobach) is being used to promote voter suppression laws. CrossCheck, a process which checks if people are voting in more than one place, has been a big thing with Kobach, but the program has been found to have a 99% error rate in matching people registered in more than one place.

Redistricting

There is no cookie-cutter approach to redistricting that will solve all of the issues with discrimination simultaneously, and solutions must involve careful analysis of data, laws, and voting patterns.

Typically, redistricting occurs decennially, following the Census. Recently, some states have been doing "mid-decade redistricting". A suit recently has been brought against Georgia for a 2015 redistricting plan which carves out minorities in places where there have been demographic shifts. As an example, State District 105 had a 550 vote margin in 2012, with the White incumbent barely edging out the Black challenger. The state redrew the lines to widen the margin protecting the incumbent.

Having representation which reflects the populace is important for social justice issues, particularly at the local level. School boards and city councils dictate important factors regarding education and policing, both areas where issues of discrimination pop up.

We also need to think about trade-offs created by constructing majority-minority districts. Doing so does provide important political power to these minority voters, but it comes at the cost of possibly creating a plan which packs minority voters and/or packs Democrats (given current political trends).

Another important question is how to handle incarcerated citizens. Where do you count them as residents for apportionment, whether or not they can vote, and the racial makeup of the prison population are all important things to think about when drawing district lines and figuring out apportionment.

MGGG Workshop at Tufts University **August 8th, 2017**

Mathematics - Compactness and Curvature

Moon Duchin (Tufts) *Zach Schutzman*

Discrete Curvature

Definition 4.8 *δ-hyperbolicity is a large-scale, metric version of sectional curvature. Formally, a space is δ-hyperbolic if the sides of every geodesic triangle are within a δ-neighborhood of each other at all points along the sides.*

We can think of this as a measurement-based analysis of how “thin” triangles are. This has a nice correspondence with the concept of negative curvature. δ is a measure of how ”thin” the triangles are. This comes from geometric group theory, and we’d like to figure out how to apply this kind of thing to finite graphs.

Definition 4.9 ***Ricci curvature*** *asks whether the distance between the neighbors of x and the neighbors of y is greater than the distance between x and y for two points x, y in a metric space.*

In the discrete case, we lose the nice pairwise correspondence between points. Instead, we think about an L_1 transportation distance from neighbors of x to neighbors of y (the cost to move one unit of weight one unit of distance is one).

Definition 4.10 *The* ***transportation distance*** *is the minimum over all transport plans.*

Inspirational Theorems

Theorem 4.11 *Gromov: (For infinite groups) A region is δ-hyperbolic if and only if its area and perimeter grow at the same rate. Otherwize, area grows at least proportionally to the square of the perimeter.*

Theorem 4.12 *Duchin:(For infinite groups) If a ball is δ-hyperbolic, then the average distance between two points is proportional to the maximum distance between any two points.*

We’d like to prove large-scale (but finite) theorems about the discrete setting.

Bottlenecks in graphs could create negative curvature, meaning that negative curvature edges are good ones to cut when we’re drawing our districts.

Optimal Partitioning

Two Morals:

- Eigenvalues tell you a lot of things about geometry

- Optimal partitioning is computationally hard and not something that produces good districts

What if we try to minimize the first eigenvalue λ_1 of the laplacian over the region? This does have a nice connection to curvature (bounds on curvature imply bounds on the eigenvalue).

Theorem 4.13 *Faber-Krahn: A ball minimizes λ_1.*

Large λ_1 corresponds to high eccentricity (in the elliptical sense).

There is also a notion for graphs, where a graph gradient looks like a discrete difference between adjacent points (we'll take squares). Boundaries of a subset correspond to those nodes outside the subset adjacent to ones in the subset.

Definition 4.14 *A collection of regions $\Omega_1 \dots \Omega_n$ is an optimal partition of Ω if the Ω_i is a proper partition, each is of equal size, and $\sum \lambda_i$ is minimal across all proper equal size partitions.*

In the redistricing setting, population plays the role of "area".

Theorem 4.15 *Ramos-Tavres-Terracini: The minimizer always exists and we get nice regularity properties, but we don't get the property that our districts are connected.*

Theorem 4.16 *Quantitative Stability: If your domain Ω is the same size as the unit ball, then λ_1 of Ω tells you how close to being a ball Ω is.*

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Race and Redistricting

Ellen Katz (UMich) *Zach Schutzman*

Modern History of Race and Voting

In Tuskegee, AL (1957), city lines were redrawn by the State Legislature to excise the Tuskegee Institute, removing many Black voters from the city. Lower courts ruled that legislatures have authority to redraw lines, but SCOTUS reversed the decision, finding that the purpose of drawing lines was explicitly racial in a way that disenfranchised Black voters, violating the 15th Amendment. In this case (and *Carr* two years later) established authority and precedent for courts to intervene in the redistricting process.

There is a tension in modern political discourse over majority-minority districts, as there is the question of whether they give minorities the ability to elect a candidate of choice or they are a form of packing minority voters.

In the early 1960s, voter registration in Alabama was heavily White-dominated. Related protests and marches led to Johnson passing civil rights legislation, including the VRA. Certain criteria subjected a jurisdiction to the Sec. 5 federal pre-clearance provisions discussed by Kristen Clarke (see Talk 1.3). This was an "intrusive remedy", as it flipped the presumption of legislative action being "good until proven otherwise" to one of "bad until proven otherwise" in the areas covered by Sec. 5. Following the VRA, Black registration and participation shot up almost immediately.

This led to a discussion about how political participation isn't just free voting access, and we needed to think about how district lines are drawn. SCOTUS heard cases about racial vote dilution. They found that just because you *can* draw a majority-minority district doesn't mean that you have to, that multimember districts in Dallas County, TX violated the Equal Protection Clause, and that evidence of intentional discrimination is necessary to make a claim of disenfranchisement. Congress responded by amending the VRA to include a results-based provision for claiming discrimination.

In 1982, Justice Brennan in *Gingles* argued that a numerous minority group living in a sufficiently compact area which voted in a cohesive bloc and the majority group consistently voted against this minority constituted a sufficient test for violation of the new Sec. 2 provisions. Within a few decades, the number of African-Americans in elected positions rose to historic levels. In *Holder*, a county which had a single elected leader and a 20% Black minority discussed changing their government to a five-member group. Thomas and Scalia decide that racial vote dilution isn't a thing and dissent in this case and all other racial voting cases. In a case arguing that Florida violated Sec. 2 by not drawing enough majority-minority districts. Souter argued that since the number of majority-minority districts was roughly equal to the proportion of minorities in the population, there were enough. He argued that majority-minority districts are a "second-best" solution, as in an ideal world, voters could form coalitions and a person being a member of a minority group doesn't preclude their ability to get elected.

Recent Events

In the *Shaw* cases in the 1990s, voters argued that the North Carolina districting was so irregular that there was no sufficient justification for it. The courts find in *Shaw* that the gerrymander was not much different than the Tuskegee case and that it was not acceptable. The courts said they would apply strict scrutiny to districting, where a strong cohesive argument is necessary to justify a districting.

The debate turned to the balance between forming majority-minority districts and racial gerrymandering in the 2000s. District-drawers decided they would use party, rather than race, to gerrymander. This cut both ways. In *Georgia v Ashcroft*, Democrats "unpacked" majority-minority districts to do a Democrat-favored partisan gerrymander. SCOTUS found that this was acceptable, as forming coalitions with White Democrats would still be a possible route to electing a candidate of choice. Congress overturned this when it reauthorized the VRA.

The court found in a Texas redistricting effort which, mid-decade, redrew lines around Laredo, dismantling a majority Latino district, that the state violated the VRA. The Roberts court takes a much narrower view of racial discrimination in voting, culminating in *Shelby County v Holder*, which struck down Sec. 5 of the VRA and opened the floodgates for some of the modern techniques of disenfranchisement. A new challenge to Sec. 2 may result in further dismantling of the VRA.

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The Quantitative Anatomy of a Sec. 2 Case

Megan Gall (LCCR) *Zach Schutzman*

Redistricting Criteria

Redistricting is a relatively quantitative process in the legal world. There are federal requirements, such as equipopulous, single-member districts and VRA provisions, each state has its own laws, and there are some local groups which also have goals and input on the process. The general process is to follow the state rules until you hit a federal violation, then take a step back. These requirements necessitate prioritization and thoughtfulness in the procedure.

Baker and *Sims* found that Congressional and State legislative districts must be equipopulous, and single-member. Federal districts must be 'substantially equal' in population, state districts must be 'as equal as practically possible'. This differs because on the scale of hundreds of thousands of voters, it's easier to draw equipopulous districts along reasonable lines as compared to small state districts which have hundreds of voters. Conversely, a 10% deviation at the state level is a difference of maybe a few dozen voters, whereas at the federal level, you might have some districts with tens of thousands more or fewer voters than others. The moral is that large deviations aren't necessarily a problem, but should be scrutinized. Most states have deviations under 10%, but a few are larger. Rulings are also squishy about who gets counted, such as non-citizens, prisoners, and nonresident military.

There are some traditional principles for redistricting. These include

- Compactness
- Contiguity
- Preservation of Political Subdivisions (don't split towns/counties/wards)
- Preservation of Communities of Interest (does not include race or voting blocs)
- Preservation of District Cores (new lines shouldn't be too far from old lines)
- Protection of Incumbents

There are many tests for compactness and very few jurisdictions where it is precisely defined. These can be useful for comparing plans.

There isn't any jurisprudence surrounding splitting political subdivisions. Do we split as few as possible, potentially many times, do we split many areas into few chunks?

Communities of interest can include pretty much anything except race. It's broad and loosely defined.

Some states prohibit, others require protecting incumbents.

There are also local rules, which do not carry the force-of-law, but can have critical influence on the process. The over 8000 jurisdictions which will be redistricting for the first time without Sec. 5 of the VRA will be an important experiment for local districting principles.

The VRA

The system of slavery is a sad, but critical piece of US history. Following the Civil War, the 13th, 14th, and 15th Amendments which granted new political power to Black citizens. Following Reconstruction, tools like literacy tests, poll taxes, and grandfather clauses were used to suppress the Black vote. Notably, in the Delta region in Mississippi, a largely poor, Black region, was becoming a less safe seat for White incumbents as Black enfranchisement grew. The State House rejected a serious cracking effort as being 'too obvious' and came up with a plan with a slight White majority, and gridlock ensued. The result was that one district that was by population a scant Black majority, but Whites still held the majority of voting-age citizens and registered voters. The VRA was designed to correct issues like this.

In *Thornburg v Gingles*, SCOTUS established preconditions to demonstrate a violation of the VRA. These three criteria are

1. Is the racial or language minority large and compact enough to be able to draw a majority-minority district?
2. Is voting racially polarized? If so, who is their candidate of choice?
3. Are the minority voters' candidates of choice usually defeated?

All three of these must be satisfied for a claim of violation.

The first criterion requires little more than Census data and voter registration information.

Since ballots are secret, getting data for the second and third criteria is not quite so straightforward. Since ballots are secret, we need to build a statistical model to figure out whether minorities are voting as a bloc for losing candidates. We need candidate vote totals, we need candidate race/ethnicity, party ID, incumbency, and other relevant features. We also need electorate breakdown by race/ethnicity. We also need the shapefiles to do the GIS analysis, and these aren't always easy to get. Getting all of this data and cleaning it is one of the biggest hurdles in the process.

There are numerous statistical methods for estimating whether minority voters vote as a bloc. One basic one is assumption of homogeneous precincts, which is a strong assumption, but is a good way of eyeballing the data to reveal some trends.

More complex is bivariate ecological regression, which relates the ethnic/racial composition of a precinct and the votes. This is more sophisticated, but can produce results which are 'out of bounds'.

Ecological inference introduces bounds with maximum likelihood estimation. This works well for places with two racial groups and also provides confidence intervals, which are nice.

Newer models are more computational and lean on Bayesian hierarchical models, which works nicely for more than two racial groups.

A Case Study

IL-4 is the 'earmuffs' district which unites a northern and southern Latino neighborhood via a stretch of highway. This could have been drawn compactly, but not without disrupting a district that was majority Black. It would be impossible to draw a plan with a majority Latino and three majority Black districts without at least one being a little weird. Illinois doesn't require compactness in congressional districting, and in this case, the state's interest in having a majority Latino district supercedes the interest in compact districts, so it holds up to court scrutiny.

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Computational Issues in Partisan Gerrymandering

Wendy Cho (UIC) *Zach Schutzman*

Preliminaries

Gerrymandering is inherently suspicious (*gerrymandering* is a loaded term) and there are not a lot of legal constraints on the process.

Most elections are non-competitive (over 90% of Congress gets reelected every cycle)

There are wide gaps in WI, NC, PA between proportion of the vote and proportion of the seats won by each party

SCOTUS has never declared a districting to be unconstitutional, partially because there isn't a good test for it.

Redistricting is a very interdisciplinary problem

Computational Issues

Even in the most basic case, redistricting is NP-Hard and the computational complexity becomes an issue as more is added to the problem

Why do we want to draw maps, as opposed to just evaluating existing ones? No map exists in isolation. Deciding if something is a partisan gerrymander depends on *what else could have been done* and what a sensible baseline is. Just saying that something has "unfairness value of 5" doens't tell you anything about how good or bad your map is. Is 5 an outlier? Is it within the expected range of your distribution? In a sense, we need to be able to look at the entire space of maps.

This problem is hard and old. Vickrey and others tried in the 1960s, but it could only be computed for toy problems. Can we draw a random sample of maps? What about a search algorithm which looks for good/optimal maps? Our goal should be a very large sample of high-quality, independent maps.

- Very large here means we want on the order of billions of maps
- High-quality means that the maps need to satisfy legal constraints and look like something a human could have done
- Independent here is in the statistical sense

Local Redistricting

Markov Chain Monte Carlo does a good job in theory, but the size of the problem is still an issue. Current approaches look at sampling around a fixed center rather than searching the whole space, i.e. *local* *redistricting*, which preserves the core of the district. Even these restrictions still yield a problem that is computationally expensive, and doesn't produce samples on the order of billions.

One big issue is that if the current map is an egregious gerrymander, does it even make sense to use that as the center for drawing our new map. The reverse can be true as well. Something can be locally very bad, but actually be a good plan with respect to the entire space of maps.

Chen and Rodden drew random maps which started with certain boundaries that were predefined to be included, but this still suffers from the same issue of bias.

The moral is that simplification isn't a solution.

Evolutionary Algorithms

Since MCMC isn't tractable as a method to search the whole space of maps, we need another approach. Wendy works on a massively parallel evolutionary algorithm which runs on a supercomputer called PEAR. PEAR doesn't require a random walk, as each processor actually runs an independent process. More about this will be discussed in the afternoon talk (next).

We can sample maps with or without weights on certain features. For example, we can tell the algorithm to ignore partisanship and then evaluate how partisan the distribution looks. We can examine how introducing partisanship then affects the draws from the distribution.

Information is changing society very quickly, but the fields of social science have been slower to respond. We can bring computational tools into these domains to affect positive change in the political process.

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Sampling the Space of Maps

Wendy Cho and Yan Liu (UIC) *Zach Schutzman*

This is the thrilling conclusion to the previous talk.

Introduction to Computational Methods

We can think of redistricting as a combinatorial optimization problem, were we want to minimize or maximize an objective subject to allocation and spatial constraints. We can attack these with industrial solvers, which use heuristic approaches. Some of these include local search, simulated annealing, genetic or evolutionary algorithms, particle swarms, ant colonies, and MCMC. We need to worry about convergence and solution quality. There is often a trade-off between speed of convergence and the quality of the solution, as implementations which converge quickly do not explore a large amount of the search space. For any of these methods, there are numerous knobs we can turn to adjust these features.

Some Details About the Genetic Algorithm

In general, GAs/EAs mimic an evolutionary process. You begin with an initial population and use operators to build a new population based on the original population. These are selection, mutation, crossover, and replacement. We have a fitness function which evaluates the quality of solutions, and we also have a stopping criterion to determine a terminating point of the algorithm.

The added constraint of spatial contiguity makes this problem a little less straightforward. In general, a crossover operation, which combines parts of two solutions to generate a new one, or mutation, which randomly flips a bit in a single solution, will probably not give you a new solution which has contiguous districts.

How do we express contiguity as a linear constraint? We can pick a reference point and direct a graph towards that point, but this isn't trivial and wasn't developed until last decade.

How we embed spatial features is also an interesting question. We can roll them into the fitness function, but classic genetic algorithms are pretty bad for spatial optimization, so we need some new ideas.

The various measures we use are important. We include population deviation, compactness, boundary preservation rate, and others. We also need to choose how we incorporate these, such as which ones we want to optimize against, and how to combine multiple measures.

Our data structure will be an adjacency graph with the boundary units of each zone. We stipulate that every solution must be hole-free and contiguous.

The first step in a GA is to pick an initial population. The authors approach is to randomly seed the districts, then grow them into equipopulous districts in a round-robin, semigreedy way. Mutation is done by making small random adjustments at the boundaries of districts. This maintains contiguity. Crossover is a little more complex. We could try to overlay the two parent solutions, then allocate the stuff that doesn't quite fit, but this doesn't work, because you might end up with a lot of small pieces that don't contribute a lot to your objective, or some big chunks that are unchanged. The current (unpublished) approach is to overlay two solutions, pick one as a reference and do a walk towards that solution, and along that walk there are many solutions you can pick.

Massive Parallelization

Now we need to think about how to parallelize this procedure in order to get a very large number of maps. We need to worry about communication at this scale, because being able to choose parents globally give a huge boost to potential solution diversity, computing in parallel the fitness evaluation can improve runtime, and we need a way to globally do replacement in a sensible way. The way we do this is by imposing a communication topology on the network of processors, and only letting processes talk to their neighbors. Every few iterations, a process can send to its neighbor a good solution, a bad solution, information about where it got stuck, or some random noise to avoid repetition in the search.

This all has some technical hurdles at the scale of hundreds of thousands of processes. Challenges include maintaining a high rate of iterations, effective initialization of the population, fast manipulation of spatial data, and efficient optimization of multiple objectives simultaneously.

MGGG Workshop at Tufts University **August 9th, 2017**

A Legal and Conceptual Primer on Political Gerrymandering

Guy-Uriel Charles (Duke) *Zach Schutzman*

Why Does the Court Think About the Problem the Way It Does?

One of the fundamental problems of American democracy is the structure of our elections and government

The big question is whether political gerrymandering cases are justiciable (can the courts hear them)? The Constitution doesn't give federal courts the authority to do so explicitly, and the devolution of rights to the states suggests that gerrymandering issues should be resolved at the state level rather than the federal level, and tradition and history has put few limitations on the ability of states to design electoral structures.

Additionally, the Constitution puts very few limits on the states. Limits on state elections are pretty much limited to the 14th, 15th, 19th, and 26th Amendments, as well as precedent from *Baker v Carr*, which upholds the principle of one-person-one-vote, derived from the 14th Amendment. For federal elections, we have the 14th, 15th, 19th, and 26th Amendments, plus *Baker* as before, plus the 24th Amendment preventing poll taxes, but there is no affirmative right to vote in federal elections. Additionally, states have some control over federal elections. Each state can regulate elections, and the 17th Amendment affects election of Senators.

There are a few federal laws imposing limitations, such as the VRA, most importantly Sec. 2 and Sec. 5, the National Voter Registration Act (1993), which requires voter registration be available when applying for a driver's license, and the Help America Vote Act (2002), which sets someelection administration standards. However, the terrain is largely lacking in federal and Constitutional constraint on elections.

The Court views redistricting as a vital state function, inherently political, and a domain where the federal government's role is limited.

The Anatomy of *Davis v Bandemer* (1986)

Indiana Democrats received 51.9% of the vote but only 43 of 100 seats in the lower house, and made a claim of vote dilution. The majority of the Court held that political gerrymandering are justiciable, and a plurality held that the district court applied an incorrect and not sufficiently high standard. The Court agrees the cases are justiciable, but they can't agree *why*.

The big question is whether there are judicially manageable and discernible standards. In other words, is there a Constitutional theory of harm which grants the federal courts the ability to address these cases? In other other words, is this an issue of constitutionality or one of courts setting policy?

Initially, we should think about these standards as a legal and constitutional question about whether the Constitution gives the Court the tools and ability to tackle this problem. In *Bandemer*, the Court relies on reapportionment cases, seeing that malapportionment and political gerrymandering both infringe on fair representation, the Court devised the one-person-one-vote principle as a result of malappotionment cases, and the Equal Protection Clause protects the right to fair representation in state legislatures. The majority opinion is based on racial gerrymandering precedent, as both cases are about vote dilution (cracking Black constituencies and cracking Democrat constituencies).

The majority falls apart in thinking about the standard to apply. The plurality opinion is that plaintiffs must prove that discrimination was intentional and that the results of the actions had a discriminatory effect. The Court says that intention of discrimination is a very low bar, stating that whoever is drawing the districts presumably know what the political breakdown of the representatives will be. The plurality also accepts that discriminatory effect is part of the political process and does not sufficiently form a constitutional violation. One single election is not a large enough sample size to prove a consistent degradation of voters' ability to influence the political process.

Justice O'Connor write an opinion concurring with the judgment and stated that partisan gerrymandering cases for major political parties are non-justiciable, and the logic of the plurality opinion leads down the slippery slope to proportional representation, which is clearly not required by the Constitution. Since major political parties have the power to defend themselves in the political process, they are not analogous to racial groups.

Justice Powell concurred with the judgment but dissented from the plurality's standard. He agrees that intentional discrimination and discriminatory effect are required, and that both standards were met in this case. However, he states that discriminatory effects should be determined by the shape of the districts, deviation from traditional districting principles, and to what extent the plaintiffs were locked out of the legislative and political process, such as whether the minority party was allowed to participate in the districting process.

Political vs Racial Gerrymandering

Political and racial discrimination are different issues. First, racial identities are protected by the 14th Amendment, whereas there is no constitutional protection for political party identification, and the 14th Amendment applies to both benign and malicious classifications. Additionally, racial minorities are discrete and insular, and we worry about the impact of majoritarian rule on these communities. Conversely, the claims of political parties are about frustration of outcome. Finally, we have federal legislation protecting racial minorities, such as the VRA, based on a long history of discrimination in this country. There are some similarities, however. Both are 'transmitted' by birth, tend to be unchanging, there is geographical sorting, and there are strong correlations between racial and party identification. Additionally, claims defending partisan gerrymandering look a lot like claims defending racial gerrymandering in terms of disguising intent.

Vieth

Plaintiffs challenged a PA gerrymander of federal House districts. The court split 4-4-1, finding that gerrymandering is a state issue, and since no cases have been adjudicated under *Bandemer*, the Court did not throw out the PA districting. Kennedy wrote that the courts shouldn't intrude on the nation's political process by regulating lines drawn, and that the Court should not adopt a standard which favors one party over another. The goal shouldn't be to find excessive or extreme gerrymanders, but rather the subtle and clever political gerrymanders. This shifts the view from the 14th Amendment (Equal Protection) to the 1st Amendment (Freedom of Affiliation). The question becomes whether the state has put an undue burden on people as a result of their party affiliation. We need to be come up with good standards of what constitutes such a burden.

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DistrictBuilder

Bob Cheetham (Azavea) *Zach Schutzman*

Azavea is a small software company in Philly which uses geospatial data for civic and social impact. Their tool DistrictBuilder is available as an open source piece of software. Their Summer of Maps program matches companies with geospatial projects to students with those skills.

The Software

DistrictBuilder came about kind of accidentally. Azavea's Cicero database of elected officials naturally has a spatial component, as it needs to track legislative districts and how they change. A research project on evaluating how bad Philly-area gerrymandering actually was led to producing a white paper and development of this tool.

Researchers interested in automated redistricting worked with Azavea to create an open source, web-based, easy-to-use, non-partisan, map generating tool. The software allows users to create, edit, and save multiple district plans, use blank, existing, or template maps to design plans, import and merge plans from other systems such as GIS software, display data, existing political boundaries, and demographics, automatically calculates relevant statistics, and integrates with other mapping systems like GoogleMaps, and ArcGIS.

DistrictBuilder has been used at the state and local level for public mapping efforts, such as the Philadelpia-area competition Fix Philly Districts. One of the teams in the Virginia competition created a plan which was used as a basis for a legal challenge against a racial gerrymander. In Minneapolis, Latino and Somali community groups drew districts to serve their local communities, and commissioners adopted these districts into the plan. Fix Philly Districts was an unofficial project born out of the government's reluctance to engage on the issue of unfair redistricting. Azavea ran a competition, where plans were scored on how many city wards were split, and it got so much attention that the city reengaged in the process.

New Census APIs and improvements to distributed architectures present new opportunities for running software like this at-scale.

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Geometry and Data: Algorithmic Approaches to Redistricting

Justin Solomon (MIT) *Zach Schutzman*

Preliminaries

Justin's group uses tools from geometry to think about application problems in areas like graphics, learning, and optimization.

The redistricting problem has a significant geometric component, so computational geometers may have something to offer. There is a wide range of views on the difficulty and approachability of the problem. Computationally, gerrymandering is NP-Complete (at least). What is the role of computation in the redistricting problem?

We can think of this gap between computers, which can do computations quickly but have no legal understanding or emotional/moral guidance. Legislatures are the opposite, with a poor ability to do computations, but a good understanding of the law and (ideally) some moral compass.

Computers are good at things like data collection and visualization, okay at things like improving and evaluating plans and sampling the space of possible plans, and not great at finding "optimal" plans (NP-Hard).

Our computational desiderata should include stability or robustness, a resilience to perturbations in the data, and efficiency, or easy to compute in the allotted time frame of the problem.

How Geometry Helps

435 districts means 435 shapes. There are millions of Census blocks, and while not every partition is a valid districting, there are still an insurmountably large number of possible plans.

Stability

Definition 11.17 *Stability generally means that a small change to the input to an algorithm should result in a small or no change in the output.*

Some calculations are resilient to noise, rounding, errors, etc. and others are not. As an example, consider the point-in-polygon problem. One way to do this is to draw a ray from your point and count the number of times your ray hits the boundary, with an odd number of intersections indicating your point is in the interior and an even number being exterior. What happens if your ray intersects at an infinite number of points? This is an edge case, and computational geometry is *all about edge cases.*

We also need to think about disconnected regions, like coastal blocks or islands, topological holes, like blocks contained entirely within another bloc, and discretization artifacts, like Census tracts not being equipopulous. We can think of the redistricting problem as one that is primarily combinatorial, rather than numerical. We can do things like give confidence intervals regarding how likely a point is to be inside a region, using tools from ϵ-geometry and optimal transport.

We have to worry about small-scale instabilities. These include things like numerical precision in computations (rounding, underflow). The Polsby-Popper score ($\frac{400\pi A}{P^2} \leq 100\%$) is a measure of circularity of a region. What if your district is very circular, but the resolution of the perimeter is very fine? You get a score that looks bad, just because you used a more detailed measurement. Pessimistically, we can think of a devious politician choosing which maps to use in order to get the best score for her proposed gerrymander. There are also discrepancies with alignment of units if one partition is done at a finer level than the other. Adjacencies are also things we have to worry about with respect to resolution, as regions which meet at things that look like corners may not actually meet at a single point at a finer resolution, and slight perturbations can dramatically change the underlying structure we are building our districting on.

Large-scale instabilities include things like inaccurate map projections, poor measurement equipment, the resolution of the map, data privacy and security, and data collection techniques.

Fairness and stability measures must be informative. If $f(x)$ is a function evaluating the fairness of a district, we hope that small changes to x only causes small changes in $f(x)$. There are multiple measures of similarity we can think of, like Frechet distance, Killing energy, and the (K, α)-Hoelder Property. These are just examples, and none/all may be informative and useful.

Tractability

Some problems are inherently difficult or impossible to do on a computer. Using reductions from $PARTITION$, $SUBSET-SUM$, or IP, we can easily show that optimal redistricting is NP-Hard in the most basic cases. Thinking about Euclidean k-means, we can even show that it is hard to approximate optimal districting.

We need to be careful about computational redistricting. Claims like "This Computer Programmer Solved Gerrymandering in his Spare Time" (WaPo) should be met with skepticism.

While searching for optimality may not be a great approach, we still have computational tools to bring to this problem. We can algorithmically evaluate and compare plans fairly easily and we can do local optimization and improvement with hill-climbing algorithms which only search a small space.

We also need to think about what 'optimal' means, and which metrics we will use to judge plans. Concepts like Pareto optimality may be useful for attacking this problem, and may be computationally tractable. Sampling the space of plans is also non-trivial, as Wendy and Yan discussed yesterday.

We need to be careful with introducing bias in our algorithms as well, both in how we sample things and how we process and evaluate solutions.

Open Problems!

(Other than P v NP ...)

How do we communicate advantages and drawbacks of computational redsitricting?

What is the role of machine learning, both in data collection and processing and in the drawing of districts?

How complicated is the energy landscape regarding these political problems?

How do we make sure our algorithms are fair, accountable, transparent, and understandable?

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Optimal Transport

Nestor Guillen (UMass) and Justin Solomon (MIT) *Zach Schutzman*

What is Optimal Transport?

What is the state of affairs? We don't really know! This is a place where math is interfacing with experimental science, so we'll throw the math at the problem while respecting the social and legal constraints and see if we get anywhere.

Let's consider a different perspective on good maps, one that considers a resource allocation problem. Thinking about a toy example like where to place a fixed number of fire stations given information about population density, we can take a few approaches. If we evenly space them, or cluster them near the dense areas, or something in between, we face different trade-offs. How can we treat this as an optimization problem?

Suppose we have N blocks in a city of M people, a distance function c_{ij} between pairs of blocks, a population for each block f_i, and a function $\sigma(i)$ which tells you which fire station is assigned to block i. One possible criterion is to minimize the average distance between any person and a station, or $\frac{1}{M}\sum_{i\sigma_i} f_i$.

Optimal transport is a field of mathematics with an array of tools which are increasingly popular. At a high level, it is concerned with ways of taking one shape and transforming it into another in a way that preserves volume or transforming one distribution into another by moving mass in an optimal way. This is fundamentally the same idea as earthmover's distance.

The problem began as a physics problem in which Gaspard Monge was interested in moving cannonballs and became of interest to economists like Leonidas Kantorovich in the 20th century. Monge considered a problem where we have X squares and Y circles in the plane and we want to move the circles to the squares, incurring a cost of $c(x, y)$ for $x \in X$ and $y \in Y$. An optimal transport map is a bijective function $T : X \to Y$ minimizing $J(T) = \sum(c(x_i, y_i))$ for each $T(x_i) = T(y_i)$. If X and Y are finite, then there is some minimizing choice of this function. We can also observe that if T is optimal, then any permutation of the elements in X must give a solution which is weakly worse than T.

The continuous version of the problem has two distributions $f(x)$ and $g(y)$, and a transport map looks like a function $T : X \to Y$ such that $\int_E f(x)dx = \int_{T(E)} g(y)dy$.

Kantorovich's variation imposed a constraint that $\sum f(x) = \sum g(y)$. The problem is now that since X and Y aren't the same size, asking to preserve measure is not going to work. We then have to allow points to split mass. The analogy is a collection of mines and factories. The output of the mines must equal the input of the factories, but we don't need each mine to send all of their output to the same factory. A transport plan $\pi(x, y)$ is now a function which sends all of the mass from X to Y such that no node sends or receives more than its capacity. The opimization problem is now to minimize $J(\pi) = \sum\sum c(x, y)\pi(x, y)$. This is a linear optimization problem, which means we can prove some things about it.

We can move to an even more general case. X, Y are now compact domains with distributions μ, ν and minimize over an integral of $X \times Y$.

Theorem 12.18 *Brenier: If we also assume that $X, Y \subset \mathbb{R}^d$, μ has compact support and is absolutely continuous with respect to its Lebesgue measure, then the optimal transport plan is unique and given by a* *map T.*

A consequence of this is that if we are trying to move a uniform distribution of mass to a distribution with finite support, we get a nice convex result, in a way that looks like Voronoi partitioning. Throwing back to Moon's talk, this method gives components with positive curvature, whereas we suspect that negative curvature may be evidence of gerrymandering.

The takeaway here is that optimal transport partitions the space.

In the discrete world, we can think of a transport plan as a matrix T such that each row sums to the mass at the corresponding source and the columns sum to the mass at the corresponding targets.

Optimal Transport and Redistricting

We introduce an idea of population bias. Now we have $\mu = \mu_B + \mu_R$ and we throw a family of constraints into our optimization which limits the proportion of Blue voters we want in each district. Not a lot is known about what things look like when we do this. We'd like a nice relationship between the unconstrained problem and this new one.

We also run into problems where our linear program gets too big to be practically solvable if we have a very large number of blocks.