Aalto University
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Antialiasing Oscillator Algorithms for Digital Subtractive Synthesis

S-89.3580/S-89.4820 Audio Signal Processing Seminar, Lecture 5

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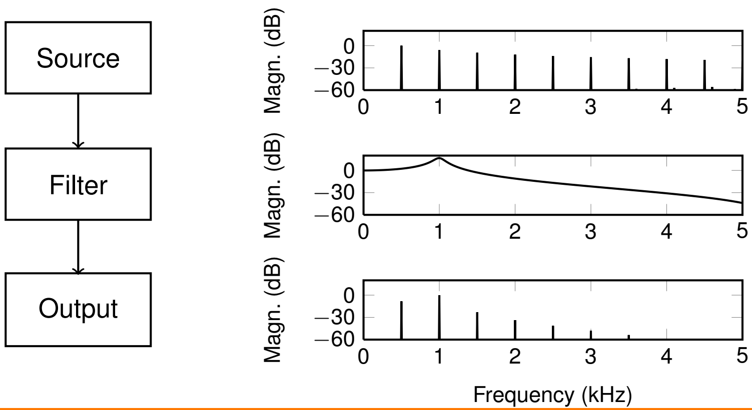
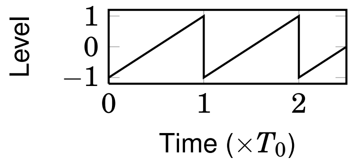
Subtractive Sound Synthesis

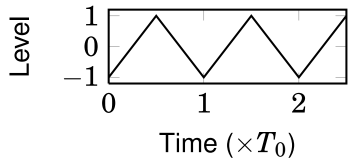


Oscillators Used in Subtractive Synthesis

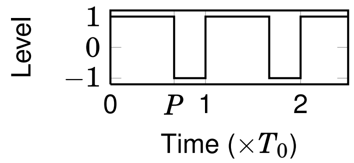
Sawtooth waveform



Triangle waveform



Rectangular waveform



P is the duty cycle or the pulse width

Discontinuous ⇒ **Aliasing!**

Contents of This Lecture

Objectives and Outline

Operation principles of oscillators that reduce/remove aliasing

Outline

1. Ideally Bandlimited Oscillators
2. Quasi-Bandlimited Oscillators

   Break

3. Alias-Suppressing Oscillators
4. Special Approaches to Classical Waveform Synthesis

Not covered: Filters (covered by Mikko in the seminar) and oscillator effects (covered by Jari next week)

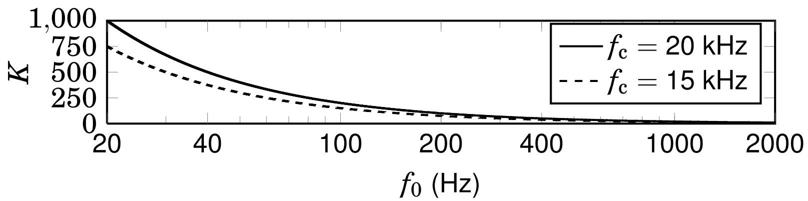
1

Ideally Bandlimited Oscillator Algorithms

1 Additive Synthesis

Chaudhary, 1998, AES 105th Convention

Synthesize the components of the waveform's Fourier series representation below a given cutoff frequency f_c (the highest harmonic index $K = \lfloor f_c / f_0 \rfloor$) and add them up



Computational complexity per sample $O(1/f_0)$

Memory requirements Depends on the sinusoidal oscillator

1 Wavetable Synthesis

Chamberlin, 1985, Book & Burk, 2004, Book

1. Precompute single cycles of the sums of Fourier series terms (like in additive synthesis)
2. Tabulate the precomputed cycles
3. On the synthesis stage read the table computed for that fundamental frequency in a loop

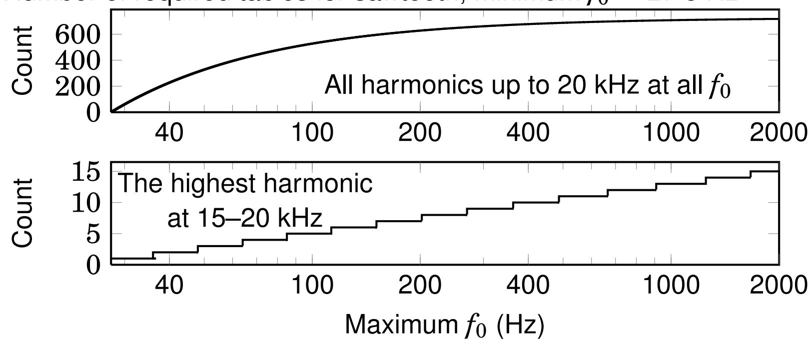
Computational complexity per sample Only control logic and table reads in the synthesis stage, hence constant $O(1)$

Memory requirements Huge! There are techniques to reduce the requirements, however, they are still large...

1 Wavetable Synthesis II

Chamberlin, 1985, Book & Burk, 2004, Book

Number of required tables for sawtooth, minimum $f_0 = 27.5$ Hz



1 Discrete Summation Formulas

Winham and Steiglitz, 1970, JASA & Moorer, 1976, JAES

Using the identities of trigonometric functions reduce a sum of sinusoids into a "simpler" expression

Example (Winham and Steiglitz, 1970):

$$\sum_{k=1}^{N} \cos(k\omega n) = \frac{\sin((2N+1)\omega n/2)}{2\sin(\omega n/2)} - \frac{1}{2}$$

Issues

- Numerical issues when the denominator is close to zero
- Amplitude mismatches – requires a post-equalizing filter

1 Summary of Idelly Bandlimited Oscillators

Additive synthesis Accurate, but computationally heavy

Wavetable synthesis Computationally light, memory requirements large, complicated control with time-varying phenomena

Discrete Summation Formulas Computationally moderate/light, numerical issues

Inverse FFT synthesis Computationally moderate, trade-off between temporal and spectral resolution, interpolation issues

Theoretical approaches useful for testing the other algorithms

1 Inverse FFT Synthesis

Deslauriers and Leider, 2009, AES 127th Convention

Compose the waveform in frequency-domain and apply inverse fast Fourier transform (IFFT) to the synthetic spectrum

Issues

- Trade-off between temporal and spectral resolution
- Data interpolation due to finite spectral resolution
- Noise due to errors in spectrum data
- Assumes linear amplitude and phase evolution within a frame

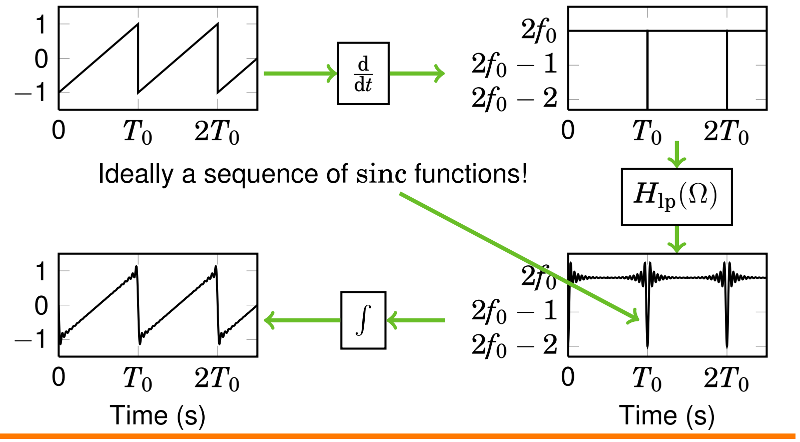
Computational complexity and memory consumption

Depend on the block size of the IFFT

2 Quasi-Bandlimited Oscillator Algorithms

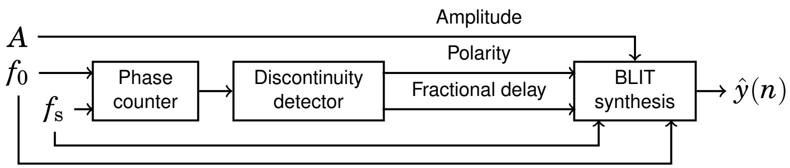
2 Bandlimited Impulse Train Synthesis (BLIT)

Continuous-Time Derivation (Stilson and Smith, 1996, ICMC)

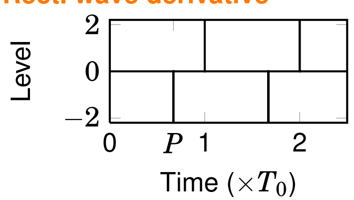


2 BLIT Algorithm

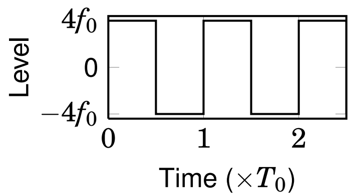
Stilson and Smith, 1996, ICMC & Stilson, 2006, PhD Thesis



Rect. wave derivative



Triangle wave derivative



2 Computational Load and Memory Requirements of BLIT

- sinc function infinitely long!
  $\Rightarrow$ Truncate to length N, window & tabulate
- Discontinuity located between sampling instants
  $\Rightarrow$ Oversampling by factor M required to get proper positioning, can be further improved by table interpolation

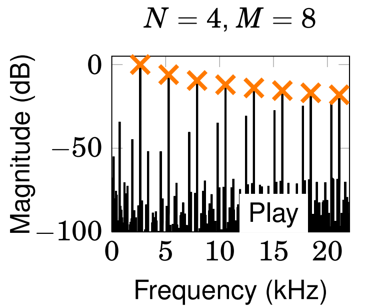
Computational Load For **a discontinuity**, the computational load is $O(N)$. Per sample the load is $O(Nf_0)$

Memory Requirements The table length is $NM(+1)$; hence the memory requirement is $O(NM)$

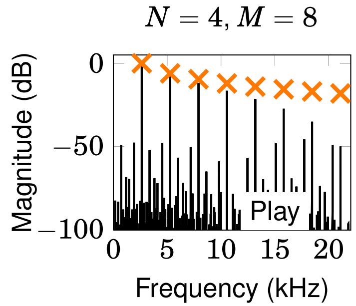
2 Alias Reduction Performance of BLIT

Pekonen et al., 2010a, DAFx

Hann-windowed sinc



Plain Hann window



The windowed sinc function is **not** the optimal!

2 Approaches to Improve the Performance of BLIT

Look-Up Table Approaches (Pekonen et al., 2010a, DAFx)

Parametric window function Use a controllable window function as the look-up table

Optimized tables Optimize the table entries according to selected criteria

Approaches Not Using Look-Up Tables

Modified FM pulses Use a modified FM synthesizer to generate bandlimited pulses (Timoney et al., 2008, DAFx)

Fractional delay filters A handy approach

2 Fractional Delay Filters in BLIT Synthesis

Pekonen et al., 2010b, ICGCS

- The purpose of fractional delay (FD) filters?
  - $\Rightarrow$ To approximate ideal bandlimited interpolation!
- The basis function of ideal bandlimited interpolation?
  - $\Rightarrow$ The sinc function!

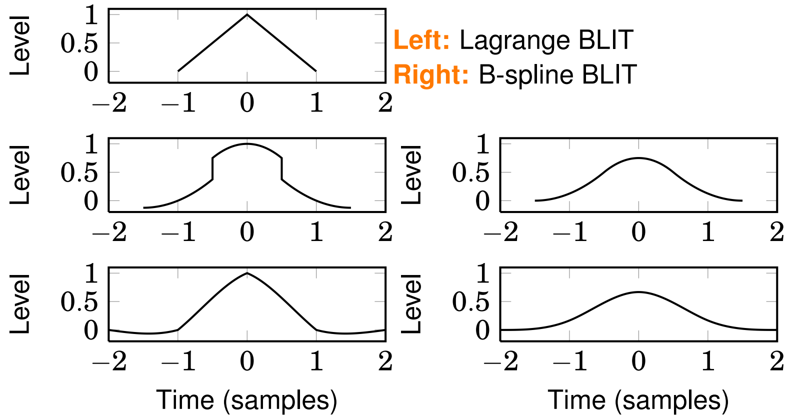
$\Rightarrow$ Use FD filters to synthesize the bandlimited impulses!

Modify the algorithm:

BLIT synthesis $\longrightarrow$ FD filter

2 Direct BLIT Synthesis Using FD Filters

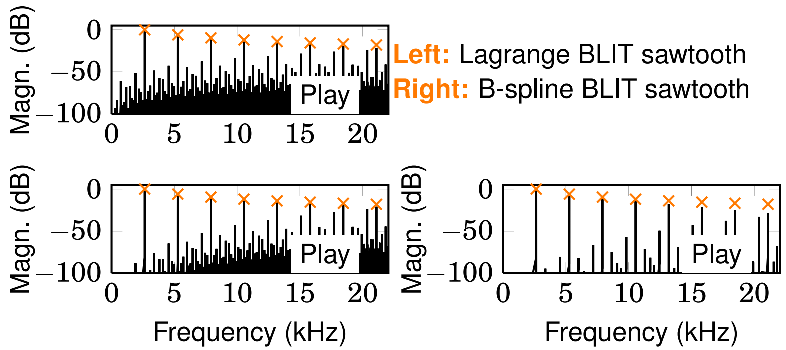
Nam et al., 2010, IEEE TransASLP



Left: Lagrange BLIT
Right: B-spline BLIT

2 Direct BLIT Synthesis Using FD Filters II

Nam et al., 2010, IEEE TransASLP

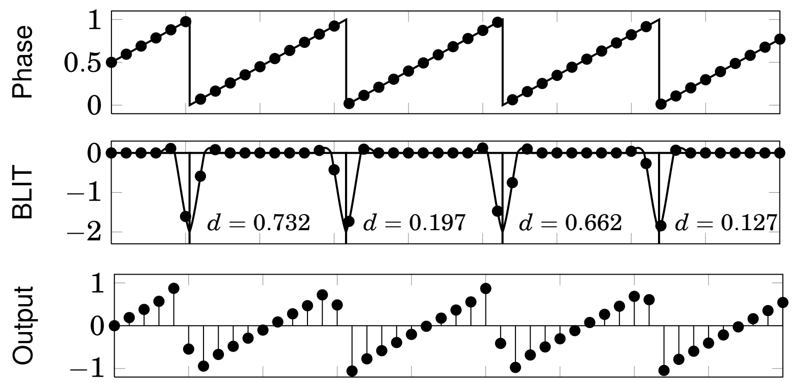


Left: Lagrange BLIT sawtooth
Right: B-spline BLIT sawtooth

Trade-off: Alias reduction vs. Amplitude drop of higher harmonics

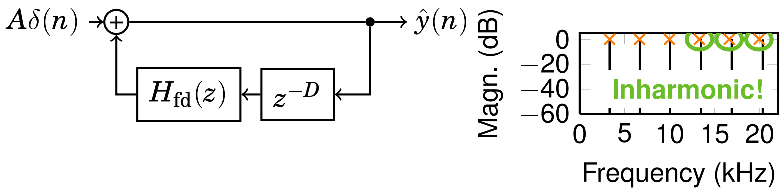
2 Example of FD-BLIT

Third-order Lagrange FD Filter

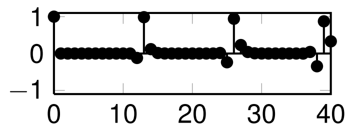


2 Feedback Loop Oscillator

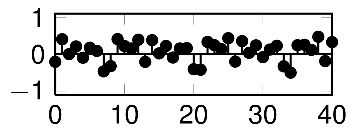
Nam et al., 2009, DAFx



In the beginning



After one second



2 Issues With BLIT

Prone to Numerical Errors

Replace the integrator with a second-order leaky integrator (Brandt, 2001, ICMC)

$$H_{\mathrm{int},2}(z) = \frac{1 - z^{-1}}{(1 - cz^{-1})^2}$$

Boosting of Aliasing at Low Frequencies

Inherent property of the algorithm, cannot be avoided

2 Bandlimited Step Function Synthesis (BLEP)

Brandt, 2001, ICMC

Avoid integration in the synthesis stage

1. Integrate the BLIT function
2. At each discontinuity, trigger the integral

In principle: Accumulate the BLIT look-up table and reading it through and output a constant one when the table size is exceeded

In practice: Compute **the difference** between the bandlimited step function and unit step function and **add** it onto the waveform around the discontinuity (Välimäki and Huovilainen, 2007, IEEE SPM & Leary and Bright, 2009, U.S. Patent)

Computational load and memory requirements the same as with BLIT

2 Summary of Quasi-Bandlimited Oscillators

Bandlimited Impulse Train (BLIT) Synthesize a sequence of bandlimited impulses and integrate, issues with the integration and boosting of aliasing at low frequencies

Bandlimited Step Function (BLEP) Synthesize a sequence of bandlimited step functions or a sequence of correction functions

3
Alias Suppressing Oscillator Algorithms

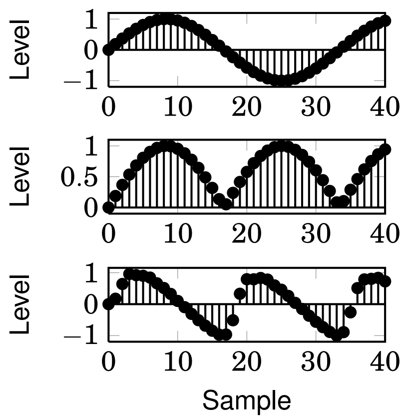
3 Oversampled Trivial Approach
Chamberlin, 1985, Book & Puckette, 2007, Book

Synthesize the trivial waveform with a high sampling rate
$\Rightarrow$ aliased components will be at lower level

Issues

- Spectral envelope of these waveforms decay gently
  $\Rightarrow$ Very high oversampling factor L required!
- Highly oversampled oscillator consumes computational power
  $\Rightarrow$ **Computational load:** $O(L)$

3 Filtering of Full-Wave Rectified Sine Wave
Lane et al., 1997, CMJ



1. Sinusoid with half of the target frequency
2. Full-wave rectify
3. Fixed lowpass filter
4. f_0-tracking highpass filter

Other waveforms with approximations (see Lowenfels, 2003, AES 115th Convention, for practical approaches)

3 Differentiation of Piecewise Polynomial Waveforms (DPW)

Välimäki et al., 2010a, IEEE TransASLP

Utilizes the following Fourier Transform properties:

$$\mathcal{F}\left(\frac{\mathrm{d}}{\mathrm{d}t}f(t)\right) = (i\omega)\mathcal{F}(f(t))$$

$$\mathcal{F}\left(\int f(t)\mathrm{d}t\right) = \frac{\mathcal{F}(f(t))}{i\omega} + C$$

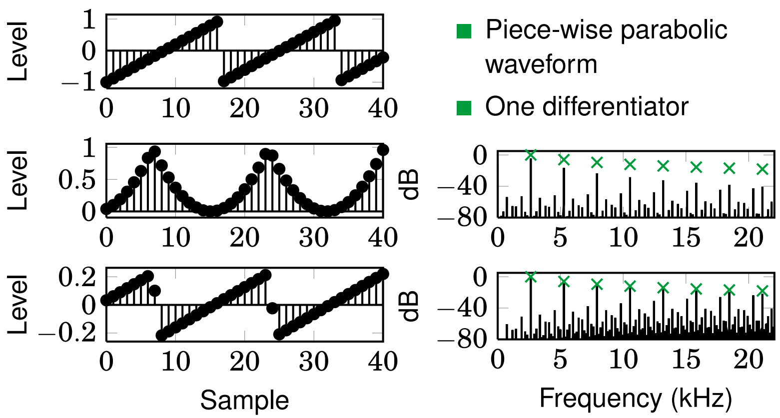
- Differentiation **increases** spectral tilt by about 6 dB per octave
- Integration **decreases** spectral tilt by about 6 dB per octave

Sawtooth waveform linear within a period

$\Rightarrow$ Analytic integration possible

3 Second-Order DPW

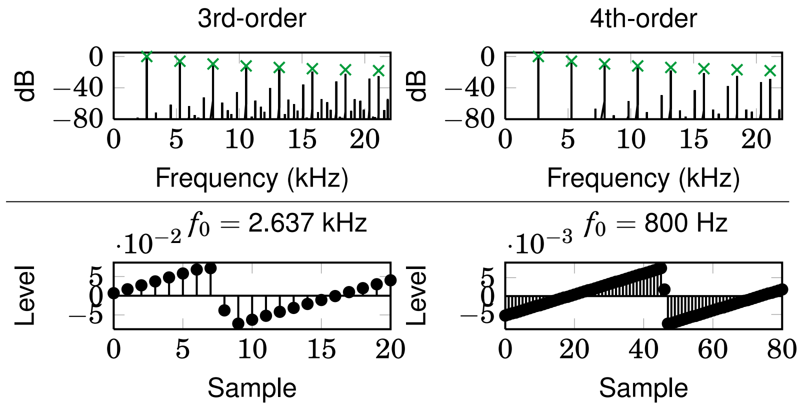
Välimäki, 2005, IEEE SPL & Huovilainen and Välimäki, 2005, ICMC



- Piece-wise parabolic waveform
- One differentiator

3 Higher-Order DPWs

Välimäki et al., 2010a, IEEE TransASLP



3 DPW Scaling

Välimäki, 2005, IEEE SPL; Välimäki and Huovilainen, 2006, CMJ & Välimäki et al., 2010a, IEEE TransASLP

The output of the differentiator(s) needs to be scaled due to nonideal differentiation

Scaling factor issues:

- Inversely proportional to the fundamental frequency!
- The fundamental frequency is in the power of the order!
- $\Rightarrow$ At low frequencies very large scaling (e.g. 200 dB) required
- $\Rightarrow$ Numerical problems...

3 Summary of Alias-Suppressing Algorithms

Sample a waveform with a tilted spectrum

Oversampling Very high oversampling factor required

Filtered full-wave rectified sinusoid Approximations, approximations. . .

Differentiated parabolic waveforms Sample integrals of linear function, problems with scaling

4 Digital Post-Suppression Algorithms

Pekonen and Välimäki, 2008, ICASSP

We have an oscillator that has aliasing, what can we do?

Below the fundamental frequency Highpass filtering

Between harmonics Comb filtering

- FIR comb filter to pass the harmonic components and to **remove some aliasing** between the harmonics, or
- IIR comb filter to pass mainly the harmonic components and to **suppress the aliasing** between the harmonics

Comb filters require the highpass filter also as they will pass DC

4 Special Approaches to Classical Waveform Synthesis

4 Distortion (Waveshaping) Synthesis

Timoney et al., 2009a, AES 126th Convention

Distrort a sinusoid with a waveshaper like in the filtered full-wave rectified sinusoid

- Different waveshapers for different waveforms (Timoney et al., 2009a, AES 126th Convention & Kleimola, 2008, DAFx)
- Not necessarily aliasing-free
- Requires control to avoid aliasing (Timoney et al., 2009a, AES 126th Convention & Lazzarini and Timoney, 2010, CMJ)
  - Example: Use Chebyshev polynomials with the number of polynomials controlled by the fundamental frequency (Pekonen, 2007, Master's thesis)

4 Phase Distortion Synthesis
Ishibashi, 1987, U.S. Patent

Like waveshaping, but for phase instead of amplitude

Papers Dealing with This Topic

Timoney et al., 2009b, ICASSP; Timoney et al., 2009a, AES 126th Convention; Lazzarini et al., 2009b, DAFx; Kleimola et al., 2009, DAFx & Lazzarini et al., 2009a, DAFx

Approaches to control aliasing discussed in Lazzarini and Timoney, 2010, CMJ

5 Summary of the Lecture

Ideally bandlimited oscillators No aliasing at all, different issues in different algorithms, useful for testing the other approaches

Quasi-bandlimited oscillators Aliasing allowed mainly at high frequencies, BLIT and BLEP approaches, integration issues in BLIT

Alias-suppressing oscillators Sample a signal that has a tilted spectrum, oversampling, filtered full-wave rectified sine wave, DPW, scaling issues in DPW

Special approaches Ad hoc approaches, post-suppression by filtering, wave- and phaseshaping, issues with aliasing in distortion approaches

5
Summary of the Lecture

5 Future of Bandlimited Oscillator Design

Oscillator with Desired Properties

1. Perceptually aliasing-free in the range of musical frequencies
2. Computationally efficient and low memory requirements
3. Does not require a division that depends on an oscillation parameter, e.g. fundamental frequency!

The first two are obtainable, the last one still unsolved problem

Modeling of Analog Oscillator Outputs

First attempts done by De Sanctis and Sarti, 2010, IEEE TransASLP, and by Kleimola et al., 2010, SMC

Appendix

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