About

These notes are from the Metric Geometry and Gerrymandering Group's (MGGG) workshop at Tufts University, August 7th-11th, 2017. For more information about the workshop itself, see `http://sites.tufts.edu/gerrymandr`*.*

As taking notes in LaTeX on-the-fly is not an easy task, I am sure this document is full of typos, sloppy notation, and small mathematical errors. If you find such an error, please send me an email at {`ianzach+notes[at]seas.upenn.edu`} *so I can correct it.*

MGGG Workshop at Tufts University **Tufts University**

Lecture 1: Test Document

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Introduction

Why theory?

1. Minimal approach to understanding the idea of **computation**.
2. What makes computation tick?
3. Theory anticipates technology.
4. Models of computation are interesting.

Mathematics!

Should know:

1. Sets
2. Functions
3. Relations
4. Logic
5. Proofs
6. Graphs

Definition 1.1 *An* ***alphabet*** *is a non-empty, finite set of characters.*

Definition 1.2 *A* ***string*** s *(over an alphabet* Σ*) is a finite ordered sequence of elements of* Σ*.*

Definition 1.3 *The* ***empty string****,* ϵ*, is the sequence of no symbols, and is in fact a valid string.*

Definition 1.4 *Let* Σ^* *be the set of all strings over* Σ*.*

Definition 1.5 *A* ***language*** *over* Σ *is any subset of* Σ^**.*

The empty set is a language. This is *not* the same as the language only containing the empty string.

Finite State Machines: A First Model

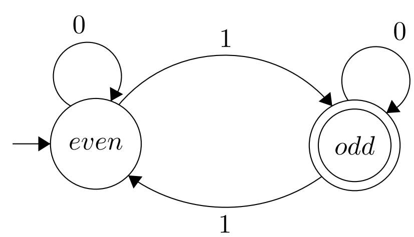
Scalability (asymptotics) is a requirement for any interesting model.

To compute on larger and larger inputs, a computer needs memory. What is the minimum amount of memory you need to do something interesting?

Definition 1.6 *A* ***finite state machine*** *will be a model with a constant amount of memory.*

The states of an FSM correspond to memory (a machine with k states can have 2^k 'bits' of memory).

Example: define the language $\mathcal{L} = \{s \in \Sigma^* | s\ has\ an\ odd\ number\ of\ 1s\}$.



Definition 1.7 *A* ***deterministic finite automaton (DFA),*** M***, is a 5-tuple*** $(Q, \Sigma, \delta, q_0, F)$***.***

Q, the set of states.

Σ, the alphabet

$\delta : Q \times \Sigma \to Q$, the transition function

q_0, the start state

F, the accept states

Definition 1.8 M ***accepts*** *a string* $s = s_1 s_2 \ldots s_k$ *if there is a sequence of states in* M *starting with* q_0 *and ending in a final state* $q_0 q_1 \ldots q_k$ *such that* $\delta(q_i, s_{i+1}) = q_{i+1}$*.*

Definition 1.9 *If* $\mathcal{L} = \{s | M\ accepts\ s\}$*, the we say* M ***recognizes*** $\mathcal{L}$*.*

Definition 1.10 *If a language* $\mathcal{L}$ *is recognized by some DFA, then it is* ***regular****.*

Definition 1.11 *A* ***nondeterministic finite automaton (NFA),*** M***, is a 5-tuple*** $(Q, \Sigma, \delta, q_0, F)$***.***

Q, the set of states.

Σ, the alphabet

$\delta : Q \times \Sigma \cup \{\epsilon\} \to \mathcal{P}(Q)$, the transition function

q_0, the start state

F, the accept states

Now, δ maps the current state and input character or ϵ to some subset of states.

Definition 1.12 *A string s is* ***accepted*** *by and NFA M if there is some path for s from the start state to a final state.*

Definition 1.13 *M* ***recognizes*** *a language $\mathcal{L}$ consisting of all strings it accepts.*

Example: let $\mathcal{L} = \{s \mid \textit{the 3rd last character is a } 1\}$.